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IP1

Orthogonally-Equivariant Dimensionality Reduction in Stiefel Manifolds

Many real-world datasets can be realized in high-dimensional Stiefel and Grassmannian manifolds, and benefit from projections onto their lower-dimensional counterparts. In this talk, I will describe Principal Stiefel Coordinates (PSC), a methodology to reduce the ambient dimensionality of data sets in Stiefel manifolds, in such a way as to be equivariant with respect to the action of the orthogonal group. Multiple numerical experiments using synthetic and real-world data will be presented; particularly, in the context of discrete and approximate vector bundles.

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IP2

Distinguishing Phylogenetic Networks

A primary goal of phylogenetics is to understand the evolutionary history of a set of species. These histories are typically represented by directed graphs, where the leaves correspond to living species and the interior nodes represent extinct ones. While evolutionary histories are often assumed to take the form of trees, networks provide a more realistic representation in the presence of events such as hybridization. However, allowing the generality of networks significantly complicates the inference process. A recent approach to phylogenetic network inference leverages computational algebraic geometry. In this talk, we will explore the role that computational algebraic geometry and symbolic computation play in addressing statistical challenges related to network inference, with a particular focus on identifiability.

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IP3

Marginal Independence and Partial Set Partitions

Independence models are widely used in probabilistic modeling, including in computational biology, statistics, and machine learning. In spite of their ubiquity, there is a large gap between the families of independence models that are easy to parametrize and completely general independence models. Focusing specifically on marginal independence models, we establish a bijection between marginal independence models on n random variables and split closed order ideals in the poset of partial set partitions. We also establish that every discrete marginal independence model is a toric variety in an appropriate coordinate system. Everything will be defined and the talk will be accessible to first year graduate students and advanced undergraduate students. This is joint work with Francisco Ponce Carrin.

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IP4

Symmetry in Machine Learning Models

Symmetry has played an influential role in machine learning, in both the data and model spaces. This talk will summarize some perspectives from our work, including modeling, theoretical and empirical observations. As a first concrete example, we will look at how to design neural networks whose inputs are eigenvectors, with applications, e.g., to positional encodings for popular graph neural networks. Second, we will take a more general theoretical complexity perspective on learning with invariances. Beyond data symmetries, symmetries are also inherent in the architecture of the neural network, e.g., certain transformations of the neural network weights result in exactly the same function being implemented. These arise, e.g., from automorphisms of the computation graph and from the form of the nonlinearities. After designing methods to remove some of these symmetries, we study the empirical effects of such parameter symmetries on training behavior and on the optimization landscape.

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IP5

Application Driven Topological Data Analysis

Persistent homology (PH) is a central tool in topological data analysis. PH provides a multiscale geometric descriptor of data that is functorial, stable to perturbations and interpretable, leading to many applications in mathematics and real-world data. Frequently one starts with point cloud data, a finite subset of a metric space (eg Euclidean space) and studies the topology arising from a filtration of simplicial complexes built on the data. While this process yields an interesting nontrivial descriptor of the "shape of data", some theoretical questions remain. In this talk, we will present two directions in application-driven point cloud persistence. First, we will focus on the spaces of point clouds with the same persistence. This inverse problem asks the following: What is the shape of the fiber of the persistent homology map? The second problem is motivated by spatial data arising in biology, which includes outliers (eg histology) or dynamic metric spaces (eg collective dynamics). We analyse such systems by presenting statistics for multiparameter persistence and then applying it to such biological data. To study these two PH problems, we adapt tools from applied and computational algebraic geometry.

