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Control and Machine Learning

By Enrique Zuazua

Two recurring questions pertain to the origin, history, and present state of mathematics. The first relates to math's incredible ability to describe natural, industrial, and technological processes, while the second concerns the unity and interconnectedness of all mathematical disciplines. Here I describe some of the gateways that link two particular mathematical branches: control theory and machine learning (ML). These areas, both of which have very high technological impacts, comprise neighboring valleys in the complex landscape of the mathematics universe.

Control theory certainly lies at the pedestal of ML. Aristotle anticipated control theory when he described the need for automated processes to free human beings from their heaviest tasks [3]. In the 1940s, mathematician and philosopher Norbert Wiener redefined the term "cybernetics"—which was previously coined by André-Marie Ampère—as "the science of communication and control in animals and machines," which reflected the discipline's definitive contribution to the industrial revolution.

Wiener's definition involves two essential conceptual binomials. The first is *control-communication*: the need for sufficient and quality information about a system's state to make the right decisions, reach a given objective, or avoid risky regimes. The second binomial is *animal-machine*. As Aristotle predicted, human beings rationally aim to build machines that perform tasks that would otherwise prevent them from dedicating time and energy to more significant activities. The close link between control and/or cybernetics and ML is thus built into Wiener's own definition.

The interconnections between different mathematical disciplines are split by conceptual and technical mountain ranges and have often evolved in different communities. As such, they are frequently hard to observe. Building the connecting paths and identifying the hypothetical mountain passes requires an important level of abstraction. Let us take a step back and consider a wider perspective.

The notion of *controllability* helps us disclose one of the gateways between disciplines. Controllability involves driving a dynamical system from an initial configuration to a final one within a given time horizon via skillfully designed and viable controls. In the framework of linear finite N-dimensional systems

$$x' + Ax = Bu,$$

the answer is elementary and classical (it dates back to Rudolf Kalman's work in the 1950s, at least) [6]. The system is controllable if and only if the matrix A that governs the system's dynamics and the matrix B that describes the controls' effects on the state's different components verify the celebrated rank condition

$$\text{rank}[B, AB, \ldots A^{N-1}B] = N.$$

The control's size naturally depends on the length of the time horizon; it must be enormous for very short time horizons and can have a smaller amplitude for longer ones.

In fact, as John von Neumann anticipated and Nobel Prize-winning economist Paul Samuelson further analyzed, the "turnpike" property manifests itself over long time horizons; controls tend to spend most of their time in the optimal steady-state configuration [5]. We apply this lesser-known principle systematically (and often unconsciously) in our daily lives. When travelling to work, for instance, we may rush to the station to take the train—our turnpike in this ride—on which we then wait to reach our final destination. Medical therapies for chronic diseases also utilize this principle; physicians may instruct patients to take one pill

See **Machine Learning** *on page 4*

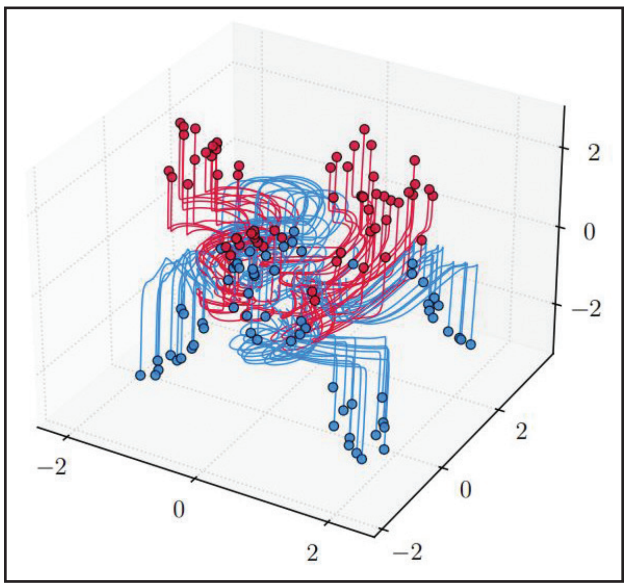

Figure 1. *Simultaneous control of trajectories of a neural ordinary differential equation (NODE) for classification according to two different labels (blue/red), exhibiting the turnpike nature of trajectories. Figure courtesy of [5].*

Multiscale Models Shed Light on Tuberculosis

By Matthew R. Francis

As demonstrated by the ongoing COVID-19 pandemic, a thorough understanding of infectious diseases requires data and models on multiple interconnected levels. Epidemiology addresses population-level issues, transmission models describe individuals within their environments, and a variety of biomedical approaches help researchers comprehend the way in which pathogens infiltrate the body — and the body's ability to fight back.

Tuberculosis (TB) is one of the deadliest infectious diseases in the world. It accounts for roughly 1.5 million deaths per year[1] and causes the most HIV-related casualties. While decision-makers know in principle how to slow the spread of certain illnesses, TB is more stubborn than most. "TB is unique compared to many other diseases and the way we treat them," Denise Kirschner, a mathematical biologist at the University of Michigan Medical School, said. During her plenary talk at the hybrid 2022 SIAM Conference on the Life Sciences[2] (LS22), which took place concurrently with the 2022 SIAM Annual Meeting[3] in Pittsburgh, Pa., this July, Kirschner described the major challenges that surround TB's characterization.

"Bacteria induce the formation of structures in the lungs known as granulomas," she said. "These structures are important, and how cells move around within the granuloma structure is important." Granulomas are roughly spherical clusters of immune cells that form around invading bacteria, most likely as an immunological response (see Figure 1). While the human body builds these structures in response to certain other types of infection as well, their presence in the case of TB partly explains why many people who are exposed to *Mycobacterium tuberculosis* bacteria remain asymptomatic.

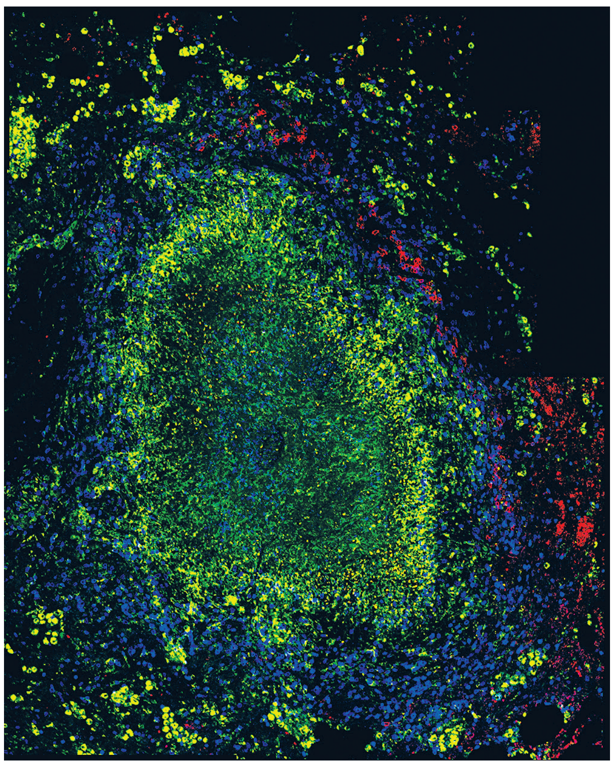

Figure 1. *Non-human primate granuloma. Figure courtesy of the laboratory of JoAnne Flynn at the University of Pittsburgh.*

TB is notoriously challenging to treat, and granulomas are partly responsible for this difficulty because most of the pathogens are isolated from the rest of the body. An infected lung might have between 10 and 30 granulomas, which generally do not seem to interact directly with each other. Medical data on patients and experiments on monkeys have revealed several distinct types of granulomas, including stable clusters wherein the surrounding cells contain bacterial growth, and broken granulomas that spill bacteria into the rest of the lung.

"Only one granuloma has to go south to cause active disease," Kirschner said. "We have to understand what 'go south' means, and usually it means the granuloma is unable to control bacterial growth. Our goal is to really determine which factors within the granuloma lead to overgrowth, control, or clearance of the bacteria."

Granular Approach to Granulomas

TB's complexity requires that Kirschner and her collaborators simultaneously build models on multiple scales that range from molecular and cellular interactions to whole-lung and whole-patient analyses. The current state of the art involves agent-based models that simulate the way in which independent "agents" organize into larger associations. In the context of Kirschner's TB research, these agents can take the form of granulomas, individual cells (bacteria and T-cells), and even regulatory proteins known as cytokines and chemokines. "We are interested in starting at the scale of most interest, where the data is most plentiful," she said. "In this case, that scale happened to be the granuloma. We let the biology drive how deep we drill down and what the biological questions are."

In other words, the unique nature of granulomas means that they are located at the metaphorical center of the picture; their formation, growth, propagation, and fate hold the key to modeling and hopefully treating TB. Many of the multiscale simulations that Kirschner and her team developed begin at the granuloma, then extend in both directions to encompass smaller- and larger-scale phenomena. The lung comprises the overall environment of these models, and the group tests the modeling outcomes against biological data from either TB patients or monkeys — which provide the closest known analog to humans.

To complicate matters, *M. tuberculosis* bacteria can also infect immune system cells. As such, complete multiscale models

See **Tuberculosis** *on page 3*

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[1] https://www.who.int/health-topics/tuberculosis

[2] https://www.siam.org/conferences/cm/conference/ls22

[3] https://www.siam.org/conferences/cm/conference/an22

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5 **GraphBLAS and GraphChallenge Advance Network Frontiers**
Many factors inspire interest in networks and graphs. Jeremy Kepner, David Bader, Tim Davis, Roger Pearce, and Michael Wolf discuss the ways in which the Graph Basic Linear Algebra Subprograms (GraphBLAS)—along with other graph innovations that appear in GraphChallenge—have enabled new capabilities for networks.

6 **The Value of Applied Mathematics Internships in Business, Industry, and Government**
Summer internships in applied mathematics and computational science provide opportunities for students to explore possible career paths and apply their coursework to real-world scenarios. Margaret Cheney, Andre Celestin, Jerett Cherry, Cole Moore, and Danny Long address various aspects of internships and share personal anecdotes.

9 **Rules, Algorithms, and Models: An Intellectual History**
Ernest Davis reviews Lorraine Daston's *Rules: A Short History of What We Live By*, which surveys the vast landscape of societal rules in terms of their appearance; evolution; creation, perception, and enforcement; and the ways in which people respond to them. The text combines conceptual and historical analysis with a wealth of examples.

10 **Virginia Commonwealth University SIAM Student Chapter Celebrates Successful Year of Activities**
The Virginia Commonwealth University (VCU) SIAM Student Chapter had a successful 2021/2022 academic year. Henry Ogu—treasurer of the VCU SIAM Student Chapter—recounts several engaging chapter activities, including an outreach visit to a local elementary school and attendance at multiple conferences.

11 **Academia or Industry… Why Not Both?**
Giovanna Guidoboni details the journey to her current position as both a faculty member and mathematical consultant. She overviews her experiences in balancing both pursuits and provides useful tips for anyone who enjoys the academic environment but would also like to explore consulting.

Living Matter and Active Cells

By Michael Shelley

Living things assemble themselves to a remarkable degree in a coordinated and hierarchical manner. A driving question in biology seeks to explore the factors that underlie and guide this astonishing capacity for self-assembly and self-organization, which is also the animating force for the multidisciplinary field of active matter. The back-and-forth conversation between cell biology and active matter physics has been fruitful and intense.

In their study of multicellular life, biologists naturally turn to its earliest stages and investigate the lively processes through which egg and embryonic cells develop. Figure 1a shows how a nematode embryo—composed of only a single cell—moves, combines, and segregates genetic material as it proceeds towards its first division. This dance is choreographed by elements of the spindle complex: a self-assembled organelle that comprises stiff biopolymers (microtubules), microtubule nucleating sites (centrosomes), molecular machines (motor proteins), and other specialized proteins (see Figure 1b). The spindle complex fulfills several tasks, and its form adapts and changes as it moves through each one. Its centrosomes negotiate the joining of the male and female nuclei and properly position the subsequent conjoined nuclei. The complex then elongates, separating the duplicated chromosomes on opposite sides of the soon-to-be divided cell.

All of this activity takes place over 10 or 15 minutes, but the complex's structural elements—its microtubules—constantly disassemble and are replaced approximately every 20 seconds. While such transience might seem weird and counterintuitive, this ephemeral quality provides the complex with some of its necessary adaptivity. Because the spindle complex moves through the cell's fluidic cytoplasm, the sequence is also a wonderfully complicated fluid-structure interaction problem. It involves mobile structures—many of which are transitory and flexible—that interact with each other hydrodynamically and through motor proteins in a confined space. Very specialized computational fluid dynamics methods, which assume that the cytoplasm is a simple Stokesian fluid, have helped unravel some of these mysteries (see Figure 1c).

Yet where does active matter come into play? First, the cell assembles the spindle complex—which is maintained and moved by a constant expenditure of energy—from many copies of the same molecules. The microtubules are coupled by molecular motors that walk along them, dragging along anything to which they are attached. For example, one motor might be attached to another molecular motor that is walking along another microtubule (see Figure 1d). At any rate, the motors need energy in order to move; the hydrolysis of adenosine triphosphate, a primary fuel source for the cell, provides this energy. Finally, Newton's third law has to be obeyed in that the total force that a motor exerts must be zero. If a motor exerts a force $\boldsymbol{F}$ on the microtubule, it must also exert a force $-\boldsymbol{F}$ somewhere else — perhaps on the cytoplasm by the drag of a payload. This puts cytoskeletal assemblies on the turf of active matter, which in its purest form concerns itself with multiscale systems whose mobile microstructure converts a local energy source into mechanical work on the system. Passive matter systems are equilibrium systems where work is performed on the system from the outside, while active matter systems perform work on themselves.

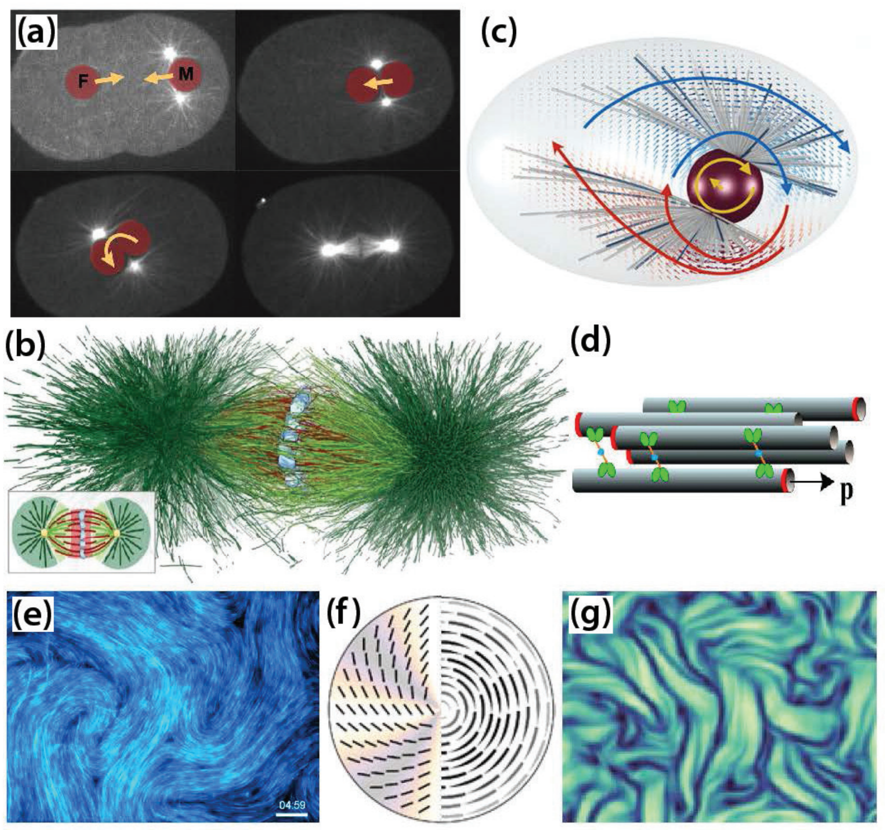

Figure 1. *Active dynamics and structures within cells, and the dynamics of their extracted components.* **1a.** *The spindle complex's dynamics in a single-cell nematode embryo.* **1b.** *Tomographic reconstruction of the spindle complex and chromosomes (blue) in a nematode embryo.* **1c.** *Simulation of a payload that is positioned by peripherally bound motors, which pull on microtubules.* **1d.** *A schematic of aligned microtubules that are pulled past each other by double-headed kinesin motor complexes.* **1e.** *An active in vitro assembly of microtubules and molecular motors.* **1f.** *The steady rotational flow that can arise from confinement of an active suspension. The left side illustrates mean particle alignment and the right side illustrates particle streaklines.* **1g.** *Detail from a large-scale simulation of active turbulence. Figure 1a adapted from [9], 1b adapted from [4], 1c adapted from [5], 1d adapted from [3], 1e adapted from [8], 1f adapted from [12], and 1g adapted from [11].*

Figure 1e depicts the outcome of the extraction and purification of these cellular ingredients [8]. Freed from the confines and regulation of the cell, microtubules and motors organize to form biologically active materials that undergo self-driven, complex, and large-scale dynamics called *active turbulence*. Applied mathematicians and physicists model these systems, sometimes based on symmetry principles and sometimes via micro-to-macro (and back again) coarse-graining methods. One fundamental idea of the physics that governs the systems is that of an "active stress" — particularly an *extensile* active stress from collective microscale activity.

