

Examining the Dynamics of Ocean Mixing

By Erica Klarreich

“The science is clear,” climate scientist Emily Shuckburgh told an audience of nearly 800 people at San Francisco’s Palace of Fine Arts on March 4. “Our collective actions have generated a climate problem that threatens our future and our

children’s future.” Shuckburgh’s talk was part of the Mathematics of Planet Earth 2013 Simons Public Lecture Series.*

The combination of mathematical modeling and observational data has created a robust understanding of climate change, as Shuckburgh demonstrated through numerous examples. The results are sober-

ing: Even under the most optimistic assumptions about our future carbon emissions, the planet is already locked into a global temperature increase that will likely reach 2 degrees Celsius over the next century, resulting in increased floods, droughts, and other climate disruptions. And if our carbon emissions continue to grow unchecked, the temperature change will almost surely exceed 4 degrees Celsius, a scenario that, in the words of World Bank president Jim Yong Kim, “can, and must, be avoided.”

In her own research, as head of the Open Oceans group of the British Antarctic Survey, Shuckburgh unites mathematics with field work to tackle one of climate science’s most fundamental challenges: elucidating the interplay between global temperatures and atmospheric carbon dioxide levels. Her work focuses on the Southern Ocean, which plays an outside role in the exchange of carbon dioxide between the atmosphere and the Earth’s waters.

Of the carbon dioxide released into the atmosphere through human activities, about half is reabsorbed by land or ocean, protecting the planet from what would otherwise be an even more pronounced greenhouse effect. And of the ocean’s portion of this carbon uptake, 40% occurs in the Southern Ocean. A vast overturning circulation connects all the Earth’s major oceans to the Southern Ocean, carrying water that has been at the bottom of the ocean for many hundreds of years to the surface around Antarctica, where it exchanges heat and carbon with the atmosphere.

When the Earth’s temperature increases, Shuckburgh explained in San Francisco, changes to the ice cover, the temperature of

the water, and the overturning circulation of the ocean prompt the release of more carbon dioxide from the Southern Ocean; this increase in greenhouse gases then causes temperatures to increase still further. It’s a feedback effect that we believe has been happening, and that is likely to continue in the future, she said.

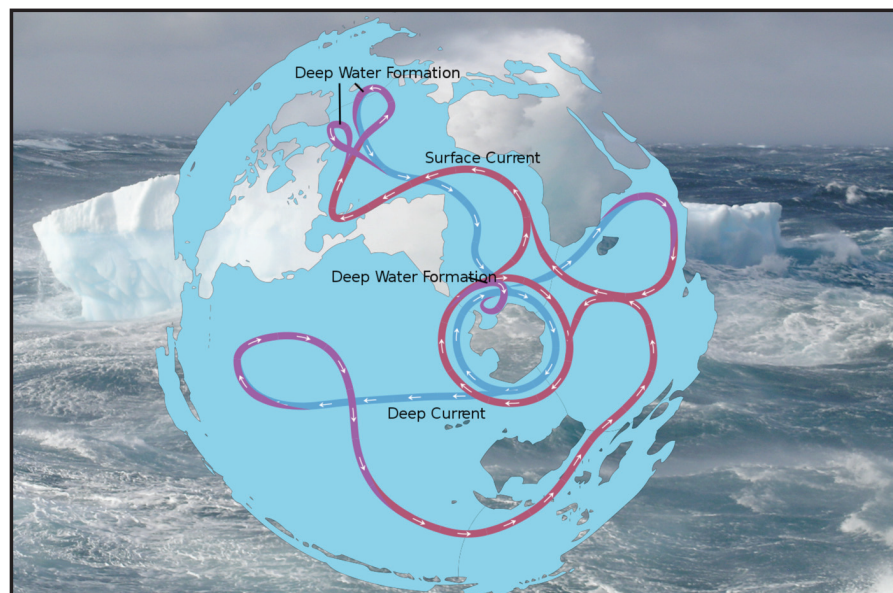
Mathematical Model of Ocean Mixing

To fully understand climate change, it is essential to explicate the workings of this “critical gateway for carbon exchange,” Shuckburgh said last summer in a different setting—an invited presentation at the 2012 SIAM Annual Meeting in Minneapolis. What drives the overturning circulation? she asked. And is it likely to change in the future?

Derivation of the differential equations governing the ocean’s currents from Newton’s second law is well understood. Computational constraints limit the resolution of the grid on which the equations can be solved, however, and any effects smaller than this resolution can only be estimated. In particular, the grid cannot show the swirling eddy formations that arise from the Earth’s rotation, and these eddies play a major role in the mixing of different ocean waters.

Shuckburgh and her collaborators have created a mathematical model for ocean mixing, including the eddies, using satellite altimetry data, which measures the height—and thus the pressure—of the ocean in different locations. From the pressure, it’s possible to model the surface currents, and thereby examine the dynamics of ocean mixing.

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A vast global circulation carries water from the Pacific, Atlantic, and Indian Oceans to the Southern Ocean, allowing water that has been at the bottom of the ocean for centuries to surface and exchange heat and carbon dioxide with the atmosphere. From Creative Commons.

*Scheduled to give lectures in the remainder of 2013 are Rupert Klein (Berlin, Germany; May 23); L. Mahadevan (Providence, Rhode Island; September 24); Martin Nowak (Minneapolis; October 8); and Emily Carter (Los Angeles; November 4). Abstracts and illustrations from previous lectures in the series can be found at <http://mpe2013.org/public-lectures/mpe2013-simons-public-lecture-series/>.

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Strength in Numbers: MPE 2013 Meets the SIAM/Moody’s Math Modeling Contest

As Mathematics of Planet Earth 2013 approaches the halfway mark, its logo has adorned a remarkable array of events and activities all over the world, at all levels—from programs at major mathematics institutes to public lecture series to activities for young students. An overarching goal of all this, along with an informed and active public, is to ensure that the activities set in motion continue well beyond 2013.

As an MPE 2013-supporting society, SIAM has highlighted the theme in a number of events. A recent example is the choice of an environmental theme—recycling—for the problem of this year’s Moody’s Mega Math Challenge, a high school modeling contest that SIAM has organized for the Moody’s Foundation for the last eight years. The 2013 problem challenged participating teams to create a model that predicts the amount of plastic waste that will be deposited in U.S. landfills in 10 years; to use this model in analyses of programs for recycling plastics and other forms of waste for three U.S. cities with different demographics; and to recommend (in the form of a report addressed to the EPA) ways in which the programs can be scaled up to the national level.