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IP6

Size (OOD) Generalization of Neural Models via Algorithmic Alignment

Size (or length) generalization is a key challenge in neural algorithmic reasoning. Specifically, when can a neural model with bounded complexity generalize to problem instances of arbitrary size? This problem, known as size generalization, is a special case of out-of-distribution (OOD) generalization. In this talk, I will present three examples of achieving such size generalization by "aligning" the neural models with algorithmic structures. In particular, the first two (quick) examples will be on the simpler "expressivity" question, where the goal is to design neural models that are capable of size generalization to solve potentially hard problems at hand. The main part of the talk will focus on the third example which goes beyond "expressivity" and tackles a much more challenging question of certifying, provably, that a trained model generalizes to input of arbitrary sizes. More precisely, we consider the problem of predicting the output of K-step Bellman-Ford (BF) procedure for computing graph shortest path. It has been observed in the literature that a special family of graph neural networks (which we refer to as BF-GNN) has a natural alignment with the BF procedure. Surprisingly, we show that we can construct a set of only a constant number of small graphs, such that if the neural Bellman-Ford model (even when over-parameterized) has low loss over these graphs, then this model will probably generalize to arbitrary graphs with positive weights. To the best of our knowledge, this is the first provable (and practical) generalization certificate for neural approximation of complex tasks. This result also has interesting implications on training neural algorithmic modules.

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IP7

Complete Monotonicity in Scattering Amplitudes

I will briefly review the concept of completely monotone functions, and then describe recent observations of how they appear in Feynman integrals and scattering amplitudes. This talk is based on reference ¡a href="https://arxiv.org/pdf/2407.05755"¿https://arxiv.org/pdf/2407.05755¡/a¿

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IP8

The Geometry of Polynomials for Log-concavity and Expansion

Log-concavity is an important feature of many functions and discrete sequences appearing across mathematics, including combinatorics, algebraic geometry, convex analysis, and optimization. In this talk I will survey recent developments in our understanding of functional log-concavity with a focus on applications to combinatorial inequalities and algorithms for approximate counting and sampling. At the heart of this story are rich classes of multivariate generating polynomials that give rise to discrete probability distributions that can be approximately sampled efficiently using Markov chains. The basis-generating polynomial of a matroid is a fundamental example. We will explore applications to matroids and extensions to other combinatorial structures.

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IP9

From Algorithms to Theorems: Some Examples From Real Algebraic Geometry

While designing better algorithms often require proving theorems – sometimes the direction of the flow is reversed. I will give several examples where careful analysis of certain exact algorithms for solving problems in real algebraic geometry lead to mathematical results of a quantitative nature that are interesting in their own right. The examples include a quantitative version of the curve selection lemma, an effective Lojasiewicz inequality, and bounds on the 'speed' of semi-algebraically defined multi-parameter persistence modules (which are important objects in topological data analysis). Based on joint works separately with Marie-Francoise Roy, Ali Mohammad Nezhad and Arindam Banerjee.

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IP10

Periodic Graph Operators for Algebraic Geometers

Understanding the spectrum of the Schrdinger operator in a periodic medium is a fundamental problem in mathematical physics that is best approached using methods from analysis. The discrete version of this concerns operators on periodic graphs. In this discrete version, the primary objects are real algebraic varieties, and thus algebraic geometry becomes relevant for the study of discrete periodic operators. The purpose of this talk will be to explain this to algebraic geometers, and describe some results obtained from this perspective, as well as some computational and combinatorial aspects of this study.

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SP1

SIAM Activity Group on Algebraic Geometry Agnes Szanto Early Career Prize Lecture: Discriminants in Particle Physics

I will discuss discriminants arising from scattering amplitudes in particle physics and their computation using methods from algebraic geometry. These discriminants detect degenerate critical points of a master function on a smooth very affine variety or capture changes in the topological Euler characteristic for a family of such varieties. They also have natural interpretations in algebraic statistics.

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CP1

Circuits and Symmetries of Paley Equiangular Tight Frames

Equiangular tight frames (ETFs) are optimal packings in projective space which also yield useful decompositions of signals. Paley ETFs are constructed using number theory. In this talk, we present the doubly homogeneous symmetry groups of the Paley ETFs and then leverage the symmetry to analyze the circuits of the associated linear matroid.