Continuum models for suspensions of active rod-like particles illustrate this concept [3, 6]. We describe the system's state with a distribution function $\Psi(\boldsymbol{x},\boldsymbol{p},t)$ of particle positions $\boldsymbol{x}$ and orientations $\boldsymbol{p}(|\boldsymbol{p}|=\mathbf{1})$, which evolve through a Fokker-Planck equation. This evolution requires conformational fluxes $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{p}}$ that capture the microscopic particle dynamics; here, they are composed of active particles that swim, or stretch, or stretch the surrounding fluid — all while they are translated and rotated by a background flow $\boldsymbol{u}$, which results from their own ensemble activity. Each active particle contributes a tensorial stress that is proportional to $\boldsymbol{pp}$. Consequently, the background velocity $\boldsymbol{u}$ solves a Stokes equation that is forced by the distributional average of $\boldsymbol{pp}$ (among other things). That is,

$$-\nabla q + \Delta \boldsymbol{u} = -\nabla \cdot (\alpha \boldsymbol{D} + \ldots)$$

$$\text{and} \quad \nabla \cdot \boldsymbol{u} = 0,$$

where $\boldsymbol{D}(\boldsymbol{x},\boldsymbol{t}) = \int_{|p|=1} dS\, \boldsymbol{pp}\, \Psi$ and α measures the strength of particle activity. An extensile active stress has $\alpha < 0$, which corresponds to fluid being stretched along the particle axis by particle activity. This stretching flow causes nearby active particles to align — eventually yielding the large-scale, self-organized flows that are commonly associated with active turbulence.

The kinetic theory outlined here first described experiments on suspensions of swimming bacteria that demonstrated similarly complex dynamics [2]. It has provided a first-principles basis for the analysis and

See **Active Cells** *on page 5*

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Stories From a Scientist Affecting Science Policy in Washington, D.C.

By Samantha Erwin

Ever since college, I have continually been involved in public service. I enjoy working towards broader community improvement, whether it be through efforts to secure dental insurance for graduate students at Virginia Tech or fundraising for facilities to rehome retired thoroughbred racehorses. As an undergraduate student at Murray State University, I joined the SIAM student chapter and served as an officer; I was also an active member of the Virginia Tech SIAM Student Chapter[1] throughout graduate school. Due to my involvement with these chapters, I received multiple travel grants to attend and present at SIAM conferences over the years. In addition, my participation in networking events and career fairs led to several job opportunities. These experiences collectively introduced me to SIAM's far-reaching impact in fostering the development of applied mathematics through publications, research, and camaraderie. When I saw the call for applications for the two-year SIAM Science Policy Fellowship Program,[2] I was thus excited at the prospect of utilizing my public service background to positively impact the SIAM community.

I applied to the SIAM Science Policy Fellowship Program because I wanted to understand the underpinnings of science policy and make meaningful contributions to the scientific endeavor both in the U.S. and globally. The SIAM Committee on Science Policy[3] (CSP) focuses on promoting the visibility and value of applied mathematics research to policymakers by personally sharing information about SIAM and its members with congressional offices. The CSP maintains a steady presence in Washington, D.C., to understand and advocate for science policy issues that affect the entire SIAM community through the creation of white papers and position statements on topics like efficient power grid design and climate change. The Committee also communicates with policymakers regarding specific issues of interest, such as funding for the National Science Foundation (NSF) or the Department of Energy's (DOE) Advanced Scientific Computing Research (ASCR) program.[4] In order to ensure that SIAM upholds its footing in Washington, D.C., Lewis-Burke Associates LLC serves as SIAM's governmental relations partner and liaison between the Society, federal agencies, and Congress members and their staff for issues that are important to applied mathematicians and computational scientists.

Samantha Erwin of Pacific Northwest National Laboratory was a SIAM Science Policy Fellowship recipient in 2020. Photo courtesy of the author.

After submitting my application, which included a personal statement and a write-up of a policy issue (I discussed biomedical data availability), I received the Fellowship in 2020 along with nine other early-career researchers. As a Science Policy Fellow, I attended CSP meetings with 21 senior members of SIAM from academia, industry, and the National Laboratories. Travel restrictions due to the COVID-19 pandemic limited the meetings in 2020 and 2021 to virtual platforms, though SIAM extended my Fellowship for an additional year — enabling me to travel to Washington, D.C., for in-person gatherings in 2022. The CSP meetings occur biannually in the spring and fall and include a half-day orientation that briefs Fellowship recipients on the basics of SIAM's history with science policy as well as the fundamentals of the federal budget, legislative process, and science policy advocacy. Staff from Lewis-Burke guided our efforts to highlight important topics in the federal budget that pertain to mathematics research. We also learned about strategies to better connect with policymakers and their staff when presenting compelling cases of the value of scientific research endeavors across the nation, ranging from the work of the Tennessee Valley Authority[5] to ongoing projects at Pacific Northwest National Laboratory.[6]

During the general CSP meetings, directors from mathematical research-focused scientific offices—such as NSF and DOE—present their current and projected budgets and share scientific priorities for the upcoming year. At the 2022 spring meeting in Washington, D.C., the CSP met with David Manderscheid and Junping Wang, respectively the Division Director and Deputy Division Director of the Division of Mathematical Sciences[7] within NSF's Directorate for Mathematical and Physical Sciences.[8] Manderscheid and Wang emphasized the need for increased funding to support initiatives for epidemiological modeling of human behavior and digital twins research. Similarly, we conversed with Barbara Helland—Associate Director of the ASCR program within DOE's Office of Science[9]—who also highlighted new focus areas in digital twins, the novel Energy Earthshot initiative,[10] and DOE's Reaching a New Energy Sciences Workforce[11] (RENEW)

See **Science Policy** *on page 6*

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[1] https://sites.google.com/vt.edu/siam

[2] https://www.siam.org/students-education/programs-initiatives/siam-science-policy-fellowship-program

[3] https://www.siam.org/about-siam/committees/committee-on-science-policy-csp

[4] https://www.energy.gov/science/ascr/advanced-scientific-computing-research

[5] https://www.tva.com

[6] https://www.pnnl.gov

[7] https://www.nsf.gov/div/index.jsp?div=DMS

[8] https://www.nsf.gov/dir/index.jsp?org=MPS

[9] https://www.energy.gov/science/office-science

[10] https://www.energy.gov/policy/energy-earthshots-initiative

[11] https://science.osti.gov/Initiatives/RENEW

Tuberculosis

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should include blood vessels and the lymph nodes that produce T cells to fight the infection. Simply put, TB simulations must span wide spatial and temporal resolutions and employ sophisticated tuning to capture the right details in every regime.

The earliest agent-based granuloma model describes bacteria, immune cell populations, and protein responses on a two-dimensional (2D) lattice [2]. This simulation also breaks time into discrete chunks via a time scale that is based on reasonable biological constraints of cell division and protein decay rates. The resulting model contains four basic entities—bacteria that reside outside of cells, T cells, macrophages, and chemokine levels—with 27 parameters that researchers either obtain from the simulation or measure experimentally. The generic form of the stepwise equations within each lattice box is

$$\omega_k(t+1)=\omega_k(t)+f_k(\omega_k,\omega_j),$$

where ω_k is one of the entities and f_k is a nonlinear term that describes the interactions between entities.

Despite the number of simplifying assumptions, this model spontaneously generates granulomas unprompted. But cell motion in three dimensions is far more complicated than in two dimensions, and hence extending this model leads to prohibitively long computer runtimes. Though 2D models are still useful in some contexts—"Should we feel guilty about this?" Kirschner quipped during her presentation—scientists must look for other possible angles to the problem if they wish to involve biological experiments.

A more recent, promising approach treats granulomas themselves as the agents on a discretized model of the lung [3]. Louis Joslyn, a former graduate student in Kirschner's group, worked with Kirschner's colleagues to model growth, reproduction, and decay of bacteria, various cell types, and proteins with 15 coupled nonlinear ordinary differential equations for each block of the lattice:

$$\frac{d\omega_k}{dt}=F_k(\omega_k,\omega_j),$$

with nonlinear interaction terms F_k. The 15 entities ω_k include cell division, cell death, bacterial growth, and protein decay, among other factors (see Figure 2).

Early results of this technique have allowed Kirschner and her collaborators to link specific biomarkers in the blood of infected subjects with early immune responses [1]. She is thus hopeful that the model might improve future therapeutic outcomes.

From Models to Treatment

Roughly 25 to 33 percent of the global population carries *M. tuberculosis* bacteria (mostly without symptoms), and poorer countries bear a heavier burden from the disease. *M. tuberculosis* also grows very slowly by bacterial standards, producing a new generation every eight to 24 hours. This relatively sedate growth rate means that researchers can only detect growth in the laboratory after one to six weeks, depending on the method of observation.

To further complicate matters, the widely used TB vaccine does not effectively prevent transmission or infection. In addition, antibiotic treatment is a lengthy and expensive process. Due to the duration of treatment and unpleasant side effects, many patients do not complete the entire course of antibiotics; in response, some strains of TB have become drug resistant. "We have to use multiple drugs for long periods of time because it's really hard to penetrate granulomas," Kirschner said. "We need to shorten the treatment, make the drugs less toxic so there are not as many side effects, and maybe reduce the number of drugs that one takes to make them more cost effective."

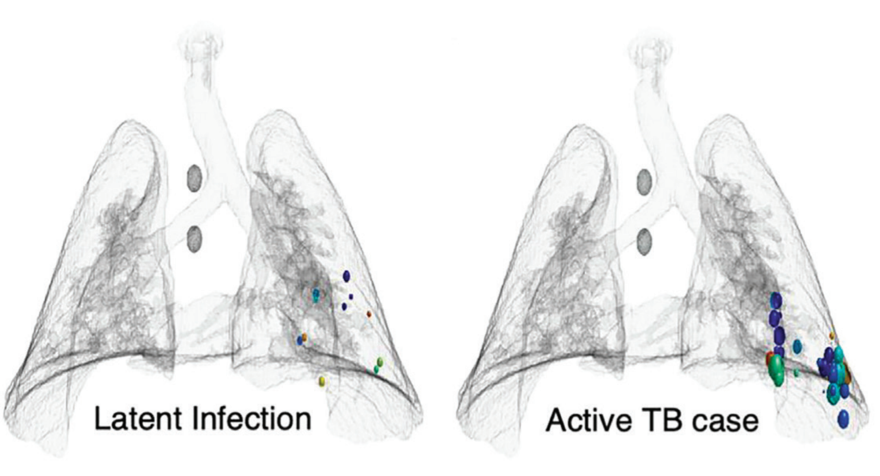

Figure 2. *Simulated lungs with tuberculosis (TB) granulomas that were created using an agent-based model. The lungs on the left have a latent asymptomatic infection, while the lungs on the right are experiencing full-blown TB symptoms because the granulomas have decayed and spilled their bacteria. Figure courtesy of [1].*

During her presentation at LS22, Kirschner noted that researchers essentially identified the current four-drug, nine-month regimen through trial and error. Mathematical models of infected lungs could help scientists determine why the four antibiotics do not work individually but are somehow effective in combination.

Kirschner reiterated that biology and medicine are the primary drivers of her mathematical models. "They're very iterative for many reasons," she said. "As we learn that certain things are important, we may go back and fine-grain the model in places where we coarse-grained it earlier. And biologists may discover the absence of something they now know is really crucial, so we want to add that in."

After all, while studying TB in the abstract is interesting and useful, the ultimate goal is to save lives and improve treatment outcomes. Catching infections early, controlling bacteria before they burst out of granulomas, and developing new therapies are ambitious goals, but the stakes are high. Kirschner hopes that biology-based whole-organ—or even whole-body—disease models will lead to new frontiers in vaccination and drug development for TB.

References

[1] Joslyn, L.R., Linderman, J.J., & Kirschner, D.E. (2022). A virtual host model of *Mycobacterium tuberculosis* infection identifies early immune events as predictive of infection outcomes. *J. Theor. Biol.*, *539*, 111042.

[2] Segovia-Juarez, J.L., Ganguli, S., & Kirschner, D. (2004). Identifying control mechanisms of granuloma formation during *M. tuberculosis* infection using an agent-based model. *J. Theor. Biol.*, *231*(3), 357-376.

[3] Wessler, T., Joslyn, L.R., Borish, H.J., Gideon, H.P., Flynn, J.L., Kirschner, D.E., & Linderman, J.J. (2020). A computational model tracks whole-lung *Mycobacterium tuberculosis* infection and predicts factors that inhibit dissemination. *PLOS Comput. Biol.*, *16*(5), e1007280.

Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Machine Learning

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a day after breakfast, rather than follow a sharper but much more complicated dosage. This property even arises in the field of economics when national banks set interest rates in six-month horizons and only revisit the policies to adjust for newly emerging macroeconomic scenarios.

Are these ideas and methods at all relevant to ML? Let us start with George Cybenko's seminal result: the so-called universal approximation theorem (UAT). The UAT states that a finite combination of rescaled and shifted activation functions (i.e., neural networks) are dense in a variety of functional classes [1]. This functional analysis result complements other fundamental outcomes in analysis, including the density of polynomials, Fourier series, and compactly supported smooth functions.

The UAT serves regression and classification purposes in the context of supervised learning (SL). Roughly, we can classify any data set by simply approximating the characteristic function—taking value 1 in one set of items and 0 in the complementary one—to ultimately allocate the correct label to each item.