On the weekend of March 2–3, 1281 teams from 29 states in the eastern U.S. settled in for the 14 hours they were given to produce their solutions. On Friday, April 5, 12 of the 151 judges who participated in this year’s contest traveled to SIAM headquarters for the semi-final round of judging, in which six finalists would be chosen

to present their results in New York City. A few of those judges are pictured here, including Ben Galluzzo of Shippensburg University, who has served as both problem writer and judge in the last few contests; as the MAA’s representative to the public awareness arm of MPE 2013, he was instrumental in connecting the Moody’s competition to MPE 2013.

In a short conversation with *SIAM News*, Galluzzo pointed out that writing a problem with a timely theme that will challenge

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Shared interests—in both the environment and the value of a public informed about mathematical modeling—drew Ben Fusaro (left) and Ben Galluzzo to SIAM headquarters for a key round of judging in the high school math modeling contest that SIAM runs for the Moody’s Foundation.



Doing their part to ensure that every one of the 1054 papers submitted to this year’s Moody’s Mega Math Challenge got fair consideration, judges (from left) Karen Bolinger, Barbara Wainwright, Joe Malkevitch, and nine others spent an April weekend in Philadelphia reading students’ proposals for effective recycling programs. Photos by Lois Sellers.

1 Examining the Dynamics of Ocean Mixing



1 Strength in Numbers: MPE 2013 Meets the SIAM/Moody's Math Modeling Contest



4 How Did Newton Go So Far Off Course?

Reviewing a meticulous new analysis of Newton's *Chronology*, Ernest Davis cites formidable obstacles faced by Newton and his contemporaries in determining any date before 600 or 700 BC. "What is surprising is Newton's extraordinary confidence," Davis writes; "he claimed that all the dates in his history were accurate to within 10 or 20 years." In fact, the *Chronology* includes major errors rarely seen before or since.

8 Benjamin Franklin: The Doctor Would Want a Master's

"There is a world shortage of culturally sensitive computer scientists, mathematicians, engineers, and scientists who are innovative and who know how to read society from the point of view of an entrepreneur," according to Charles Van Loan, this issue's Careers columnist. Readers who "would like to become part of this group and are at the stage where getting a master's degree is an option" will find Van Loan to be an astute and insightful guide.

8 The Role of Linear Algebra in the Computer Science Curriculum

7 Professional Opportunities

7 Announcements

Was Pythagoras the First to Discover Pythagoras's Theorem?

By Anna Barry

According to Brown University mathematician David Mumford, the answer to the question is an emphatic "No!" On February 27, 2013, in a public lecture at the Institute for Mathematics and its Applications at the University of Minnesota, Mumford showed how ancient cultures, including the Babylonians, Vedic Indians, and Chinese, all proved the beloved formula long before the Greeks. He argued that the theorem is ultimately the rule for measuring distances on the basis of perpendicular coordinates. This comes up naturally in calculations of land area for purposes like taxation and inheritance, as shown in Figure 1. He further suggested that the Greeks' love of formal proof may have contributed to the Western belief that they discovered what Mumford calls the "first nontrivial mathematical fact."



Figure 1. An Egyptian surveyor set out with ropes and rods to measure land area.

Along with Pythagoras's theorem, Mumford discussed the discovery and use of algebra and calculus in ancient cultures. One of his key points is that deep mathematics was developed for different reasons in different cultures. Whereas in Babylonia algebraic "word" problems were posed seemingly just for fun, the *Nine Chapters on Computational Methods*, considered the Chinese equivalent of Euclid's *Elements*, was compiled in about 180 BCE for very practical applications—among them Gaussian elimination for solving systems of linear equations, which the Chinese carried out using only counting rods on a board (Figure 2). Riemann sums grew naturally out of the necessity for estimating volume. Mumford suggested that Vedic Indians even pondered problems of limit in integral calculus.

Contrary to Western historical belief, Mumford showed, the West did not always lead in mathematical discovery. Apparently, the origins of calculus sprang up totally independently in Greece, India, and China. Original concepts included area and volume, trigonometry, and astronomy. Mumford considers the year 1650 a turning point, after which mathematical activity shifted to the West.

Mumford's presentation runs counter to current texts on the history of mathematics, which often neglect discoveries occurring



Figure 2. Gaussian elimination, performed in China during the Han dynasty, with only rods and a board.

outside the West. He showed that purposes for which mathematics is pursued can be very culturally dependent. Nevertheless, his talk points to the fundamental fact that the mathematical experience has no inherent cultural boundaries.

Mumford, a professor emeritus in the Division of Applied Mathematics at Brown University, has worked predominantly in the area of algebraic geometry and is a leading researcher in pattern theory. Mumford received a Fields Medal in 1974; his more recent awards include the Shaw Prize (2006), the Steele Prize for Mathematical Exposition (2007), the Wolf Prize (2008), and the National Medal of Science (2010).

Anna Barry is a postdoctoral fellow at the Institute for Mathematics and its Applications at the University of Minnesota. Following up on her coverage of David Mumford's IMA lecture for SIAM News, she conducted the interview transcribed below.

An Interview with David Mumford

You have done highly distinguished work in both pure and applied mathematics, from algebraic geometry to computer vision, and now the history of mathematics. How did your career path evolve?

I've always been interested in things besides math. Growing up, I was very interested in physics and worked for Westinghouse on the atomic submarine reactor while a student. Around 1956, listening to the lectures of George Mackey, Lars Ahlfors, and Oscar Zariski, I fell in love with abstraction, with the idea of a secret garden containing amazing things that could only be seen with the mind's eye but never touched: pure math. However, I always had an interest in other questions. I proved a theorem in algebraic geometry in the 1980s that was the culmination of a long series of efforts, and I thought, If I'm ever going to pursue these other things that I had wanted to as a student, then now would be the time.

During the 1950s, I had read everything I could about the brain and how it operates. It felt as though there was a real possibility of taking a limited aspect of the capabilities of the brain, specifically of understanding two-



dimensional images and making a model of a three-dimensional world with them.

As for history, there was a great group of mathematicians in India that I visited in Bombay quite a lot. In the 1990s I talked to some people about the history of Indian math, read about it, and one thing led to another. The lecture I gave last night was a culmination of trying to put together the

side of history that I find most fascinating—that similar things were discovered in many cultures but always in a different context. The culture made a very strong mark on the mathematics.

Do you believe that cultural differences affect the way that mathematicians communicate even today?

No, now we all speak the same language on a scientific level. I think it's fantastic that these barriers have been removed. The whole intellectual enterprise is so vast these days.

What are some of the difficulties you faced as you moved from one area of mathematics to another?

There was a period of about ten years before I felt like I had my feet on the ground in computer vision. Trying to find the right mathematical tools was difficult, but I gradually got a sense for what these might be. I've always thought that my skills were those of a mathematician, but I loved the idea that they could be used to model something like a cognitive skill.