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CP1

An Attack on the Elliptic Curve Discrete Loga-

rithm Problem

The elliptic curve discrete logarithm problem is of fundamental importance to public-key cryptography. We explore ways to solve the elliptic curve discrete logarithm problem using mostly computational results. However, it seems, the methods we develop and directions we pursue can provide a potent attack to this problem. Let $\mathbb{F}$ be a finite field of arbitrary characteristic. Let

$$\mathcal{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1d} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2d} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{l1} & a_{l2} & a_{l3} & \dots & a_{ld} \end{pmatrix} \tag{1}$$

be a $l \times d$ matrix over $\mathbb{F}$. Let α, β be two ordered subsets of $\{1, 2, \dots, l\}$ and $\{1, 2, \dots, d\}$ respectively and are of the same size k. Then we define the sub-matrix $\mathcal{A}[\alpha|\beta]$ of $\mathcal{A}$ to be the square matrix of size k consisting of elements that are in the intersection of the rows in α and columns in β in $\mathcal{A}$. The ordering in the sub-matrix is the same as the ordering in the matrix $\mathcal{A}$. The determinant of $\mathcal{A}[\alpha|\beta]$ is a minor of $\mathcal{A}$. In this talk, we show that if one can find a zero minor in a matrix A similar to the above obtained from the elliptic curve $\mathcal{E}$, the elliptic curve discrete logarithm problem is solved. We show some results obtained through experiments.

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CP1
Degree k Column Relations for Moment Matrices

Let $M(k)$ be the moment matrix associated with a complex moment sequence $\gamma^{(2k)}$. Let $V(M(k)) \equiv V$ be the algebraic variety associated with $M(k)$. That is, if $p(z, \overline{z})$ is a polynomial which corresponds to a column relation of $M(k)$, then $V(M(k))$ is the intersection of the zero locus of $p(z, \overline{z})$ and $\overline{p(z, \overline{z})}$. If the rank of $M(k)$ is equal to the cardinality of V , we say that the problem is extremal. In this talk, we show that if G is the Gröbner basis for the ideal $I(V)$ associated with V by the Nullstellensatz, then G contains exactly the polynomials which correspond to the rest of the column relations of $M(k)$. Moreover, we can find a numerical condition on the level of the moments which is equivalent to the existence of a representing measure.

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CP2
Polynomial and Analytic Methods for Classifying Complexity of Planar Graph Homomorphisms

We introduce some polynomial and analytic methods in the classification program for the counting complexity of planar graph homomorphisms. It is known that the problem of computing the partition function from an input graph G to a complex valued symmetric matrix M (denoted as $\texttt{GH}(M)$) GH is either poly-time tractable or #P-Hard. A similar dichotomy is hypothesized when the input graphs are restricted to be planar (this problem is denoted as $\texttt{Pl-GH}(M)$), but has only been proved for matrices of size ≤ 3. Interestingly, it is known that there exist non-trivial M such that $\texttt{GH}(M)$ is #P-Hard, but $\texttt{Pl-GH}(M)$ is poly-time tractable. It is further hypothesized that such matrices are precisely those for which $\texttt{Pl-GH}(M)$ can be solved using Valiant's holographic reduction to the FKT algorithm (for computing the number of perfect matchings on planar graphs). We introduce techniques that leverage the properties of polynomial and analytic functions to isolate these poly-time tractable matrices represented by tensor products of matchgates, and indicate how we may prove that for all other matrices M, $\texttt{Pl-GH}(M)$ is #P-Hard. We use these methods to prove a complexity dichotomy for 4×4 matrices that shows that Valiant's holographic algorithm is indeed universal for planar tractability in this setting. More details can be found in the full version of this paper: https://arxiv.org/abs/2412.17122

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CP2
On the Gcd Graphs over Polynomial Rings

Gcd-graphs over the ring of integers modulo n are a natural generalization of unitary Cayley graphs. The study of these graphs has foundations in various mathematical fields, including number theory, ring theory, and representation theory. Using the theory of Ramanujan sums, it is known that these gcd-graphs have integral spectra; i.e., all their eigenvalues are integers. In this work, inspired by the analogy between number fields and function fields, we define and study gcd-graphs over polynomial rings with coefficients in finite fields. We establish some fundamental properties of these graphs, emphasizing their analogy to their counterparts over Z.