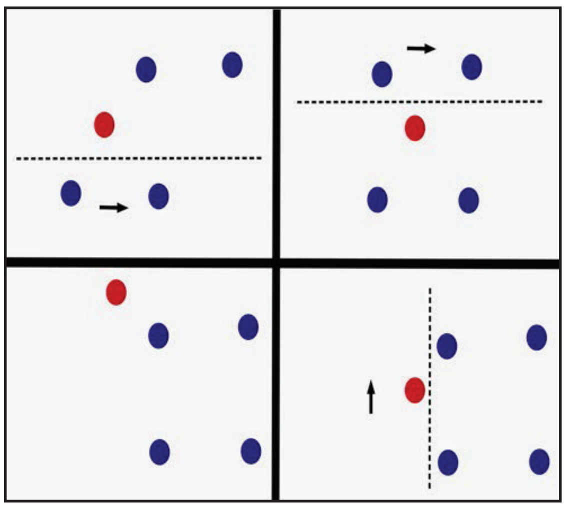

Figure 2. *Canonical iterative motion leads to classification by induction with piecewise-constant switching controls. Figure courtesy of Domènec Ruíz-Balet of Imperial College London.*

Cybenko's beautiful result, which was proven as a corollary of the Hahn-Banach theorem, opened the door for a variety of methods that now play an essential role in ML. Because the UAT guarantees the achievement of all goals by simply identifying the right parameters in Cybenko's ansatz

$$f(x)=\sum_{k=1}^{n} W_k\sigma(A_k\cdot x+b_k),$$

we may adopt the least squares point of view and search for parameter values that minimize the distance to the needed function during the so-called training phase. Of course, such a naive and natural approach leads to great challenges — we must simultaneously face the devil of the lack of convexity and the curse of dimensionality!

Eduardo Sontag and Hector Sussmann explored the control consequences of dynamical systems properties of the form

$$\dot{x}(t)=W(t)\sigma(A(t)\cdot x(t)+b(t)),\quad t\geq 0,$$

a concept that Weinan E later revisited [2, 9]. This form is a neural ordinary differential equation (NODE) that is driven by activation functions σ, like the sigmoid functions (monotonic continuous functions that take value 0 at $-\infty$ and 1 at $+\infty$). Cybenko introduced these functions—which are rather atypical in mechanics—for approximation purposes.

It took the mathematical control community many years to understand how to apply control methods to the real challenges of SL. Recent works have finally proved that deep residual neural networks (ResNets)—time discretizations of NODEs—enjoy the amazing and unexpected property of simultaneous or ensemble control [8]. We can build controls (i.e., train parameters) in such a way that an arbitrarily large number of trajectories simultaneously arrive almost exactly at their targets: the labels that correspond to the items of the data set to be classified (see Figure 1, on page 1).

This dynamical systems perspective presents some interesting advantages by offering better dependence of available data and the opportunity to tune classification methods for improved stability properties. It can also exploit plenty of the existing knowledge in more mature areas of applied mathematics. In fact, the activation function σ's very nature is responsible for the exceptionally powerful property of simultaneous control that ensures the requirements of SL. The most paradigmatic example is the Rectified Linear Unit (ReLU) activation function, which simply takes the value 0 when $x<0$ and 1 when $x>0$. When driven by the ReLU, a NODE behaves like a Rubik's Cube — it is solvable via a finite number of smart operations for which part of the cube is frozen while the other part rotates in the appropriate direction and sense. The goal of a Rubik's Cube is to ensure that all faces are homogeneous in color. This objective is similar to the task of a NODE, which drives each initial item to a given distinguished reservoir according to its label.

The proofs in previous studies are inductive, and researchers build the controls (or parameters) to be piecewise constant in order to exploit the ReLU's essence [8]. At each time instant, the ReLU splits the Euclidean space into two half-spaces: (i) one that is frozen along the dynamics because the nonlinearity vanishes and (ii) one that evolves exponentially where the ReLU is active. A strategic, inductive choice of the different hyperplanes/equators (via selection of the values of the controls/parameters A and b) and the direction of the dynamics/wind (via the control W) guarantees classification in a finite number of steps (see Figure 2).

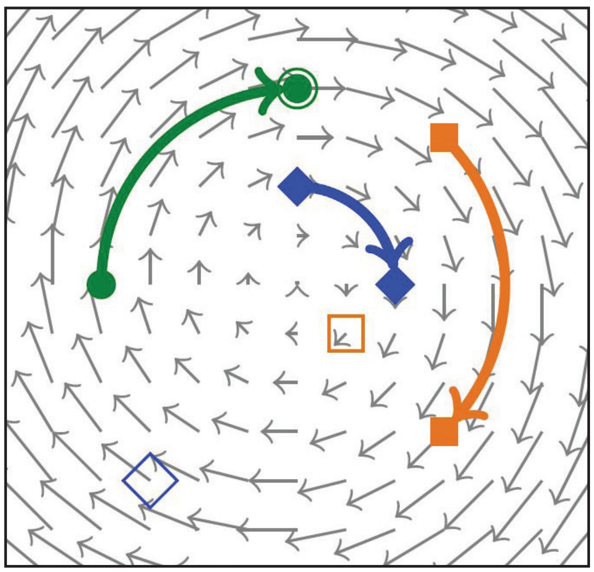

Figure 3. *Motion generated by a linear control system. All points move simultaneously without the possibility of classification according to their labels. Figure courtesy of Daniël Veldman of Friedrich-Alexander-Universität Erlangen-Nürnberg.*

These results provide the backbone theory to ensure that NODEs fulfill the ensemble controllability properties that are necessary for classification. Of course, the controls that we observe in numerical simulations are often less complex, since they are computed as minimizers of a suitably penalized loss functional. Such findings rely fundamentally on the nonlinearity of the activation functions σ. Indeed, the ensemble controllability property is impossible for a linear system that would rather behave like the system in Figure 3, unable to classify items according to labels.

We can transfer this control result for ResNets and NODEs to the framework of transport equations (advection, convection, and so forth) via the classical principle that the trajectories of the first equations constitute the characteristics of the latter:

$$x'(t)=\sigma(x(t),t)\rightarrow$$
$$\partial_t v+\mathrm{div}_x(\sigma(x,t)v)=0.$$

Approximating the distributions of masses to be transported with atomic measures—whose supports play the role of items in classification—accomplishes this transfer. Control and ML also come together in the traditional problem of mass transport, though not exactly in the same way as in optimal transport or the Monge-Kantorovich problem. Rather, these disciplines align by means of time-dependent vector fields with the oversimplified geometry of the activation function.

My colleagues and I are not the first researchers to claim the tight connections between control and ML [4, 7]. But now that we have been working on this topic for several years, we realize that there is still much to discover in the vast forest that connects these two areas. Although finding the paths through the dense grove will be intellectually challenging, doing so may add additional detail to the fascinating global map of the mathematical sciences. These paths will likely take a zigzagged course that resembles the strategies for solving a Rubik's Cube or the trajectories that assure the needs of learning through ResNet control.

Many other unexplored realms merit the attention of the applied mathematics community as well. One such topic is federated learning — a subject that is closely related to the classical splitting and domain decomposition methods in numerical analysis. We are currently investigating federated learning in cooperation with the artificial intelligence company Sherpa.ai,[1] but we leave this topic for another occasion.

This article is based on Enrique Zuazua's W.T. and Idalia Reid Prize Lecture[2] at the 2022 SIAM Annual Meeting,[3] which took place this July in Pittsburgh, Pa.

References

[1] Cybenko, G. (1989). Approximation by superpositions of a single function. *Math. Control Signals Syst.*, *2*, 303-314.

[2] E, W. (2017). A proposal on machine learning via dynamical systems. *Commun. Math. Stat.*, *5*, 1-11.

[3] Fernández-Cara, E., & Zuazua, E. (2003). Control theory: History, mathematical achievements and perspectives. *Bol. Soc. Esp. Mat. Apl.*, *26*, 79-140.

[4] Fradkov, A.L. (2020). Early history of machine learning. *IFAC-PapersOnLine*, *53*(2), 1385-1390.

[5] Geshkovski, B., & Zuazua, E. (2022). Turnpike in optimal control of PDEs, ResNets, and beyond. *Acta Numer.*, *31*, 135-263.

[6] Kalman, R.E. (1960). On the general theory of control systems. *IFAC Proc. Vol.*, *1*(1), 491-502.

[7] LeCun, Y. (1988). A theoretical framework for back-propagation. In D. Touretzky, G.E. Hinton, & T. Sejnowski (Eds.), *Proceedings of the 1988 connectionist models summer school* (pp. 21-28). Pittsburgh, PA: Carnegie Mellon University.

[8] Ruiz-Balet, D., & Zuazua, E. (2021). Neural ODE control for classification, approximation and transport. Preprint, *arXiv:2104.05278.*

[9] Sontag, E., & Sussmann, H. (1997). Complete controllability of continuous-time recurrent neural networks. *Syst. Control Lett., 30*(4), 177-183.

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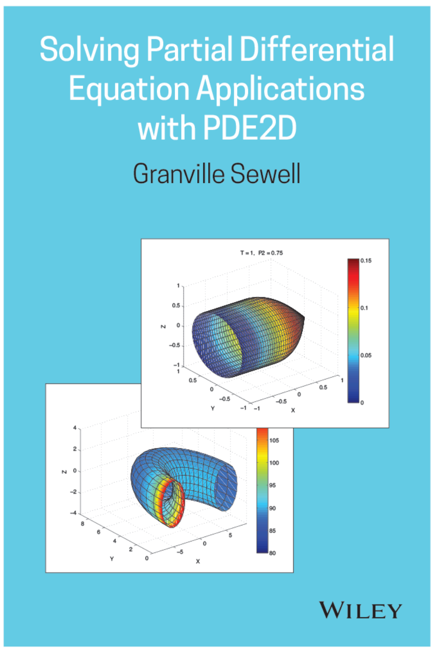
[1] https://www.sherpa.ai

[2] https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=74981

[3] https://www.siam.org/conferences/cm/conference/an22



GraphBLAS and GraphChallenge Advance Network Frontiers

By Jeremy Kepner, David A. Bader, Tim Davis, Roger Pearce, and Michael M. Wolf

Many factors inspire interest in networks and graphs. The Internet is just as important to modern-day civilization as land, sea, air, and space; it connects billions of humans and is heading towards trillions of devices. Deep neural networks (DNNs)—which are also graphs—are key to artificial intelligence, and biological networks underpin life on Earth. In addition, graph algorithms have served as a foundation of computer science since its inception [3]. One can represent and operate on graphs in many different ways. A particularly attractive approach exploits the well-known duality between a graph as a collection of vertices and edges and its representation as a sparse adjacency matrix.

Graph Algorithms in the Language of Linear Algebra,[1] which was published by SIAM in 2011 [6], provides an applied mathematical introduction to graphs by addressing the foundations of graph/matrix duality (see Figure 1). This fundamental connection between the core operations of graph algorithms and matrix mathematics is quite powerful and represents a primary viewpoint for DNNs. Yet despite its widespread use in graph analysis, basic graph/matrix duality is still only a starting point. For instance, the final chapter of *Graph Algorithms in the Language of Linear Algebra* posed several fundamental questions about the analysis of large graphs in ontology/data modeling; time evolution (or streaming); detection theory (or graph modeling in general); and algorithm scaling [6]. These questions—along with the emergence of important applications in privacy, health, and cyber contexts—set the stage for the subsequent decade of work.

SOFTWARE AND PROGRAMMING

Since 2011, researchers have written thousands of papers that explore the aforementioned topics from a graph/matrix perspective. Interestingly, previous prototyping efforts that began in the mid-2000s recognized that existing computer architectures were not a good match for a variety of graph and sparse matrix problems [10]. The prototypes introduced several innovations: high-bandwidth three-dimensional networks, cacheless memory, accelerator-based processors, custom low-power circuits, and—perhaps most importantly—a sparse matrix-based graph instruction set. Today, many of these innovations are present in commercially available systems like Cerebras,[2] Graphcore,[3] Lucata,[4] and NVIDIA.[5]

The challenges associated with graph algorithm scaling led multiple scientists to identify the need for an abstraction layer that would allow algorithm specialists to write high-performance, matrix-based graph algorithms that hardware specialists could then design to without having to manage the complexities of every type of graph algorithm. With this philosophy in mind, a number of researchers (including two Turing Award winners) came together and proposed the idea that "the state of the art in constructing a large collection of graph algorithms in terms of linear algebraic operations is mature enough to support the emergence of a standard set of primitive building blocks" [8]. The centerpiece of this abstraction is the extension of traditional matrix multiplication to semirings

$$\mathbf{C} = \mathbf{AB} = \mathbf{A} \oplus . \otimes \mathbf{B}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are (usually sparse) matrices over a semiring with corresponding scalar addition $\oplus$ and scalar multiplication $\otimes$. Particularly interesting combinations include standard matrices over real or complex numbers $(+.\mathrm{x})$, tropical algebras $(\max.+)$ that are important for neural networks, and set operations $(\cup.\cap)$ that form the foundation of relational databases like SQL. One can build countless graph algorithms with these combinations of operations, and the Graph Basic Linear Algebra Subprograms (GraphBLAS) mathematical specification, `C` specification, and high-performance implementation subsequently emerged [1, 4, 5]. These programs are now part of some of the world's most popular mathematical software environments.

Many innovations in graph processing occurred during this time, inspiring new venues to highlight these developments. MIT, DARPA, Amazon, IEEE, and SIAM collaborated to establish GraphChallenge,[6] which consists of several hundred data sets and

See **GraphChallenge** *on page 7*

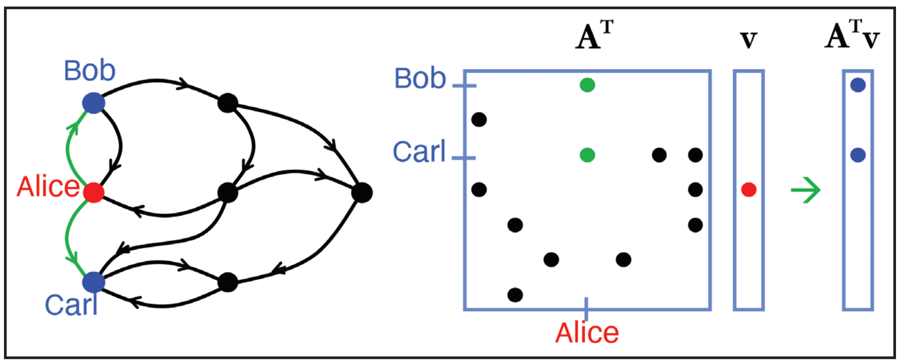


Figure 1. *Graph/matrix duality as depicted via a breadth-first search from starting point Alice to neighbors Bob and Carl and its adjacency matrix multiplication equivalent. Here,* $\mathbf{A}(i,j) > 0$ *implies an edge between vertices* i *and* j. *Figure courtesy of Jeremy Kepner.*

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[1] https://epubs.siam.org/doi/book/10.1137/1.9780898719918

[2] https://www.cerebras.net

[3] https://www.graphcore.ai

[4] https://lucata.com

[5] https://www.nvidia.com

[6] https://graphchallenge.mit.edu

Active Cells

Continued from page 2

simulation of active suspensions, shedding light on their peculiar flow instabilities [7] and the way in which confinement sculpts their behaviors (see Figure 1f, on page 2). Recently, scientists have used reductions that are based on thermodynamically consistent moment closures to simulate large-scale active turbulence (see Figure 1g, on page 2). Looping back to the cell, similar active matter theories that stem from symmetry principles have successfully described the internal structure of the spindles themselves [1].

Another example with the same ingredients of microtubules and molecular motors occurs in egg cells, or oocytes: the largest cells that animals produce. Raising an egg is a community effort, and its production can require the movement of specialized proteins—supplied by other cells—around and across the oocyte. Molecular diffusion does so efficiently in small cells, but diffusive transport can be exceedingly slow in large ones. How might an oocyte overcome this supply chain issue? The shortcut is flow. Figure 2a shows the streamlines of a cell-spanning cytoplasmic vortex in the 300 micrometer-scale oocyte of the common fruit fly. With speeds of roughly 100 nanometers per second, these flows can transport proteins from one end of the cell to the other in 30 minutes; diffusion alone takes about a day.

What drives this little hurricane in a nearly microscopic egg? Fluorescent microscopy reveals that microtubules are attached to the cell wall and bent sideways, like seaweed in a running tide. Here, however, the seaweed pushes the flow. Figure 2b provides the basic physics. Kinesin-1 motor proteins carry payloads along microtubules towards their free "+" ends. According to Newton's third law, the motors must push down on the microtubules—perhaps causing them to bend—and simultaneously push the cargo up through the fluid, causing it to flow.