What advice can you share with other mathematicians attempting to break into new areas of mathematics?

I think it's a matter of personality and a question of comfort zone in many ways. I think I've had an adventurous streak. Perhaps I'm less inhibited. I acknowledge that I didn't make the same impact on vision that I did in algebraic geometry. Vision is a much bigger field, with people pursuing it from all sorts of different directions. I felt my contribution was to raise questions the way mathematicians would, and this point of view was useful. I try to understand on a deeper level as much as possible what the fundamental problems are.

The IMA also works to bring applications to the attention of pure mathematicians. At Brown, we have started a similar institute, ICERM; one of the narratives that started ICERM was that arithmetic algebraic

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Nominate a colleague for the 2014 Class of Fellows. Nominations close November 4, 2013.

How Did Newton Go So Far Off Course?

Newton and the Origin of Civilization. By Jed Z. Buchwald and Mordechai Feingold, Princeton University Press, Princeton, New Jersey, 2012, 544 pages, \$49.50.

Isaac Newton’s book *The Chronology of Ancient Kingdoms Amended* was published in 1728, a year after his death. Like much else in Newton’s career, the appearance of the book was mired in unedifying controversy. It was published largely at the urging of William Whiston, Newton’s successor

as Lucasian Professor of Mathematics at Cambridge, once an acolyte, now a bitter enemy, in order that he could respond with a crushing refutation. Three years earlier, an unauthorized, abridged version of the book had been published in France, together with a short refutation; the aged Newton was, reasonably, furious.

In the course of the inevitable controversy over the pirated edition, Newton minimized the significance of his own work, saying that he “had occupied himself agreeably with history and chronology when fatigued with other work.” In fact, he had had a serious interest in chronology from a fairly early age, and since about 1700 had applied himself intensely to it. The subject constitutes a significant fraction of the immense body of manuscript notes that he left behind. He had compiled a large library of both primary and secondary sources. Besides Latin, he had a good knowledge of Greek and some knowledge of Hebrew. He analyzed the sources critically, taking into account reliable information that would have been available to the authors, and what the authors’ biases would have been. And, of course, Newton was a genius.

Nonetheless, as viewed with 20/20 hindsight after nearly 300 years, the *Chronology* made no useful contribution whatever to historical research, either factual or methodological. As far as I can determine, the *Chronology* has nothing new in it that is right, and it includes major errors that practically no one else, before or since, has made. Most egregiously, Newton claimed, on the flimsiest of grounds and in the face of massive contrary evidence, that pharaonic Egypt was not a kingdom of major significance until after the time of King Solomon. The *Chronology* is also extremely dull and inhuman; people live and die, cities are founded and destroyed, kingdoms rise and fall with no trace of individuality or character, only intricate argumentation about dates.

How did Newton go so far off course? To begin with, in the early 18th century, a scholar trying to determine reliably any date prior to about 600 or 700 BC faced formidable, perhaps insuperable, obstacles. Archeological evidence was limited to a few undatable monuments, like the Pyramids; the only evidence was textual. The surviving histories of the early period were written many centuries after the events they described; they combined history, myth, and sheer fiction; and they were often polemics or propaganda aimed at establishing the antiquity or early supremacy of whichever group the author favored. It was essentially impossible for Newton or his contemporaries to establish a clear history. Viscount Bolingbroke, a generation younger than Newton, worked avidly on ancient chronology for a while, and then gave up in despair, writing “Who can resolve to build with great cost and pain when he finds, how deep soever he digs, nothing but loose sand?” What is surprising is Newton’s extraordinary confidence; he claimed that all the dates in his history were accurate to within 10 or 20 years.

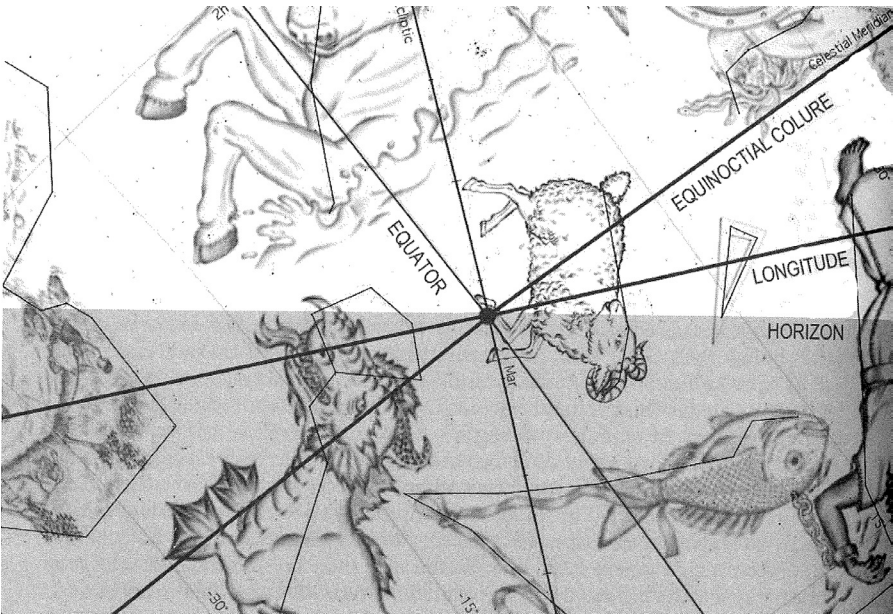
The argumentation in the *Chronology* is so intricate and so intertwined that it is hard to separate out particular errors, but a few stand out. Newton was a firm believer

in the Euhemerist theory of mythology, which posits that mythological figures all derived from historical persons. This was a dangerous starting point for a chronologer, because it put a premium on the vain pursuit of dates for the originals of Hercules, Osiris, and so on. Much worse, Newton tried to identify corresponding historical and mythological figures—Saturn and Noah, Jupiter and Shem, for instance. In particular, the identification of the Egyptian pharaoh Sesostris with Osiris, Dionysus, and the Biblical

figure Sesak placed a huge strain on the chronological structure.

Newton was quite skeptical of the written sources; Buchwald and Feingold argue that he became more so after, as Master of the Mint, he had to conduct investigations into cases of counterfeiting and to sift the lies that both culprits and witnesses told with abandon. But his skepticism deserted him at exactly the wrong times. He threw out as unreliable many ancient historical texts, such as the chronology of Egypt written by Manetho in (probably) the third century BC, but he was entirely confident of his own unfounded theory that all ancient civilizations derived from the Israelites. He was willing to reinterpret some parts of the Biblical text; yet he took the flood of Noah at face value, and therefore set the dates of the creation of large kingdoms late enough to allow for sufficient population growth from the eight people who survived the flood.