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CP2
Homology and Homotopy Properties of Scale-Free Networks

Many real-world networks are believed to be scale-free. We study the random model of preferential attachment for such networks. Our results show that preferential attachment favors higher-order connectivity, in the sense that it drives the growth of Betti numbers in the finite-graph setting, and it annihilates homotopy groups in the infinite-graph setting. More precisely, we determined the high-probability growth rates of the Betti numbers of the clique complexes of finite preferential attachment graphs, as well as the sharp threshold at which the infinite clique complex becomes homotopy-connected almost surely. This is joint work with Gennady Samorodnitsky, Christina Lee Yu, and Rongyi He. The talk is based on the preprints [https://arxiv.org/abs/2305.11259] and [https://arxiv.org/abs/2406.17619].

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CP3

Maximum Likelihood Degree of the β-Stochastic Blockmodel

Log-linear exponential random graph models are a specific class of statistical network models that have a log-linear representation. This class includes many stochastic blockmodel variants. In this talk, we focus on β-stochastic blockmodels, which combine the β-model with a stochastic blockmodel. Here, using recent results by Almendra-Hernandez, De Loera, and Petrovic, which describe a Markov basis for β-stochastic block model, we give a closed form formula for the maximum likelihood degree of a β-stochastic blockmodel. The maximum likelihood degree is the number of complex solutions to the likelihood equations. In the case of the β-stochastic blockmodel, the maximum likelihood degree factors into a product of Eulerian numbers.

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CP3

Applications of Persistent Homology to Fluorescent Microscopy

Microscopy images are rich with complex information that poses significant challenges for analysis and interpretation. Advances in machine learning, particularly convolutional neural networks (CNNs), have significantly improved image segmentation and classification tasks. However, understanding how these pipelines extract and interpret features from training data remains a critical challenge. To address this, we introduce a novel set of image analysis features derived from topological data analysis (TDA), specifically persistent homology and persistence landscapes. TDA provides a mathematical framework to quantify the shape of data and has shown promise in machine learning applications. Building on these concepts, we present TDAExplore, an innovative pipeline for image classification and segmentation. The weakly supervised segmentation model identifies regions whose shape features most strongly characterize image classes, requiring only image labels for training. Across multiple datasets, TDAExplore demonstrates high classification accuracy, robustness to hyperparameter changes, and efficient computational performance. Additionally, the method excels in recognizing diverse subcellular structures, offering biologically meaningful insights from complex image data.

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CP3

Tropical Representations of Transformer Neural Networks

A 2018 paper by Zhang, Naitzat, and Lim showed that neural networks under ReLU activation can be represented algebraically as tropical rational functions, demonstrating the potential for these architectures to be studied geometrically using the language of tropical geometry. Here, I will present an extension of this work to a particular class of networks known as transformers, which form the backbone of large language models such as ChatGPT. I will show that certain transformers can also be represented as tropical rational functions and use this fact to discuss the geometric nature of these structures. We'll see that the tropical function associated to a transformer admits a geometric representation comprised of zonotopes, and also that the tropical hypersurface of such a function contains the decision boundary of the associated transformer. I'll discuss the combinatorics of this hypersurface and conclude by explaining how tropical representations of a transformer could provide a framework for constructing a rigorous theory of the behavior of these network models.

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CP4

LLM Steganography

Large language model-based steganography encodes hidden messages into model-generated tokens. The key tradeoff is between how much hidden information can be introduced and how much the model can be perturbed. To address this tradeoff, we show how to adapt strategies previously used for LLM watermarking to encode large amounts of information. We tackle the practical (but difficult) setting where we do not have access to the full model when trying to recover the hidden information. Theoretically, we study the fundamental limits in how much steganographic information can be inserted into LLM-created outputs. We provide practical encoding schemes and present experimental results showing that our proposed strategies are nearly optimal.

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CP4

Extending Class Group Action Attacks Via Sesquilinear Pairings

We introduce a new tool for the study of isogeny-based cryptography, namely pairings which are sesquilinear (conjugate linear) with respect to the $\mathcal{O}$-module structure of an elliptic curve with CM by an imaginary quadratic field $\mathcal{O}$. We use these pairings to study the security of problems based on the class group action on collections of oriented ordinary or supersingular elliptic curves. This extends work of both (Castryck, Houben, Merz, Mula, Buuren, Vercauteren, 2023) and (De Feo, Fouotsa, Panny, 2024).