To better understand how this phenomenon might work, we developed a continuum model of the microtubule bed as an active porous medium [10]. A microtubule with position $\boldsymbol{X}(s,t)$ (s is the arclength from the base) evolves relative to a background flow $\boldsymbol{u}$ under its internal elastic forces and under a tangentially aligned motor load $-\sigma \boldsymbol{X}_s$, which is a distributed follower force. Assuming that the microtubule bed is locally well aligned yields a coarse-grained bed velocity $\boldsymbol{v}$, which itself creates the background fluid flow $\boldsymbol{u}$ as the solution to a Brinkman-Stokes equation:

$$-\nabla q + \Delta \boldsymbol{u} = \rho J \left[\boldsymbol{\Lambda} (\boldsymbol{u} - \boldsymbol{v}) - \sigma \boldsymbol{X}_s \right]$$

$$\text{and} \quad \nabla \cdot \boldsymbol{u} = 0.$$

Here, ρ is the areal density of anchored microtubules, $\boldsymbol{\Lambda}$ is a geometric tensor that captures the effect of microtubule orientation on fluid drag, and J is a Jacobian that handles the transition from the Lagrangian bed frame to the Eulerian fluid frame. Our model shows that the motors' compressive load upon the microtubule bed can drive a novel, collectively organized buckling instability of the bed wherein the microtubules all bend as one, inclining themselves and letting the motor proteins drive the flow (see Figure 2c). The instability's collective nature is revealed by the fact that it can only occur at sufficiently high microtubule density ρ (see Figure 2d).

While this fluid-structure problem yields new physics, models, and simulations, our contributions still leave many questions unanswered. What is the effect of the three-dimensional cell shape and bed inhomogeneity? Which cellular determinants initiate a transition to flow? What precise biological purpose is being served? We are working with experimentalists on these queries and hope to ultimately solve them. Many of the tools that we discuss here—i.e., multiscale modeling and coarse graining, continuum mechanics, partial differential equations that evolve in evolving domains, large-scale simulations, and stability analyses—have applications in other problems where cell biology and development meet active matter physics.

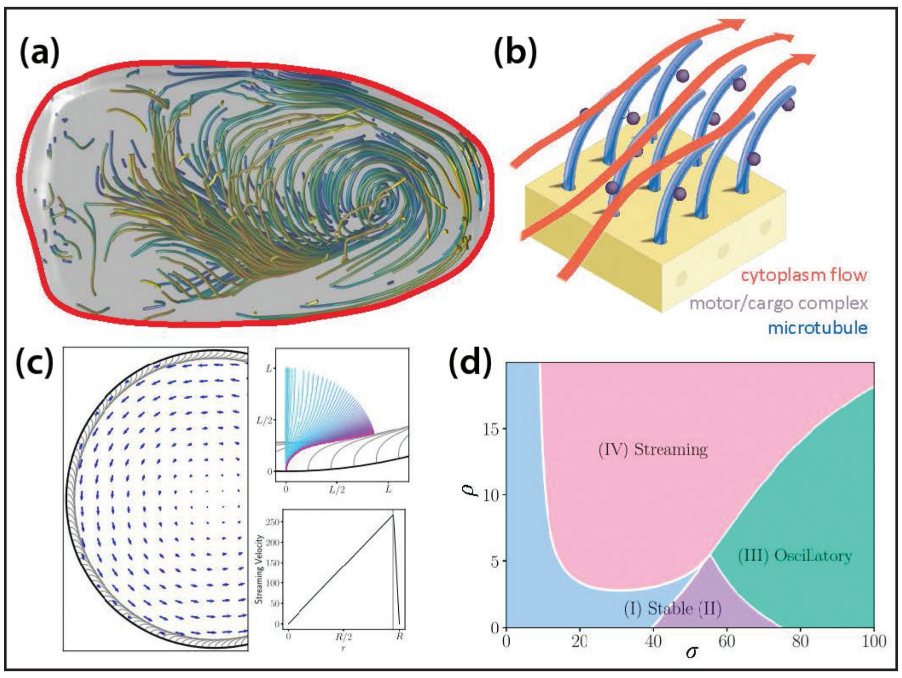

Figure 2. *Internal flows within the developing egg (oocyte) of the fruit fly.* **2a.** *Measured streamlines of cytoplasmic flows in the oocyte.* **2b.** *A schematic of kinesin-1 motors carrying cargos up peripherally bound microtubules and generating flow.* **2c.** *A bed of motor-laden microtubules undergoes a collective bending instability and produces a steady streaming flow.* **2d.** *Phase diagram that shows the nature of flow instabilities as a function of dimensionless motor forcing* σ *and microtubule density* ρ. *Microscopy data for 2a supplied by Wen Lu and Vladimir Gelfand and analyzed by Reza Farhadifar and Sayantan Dutta. Figure 2b courtesy of Reza Farhadifar, and Figures 2c and 2d adapted from [10] and courtesy of David Stein.*

References

[1] Brugués, J., & Needleman, D. (2014). Physical basis of spindle self-organization. *Proc. Natl. Acad. Sci.*, *111*, 18496.

[2] Dombrowski, C., Cisneros, L., Chatkaew, S., Goldstein, R.E., & Kessler, J.O. (2004). Self-concentration and large-scale coherence in bacterial dynamics. *Phys. Rev. Lett.*, *93*, 098103.

[3] Gao, T., Betterton, M., Jhang, A.-S., & Shelley, M. (2017). Analytical structure, dynamics, and coarse graining of a kinetic model of an active fluid. *Phys. Rev. Fluids*, *2*, 093302.

[4] Lindow, N., Redemann, S., Brünig, F., Fabig, G., Müller-Reichert, T., & Prohaska, S. (2018). Quantification of three-dimensional spindle architecture. *Methods Cell Biol.*, *145*, 45-64.

[5] Nazockdast, N., Rahimian, A., Needleman, D., & Shelley, M. (2017). Cytoplasmic flows as signatures for the mechanics of mitotic positioning. *Mol. Biol. Cell*, *28*(23), 3261-3270.

[6] Saintillan, D., & Shelley, M. (2008). Instabilities and pattern formation in active particle suspensions: Kinetic theory and continuum simulations. *Phys. Rev. Lett.*, *100*, 178103.

[7] Saintillan, D., & Shelley, M. (2013). Active suspensions and their nonlinear models. *Comp. Rend. Phys.*, *14*(6), 497-517.

[8] Sanchez, T., Chen, D.T.N., DeCamp, S.J., Heymann, M., & Dogic, Z. (2012). Spontaneous motion in hierarchically assembled active matter. *Nature*, *491*, 431-434.

[9] Shinar, T., Mana, M., Piano, F., & Shelley, M.J. (2011). A model of cytoplasmically driven microtubule-based motion in the single-celled *Caenorhabditis elegans* embryo. *Proc. Natl. Acad. Sci.*, *108*(26), 10508-10513.

[10] Stein, D.B., De Canio, G., Lauga, E., Shelley, M.J., & Goldstein, R.E. (2021). Swirling instability of the microtubule cytoskeleton. *Phys. Rev. Lett.*, *126*, 028103.

[11] Weady, S., Shelley, M.J., & Stein, D.B. (2022). A fast Chebyshev method for the Bingham closure with application to active nematic suspensions. *J. Comp. Phys.*, *457*, 110937.

[12] Woodhouse, F.G., & Goldstein, R.E. (2012). Spontaneous circulation of confined active suspensions. *Phys. Rev. Lett.*, *109*, 168105.

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The Value of Applied Mathematics Internships in Business, Industry, and Government

By Margaret Cheney, Andre Celestin, Jerett Cherry, Cole Moore, and Danny Long

Summer internships are tremendously valuable experiences for undergraduate and graduate students in applied mathematics and computational science. Interns have the opportunity to explore possible postgraduate career paths and engage in important real-world applications of their coursework. They return to school freshly motivated and excited to delve further into relevant material. Internships can also help participants—particularly graduate students—build a small financial reserve and may lead directly to employment after graduation. Even if graduates ultimately pursue positions elsewhere, internship experience on a resume is particularly attractive to prospective employers. In addition, summer internships allow students to explore different geographical locations and diverse types of working environments.

Many workplaces in business, industry, and government offer internships. Students can find postings for openings at government research laboratories through the Department of Defense,[1] Department of Energy,[2] National Aeronautics and Space Administration[3] (NASA), and various other agencies. They can also explore federally funded research and development centers (FFRDCs)—such as the Massachusetts Institute of Technology Lincoln Laboratory,[4] The Aerospace Corporation,[5] and MITRE[6]—as well as university-affiliated research centers (UARCs) like the Georgia Tech Research Institute,[7] Johns Hopkins University Applied Physics Laboratory,[8] and the University of Maryland's Applied Research Laboratory for Intelligence and Security.[9]

Various laboratories and sections within each laboratory have distinctive cultures and different emphases on research versus development. Working environments can vary widely depending on the type of organization. For example, government settings typically provide good job security (often with civil service tenure) but generally lower pay and a potentially frustrating bureaucracy. In contrast, small companies commonly have very little bureaucracy and offer higher pay, but they can be riskier in that employees frequently must help bring in funding in order to get paid. The key is to find a compatible match between one's working style and the job environment. Many employees find that intermediate choices like FFRDCs, UARCs, and medium-sized companies are a good fit.

Companies and laboratories enjoy having interns, as internship programs allow them to accomplish work with less expense and develop relationships with students and universities. Many organizations think of internships as extended job interviews. Some companies and labs hire year-round interns, or encourage students to work with them during vacations or conduct part-time remote work during the academic year. Such projects can even become master's theses or parts of doctoral dissertations. In some cases—and with appropriate coordination between an advisor and a willing partner at the organization in question—companies or labs might be able to pay students a full salary to work on projects that become their doctoral dissertations while simultaneously serving as company/lab deliverables. In this scenario, the student's boss is typically on their Ph.D. committee and the student is in residence at the company/lab. Such a situation can be tricky to coordinate but benefits everybody when it is successful. The student receives better pay, completes their dissertation, and has a guaranteed job upon graduation; the company/lab accomplishes necessary work and gains a well-trained employee; the sponsor/customer is pleased with the project's progress and the enlarged pool of trained personnel; and the university learns about important problems, can use the original student's salary for another student, and establishes a better connection to the company/lab.

Below, several students from Colorado State University describe their internship experiences in the summer of 2022.

Andre Celestin: NASA Jet Propulsion Laboratory

I interned at NASA Jet Propulsion Laboratory[10] (JPL) in the Algorithm Development Group within the Radar Science and Engineering Section.[11] Project scientist Paul Rosen served as my mentor. We constructed a mathematical framework for an imaging satellite that will be sent to Enceladus, one of Saturn's moons. The satellite will form images via synthetic aperture radar (SAR) interferometry, which employs image processing techniques to combine many measurements of a target. Due to time constraints, I spent most of my internship familiarizing myself with SAR interferometry by building simulators and image processing code. On my first day, I received a massive 64-core computer with two 3090 NVIDIA graphical processing units (GPUs) for writing my code. I utilized the machine's full power by learning how to parallelize code and validate the results. I hope to return to JPL and work with Paul again because the experience was incredible; everyone at JPL is eager to talk about their ongoing projects and encourages you to succeed.

Jerett Cherry: U.S. Naval Research Laboratory

I interned at the U.S. Naval Research Laboratory's[12] (NRL) Radar Division[13] in Washington, D.C., where I collaborated with engineers on research and development for radar systems. I learned about radar digital signal processing and wrote data analysis software for use in conjunction with a radar testbed. Data from this testbed is then pulse compressed and Doppler processed to form a range-Doppler map. This map plots reflections from moving and stationary objects in terms of range and Doppler shift (see Figure 1).

See **Mathematics Internships** *on page 8*

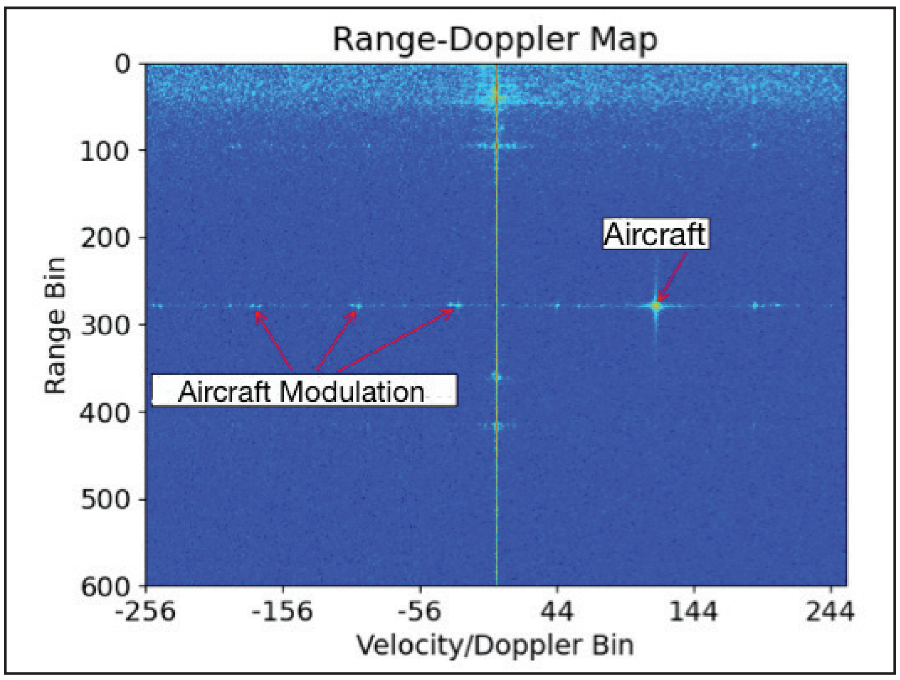

Figure 1. *A range-Doppler map, which plots radar reflections in terms of range and velocity. Figure courtesy of Jerett Cherry.*

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[1] https://dodstem.us/participate/opportunities

[2] https://www.energy.gov/eere/education/internships-fellowships-graduate-and-postdoctoral-opportunities

[3] https://intern.nasa.gov

[4] https://www.ll.mit.edu/careers/student-opportunities/summer-research-program

[5] https://aerospace.org/students-and-recent-graduates

[6] https://careers.mitre.org/us/en/student-programs

[7] https://www.gtri.gatech.edu/careers/student-positions

[8] https://www.jhuapl.edu/careers/internships

[9] https://www.arlis.umd.edu/student-research-opportunities

[10] https://www.jpl.nasa.gov/edu/intern

[11] https://communicationstrackingradar.jpl.nasa.gov/sections/sec-334

[12] https://www.nrl.navy.mil/Careers/Students/SSEP

[13] https://www.nrl.navy.mil/Our-Work/Areas-of-Research/Radar

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Science Policy

Continued from page 3

solicitation. RENEW seeks to build workforce capacity at institutions that are historically underrepresented in the Office of Science research portfolio. We likewise met with Carrie Wolinetz, the Deputy Director for Health and Life Sciences in the White House Office of Science and Technology Policy,[12] who discussed the National Institutes of Health's (NIH) new Advanced Research Projects Agency for Health[13] and overviewed ways in which applied mathematicians can engage.

The CSP wraps up each meeting with discussions about future scientific priorities and specific topics and initiatives that take precedence within the SIAM community. During the spring CSP meetings, committee members also convene directly with congressional staffers. At the 2022 spring gathering, I had the opportunity to meet with members of Congress who were especially passionate about health policy, which is the focus of my own work. I was able to communicate the impacts of DOE budgetary choices on health research. For example, ongoing collaborative research projects between DOE and NIH utilize novel algorithms that SIAM members are developing. By linking the importance of mathematical research to initiatives about which congressional staffers are passionate, Fellowship recipients can yield a positive policy outcome for the entire SIAM community.