Because texts and textual analysis were unreliable, Newton put his faith in a number of astronomical arguments. The most important for his system was this: The Greek astronomer Hipparchus quoted a pas-



The sun (black circle) setting in southern Grecian latitudes on the day of the vernal equinox in 939 BC, the year, according to Newton, in which Chiron delineated the first celestial sphere—one of several arguments in Newton’s *Chronology* that proved to be inaccurate. From Newton and the Origin of Civilization.

sage from the earlier astronomer Eudoxus that lists the constellations that lie on the great circles connecting the poles with the equinoxes and with the solstices. Hipparchus found the list of Eudoxus incorrect. Newton conjectured that Eudoxus was in fact describing a celestial globe created by the much earlier astronomer Chiron, a contemporary of the Argonauts, and moreover that what Hipparchus had considered a mistake was entirely a result of the change in the position of the equinoxes and solstices due to precession in the centuries between Chiron and Hipparchus. A straightforward astronomical calculation makes it possible to determine the date at which the description in Eudoxus’ text would have been

accurate. Newton concluded that the sphere was constructed around 939 BC and that the Argonauts’ voyage was about a generation earlier; the standard date given for the voyage in chronologies based on Greek historical texts was three hundred years earlier than that.

Serious weaknesses in the argument were gleefully picked apart by his opponents in the years following 1728. First, the text of Eudoxus states only that the great circles pass through the constellations, not where they pass through. Different placement of the great circles within the specified constellations would change the results of Newton’s calculations by centuries. Newton

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Ocean Mixing

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There’s no direct way to measure mixing in the real ocean to see how well the computer model performs. So as a first step toward testing the model, Shuckburgh’s team has used a proxy for the mixing: a dye that the researchers released just west of the Drake Passage between Antarctica and South America in February 2009, and which they have since tracked annually as it flows through the passage.

If you stir cream into coffee, the more vigorously you stir, the more complicated the cream formations that initially arise; these formations increase the surface area over which the cream diffuses into the coffee, making the cream assimilate more quickly. In much the same way, when the

surface area allows for greater diffusion and increased mixing. Shuckburgh and her collaborators have been tracking the evolution of the dye’s contours in the ocean.

“We’ve sampled it in some locations to see if it has the same structure” as in the computer model, Shuckburgh said. The model “has done remarkably well.”

Dye Maps and Unstable Manifolds

In the computer model, the team released dye in many slightly different locations. As might be expected in a chaotic system, the dye contours that developed over time differed significantly from each other. At the same time, the various dye maps display a structural coherence that is clear to the eye, and is common to all the different releases.

“It’s slightly different in each case, but they’re all falling on the same underlying structure, something related to the flow

Shuckburgh’s team turned to the theory of dynamical systems to try to elucidate this underlying skeleton. By calculating the velocity field of the surface currents, the researchers were able to identify the hyperbolic points—the fixed points of the flow that attract water along one direction (the stable manifold) and repel it along another (the unstable manifold). And by calculating the stretching rates (the Lyapunov exponents) as nearby points were separated by the flow, the team could determine the direction of maximal stretching—that is, the unstable manifold. It is this manifold that forms the skeleton onto which the dye maps, Shuckburgh said.

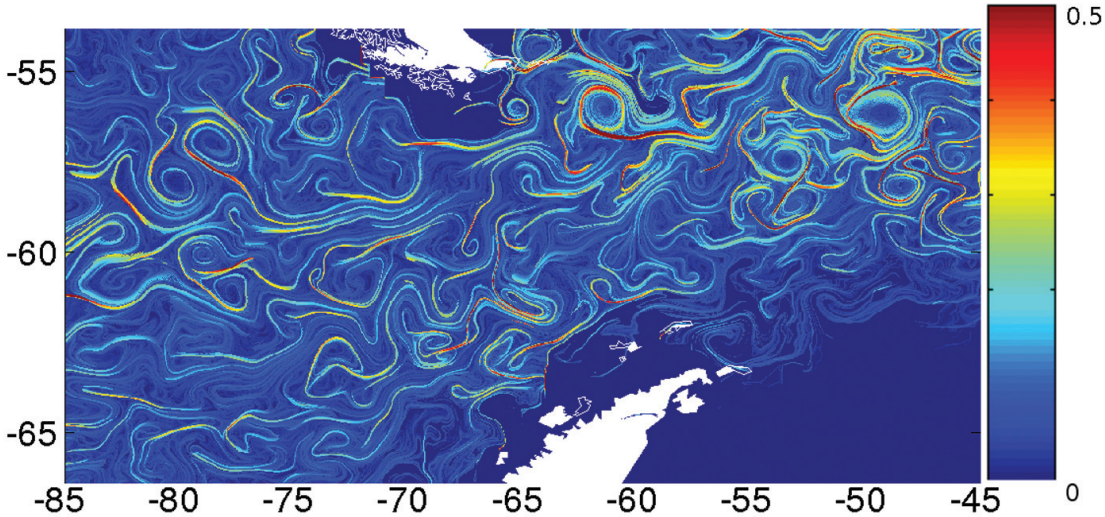
“If a blob of dye starts out close to one of the hyperbolic points, it moves toward the point along the stable manifold and then gets pulled out along the unstable manifold,” she said.

To get a better sense of how well the unstable manifolds and Lyapunov exponents estimated from satellite data mirrored the reality in the ocean, Shuckburgh’s team released an assortment of floats in the Southern Ocean near the estimated hyperbolic points, and then tracked the floats’ paths and measured how quickly they separated.

The ship’s crew didn’t quite know what to make of the float releases, Shuckburgh recalled in the Minneapolis talk. “If you’re sitting in the Southern Ocean telling the ship’s captain, who is

an old sea dog, that you’re going off looking for unstable manifolds, you might as well

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Surface flow through the Drake Passage between South America and Antarctica at one instant in time, as estimated by Shuckburgh’s team from satellite altimetry data. By calculating the flow’s Lyapunov exponents, the researchers were able to home in on the unstable manifolds of this estimated flow. Image courtesy of Emily Shuckburgh, British Antarctic Survey, Natural Environment Research Council.

dye reaches an eddy—whether in computer simulations or in the actual ocean—it swirls into a more complex structure, whose large

itself,” Shuckburgh said. “Wherever you put the dye, it’s like an underlying skeleton that the dye maps onto.”

Ocean Mixing

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tell him you're looking for the Loch Ness monster," she said. "They were deeply, deeply skeptical."

The research team found, however, that the floats do, by and large, follow the unstable manifolds predicted by the satellite data.

"We're applying these mathematical ideas well outside the bounds where they're strictly rigorous, but nevertheless they seem to give us important insights," Shuckburgh said. "We have reasonable confidence that this is now a good diagnostic of the amount of transport and mixing."