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CP4

Polynomial Characterization and Existence of Qsds for Markov Processes

In this talk, we study a class of Markov processes on $\mathbb{N}_0$, that includes Birth-Death processes. We are interested in characterization and existence of Quasi-Stationary Distributions (QSDs) of such processes, provided the process is absorbed almost surely. We give a polynomial characterization of QSDs and show that an extremal QSD always exists for Kingman's parameter θ_K. Furthermore, we show that θ_K might be found numerically as the limit of a sequence of smallest roots of concretely given polynomials of increasing degree. Thereby, we connect to Karlin and McGregor seminal paper from 1957. For the latter part, the method of proof is Perron-Frobenius theory of infinite matrices and their finite truncations.

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MS1

Trace Tests and Monodromy Groups

The trace is the sum of the zeros of a polynomial system. In algebraic geometry, trace tests are used after solving to check for completeness of the solution set. A trace test for sparse polynomial systems relies on finding a subset of coefficients on which the trace relies linearly. When coefficients of a polynomial system are treated as parameters and allowed to vary, this induces permutations of the zeros of the system. The monodromy group is the group generated by these permutations. We study the monodromy group resulting from treating different sets of coefficients as parameters. In order to completely characterize systems under the sparse trace test, we are interested in cases when the monodromy group is the full symmetric group. We discuss extensions involving the monodromy group of a branched cover, where monodromy is an action of the fundamental group on a fibre, and we provide examples.

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MS1

Monodromy in the Space of Symmetric Cubic Surfaces with a Line

We explore the enumerative problem of finding lines on cubic surfaces defined by symmetric polynomials. We prove that the moduli space of symmetric cubic surfaces is an arithmetic quotient of the complex hyperbolic line, and determine constraints on the monodromy group of lines on symmetric cubic surfaces arising from Hodge theory and geometry of the associated cover. This interestingly fails to pin down the entire Galois group. Leveraging computations in equivariant line geometry and homotopy continuation, we prove that the Galois group is the Klein 4-group.

This is the first computation in what promises to be an interesting direction of research: studying monodromy in classical enumerative problems restricted by a finite group of symmetries.

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MS1

Power-flow System and Semi-Bernshtein-generalness

We explore the concept of semi-Bernshtein-generalness, a relaxation of the classical notion of Bernshtein-general polynomial systems. While a (Laurent) polynomial system is Bernshtein-general when it has isolated solutions whose total count, with multiplicity, equals the BKK bound, we investigate a weaker condition motivated by practical applications. Specifically, we examine power-flow equations from electrical engineering as a motivating example for this generalization. Our discussion focuses on the numerical challenges that arise when solving semi-Bernshtein-general systems, particularly through homotopy-based methods, and presents several partial solutions to these computational obstacles. This is a joint work with Evgeniia Korchevskaia andJulia Lindberg.

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MS1

Randomized Algorithms for Verifying Monodromy Groups

Certifying the computation of monodromy groups via certified homotopy continuation is computationally difficult. In practice, one computes it by taking the group generated by a few random monodromy permutations, a method subject to numerical and probabilistic errors. Under some monodromy assumptions, we analyze the reliability of this method and develop an improved version which can detect errors in both sampled permutations and computed candidate groups. Through examples, we illustrate this improvement and the maintained efficiency of our verification method.

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MS2

Hopf Bifurcation Points in Mass Action Kinetics Networks

We present a method for identifying Hopf bifurcation points in parametric ordinary differential equations (ODE) systems modeling reaction networks with n species. The method is based on a Hopf bifurcation theorem for parametric systems, algebraic geometry, majorization theory and convex analysis. Selecting parameter values such that the next to last Hurwitz determinant $\det H_{n-1}$ is zero is a condition of the Hopf bifurcation theorem. The existence of a vertex of the Newton polytope of $\det H_{n-1}$ associated with a negative monomial usually guarantees finding parameter values such that $\det H_{n-1}$ is zero. We show that a vertex of the Newton polytope of $\det H_{n-1}$ exists among the monomial exponents in the product of determinants of diagonal submatrices of H_{n-1}.