In addition to my participation on the CSP, I also completed an independent policy project that involved meeting with staff members from the U.S. House of Representatives' Committee on Science, Space, and Technology's Subcommittee on Energy,[14] which has jurisdiction over DOE research programs. While recent large-scale initiatives have attempted to utilize artificial intelligence (AI) and mathematics in health research, the field is plagued by a lack of available biomedical data. In my discussions with members of the Subcommittee on Energy, I highlighted areas wherein AI technologies with healthcare applications could assist in the detection of a new opioid over-prescription zone, provide probabilistic early warning analyses of COVID-19 outbreaks, and identify regions of environmental exposure to radon. Several recent and ongoing initiatives aim to develop novel algorithms and applied mathematics research to drive biomedical advancement, though an understandable hesitancy surrounds the sharing of medical data. As such, I accentuated recent computing advances for the handling of private data, such as DOE's CITADEL security framework.[15] We must ensure that researchers have both the funding to complete their projects and the appropriate mechanisms for curating the data that are required for large-scale biomedical data science.

The opportunity to learn about research priorities and initiatives directly from agency directors was invaluable, and firsthand knowledge about the creation of program initiatives is crucial to furthering our understanding of the way in which public need influences and pivots research directions. Recognizing the impact of these choices on the SIAM community—from undergraduate fellowship programs to large-scale initiatives—underscores the significance of the CSP. By participating in the SIAM Science Policy Fellowship Program, I gained a deeper appreciation of the CSP's importance in the context of science policy. It is vital that the SIAM community continues to have a voice in Washington, D.C. Budget and policy decisions are constantly underway, and the future of the scientific community hinges on these choices. It has been a privilege to participate on the CSP as a Science Policy Fellowship recipient, and I look forward to serving SIAM in future science policy ventures.

Are you interested in applying for the SIAM Science Policy Fellowship Program? The application deadline for the next round of Fellowship recipients is ***November 15, 2022****. Three to five postdoctoral and/or early-career researchers will be selected to serve a two-year term that includes training, attending biannual SIAM Committee on Science Policy meetings, interfacing with federal officials, and participating in an advocacy day on Capitol Hill in Washington, D.C. Learn more and apply online.*[16]

Samantha Erwin is a data scientist in Applied AI Systems at Pacific Northwest National Laboratory. Her research focuses on the creation of mathematical models to elucidate biological mechanisms. She has led teams to develop data analysis pipelines that integrate natural language processing, graphical models, machine learning, and statistical analysis.

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[12] https://www.whitehouse.gov/ostp

[13] https://www.nih.gov/arpa-h

[14] https://science.house.gov/subcommittees/energy-117th-congress

[15] https://www.olcf.ornl.gov/2021/05/05/nccs-introduces-citadel-security-framework

[16] https://www.siam.org/students-education/programs-initiatives/siam-science-policy-fellowship-program

How Do Snakes Slither? A Recipe for Reptation

MATHEMATICAL CURIOSITIES

By Mark Levi

While biking uphill one day, I saw a snake crossing the path ahead of me. The pavement was a bit slippery from a recent rain shower, and the snake slid rather inefficiently towards the grass on the opposite side of the path while also sliding down the slope. But once in the grass, the snake shot almost as an arrow and slithered perfectly with virtually no sideways motion (see Figure 1). Its body sliced through the grass as if constrained to a tight channel. A slow motion video of all this is available on my website.[1] The velocity constraint imposed by the grass must have been key to the efficient locomotion; the constraint was absent on the asphalt and the snake was almost helpless. This is akin to skating on ice, where the zero sideways velocity constraint on the skate is key to propulsion.

The snake slithering in the grass converts the bending/unbending effort of the body—the only thing it can do, I think—into forward motion. How exactly does the snake do it? Which muscles must it contract, and which ones must it relax? In other words, what is the recipe for such reptation?[2] Our simplified snake has a negligibly thin body that is confined to a prescribed channel, which allows only tangential motion.

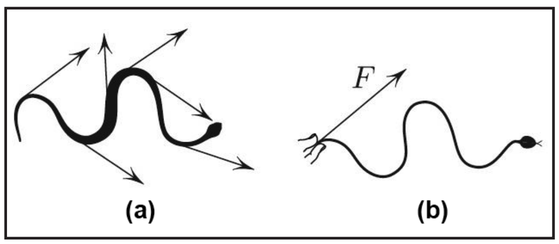

Figure 1. *The snake in motion and static.* **1a.** *The velocity of the snake in the grass is tangential.* **1b.** *The propulsion force F is given by (1). Instead of pulling the fingers, the snake can use F to overcome friction with the grass when slithering. This figure is only a thought experiment. I did not touch the snake.*

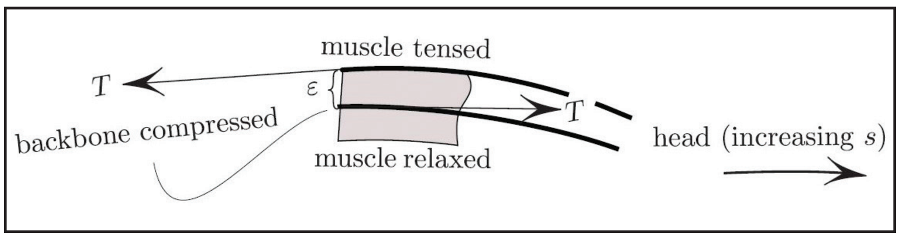

Figure 2. *The torque $\tau=\varepsilon T$, which is created by compression T of the backbone and the equal tension of the muscle, tries to rotate the shaded section counterclockwise. Here, ε is small (the snake is thin), T is large, and $\tau=\varepsilon T$ is finite. The torque is counterclockwise, so $\tau>0$.*

After watching the snake, I realized that the pulling force F in Figure 1b is given by

$$F=\int_0^L \tau(s)k'(s)ds, \qquad (1)$$

where s is the arclength parameter, τ is the bending torque (see Figure 2), k is the curvature, and L is the length of the snake.

Taking (1) for granted, how should the snake flex its muscles? To fix the assumptions, we take the channel in which the snake moves as given. Presumably, the very least the snake should do is make sure that $\tau(s)k'(s)>0$ for all s in order to avoid cancellation in (1) — i.e., a wasteful competition between pulling forward ($\tau k'>0$) and backward ($\tau k'<0$) with different parts of its body. Figure 3 illustrates this recipe for reptation: *bend the body to the right (or left) if k is increasing (or decreasing).*

A Dynamical Recipe for Reptation

The aforementioned recipe is good for a mathematician, but detecting $k'(s)$ seems like a physiologically formidable task for the snake, since doing so involves the apple-versus-orange type comparison of k between two different parts of the body. How can the snake then estimate k', which it must do in some way because (1) captures the physics of slithering? Here is a possible answer.

As the snake slithers along the channel, the curvature at a fixed point of the body changes from $k(s)$ at time $t=0$ to $k(s+vt)$ at time t (here, v is the speed and s is measured along the channel). Therefore,

$$k'(s)=v^{-1}\partial k/\partial t;$$

$k'(s)$ is thus determined by $\partial k/\partial t$, which feels like bending/unbending and hence is much easier to sense than $k'(s)$. We can consequently restate the recipe for reptation as follows: *the bending effort must coincide with the direction of bending change*. That is, τ must attempt to enhance the deformation that is imposed by travel along the channel. Or more palpably, *tense the muscles that are contracting* (due to bending/unbending while the snake slithers along the channel) and *relax the muscles that are stretching*.

Under this recipe, the muscles do positive work — which then must go into locomotion (e.g., into overcoming friction, into acceleration, or both).

See **Recipe for Reptation** *on page 10*

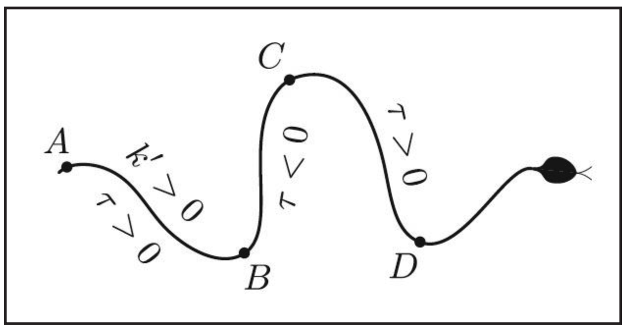

Figure 3. *The recipe for reptation is to change the direction of the bending effort τ at points A, B, C, and D of extreme curvature. Here, k is minimal (negative) at A and C and maximal (positive) at B and D.*

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[1] https://www.marklevimath.com/sinews

[2] Some snakes, such as sidewinders, reptate by different mechanisms without obeying the velocity constraint. Here I only address one specific type of reptation.

GraphChallenge

Continued from page 5

well-defined mathematical graph problems in the areas of triangle counting, clustering of streaming graphs, and sparse DNNs. Since its debut in 2017, GraphChallenge has seen an abundance of submissions — contestants have even integrated parts of their work into a wide range of research programs and system procurements.

GraphChallenge has revealed that due to resulting improvements in graph analysis systems, many graph problems are now fundamentally bound by computer memory bandwidth (as opposed to processor speed or memory latency) [9]. It has also provided clear targets for those who are trying to advance computing systems scaling for the solution of graph problems. Additionally, innovations to improve the performance of graph algorithms have fed back into sparse linear algebra libraries to benefit scientific computing applications like Kokkos Kernels [11].

Questions in ontology/data modeling pertain to the way in which researchers handle more diverse data. The data that we want to manage with graphs involve more than just simple vertices and edges; they often include a large variety of very diverse metadata that are stored in SQL and NoSQL databases. Unsurprisingly, many folks in the database community have also been working on graph databases. To mathematically encompass these concepts, we must generalize the idea of a matrix into something called an *associative array*. For example, one can view a matrix as a mapping

$$\mathbf{A}: I\times J\rightarrow V$$

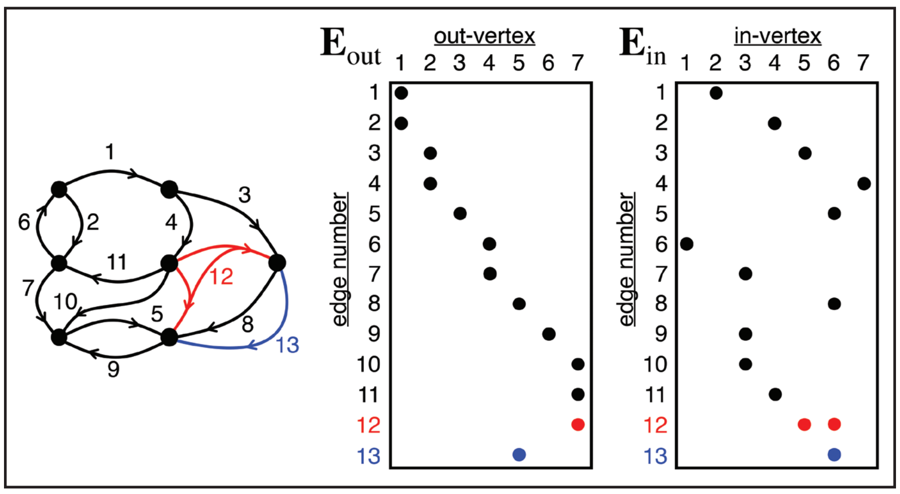
where $I=\{1,\ldots,M\}$, $J=\{1,\ldots,N\}$, and V is complex. In an associative array, I and J are now any strict totally ordered set (e.g., a set of strings) and V is a semiring [7]. This concept was first implemented in the D4M software system,[7] which links matrix mathematics and databases. It is now present in a number of database systems that utilize GraphBLAS as their underlying mathematical engine [2].

Time-evolving or streaming graphs have become one of the most important problems in graph analysis, and GraphBLAS has a natural way of addressing streaming graphs with diverse data via edge (or incidence) matrices (see Figure 2). Traditional adjacency matrices are limited in the types of graphs that they can represent. Adjacency matrices typically represent directed weighted graphs—which are a very important class of problems—but real data tend to be much more dynamic and diverse, with multiple edges and hyper-edges (edges that are connected to multiple vertices). In an edge matrix representation, one can easily adjust for this type of graph by simply adding rows to the end of the matrix. Furthermore, researchers can compute the corresponding adjacency matrix via

$$\mathbf{A}=\mathbf{E}_{\text{out}}{}^{\mathrm{T}}\mathbf{E}_{\text{in}}$$

where $^{\mathrm{T}}$ denotes the matrix transpose.

Ultimately, the aforementioned capabilities—enabled by GraphBLAS along with other graph innovations that are highlighted by GraphChallenge—have yielded new tools for tackling some of the most difficult and important problems in health data, privacy-preserving analytics, cybersecurity, and DNNs.



$\mathbf{E}_{\text{out}}$

edge number \ out-vertex	1	2	3	4	5	6	7
1	●						
2	●						
3		●					
4		●					
5			●				
6				●			
7				●			
8					●		
9						●	
10							●
11							●
12							●
13					●		

$\mathbf{E}_{\text{in}}$

edge number \ in-vertex	1	2	3	4	5	6	7
1		●					
2				●			
3					●		
4							●
5						●	
6	●						
7			●				
8						●	
9			●				
10			●				
11				●			
12					●	●	
13						●	

Figure 2. *Streaming multi-hyper-edge graph. Edge (or incidence) matrices assign a row to each edge and naturally handle the dynamic addition of identical edges (multi-edges) and edges that connect more than two vertices (hyper-edges). $\mathbf{E}_{\text{out}}(e,i)>0$ and $\mathbf{E}_{\text{in}}(e,j)>0$ imply an edge e between vertex i and vertex j. Figure courtesy of Jeremy Kepner.*

References

[1] Buluç, A., Mattson, T., McMillan, S., Moreira, J., & Yang, C. (2017). Design of the GraphBLAS API for C. In *2017 IEEE international parallel and distributed processing symposium workshops (IPDPSW)*. Lake Buena Vista, FL: IEEE.

[2] Cailliau, P., Davis, T., Gadepally, V., Kepner, J., Lipman, R., Lovitz, J., & Ouaknine, K. (2019). RedisGraph GraphBLAS enabled graph database. In *2019 IEEE international parallel and distributed processing symposium workshops (IPDPSW)*. Rio de Janeiro, Brazil: IEEE.

[3] Cormen, T.H., Leiserson, C.E., Rivest, R.L., & Stein, C. (2022). *Introduction to algorithms* (4th ed). Cambridge, MA: MIT Press.

[4] Davis, T.A. (2019). Algorithm 1000: SuiteSparse:GraphBLAS: Graph algorithms in the language of sparse linear algebra. *ACM Transact. Math. Software*, *45*(4), 1-25.

[5] Kepner, J., Aaltonen, P., Bader, D., Buluç, A., Franchetti, F., Gilbert, J., … Moreira, J. (2016). Mathematical foundations of the GraphBLAS. In *2016 IEEE high performance extreme computing conference (HPEC)*. Waltham, MA: IEEE.

[6] Kepner, J., & Gilbert, J. (Eds.) (2011). *Graph algorithms in the language of linear algebra*. Philadelphia, PA: Society for Industrial and Applied Mathematics.

[7] Kepner, J., & Jananthan, H. (2018). *Mathematics of big data: Spreadsheets, databases, matrices, and graphs*. In *MIT Lincoln Laboratory Series*. Cambridge, MA: MIT Press.

[8] Mattson, T., Bader, D., Berry, J., Buluç, A., Dongarra, J., Faloutsos, C., … Yoo, A. (2013). Standards for graph algorithm primitives. In *2013 IEEE high performance extreme computing conference (HPEC)*. Waltham, MA: IEEE.