While the satellite data does a good job of identifying the unstable manifolds in the real ocean, it appears to underestimate the Lyapunov exponents by as much as 50%, presumably because of limitations on the resolution of the satellite data, Shuckburgh said. It may be possible, by releasing floats in additional locations, to figure out whether the exponents derived from satellite data are always off by the same amount, and to make a correction. Or, she said, the solution may simply be to wait a few years until a planned higher-resolution satellite starts to operate.

Science Policy Implications

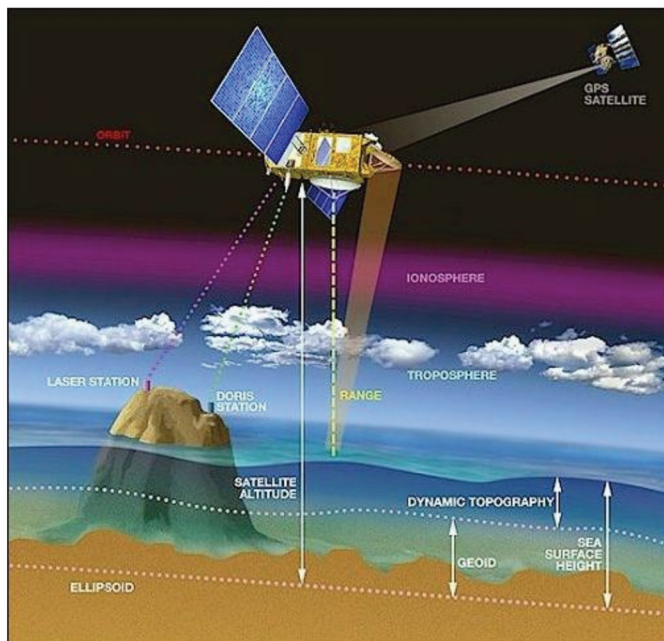
Even this theoretical work has a direct link to policy decisions, Shuckburgh said,



Shuckburgh and her colleagues released floats in the Southern Ocean near the estimated hyperbolic points of the flow, and then tracked their paths. "We literally threw them over the back" of the ship, Shuckburgh said at the 2012 SIAM Annual Meeting in Minneapolis. "It was good fun." Photo courtesy of Emily Shuckburgh, British Antarctic Survey, Natural Environment Research Council.

because the amount of carbon dioxide taken up by the Southern Ocean is so important for understanding the Earth's future climate. "There are not many steps between mathematics and policy in a lot of the work we do."

Shuckburgh dedicates significant time and energy to explaining the policy ramifications of both her own work and other climate research. She has a part-time appoint-



Satellite altimetry measures with great accuracy the height, and thereby the pressure, of the ocean in various locations, enabling Shuckburgh's team to estimate the surface currents. From Centre National d'Etudes Spatiales.

ment at the UK Department of Energy and Climate Change, and leads the Climate Science Communications Group at the Royal Meteorological Society. She is a regular speaker at climate change workshops for policy makers and for general audiences.

In her public lecture in San Francisco in March, Shuckburgh painted a grim picture of the planet's future if we fail to curb carbon emissions, and emphasized that we have years, not decades, in which to come up with solutions to this massive problem. At the same time, she portrayed climate change not just as a threat, but also as an opportunity. "If our collective actions have generated this problem, our collective actions can also help address this problem," she said.

Shifting her focus to the mathematical sciences community, Shuckburgh points out that "right now, a team is down in the Southern Ocean taking more measurements." This is just one of the many problems in climate science to which applied mathematicians could bring insight and expertise. "And Mathematics of Planet Earth 2013 will hopefully highlight some of the opportunities and encourage applied mathematicians to get involved."

Emily Shuckburgh's invited talk in Minneapolis can be viewed on the web at <http://www.siam.org/meetings/presents.php>.

Erica Klarreich is a freelance writer based in Berkeley, California.

Contest

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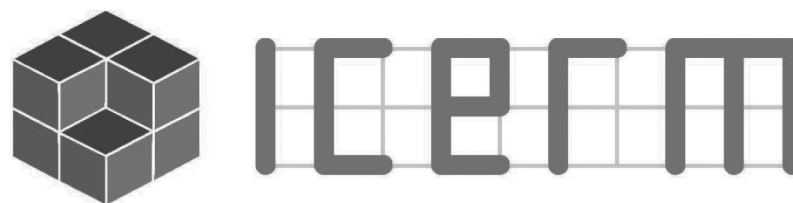
high school students, but not inordinately, and that is amenable to solution within the 14-hour contest period, is itself a substantial challenge. (One of the main ways in which the SIAM community can help with future contests is in submitting and then helping to edit problems into a final workable form.)

Galluzzo, whose research is in subsurface fluid flow, has long been persuaded of the value of undergraduate research and, in particular, of the involvement of undergraduates in mathematical modeling. At Shippensburg, he teaches mathematical modeling, in large part to future mathematics teachers, and as a graduate student he created a (still ongoing) 24-hour regional modeling competition for college students. It was through the latter that he met Ben Fusaro, of MCM and, more recently, Moody's competition fame. For both Fusaro

and Galluzzo, several important threads were tied up when the SIAM/Moody's contest and MPE 2013 joined forces.

An awareness of mathematical modeling among people of all ages—"How many high school students even know what modeling is?" Galluzzo wonders—is one characteristic of the informed and proactive public that is needed to halt and begin to turn back the destruction of the planet. Accordingly, both he and Fusaro have thrown themselves into activities designed to reach the largest possible numbers of people—by teaching teachers, introducing students to modeling, running competitions, with the emphasis on environmental issues. For his part, Galluzzo is thinking of directing the efforts to younger and younger students. "If high school students can be reached," he muses, "what about middle school, and then . . . ?"—GRC

Information about the Moody's Mega Math Challenge can be found at <http://m3challenge.siam.org/>.



Institute for Computational and Experimental Research in Mathematics

Upcoming Topical Workshop

Issues in Solving the Boltzmann Equation for Aerospace Applications

June 3 – 7, 2013

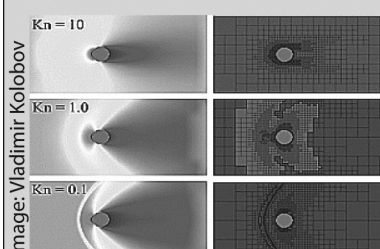


Image: Vladimir Kolobov

The goal of this five-day workshop is to facilitate the development of high-fidelity computational capabilities for the solution of the Boltzmann equation in application to simulation of non-continuum flows. This will be accomplished

by addressing the gaps in communication between mathematicians, engineers and researchers in various fields of research.