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MS2

Characterization of generic ACR and local ACR in Reaction Networks

A dynamical system modeling the evolution of the concentrations of the species in a network has Absolute Concentration Robustness (ACR) for one species if there is a fixed value for the concentration of such species at steady state (assuming that there is some steady state). We say that the system has local ACR instead when there is not only one but a finite set of values that this concentration can achieve at steady state. The existence of these two properties, as well as the specific values, can depend on the reaction rates. We will discuss how to characterize networks that have these properties generically, that is, for most values of reaction rates, based on the geometry of steady state varieties, using the results developed in [The generic geometry of steady state varieties, E. Feliu, O. Henriksson, B. Pascual-Escudero. (arXiv:2412.17798)].

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MS2

Unveiling Surprising Connections Between the Classical Theory of Reaction Networks and Generalized Lotka-Volterra Systems

We explore the relationship between *Reaction Networks* and *Population Dynamics*, with a specific focus on *Generalized Lotka-Volterra systems* (also called *Kolmogorov systems*). Surprisingly, we find strong analogies between classical Mass Action Kinetics results (like the Horn-Jackson theorem and the deficiency-zero theorem) and new counterparts for Generalized Lotka-Volterra (GLV) systems, hinting at a deep connection, where previously none was known. Notably, in the GLV setting, we can prove that complex-balanced equilibria (properly defined) are globally attractive (which corresponds to the global attractor conjecture" in the Reaction Networks setting). As an example, we show how to apply this new theory to characterize global stability for a large class of cooperative GLV systems. We also extend our results to analyze the properties of variable-k systems, an area not fully explored in the context of GLV systems. We especially focus on versions of the *Persistence Conjecture* and *Permanence Conjecture* for GLV systems.

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MS3

The Eigenvalue Method in Coding Theory

We lie the foundations of the spectral method in coding theory. The method uses modern algebraic graph theory to derive upper bounds on the size of error-correcting codes for various metrics, addressing major open questions in the field. We identify the core assumptions that allow applying the eigenvalue method, test it for some of the best-known classes of error-correcting codes, and compare the results with the best bounds currently available. By applying the eigenvalue method, we obtain new bounds on the size of error-correcting codes that often improve the state of the art. Our results show that spectral graph theory techniques capture structural properties of error-correcting codes that are missed by classical approaches.

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MS3

Properties and Applications of Reed-Solomon Composition Codes

Reed-Solomon codes are error-correcting codes defined using evaluations of polynomials of bounded degree over finite fields. They are known for being elegant, useful, and for possessing the maximum possible minimum distance for a fixed codeword length and dimension. One drawback of Reed-Solomon codes is that the codeword length is bounded by the field size. Consequently, there is no non-trivial binary Reed-Solomon code. One could use this limitation to motivate codes over algebraic curves. There are also various approaches that start with a nonbinary code and transform it into a binary one. In this talk we will present properties and applications of binary images of Reed-Solomon codes, primarily by using the codeword images as columns of a parity-check matrix. We will focus on the overlap of common symbols in pairs of codewords and present bounds on this parameter.

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MS3

Permutation Group of Cartesian Codes

A decreasing Cartesian code is defined by evaluating a monomial set closed under divisibility on a Cartesian set. Some well-known examples are the Reed-Solomon, Reed-Muller, and (some) toric codes. The affine permutations consist of the permutations of the code that depend on an affine transformation. In this work, we study the affine permutations of some decreasing Cartesian codes, including the case when the Cartesian set has copies of multiplicative or additive subgroups.