[9] Samsi, S., Kepner, J., Gadepally, V., Hurley, M., Jones, M., Kao, E., … Monticciolo, P. (2020). GraphChallenge.org triangle counting performance. In *2020 IEEE high performance extreme computing conference (HPEC)*. Waltham, MA: IEEE.

[10] Song, W.S., Kepner, J., Nguyen, H.T., Kramer, J.I., Gleyzer, V., Mann, J.R., … Mullen, J. (2010). *3-D graph processor*. Presented at the fourteenth annual high performance embedded computing workshop (HPEC 2010). Lexington, MA: MIT Lincoln Laboratory.

[11] Trott, C., Berger-Vergiat, L., Poliakof, D., Rajamanickam, S., Lebrun-Grandie, D., Madsen, J., … Womeldorff, G. (2021). The Kokkos EcoSystem: Comprehensive performance portability for high performance computing. *Comput. Sci. Eng.*, *23*(5), 10-18.

Jeremy Kepner is head and founder of the Massachusetts Institute of Technology (MIT) Lincoln Laboratory's Supercomputing Center, a founder of the MIT-Air Force AI Accelerator, and a founder of the Massachusetts Green High Performance Computing Center, with appointments in MIT's Department of Mathematics and MIT Connection Science. He is a SIAM Fellow. David A. Bader is a distinguished professor and founder of the Department of Data Science in the Ying Wu College of Computing, as well as director of the Institute for Data Science at the New Jersey Institute of Technology. He is a SIAM Fellow. Tim Davis is a professor of computer science and engineering at Texas A&M University. He is also a SIAM Fellow. Roger Pearce is a computer scientist in the Center for Applied Scientific Computing at Lawrence Livermore National Laboratory and an adjunct Computer Science and Engineering Associate Professor of Practice at Texas A&M University. Michael M. Wolf manages the Scalable Algorithms Department at Sandia National Laboratories.

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[7] https://d4m.mit.edu

Mathematics Internships

Continued from page 6

Radar signal processing utilizes the fast Fourier transform, convolution, and least squares problems. Our running joke for the summer was that convolution was the answer whenever we wanted to do something with the data, which was not far from the truth. I felt mostly at home as a mathematician working with engineers, though difficulties did sometimes arise mainly due to problems in translation. Engineers speak a dialect of math that is just different enough to cause confusion for mathematicians (imagine using j for an imaginary number).

Beyond the lab-based learning, I was able to network, explore a new city, and earn more money. Working for the government also comes with useful benefits like gaining a security clearance, which is an attractive qualification for many job openings. Experiencing the thriving dynamic range of experimental and theoretical projects at NRL has even helped me consider additional topics for my own research. Interning is the best use of your summer.

Cole Moore: U.S. Naval Research Laboratory

Like Jerett, I also interned with the NRL's Radar Division. This internship served as an amazing opportunity for me to learn, meet new people, and explore an unfamiliar city. Most of my efforts focused on inverse electromagnetic scattering, and I specifically worked to develop faster methods for image reconstruction by using scattered microwave data (see Figure 2). This project involved the use of GPUs to accelerate computations and the implementation of multi-level algorithms to increase efficiency. In addition, I met and learned from many smart and passionate people; spending my summer in a new place provided ample occasions to get out and have fun. I would highly recommend interning to all of my fellow mathematics students.

Danny Long: Lawrence Livermore National Laboratory

I was fortunate enough to intern at Lawrence Livermore National Laboratory[14] (LLNL). I broadly work within the field of uncertainty quantification, which is quite useful for many applications at LLNL. My project involved the use of adjoint methods to approximate derivatives for functions that require the solution of partial differential equations (PDEs), which in turn help efficiently propagate uncertainty throughout the calculations. I contributed to a codebase that utilizes GPUs to solve the PDEs. Doing so allowed me to experience the state of the art in high-performance computing and interact with a variety of experts in mathematics, physics, and engineering.

LLNL was very supportive and clearly wanted interns to have a valuable experience. I spoke with research scientists outside of my project group on numerous occasions to hear about other research areas and establish professional connections. I also learned about LLNL-funded Ph.D. opportunities and the LLNL Postdoc Program,[15] for which summer interns are encouraged to apply.

Finding and Applying for Internships

Identifying and applying for internships naturally requires some effort. One way to begin the process is through networking, which involves reaching out to someone—such as an acquaintance from a conference—who has connections at an organization of interest. SIAM's online Career Center[16] provides a helpful list of internships and career information, and many organizations list internship opportunities in the "careers" section of their websites. Students can also email their resumes directly to anyone who conducts intriguing research and inquire about summer internships. It is best to start this process early so that the researcher in question has ample time to arrange funding.

A resume should help hiring managers or potential mentors identify an applicant's skills and interests in order to best align those qualities with the tasks at hand. Listing technical courses and other relevant topics of study—including physics, computer science, statistics, and engineering (as well as math)—is also beneficial. In some cases, course projects can demonstrate practical experience. Computer skills are important as well, as many internships involve programming tasks. Hiring managers mainly look for evidence that applicants are interested in learning about their institution's endeavors and enthusiastic about contributing.

The timing for internship applications varies widely. Deadlines can fall unexpectedly early (especially if a security clearance is needed) and may even occur before the end of October. For instance, Lincoln Laboratory[17] and Sandia National Laboratories[18] posted advertisements in late summer 2022 for internship positions in the summer of 2023. On the other hand, smaller companies may not finalize their openings until the spring if they are waiting for available funding. Furthermore, internship positions at large organizations often remain open until they are filled — so it is always worth seeking out an exciting opportunity, no matter when one's internship search begins. Happy hunting!

Margaret Cheney is a professor of mathematics at Colorado State University. She is also a SIAM Fellow. Andre Celestin, Jerett Cherry, Cole Moore, and Danny Long are graduate students in the Department of Mathematics at Colorado State University.

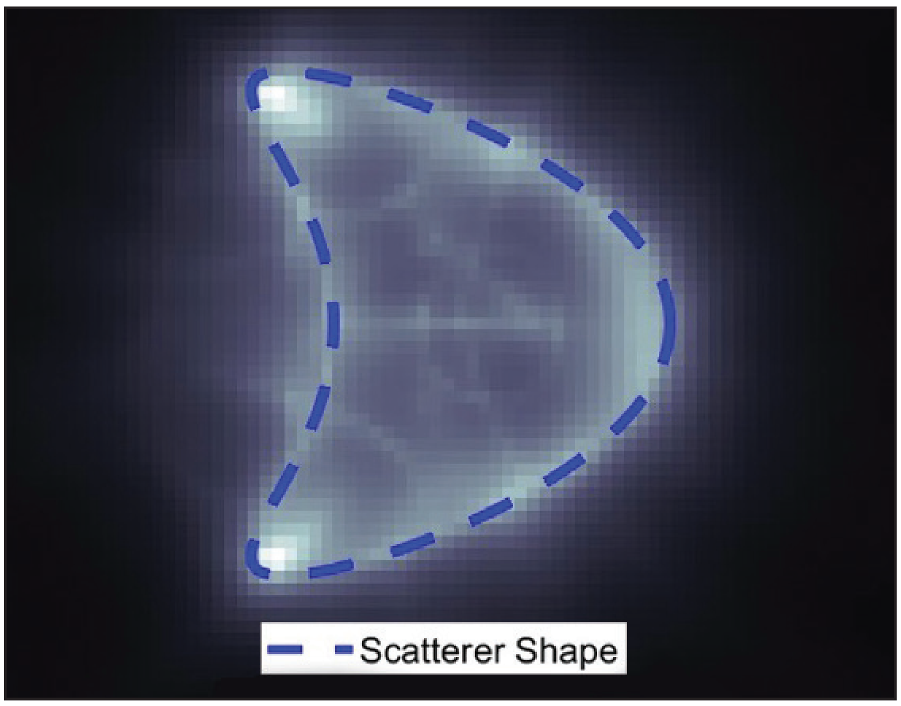

Figure 2. *Image reconstruction of a kite-shaped scattering target. The dashed line overlay illuminates the true target outline. Figure courtesy of Cole Moore.*

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[14] https://www.llnl.gov/join-our-team/careers/students

[15] https://st.llnl.gov/opportunities/postdocs/postdoc-program

[16] https://www.siam.org/careers/internships

[17] https://careers.ll.mit.edu/search/?q=%22Summer%22

[18] https://www.sandia.gov/careers/career-possibilities/students-and-postdocs/internships-co-ops

Rules, Algorithms, and Models: An Intellectual History

BOOK REVIEW
By Ernest Davis

Rules: A Short History of What We Live By. *By Lorraine Daston. Princeton University Press, Princeton, NJ, July 2022. 384 pages, $29.95.*

Our lives are permeated with rules, both large and small. They range from substantial precepts like "Thou shalt not kill" and "Congress shall make no law...abridging freedom of speech" to lesser guidelines like "Don't wear white to a wedding" and "In simplifying a mathematical equation, be careful not to divide both sides by zero." In *Rules: A Short History of What We Live By*, author Lorraine Daston surveys the vast landscape of societal rules in terms of their appearance; their evolution; their creation, perception, and enforcement; and the ways in which people obey, internalize, or flout them.

Daston is an eminent historian of early modern science and philosophy whose previous works include *Classical Probability in the Enlightenment* and *Wonders and the Order of Nature, 1150-1750* (coauthored with Katherine Park). Her new book, *Rules*—which arose from her 2014 Lawrence Stone Lectures at Princeton University—combines conceptual and historical analysis with a wealth of remarkable historical examples. The text focuses primarily—but by no means exclusively—on Western Europe from roughly 1650 to the present.

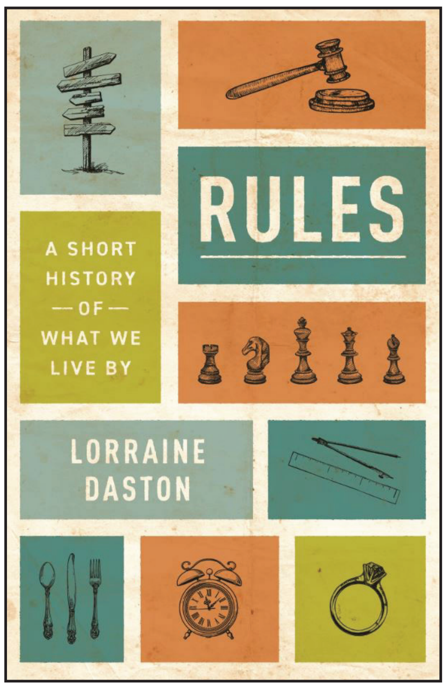
The book taxonomizes the space of rules across a number of dimensions. Rules can take the form of *laws* (with *regulations* as a subcategory), *algorithms*, or *models*. They can be flexible or rigid, general or specific, and thick or thin. A *thick* rule is accompanied by a rich body of justifications, specific cases, caveats, and exceptions; application of a thick rule to a particular case thus requires knowledge, judgment, and discretion. In contrast, a *thin* rule is a bare, unelaborated statement of a principle or procedure.

Daston is particularly interested in the idea of a model as a kind of rule. She argues that this view was common in the early modern period[1] but fell into disfavor around 1800, after which philosophers came to view rules and models in opposition to one another. She prefers the earlier viewpoint, writing that "In the end, the one ancient meaning of rules that seemed to go extinct around 1800 may prove to be the most enduring. Rules-as-models are the most supple, nimble rules of all, as supple and nimble as human learning."

A multitude of historical examples illustrate Daston's theoretical analysis and leaven her writing, including the rules of the Benedictine monastic order, sumptuary laws and dress codes, 18th-century cooking recipes, Parisian police traffic ordinances, and proposals for spelling regularization in English, French, and German. The detailed description of human computers' construction of large astronomical and actuarial tables between roughly 1790 and 1950—as well as the influence of the introduction of mechanical calculators around 1870—will likely be of particular interest to *SIAM News* readers.

The book is quite wonderful in many respects. Daston is an exceptionally fine writer, and her theoretical analysis is generally outstandingly clear, insightful, and deep. Her historical vignettes are fresh, remarkable, and entertaining; full of fascinating details; and told with an appealing balance of sympathy and ironic distance. I repeatedly encountered passages that I will long remember and ponder, such as the following:

> All of these contrasts boil down to one big contrast: a world of high variability, instability, and unpredictability versus one in which the future can be reliably extrapolated from the past, standardization insures uniformity, and averages can be trusted. Although the episodes recounted in this book trace a rough historical arc from the former world to the latter, there is no inexorable dynamic of modernity at work here. An island of stability and predictability in a tumultuous world, no matter what the epoch or locale, is the arduous and always fragile achievement of political will, technological infrastructure, and internalized norms. At any moment it can suddenly be overwhelmed by war, pandemic, natural disaster, or revolution. In such emergencies, thin rules suddenly thicken, rigid rules become rubbery, [and] general rules wax specific.



Rules: A Short History of What We Live By. By Lorraine Daston. Courtesy of Princeton University Press.

However, *Rules* does contain a few significant flaws and gaps. No doubt these issues are partially the result of the text's origin as a series of lectures and the author's effort to squeeze an enormous topic into a comparatively short book. Nevertheless, I found them troubling.

The analysis and historical episodes are all excellent on their own, but I am not sure that they mesh particularly well — i.e., that the episodes actually shed significant light on the analysis. For instance, Daston's discussion of human computers is a splendid piece of historical narrative, but does it actually tell the reader anything about rules in general? Someone who needs to compute large mathematical tables via human labor will probably have to employ a rigid and systematic organization, and the labor will be both staggeringly tedious and require close mental attention. However, it is unclear whether this reality either reflects or affects the role or form of any other kind of rule.

The opposition of rules versus models has been the focus of a substantial amount of research in cognitive psychology that examines the extent and circumstances in which human cognition uses one or the other mode of reasoning. Daston might have reasonably thought that this aspect of the question was too large and peripheral for inclusion at any length, but it is at least worth mentioning.

The book has one outright error that I thought was significant. In chapter four, Daston writes "Nowadays, virtuoso mental calculators like the great eighteenth-century mathematician Leonhard Euler (1707-1783) would be more likely to be labelled idiot savants than brilliant mathematicians, the latter often making a point of emphasizing how hopeless they are at arithmetic." She later repeats this idea in a softened but more general form:

> The history of eighteenth- and early nineteenth-century mathematics boasts several calculating prodigies who later became celebrated mathematicians, including Leonhard Euler, Carl Friedrich Gauss (1777-1855), and André-Marie Ampère (1775-1836). Anecdotes circulated about their precocious feats of mental arithmetic as early signs of mathematical genius. But by the late nineteenth and early twentieth centuries, psychologists and mathematicians had come to believe that such cases were anomalous. Great mathematicians were rarely calculating virtuosi, and calculating virtuosi were even more rarely great mathematicians.

Of course, the statement about Euler—who is indisputably the most creative and inspired mathematician between Isaac Newton and Gauss—is just wrong; it is akin to claiming that present-day society would now dismiss Mozart as merely a virtuoso pianist. More importantly, the second claim about mathematicians versus calculating prodigies is hardly truer than the hypothetical corresponding claim about musical composers versus virtuoso instrumentalists. In the 18th century (as in the 20th), the categories of creative mathematician and virtuoso calculator were different and understood to be so. And in the 20th century (as in the 18th), many important mathematicians—including Srinivasa Ramanujan, Norbert Wiener, John von Neumann, Paul Erdős, and Terence Tao—were child prodigies with extraordinary capabilities for mental mathematical problem-solving (it is said that when von Neumann was six years old, he could divide two eight-digit numbers in his head). There has probably been some historical shift in the direction of Daston's claim—a figure like Nathaniel Bowditch might have been less successful in the mathematical world of today, for instance—but it is not nearly as large as she suggests.

Yet despite these minor flaws, *Rules* is ultimately one of the best written, most profound, and most far-reaching works of intellectual history that I have ever read.