Topics of the workshop include but are not be limited to: different forms of the Boltzmann equation; reduced order models for the Boltzmann equation; mesh adaptation in velocity space; fast evaluation of the Boltzmann collision integral; simulations that account for real gas effects and chemical and electromagnetic interaction of particles; complex geometry simulations; coupling of continuum and non-continuum models; and quantification of numerical error and uncertainty of simulations.

Presenters will incorporate in their lectures at least one of the following three common topics:

- Communication of issues related to high computational costs of simulations;
- Communication of issues related to accuracy of models that is the accuracy in approximating the solutions to the Boltzmann equation and the accuracy in approximating physics of gas flows;
- Communication of progress in the analysis of numerical errors.

Organizing Committee:

Alex Alekseenko, California State University, Northridge/Air Force Research Laboratory at Wright-Patterson Air Force Base
Jose Camberos, Air Force Research Laboratory at Wright-Patterson Air Force Base
Irene Gamba, University of Texas at Austin
Sergey Gimelshein, University of Southern California
Prakash Vedula, University of Oklahoma, Norman
Ingrid Wysong, Air Force Office of Scientific Research

To learn more about this and other ICERM programs, organizers, confirmed program participants, and to submit an application, please visit our website:

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Participation: Applications are now being accepted. Support for local expenses may be provided. Decisions about online applications are typically made 1–3 months before the program, as space and funding permit. ICERM encourages women and members of underrepresented minorities to apply.

About ICERM: The Institute for Computational and Experimental Research in Mathematics is a National Science Foundation Mathematics Institute at Brown University in Providence, Rhode Island. Its mission is to broaden the relationship between mathematics and computation.



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Institute for Applied Mathematics and its Applications

Nominations for Board of Governors

The Institute for Applied Mathematics and its Applications invites nominations for its Board of Governors. Nominees can either self-nominate, or they can be nominated by others.

The IMA's board consists of 15 distinguished members from academia, industry, and government. The board is the principal governing body of the IMA. Incoming members of the board will serve a five-year term, beginning on January 1, 2014.

The role of the board is twofold: first, to provide oversight and advice on matters of institute management, development, and institutional relationships; and second, for its members to play an active scientific role in planning and developing annual program themes, as well as identifying lead program organizers. The board meets for two days annually, and subcommittees meet several times annually via conference call.

Submission of Nominations: Nominations for prospective members of the board should be submitted via the online form at <http://www.ima.umn.edu/bog>. All nominations will be reviewed by the Nominations Committee. Nominees will be notified of the committee's decision no later than December 1, 2013.

Closing Date: Nominations are due no later than July 31, 2013.

Readers with questions should contact: Fadil Santosa, santosa@ima.umn.edu, the IMA director, or Thomas Hou, hou@cms.caltech.edu, chair of the IMA Board of Governors.

New Journal: Geometry, Imaging and Computing

International Press of Boston is pleased to announce its forthcoming journal *Geometry, Imaging and Computing*, which will cover topics in applied geometry, imaging sciences, and their computational aspects. The journal's main theme is differential geometry-based modeling/computation in 3D and higher dimensions, with applications to imaging, computer vision, and graphics. The journal will publish high-quality papers over a broad range of topics, including computational differential geometry, geometry processing, shape analysis, shape registration, image processing, image analysis, image understanding, computer graphics, vision, and visualization, with applications to science, medicine, engineering, and other fields.

Editors-in-chief: Xianfeng Gu (SUNY Stony Brook), Stanley Osher (UCLA), Chi-Wang Shu (Brown University), Stephen Wong (Methodist Hospital Research Institute), and Shing-Tung Yau (Harvard University).

Ben Franklin

continued from page 8

but that is only one measure of success. For an undergraduate, exposure to an unstructured learning environment that is filled with dead ends and in which the hard part is coming up with the right questions is an incredibly rewarding experience in its own right.

Conversely, the opportunity for PhD students to serve as teaching assistants in elementary courses can be a very valuable component of the research environment. What could be more important for a researcher-in-training than to practice explaining complex ideas to the non-expert? It will pay off later in situations in which you have to sell your research, e.g., in giving a job interview talk or in writing a proposal to a funding agency.

A degree program that stands in isolation from its neighbors is missing something. I conclude from Jay Walker's remarks that a master's program that has an entrepreneurial slant has much to gain by mixing it up with the undergraduate and PhD scenes, and vice versa. For example, a tight coupling between an academically driven four-year bachelor's program and an industrially driven one-year master's program imparts a sense of unity to the overall educational package. (Cornell had various "3-plus-2" opportunities for engineering students back in the 1940s that reflect this ideal.) More generally, a properly structured liberal education can set the stage for that extra year of professional education. Thus, a history major who values quantitative thought and logic and takes four or five CS courses as electives is more than ready to jump into a CS M.Eng. By the way, computer science intersects the seven original liberal arts more than any other field, a point that I love to bring up with my colleagues in the humanities!

Regarding the interface between the entrepreneurial master's and the traditional PhD, Ben Franklin would be ticked off

to hear us quibbling about the relative merits of "product-driven" and "curiosity-driven" research. Those debates promote the idea that some research styles are more in tune with innovation than others. Another reason for promoting the adjacency of these two degree programs has to do with mentally preparing the student for the careers that ensue. Researchers and entrepreneurs are risk-takers who require character as ballast to handle the ups and downs of the roller coaster. Students who contemplate these professional lifestyles need to be uniformly advised. Borrowing from my Disney World experiences as a parent, "You must be at least 42 inches tall to go on this ride!"

If you are attending a four-year college, you might find this discussion about the intermingling of the best of undergraduate, master's, and PhD programs at your school irrelevant. Not so. Seek out professors who sponsor independent study, or who are actively involved in research, or who have industrial experience. By virtue of having a broader set of teaching responsibilities than their colleagues at the research universities, these faculty often have a superior sense of liberal education and a wider range of technical expertise to draw upon. They can help you set a brilliant stage for graduate study in the directions that I have been considering.

What Is Next for You?

There is a world shortage of culturally sensitive computer scientists, mathematicians, engineers, and scientists who are innovative and who know how to read society from the point of view of an entrepreneur. If you would like to become part of this group and are at the stage where getting a master's degree is an option, then here is what I would think about when evaluating a program.