Ernest Davis is a professor of computer science at New York University's Courant Institute of Mathematical Sciences.

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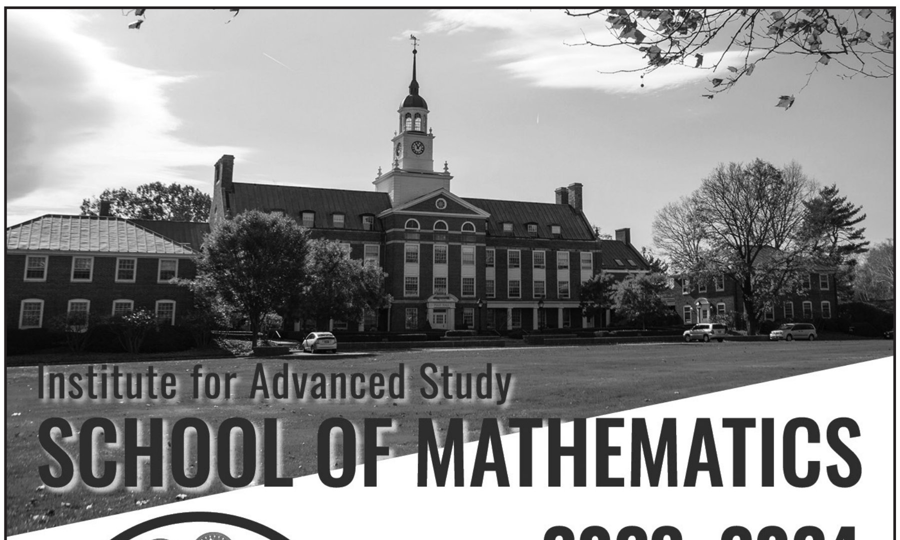
[1] At the time, the word "rule" was often used to mean "model." For example, the Oxford English Dictionary quotes "He was looked on as a living rule of religious perfection" from 1756. The idea of a model as a rule is central in Platonism and Christian thought.

SIAM Makes Award-winning Student Papers Freely Accessible

In an effort to honor our Student Paper Prize recipients and their excellent research, SIAM is excited to announce an update to the availability of winning papers. Starting with the 2021 and 2022 awardees, all winning papers will be freely accessible for one full year after the paper receives the award. SIAM is proud to support young mathematicians at the outset of their careers and hopes to share their impressive works with as wide an audience as possible. The 2021 and 2022 winners' associated papers are currently freely accessible and will remain so until August 20, 2023. Information about the 2022 papers, which were acknowledged at the 2022 SIAM Annual Meeting, is listed below:

– **William Anderson:** "Evolution of Nonlinear Reduced-order Solutions for PDEs with Conserved Quantities" (*SIAM Journal on Scientific Computing*)
– **Barbara I. Mahler:** "Analysis of Contagion Maps on a Class of Networks That Are Spatially Embedded in a Torus" (*SIAM Journal on Applied Mathematics*)
– **Ruoxuan Yang:** "Shock Formation of the Burgers--Hilbert Equation" (*SIAM Journal on Numerical Analysis*)



Virginia Commonwealth University SIAM Student Chapter Celebrates Successful Year of Activities

By Henry A. Ogu

Several Virginia Commonwealth University (VCU) SIAM Student Chapter members pose at the 2022 Biology and Medicine Through Mathematics Conference, which took place at VCU in May 2022. Photo courtesy of the VCU SIAM Student Chapter.

The Virginia Commonwealth University (VCU) SIAM Student Chapter[1] maintains a strong and well-known presence within VCU's College of Humanities and Sciences. In fact, a respectable number of professors in the College are SIAM members who provide a wonderful source of information and inspiration for students.

Both undergraduate and graduate students are involved with the VCU SIAM Student Chapter, with leadership from elected officers and a faculty advisor. Our chapter experienced a successful 2021/2022 academic year, which began with the election of a new president, vice president, secretary, and treasurer in September 2021.

In March 2022, the chapter embarked on an outreach visit to Pinchbeck Elementary School in Henrico, Va., as part of our annual volunteer program. During this visit, we taught the class how to make Möbius strips with paper, delivered informal motivational talks about mathematics, provided small gifts for the students, and conversed with students and teachers. These activities encouraged the students to engage in critical thinking, appreciate their education, and develop an interest in science, technology, engineering, and mathematics (STEM) — potentially helping them to imagine a career in applied mathematics or another STEM field. We hope to return to Pinchbeck for a follow-up visit and further volunteering sessions.

Our members also partake in a variety of intellectual development activities. They enrich their academic lives and research by organizing various mathematical and scientific-minded events, including screenings of educational films, mind-sharpening games, and discussions about different research activities or areas. VCU's Department of Mathematics and Applied Mathematics and Department of Statistical Sciences and Operations Research host weekly seminars on biomathematics, analysis and linear algebra, and discrete mathematics/combinatorics. This platform affords students and faculty the opportunity to showcase their research and learn from each other. Members of the VCU SIAM Student Chapter took full advantage of these seminars last year by both listening to faculty talks and giving presentations to develop their own career goals.

In addition, students routinely attend national and international conferences in the mathematical sciences, including meetings by SIAM and the American Mathematical Society. Their participation in these events often includes going to student sessions, delivering poster presentations, and networking at career fairs. During the 2021/2022 school year, student chapter members also presented posters at the Biology and Medicine Through Mathematics Conference,[2] which commenced at VCU in May. Additionally, I personally presented a paper on "Long-term Dynamics of the Kidney Disease Epidemic Among HIV-infected Individuals" [1] at the 47th Annual New York State Regional Graduate Mathematics Conference[3] at Syracuse University in April. Several of our members even went to the 2022 SIAM Annual Meeting[4] (AN22), which took place in Pittsburgh, Pa., this July. I received compensation from SIAM to attend AN22 as the VCU SIAM Student Chapter representative; in fact, every student chapter is eligible for funding to send a representative to the SIAM Annual Meeting. While there, I participated in the Student Chapter Breakfast with SIAM staff, leadership, and other chapter representatives to brainstorm ideas for future chapter engagement.

Our last formal activity of the 2021/2022 academic year was a picnic at Monroe Park on VCU's campus. The student chapter officers organized this event, which featured food, drinks, games, interesting conversation, and the sharing of ideas and life experiences.

In conclusion, the VCU SIAM Student Chapter is doing quite well. The officers keep the chapter active by hosting enriching student-oriented programs and encouraging members to participate in SIAM conferences. Special thanks are owed to student members and our faculty advisor, who makes our chapter's success possible.

Officers of the Virginia Commonwealth University (VCU) SIAM Student Chapter visited Pinchbeck Elementary School in Henrico, Va., in March 2022 as part of the chapter's annual volunteer program. Photo courtesy of the VCU SIAM Student Chapter.

References

[1] Gudaz, H., Ogu, H.A., & Schwartz, E.J. (2020). Long-term dynamics of the kidney disease epidemic among HIV-infected individuals. *Spora: J. Biomath., 6*(1), 52-60.

Henry A. Ogu is treasurer of the Virginia Commonwealth University (VCU) SIAM Student Chapter. He is an applied mathematics Ph.D. student at VCU with a special focus on dynamical systems, biomathematics, and mathematical modeling with ordinary differential equations and partial differential equations.

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[1] https://siam.vcu.edu

[2] https://siam.vcu.edu/bamm

[3] https://mgo.syr.edu/conferences/past-conferences/2022-conference

[4] https://www.siam.org/conferences/cm/conference/an22

Recipe for Reptation

Continued from page 7

Some Consequences of (1)

1. If the snake can exert effort $|\tau| \leq 1$ for all s, then it must choose $\tau = \text{sign}\, k'$ for the maximum pull; the resulting pull

$$\int_0^L |k'(s)|ds$$

is the total variation of k. For the snake, it thus pays to be wiggly.

2. In contrast, adopting the shape of a circular arc is bad because $k'(s) = 0$ results in $F = 0$, regardless of the bending effort $\tau(s)$.

3. Figure 3 (on page 7) illustrates the fact that τ *must change sign at the points of maximal curvature* in order to avoid having some sections of the body pull backwards.

4. How would Euler's elastica behave if confined to a channel? The elastica tries to straighten with moment $\tau = -k$ (the minus sign is there because the torque *resists* bending if $k > 0$, thus making $\tau < 0$). From (1), we get the propulsion force

$$\int_0^L (-k)k'(s)ds = \frac{1}{2}(k^2(0) - k^2(L)).$$

In this case, the force depends only on the curvature at the ends.

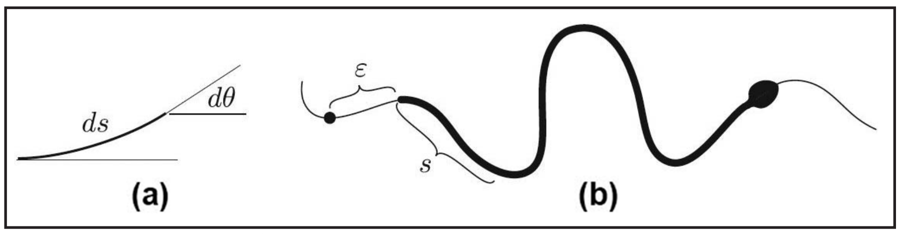

Figure 4. *For this snake, $\tau(s)$ does not change as the snake bends or unbends.* **4a.** *The potential energy of the element ds is $-\tau d\theta$.* **4b.** *The snake advances by ε along the channel.*

Derivation of (1) Via Potential Energy

Let us replace the live snake with an elastica that tries to bend with an intensity $\tau(s)$ that is independent of its curvature. This is unlike Euler's elastica, which wants to be straight with an intensity that is proportional to k. In contrast with Euler's, each element of our elastica wants to curl up with a fixed intensity $\tau(s)$ — either left or right depending on the sign of $\tau(s)$. The potential energy of such an element is, by definition, the work that we must do to bend the element from the straight shape to its current one (see Figure 4a). To that end, we must apply torque τ in opposition to the one that this element applies to our hand, i.e., $-\tau$. The potential energy of an element ds in Figure 4a is therefore

$$-\tau d\theta = -\tau k ds.$$

For example, if $\tau > 0$ in Figure 4, then the element tries to curl up from the straight shape and hence does work for us (so that we do negative work). The potential energy of the "snake" in Figure 4b is therefore

$$P(\varepsilon) = -\int_0^L \tau(s)k(s+\varepsilon)ds,$$

where $s = 0$ corresponds to the tail and ε is the distance by which we advance the "snake" along the channel. Indeed, as the snake advances by ε, its curvature changes to $k(s+\varepsilon)$ while τ remains the same by the assumption. The potential energy determines the force

$$F(\varepsilon) = -P'(\varepsilon) = \int_0^L \tau(s)k'(s+\varepsilon)ds,$$

which agrees with (1) when $\varepsilon = 0$.

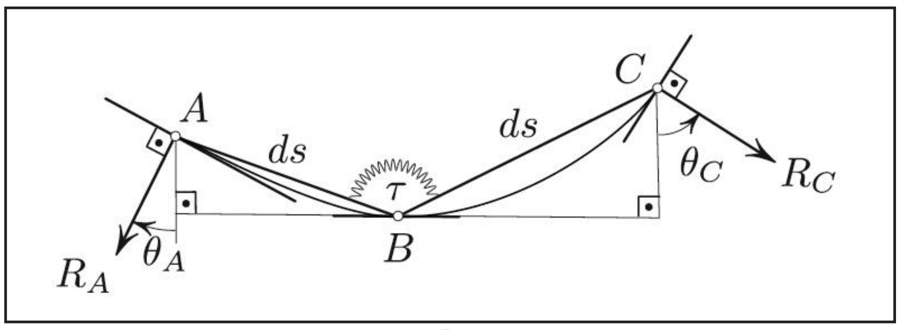

Figure 5. *Here, $k' > 0$ and the "muscle" at B contracts, i.e., $\tau > 0$. There are two different explanations of propulsion: (i) Projection of R_C onto the tangent at B wins over that of R_A due to $k' > 0$, and (ii) a small displacement of the "train" ABC to the right decreases $\angle ABC$ and also the potential energy of the system.*

A Geometrical Explanation of (1)

The previous derivation does not explain *exactly how* the bending propels the snake forward. Here is an attempted explanation.

Figure 5 shows a segment of the snake that is discretized, i.e., replaced by two rods of fixed equal lengths with the ends A, B, and C attached to skates that can slide on the prescribed channel but cannot move transversally to it. The "muscle" at B tries to contract, i.e., $\tau > 0$. This attempted contraction results in reaction forces R_A and R_C. Consider the projections of these forces onto the tangent at B. If $k' > 0$, then $\theta_C > \theta_A$, thus suggesting that R_C's projection—which pulls B to the right—wins over its counterpart at A, which pulls B to the left. This is indeed the case since $|R_C| \approx |R_A|$ (with sufficient precision, as it turns out).

As an alternative explanation, an infinitesimal displacement to the right of the "train" ABC in Figure 5 results in a decrease of $\angle ABC$ because $k(s)$ is an increasing function, and decreasing $\angle ABC$ is exactly what the "muscle" at B attempts to do.

The figures in this article were provided by the author.

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Academia or Industry…Why Not Both?

CAREERS IN MATHEMATICAL SCIENCES

By Giovanna Guidoboni

Children are often asked, “What would you like to do when you grow up?” Personally, I hope that I will never stop growing and will be able to do many different things throughout my life. The notion that I had to choose only one career always bothered me. When prompted with questions like “Do you want to be a mathematician or an engineer?” or “Would you prefer to work in academia or industry?”, I routinely wonder: “Why not both?”

I always dreamed of conducting academic research, though I never had a clear sense of the specific field on which I wanted to focus. Initially, I studied materials engineering and appreciated engineering’s capacity to solve problems with concrete, direct impacts on society. I admired the creative and applied sides of the field, as well as the use of mathematics and physics to address complex scenarios. In response, I pursued a Ph.D. in mathematics and a postdoctoral appointment in scientific computing. The learning curve was steep but certainly worthwhile.

I first considered a career outside of academia when I had to make important decisions about my job and personal life. As my family and I prepared to move between cities, I felt compelled to reevaluate my career. I loved the intellectual agility that academia provides and truly enjoyed interacting with students and colleagues from all disciplines. Yet because the prospect of attaining an academic appointment at my next destination seemed unlikely, I pondered the idea of opening my own company. After all, I was building my career on the fact that mathematics can be applied; why not apply it directly with organizations in need of mathematical services?

The thought of becoming a consultant was simultaneously exciting and scary. I worried about how to find clients and draft contracts and fretted over my ability to maintain a continuous inflow of revenue. I also wondered whether my consulting endeavors would generate intellectual property and if legal troubles could ensue. These thoughts and others began haunting me in 2009, when we relocated from Houston, Texas, to Indianapolis, Indiana. I was therefore greatly relieved when I finally secured a faculty position at Indiana University–Purdue University Indianapolis.

Even though I had shelved the consulting idea for the time being, the sole fact that I had entertained the thought made me more aware of my research’s potential impact in a broader sense. I could publish articles to communicate new discoveries, but I could also file patents for ideas that might lead to marketable services or products. In 2015, I coauthored my first patent on the use of mathematical models to diagnose disease conditions in the eye and develop new therapies.

Filing a patent was actually quite easy. Since the discovery occurred in an academic setting, the Indiana University Research and Technology Corporation (IURTC) handled all of the paperwork and covered the costs. As the inventor, I filled out an invention disclosure form that described the contents of my work. The IURTC personnel—along with an external group of specialized attorneys—then drafted and filed the patent.

Consulting Is What Happens When You Make Other Plans

My prior worries about finding work resurfaced once more when my family relocated yet again, first from Indianapolis to Strasbourg, France in 2014—where I found employment at Université de Strasbourg—and then from France to Columbia, Missouri, in 2017 — where I joined the faculty at the University of Missouri. My colleagues in Missouri introduced me to the world of sensors for noninvasive monitoring of cardiorespiratory function. We investigated signal shapes and built a mathematical model based on differential equations that explained these shapes according to fundamental principles of cardiovascular physiology. While publishing an article about our findings in 2018, I filed another patent through the University of Missouri’s Technology Advancement Office (TAO).

Shortly thereafter—at a time when I thought I had put my consulting ambitions to sleep—a company approached me that was interested in my cardiovascular monitoring patent. That was when I finally realized that I could truly become a consultant. I contacted the TAO and the University’s Conflict of Interest Committee (COIC) to ensure that I was allowed to consult for external companies while also serving as faculty. I received approval but discovered that there is a limit to the number of hours per month that staff can devote to such activities. It is also important for consultants to clarify who will own new intellectual property that their activities could potentially generate.

I hired an attorney to affirm that I was doing things correctly. Starting a company is surprisingly simple; one only needs a name, a bank account, and an Employer Identification Number. The attorney filed the necessary documents to the State of Missouri and checked that the name I chose was available. Just like that, Gspace LLC was born in October 2019. When a company approached me to enlist my services, my attorney helped me review the contract to confirm that I was covered from all angles. I recovered all of the legal fees within the first month.

Consulting in Practice

My first consulting job required that I provide guidance on the design and optimization of sensors for cardiopulmonary monitoring in assisted living facilities to verify that the acquired signal actually contained the physiological data of interest. I employed my expertise in mathematical modeling of the cardiovascular system but also learned many new concepts that pertained to signal processing, hardware solutions, and—most importantly—human interactions with the devices. I continue to meet weekly with a team from the sensor company, which sets work goals according to the company’s strategic development plan. We then define specific tasks based on these goals that will not exceed my allotted monthly time. I express my opinion, but the company makes all final decisions regarding future directions for the project. I can thus simply enjoy solving real problems using mathematics, and our arrangement allows me to effectively balance my time between academia and consulting.

I was recently approached by another company that wants to apply mathematical modeling to evaluate the outcomes of certain drugs. This time, setup was very easy; I received approval from the COIC, enlisted my attorney to help review the consulting contract, and now have a second client.

To be honest, I have not been actively looking for more customers. My academic position serves as my primary means of income and allows me the privilege of being selective in the consulting projects


Giovanna Guidoboni (right) and Marge Skubic of the University of Missouri discuss the design and optimization of sensors for cardiopulmonary monitoring in assisted living facilities. Guidoboni’s involvement in this project—which constituted her first consulting job—was based on her cardiovascular monitoring patent. Photo courtesy of the University of Missouri.


During her time at Indiana University–Purdue University Indianapolis (IUPUI), Giovanna Guidoboni (right) and Alon Harris of the Indiana University School of Medicine used ultrasound technology to measure blood flow in the eye and ultimately model risk factor behavior in diseases like glaucoma and diabetes. Photo by School of Science at IUPUI.

See **Academia or Industry** *on page 12*

A Mathematical Tale of Fibers, Fluids, and Flagella

By Matthew R. Francis

Under a microscope, a cell scoots along by its own power and hoovers up small crumbs of nutrition from the water around it. An example of such an organism is a choanoflagellate, which has a thin, whip-like appendage called a flagellum that controls its feeding and motion. While similarly proportioned apparatuses would be useless on a human scale, flagella are common among single-celled organisms like bacteria, the sometimes-toxic dinoflagellate algae, and even human sperm cells.

Motion in the microscopic world—particularly in fluids—involves an entirely different set of forces than those that govern macroscopic environments. Flagella operate efficiently under these forces and allow microscopic life to move around in fluids, where large viscous forces are present even in substances such as water. The motion of choanoflagellates and the way in which flexible fibers or strands of cells passively respond to liquid flow all constitute a set of complex problems with many potential applications in engineering and medicine.

"With the advent of microfluidic devices and computational technology, there has been an incredible resurgence in studies of the flow of tiny creatures at the microscale," Lisa Fauci, an applied mathematician at Tulane University and a former president of SIAM, said. "There are possibilities of creating nanorobots that can be guided with external magnetic fields to break up blood clots or deliver drugs to a tumor."

However, potential applications are not the only reason that Fauci is drawn to flexible fibers in fluids. "From the start, I was interested in problems in biological fluid dynamics," she said. "What is really interesting and hard about fluid mechanics in a biological setting rather than an engineering setting [is that] the structure interacting with the fluid is elastic. It's flexible, and its shape responds to the fluid flow."

The fluids themselves in microscopic biological systems are also often complicated, regardless of whether the system of interest is blood, a reproductive tract, or the inside of a cell. Even free-swimming cells in water inhabit a world full of proteins and other fibrous biological detritus that certainly affects their movement. Fauci elaborated on these concepts during her Past President's Address at the hybrid 2022 SIAM Annual Meeting,[1] which took place in Pittsburgh, Pa., this July.

Fibers on Fibers

Fauci was among the pioneers who modeled microfluid environments via computers at a time when machines were becoming powerful enough to run increasingly realistic simulations. "There was interest in studying processes like sperm motility in the female reproductive tract and phytoplankton motion in the ocean through the lens of fluid mechanics," she said. "I started working on these types of problems using computer simulations and new methodologies."

In the microscopic regime dominated by viscous forces, Fauci and her collaborators often start with an incompressible Newtonian fluid that is governed mathematically by the linearized Navier-Stokes equations. This approximation results from the assumption that the fluid's viscosity dominates over both its inertia and macroscopic external forces such as gravity. In fluid dynamics terms, this is the limit at which the Reynolds number—inversely proportional to viscosity—is zero. The resulting equations that govern the fluid are

$$-\nabla\rho+\mu\nabla^2\mathbf{u}+\mathbf{g}=0$$
$$\nabla\cdot\mathbf{u}=0,$$

where $\mathbf{u}$ is the velocity field for the fluid, μ is the viscosity, ρ is the pressure, and $\mathbf{g}$ is the force per unit volume from non-fluid influences like flagella.

The forms of the solutions to this equation naturally depend on forces $\mathbf{g}$. To describe flexible fibers in Newtonian fluids, Fauci's colleague at Tulane and frequent collaborator Ricardo Cortez introduced highly localized forces that can model both the fibers' response and—where applicable—polymers that are suspended in the fluid:

$$\mathbf{g}=\mathbf{f}\varphi_\epsilon(r),\ \text{where}\ \ r=|x-x_\mathbf{0}|\ \text{and}$$
$$\varphi_\epsilon(r)=\frac{15\epsilon^4}{8\pi(r^2+\epsilon^2)^{7/2}}$$

is a regularized version of a Dirac delta function for small parameter ϵ. The exact solutions to the linearized Navier-Stokes equation with this force are

$$\mathbf{u}(x)=\mathbf{S}(x;\epsilon)\cdot\mathbf{f}=\frac{1}{8\pi\mu(r^2+\epsilon^2)^{3/2}}\times$$
$$[\mathbf{f}(r^2+2\epsilon^2)+(\mathbf{f}\cdot(x-x_0))(x-x_0)],$$

where the matrix $\mathbf{S}$ is a "regularized Stokeslet" [3]. The Newtonian fluid that these solutions describe serves as either the background for the simulation of passive fiber motion or the environment for a complex fluid of polymers.

For the latter, Fauci and her collaborators developed a mechanical model on a lattice that treats the molecules like springs and dashpots: a mechanical damper of the same type that prevents slamming in automatic door-closing mechanisms[2] [4]. Cells lash their flagella within this two-layered background to move and feed (see Figure 1). "We're using computational methods to model flagellar fibers that are moving through a fibrous network," Fauci said. "So, fibers on fibers."

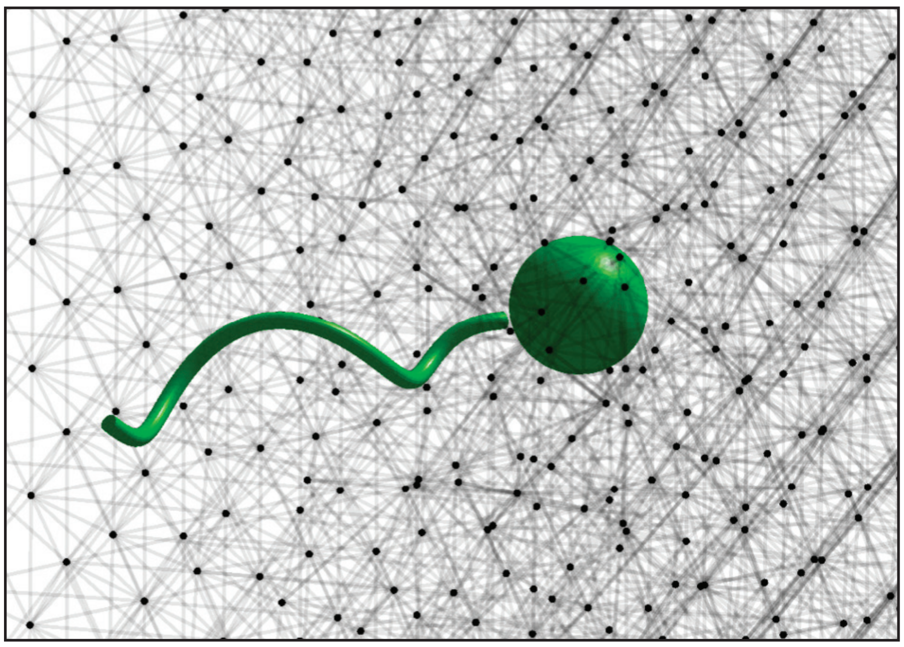

Figure 1. *Simulation of a cell that has a helical flagellum within a fluid consisting of a continuous Newtonian component and a lattice that represents suspended polymers. The simulation uses simple mechanical models to describe both the flagellum and the polymers. Figure courtesy of [2].*

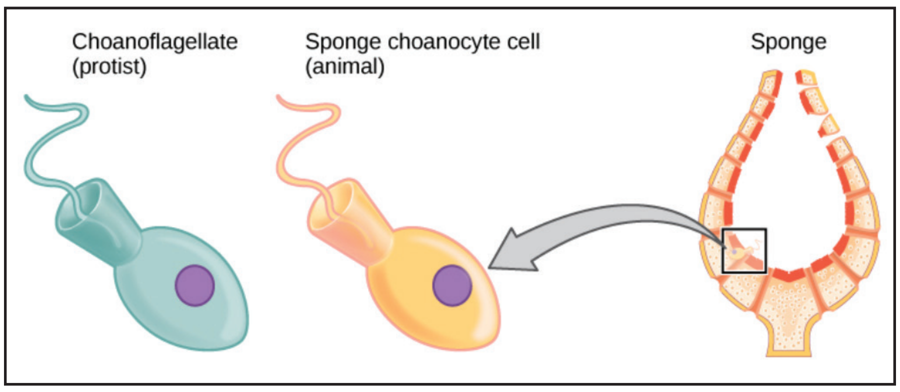

Figure 2. *A choanoflagellate cell's morphological similarity to a sponge cell shows a probable evolutionary connection. Both types of cells have a flagellum that is surrounded by a collar of hair-like cilia, which help capture food. Understanding the flagella's operation could elucidate the reason that sponges took up communal life hundreds of millions of years ago. Figure courtesy of Mary Ann Clark, Jung Choi, and Matthew Douglas under Creative Commons Attribution 4.0 International license.*

Just Keep Swimming

The active fiber motion of flagella is not the only flexible-fiber system worth studying; microscopic biological environments are full of stringy proteins and even cells that are borne on microcurrents [1]. "Diatom chains are phytoplankton that are just swept around," Fauci said. Phytoplankton—photosynthesizing, water-dwelling microorganisms—as a group are responsible for the production of roughly half of the planet's oxygen. In other words, the passive motion of diatoms around the globe contributes to the chemistry of the atmosphere and oceans and links the microscopic to climate change.

Yet active flagella hold a special place in Fauci's research program, and not just in the context of motion. Choanoflagellates sometimes anchor themselves to surfaces and use their flagella to snag food, reversing the same mechanisms that they originally evolved for swimming purposes [2]. This behavior, as well as their general shape, links choanoflagellates to the oldest surviving lineage of multicellular animals: the sponges (see Figure 2).

"As a mathematician, you have your toolkit with which you can solve these equations, figure out velocity fields and forces, and so forth," Fauci said. "But it involves working with evolutionary biologists as well, who frame a very simple question [such as] is it better for me to be stuck to a wall? Or is it better for me to be free swimming?"

Choanoflagellates obviously do both, but sponges and more structurally complex animals forwent the single, free-swimming life in favor of communal grouping. More efficient food capture might have played a role in this development, in addition to the obvious advantages of collective safety in numbers. Such biological curiosities have driven Fauci as much as anything. "It's really fabulous for me to spend my career in a mathematics department," she said. "We can study whatever we want. It's not 'let's get some technology out there.' It's basic science. It's 'how is this working?'"

References

[1] Musielak, M., Karp-Boss, L., Jumars, P.A., & Fauci, L.J. (2009). Nutrient transport and acquisition by diatom chains in a moving fluid. *J. Fluid Mech., 638*, 401-421.

[2] Nguyen, H., Koehl, M.A.R., Oakes, C., Bustamante, G., & Fauci, L. (2019). Effects of cell morphology and attachment to a surface on the hydrodynamic performance of unicellular choanoflagellates. *J. Royal Soc. Interface, 16*(150), 20180736.

[3] Schuech, R., Cortez, R., & Fauci, L. (2022). Performance of a helical microswimmer traversing a discrete viscoelastic network with dynamic remodeling. *Fluids, 7*(8), 257.

[4] Wróbel, J.K., Cortez, R., & Fauci, L. (2014). Modeling viscoelastic networks in Stokes flow. *Phys. Fluids, 26*, 113102.

Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

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[1] https://www.siam.org/conferences/cm/conference/an22

[2] Interestingly, 19th-century physicist James Clerk Maxwell developed this type of mechanical lattice to describe the since-debunked "luminiferous aether."

Academia or Industry

Continued from page 11

that I undertake. I also try to work in areas that are somewhat aligned with my academic research interests in order to use my time effectively. Therefore, I do not currently have a company website or a marketing strategy — though I know that I could step up my game and change my career priorities if I so desired.

Useful Tips

If you also enjoy the academic environment but would like to apply your research to real-world scenarios as a consultant, the following tips might help you get started:

- Consider filing patents in addition to writing articles; ask yourself whether some aspects of your research could lead to marketable services or products.
- Get to know the people in your technology transfer office and familiarize yourself with invention disclosure forms.
- Know the requirements for your institution's conflicts of interest and commitment, as requirements can vary from place to place.
- Hire external help for specific needs, such as an attorney to review contracts and a company to assist with tax returns.
- Find a balance that works for you, in terms of time *and* intellectual engagement.

My experience as a consultant has been extremely rewarding so far, and I personally feel more alive when I do multiple things simultaneously. My family and I receive an additional revenue stream, I can save more money for my retirement since I own a business, and I get to actively see people benefit from my mathematical research. The sensors that I helped to design currently monitor hundreds of elderly individuals in assisted living facilities. They provide early warning of deteriorating cardiopulmonary conditions and have already saved many lives. Math can genuinely improve people's quality of life.

Academia or industry: why not both? I truly believe that my academic research and experience make me a valuable consultant, and my consulting activities make me a better scientist. I have no idea what's next, but I am happy to say that I am still in the process of figuring out what I want to do when I grow up.

Giovanna Guidoboni is Associate Dean of Research in the College of Engineering and a professor with joint appointments in electrical engineering and computer science and mathematics at the University of Missouri. Her research focuses on mathematical modeling and data science in the context of engineering and the life sciences. She collaborates nationally and internationally in areas such as ocular blood flow and risk factors.