Support for Different Types of Entrepreneurship. The creation of a successful start-up company that creates jobs and raises the standard of living is rightfully held up as the "gold standard" when it comes to entrepreneurship. If a master's program fixates on this

Students (and others) in search of information about careers in the mathematical sciences can click on "Careers and Jobs" at the SIAM website (www.siam.org) or proceed directly to

www.siam.org/careers

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The *Institute for Computational Engineering and Sciences (ICES)* at The University of Texas at Austin is searching for exceptional candidates with expertise in computational science and engineering to fill several Moncrief endowed faculty positions at the Associate Professor level and higher. These endowed positions will provide the resources and environment needed to tackle frontier problems in science and engineering via advanced modeling and simulation. This initiative builds on the world-leading programs at ICES in Computational Science, Engineering, and Mathematics (CSEM), which feature 16 research centers and groups as well as a graduate degree program in CSEM. Candidates are expected to have an exceptional record in interdisciplinary research and evidence of work involving applied mathematics and computational techniques targeting meaningful problems in engineering and science. For more information and application instructions, please visit: www.ices.utexas.edu/moncrief-endowed-positions-app/. This is a security sensitive position. The University of Texas at Austin is an Equal Employment Opportunity/Affirmative Action Employer.

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GIC will publish four issues per year, tentatively beginning in September 2013. The first issue will be dedicated to David Mumford of Brown University, in honor of his 75th birthday and in recognition of his tremendous contributions in applying geometry to imaging and computer vision.

International Press of Boston invites the submission of papers that address the relationship between geometry and imaging, and their numerical/computational aspects. All submitted papers will be peer reviewed. For paper submission or more information, readers should e-mail gic@intlpress.com or visit <http://intlpress.com/GIC>.

metric, however, it may not fully support the development of other types of entrepreneurial instinct. If you are altruistic, you may be interested in social entrepreneurship, where the overriding ambition has less to do with profit and more to do with bringing about positive social change. (Think Ben Franklin.) Or you may be apprehensive about taking the kinds of risks that are associated with daring start-ups. If so, you may be more interested in developing skills that make you an outstanding intrapreneur. In this setting you will be an employee of an established firm and have the wherewithal to turn ideas into products on behalf of your employer. Does the master's program that you are considering support the "brand" of entrepreneurship that excites you?

Outside Coursework. You need to know about business models, venture capital, negotiation, and related matters. Are suitable business courses available? Courses on the history of technology, science and society, the sociology of change, technical writing, intellectual property, patent law, and international trade may also be relevant. Does the program you are considering have space for a couple of non-technical courses? There is nothing "professional" about a narrow education.

The "Real World" Component. Industrial mentorship and on-campus incubators obviously add to the educational experience in that they give students a snapshot of the city streets before they head out on their own. Does the master's program have these features?

The Flexibility Factor. It is most natural for a master's program in technical field X to focus on recruiting undergraduates who have majored in X. But suppose your major was in some other field and that you simply minored in X. Assuming that your math and computing skills are at the required level, how welcoming is the program to students with your background? Does the program revel in the idea that the best entrepreneurs have multiple talents? How friendly would the program be if your avowed intention is to pursue a PhD afterward?

The Thesis/Time Factor. The one-year master's without thesis is not the only show in town. Many schools offer two-year programs that require the writing of a master's thesis. Such a program can work for students with entrepreneurial interests, and sometimes there are opportunities for teaching assistantships to offset the costs of tuition. In any case, how do you feel about one versus two years? And would you benefit from a writing component?

The Self-Esteem Factor. The entrepreneurial spirit requires a certain measure of self-confidence and self-esteem. Undergraduate programs and PhD programs pay close attention to these emotional factors. What can you say about the master's program that you are considering?

To sum up, look for clues that the program appreciates the complexity of what it means to be an entrepreneur, and do not be misled by exaggerated claims that you are destined to become the next Bill Gates, even though it may be fun to think about the dollars!

Conclusion

Nowadays, master's-level education is one of the most exciting venues across the higher education landscape. It is where all that is hot about entrepreneurship is mixing it up with all that is cool about liberal education and basic research. Big systems are colliding and I hope that you are as fascinated by the superstorm as I am. If you are proactive as a student, then this is where you can refine your sense of culture, business, and research as you proceed to solidify your technical base.

That's how Ben Franklin sees it. And that's how I see it too with my bifocals and lightning rod!

Charles Van Loan is a professor in the Department of Computer Science at Cornell University.

Sue Minkoff (sminkoff@utdallas.edu), of the University of Texas at Dallas, is the editor of the *Careers in the Math Sciences* column.

Benjamin Franklin: The Doctor Would Want a Master’s

Whether you are in school or just starting out, the height of your trajectory may very well be determined by how much you appreciate the connections that exist between liberal education, entrepreneurship, and basic research. Contrary to popular opinion, these are not strange bedfellows, an observation based on what I have learned while serving at various times as director of the undergraduate, master’s, and PhD programs in computer science at Cornell University. These positions have given me the opportunity to think hard about how to recruit and advise students, i.e., how to relate this or that degree program to the “big picture” and the careers that may follow.

Currently, as director of our one-year Master of Engineering (M.Eng.) program, I am preoccupied with the notion of entrepreneurship. One reason for this has to do with the job market. Another is that Cornell is in the process of setting up a new campus in New York City, with master’s-level degree programs that have a strong and very explicit entrepreneurial component. Driven by the need to advise students who are thinking about entrepreneurship, the NYC initiative has prompted me to rethink the connection between what we do here on the Ithaca campus through our traditional degree programs. Let me state at the outset that I have never worked for a company and have never been directly involved in the commercialization of a research idea! Moreover, I am a believer in undirected basic research and a gigantic fan of liberal education, a pair of enthusiasms that would appear to disqualify me from any discussion of the entrepreneurial master’s.

Nevertheless, I have a game plan for discussing the issues. It is to peek out

from behind the life of Benjamin Franklin, America’s greatest entrepreneur–scientist–journalist–statesman. He shows us how to think about liberal education even though he had just two years of formal schooling. He shows us how to think about entrepreneurship even though he “gave away” the patents to all his inventions. He shows us how to think about basic research even though many of his scientific exploits were driven by practical concerns. Ben is my entrepreneurial master’s cover story—he is exactly the kind of student I would like to see in our program. Of course, I will have to speak to the Admissions Committee about waiving the bachelor’s degree requirement and about accepting his honorary doctorates from Oxford, St. Andrews, Harvard, and Yale in lieu of GRE scores!

A Venue for Change

The one-year master’s degree (typically without thesis) is increasingly popular across engineering, where there is considerable sentiment that four years of undergraduate study is just not enough. Indeed, many undergraduate programs in engineering expanded to five years just after World War II as the discipline became more science-based and rigorous. Then, beginning in the 1960s, the “5” became “4” for a variety of reasons. For example, it is handy in a university setting for all bachelor’s programs to have the same eight-semester volume. The streamlining also made it easier for students to choose from an expanding list of postgraduate options: the PhD, the MBA, or the extra year of coursework to bolster one’s credentials as a professional engineer.

An important observation to make from this thumbnail history is that the master’s

degree has evolved and diversified over the past 70 years. It sits in a crowded space between the undergraduate experience, industry, and other advanced degrees. It is even more crowded now that entrepreneurship is part of the scene.

OK—What Is Entrepreneurship?

Can it be taught? Is it a frame of mind? Is it a special type of problem solving? How does it relate to basic research and the serendipity of discovery? Is it just about start-ups? What are the required social skills? After looking at all the one-liners on Wikipedia and reading a few scholarly articles, I buy into the idea that entrepreneurship is all about the distance from research to product realization and that entrepreneurs are preoccupied with the paths that lead from one to the other. It is productive to think in terms of “distance” rather than “time”: The distance metaphor focuses attention on the landscape of entrepreneurship and the problem-solving mentality that is required to navigate the terrain. In this regard I found a recent speech by Jay Walker to be particularly insightful. Walker is an entrepreneur, inventor, and chairman and curator of TEDMED. In a 2012 address to the Cornell Entrepreneurship Summit, he made several key points, which are roughly as follows:

1. The entrepreneur’s job is to identify a problem worth solving.
2. Problem complexity is changing faster than technology.
3. Great entrepreneurs are able to describe a problem clearly, precisely, and with an economic description that talks about a customer and a price.
4. As an entrepreneur, you need humility to know what you don’t know because cus-

tomers think differently, often in ways that have nothing to do with science, logic, or evidence.


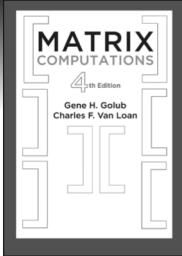
What strikes me about this list is how well it resonates with life in the “Ivory Tower”:

- 1.’ The PhD student’s job is to find a research problem worth solving.
- 2.’ Research problems are changing faster than field-specific education and can no longer be solved by homogeneous teams of look-alike experts.
- 3.’ Great researchers are able to describe “the nut they cracked” in terms that can be understood by the public.
- 4.’ As a researcher, you need humility to know what you don’t know because colleagues outside your area often think in ways that are orthogonal to the traditions of your field.

It appears to me that entrepreneurship has much in common with research and that society is served well by liberally educated entrepreneurs and researchers who reject the know-it-all mentality.

Degree Programs Need Neighbors

The bachelor’s and the PhD provide a case study that showcases just how much the presence of one degree program can positively affect what goes on in the other. The past twenty or so years has seen an explosion in undergraduate research. Partly to attract students to graduate-level study, and partly to offer an alternative to the large-class experience, many universities have promoted the idea of independent study with specially structured opportunities to mingle with faculty. Sometimes the result of an independent study is a research paper, See Ben Franklin on page 7



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The Role of Linear Algebra in the Computer Science Curriculum*

As the field of computer science has expanded to touch almost every facet of our lives, the computer science curriculum is under enormous pressure to deliver a rigorous core while also allowing students to follow their interests along the many diverse and productive paths offered by the discipline.

At the same time, the growth of science and engineering disciplines has been accompanied by expanded use of mathematics, as new mathematical problems are encountered and new mathematical skills are required. In this respect, linear algebra has come to play a particularly significant role in many important computer science undertakings. A few well-known examples are Internet search, graph analysis, machine learning, graphics, bioinformatics, scientific computing, data mining, computer vision, speech recognition, compilers, and parallel computing.

The broad utility of linear algebra in computer science reflects deep connections arising from the discrete nature of matrix mathematics and digital technology.

Mathematics in the Computer Science Curriculum

The ACM/IEEE task force identified three

*This article is a slightly edited version of a position paper submitted to the joint ACM/IEEE task force Computer Science Curricula 2013. The authors are Jeremy Kepner (chair), MIT; Tim Davis, University of Florida; James Demmel, UC Berkeley; Alan Edelman, MIT; Howard Elman, University of Maryland; John Gilbert, UC Santa Barbara; Michael Heath, University of Illinois; Dianne O’Leary, University of Maryland; Michael Overton, Courant Institute, New York University; Yousef Saad, University of Minnesota; Ahmed Sameh, Purdue University; Michael Stonebraker, MIT; Gilbert Strang, MIT; Robert van de Geijn, University of Texas; Charles Van Loan, Cornell University; and Margaret Wright, Courant Institute.

specific mathematical subjects as core to computer science: calculus, differential equations, and linear algebra. In addition to these courses, many computer science curricula require statistics and discrete mathematics. At our own institutions, requirements include the following:

- Algorithms I & II, AI; Calculus I, Discrete Math OR Probability, Linear Algebra OR Calculus II
- Algorithms I; Calculus I & II, Linear Algebra OR Physics II
- Algorithms I & II; Calculus I & II & III, Probability, Linear Algebra
- Algorithms I, Discrete Math; Calculus I & II, Statistics, Linear Algebra OR Calculus III
- Algorithms I, Discrete Math; Calculus I
- Algorithms I, Discrete Math; Calculus I & II, Statistics, Linear Algebra
- Algorithms I, Discrete Math; Linear Algebra
- Algorithms I, Discrete Math; Calculus I & II & III, Statistics, Linear Algebra
- Algorithms I, Discrete Math; Calculus I & II, Statistics, Linear Algebra
- Algorithms I, Discrete Math; Calculus I & II & III, Linear Algebra

The above sample is not a complete survey of computer science curricula; nevertheless, we can make a few broad observations:

- Linear algebra is required in about half of the computer science curricula and is optional or not required in the other half.
- Linear algebra comes near the end of the mathematics sequence (usually following calculus).
- Furthermore, we can infer that:
- Computer science courses cannot reliably

assume that their students have an understanding of linear algebra.

■ Linear algebra courses can reliably assume that their students have an understanding of calculus.

Based on these observations, we have a variety of recommendations to offer with regard to the role of linear algebra in the computer science curriculum.

Recommendations

The following recommendations represent the views of the majority of contributors.

Computer Science Recommendations
CS1 Encourage that computer science students be made more aware of the importance of linear algebra in various areas of computer science (e.g., Internet search, computer graphics, and machine learning).

CS2 Encourage the inclusion of linear algebra in computer science theory, algorithms, and data-structures courses (e.g., matrix multiply algorithms, adjacency matrix data structures, and SVD data analysis).

CS3 Encourage making linear algebra a requirement for computer science majors, particularly those who are interested in advanced study.

Mathematics Recommendations
MA1 Encourage the inclusion of common computer science examples in linear algebra classes (e.g., graph analysis, 3D transformations, and speech recognition).

MA2 Encourage the use of software in linear algebra classes to satisfy computer science “second language” goals (e.g., Python, R, or Matlab).

MA3 Encourage that a version of linear algebra be taught earlier without a calculus prerequisite.