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MS3

Irreducible Ferrers Diagrams in the Etzion-Silberstein Conjecture

In 2009 Etzion and Silberstein provided a combinatorial upper bound on the largest dimension of a rank-metric code over a finite field whose nonzero matrices are supported on a given Ferrers diagram and all have rank lower bounded by a fixed positive integer d. In the same paper, they also conjectured that such an upper bound is always tight: that is, for every d and Ferrers diagram D there exists an optimal rank-metric code supported on D Since then, their conjecture has been verified in a number of cases, but as of today it still remains widely open. In this talk, we investigate the notion of reducibility of Ferrers diagrams: a diagram D reduces to D' if an optimal code supported on D can be obtained by shortening and/or inclusion of an optimal code supported on D'. This induces a natural notion of irreducibility of Ferrers diagrams, and the validity of the conjecture for irreducible diagrams implies the validity of the full conjecture for all diagrams. Moreover, following the notion of irreducibility, we can provide the Hasse diagram of Young's lattice with an orientation. This produces a directed graph in which sources correspond to irreducible diagrams. As our main result, we give a combinatorial characterization of irreducible diagrams, as the integer points of a special polytope in R^{2d-4}.

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MS4

On the Isomorphism Classes and Automorphism Groups of a Family of Maximal Curves

Maximal function fields, or equivalently maximal curves, over finite fields are interesting both in their own right and because they give rise to algebraic geometry codes with good parameters. It is a big open problem to classify all maximal function fields. In this talk, we determine isomorphism classes among a certain family of maximal function fields. For an odd prime power q, we define $d := (q+1)/2$ and consider function fields of the form $F_i := \mathbb{F}_{q^2}(x, y)$, where

$$y^{q+1} = x^{2i}(x^2 + 1),$$

for $i \in \mathbb{Z}$ satisfying $\gcd(i(i+1), d) = 1$. These function fields have genus $q-1$ and are $\mathbb{F}_{q^2}$-maximal since they are known to be subfields of the Hermitian function field. In the case where d is a prime, the isomorphism classes and automorphism groups were determined by Giulietti et al. (*Finite Fields Appl.* **12** (2006), 539–564). We describe how to determine the isomorphism classes and automorphism groups also when d is not a prime. In doing so, we obtain more examples of non-isomorphic maximal function fields with the same genus and the same automorphism group.

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MS4

Division Algebras and Optimal Codes from Skew Polynomial Rings

In recent years, rank-metric codes have gained growing popularity due to their several applications. Additionally, rank-metric codes present very interesting mathematical structure. Indeed, these codes have been linked to a variety of algebraic and combinatorial objects. Among these codes, those that attain the largest size for a specified minimum distance are called Maximum Rank Distance (MRD) codes, and they have attracted particular interest in the context of Random Linear Network Coding. (Finite) semifields are nonassociative division algebras over a field. Existence of non-trivial examples was established by Dickson in 1906. They are related with combinatorial objects, such as projective planes, spreads, and flocks. Moreover, there is a well-known correspondence between semifields and MRD codes with specific parameters, through the multiplication maps they induce. In this talk, we introduce a new family of MRD codes and semifields by studying subsets of quotient spaces of skew polynomial rings. This family incorporates previously distinct constructions. We are able to determine the nuclei/idealizers of the resulting semifields/MRD codes, proving that this family contains new constructions of such objects.

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MS4

Some Interesting Combinatorial Structures and Related Codes

In this talk, we will present constructions of self-orthogonal codes from orbit matrices of Deza graphs and Deza digraphs of type II introduced in [D. Crnković, H. Kharaghani, S. Suda, A. Švob, New constructions of Deza digraphs, Int. J. Group Theory 13 (2024), 225-240.] These constructions can also be applied to adjacency matrices of the above mentioned (di)graphs. This talk is a joint work with D. Crnković.

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MS5

New Invariants of Linkoids and Measures of Entanglement of Open Curves in 3-space

Measuring the entanglement complexity of collections of open curves in 3-space has been an intractable, yet pressing mathematical problem, relevant to a plethora of physical systems. In this talk, we describe a novel definition of the Jones polynomial that generalises the classic Jones polynomial to collections of open curves in 3-space. More precisely, first we provide a novel definition of the Jones polynomial of linkoids (open link diagrams) and show that this is a well-defined single variable polynomial that is a topological invariant, which, for link-type linkoids, it coincides with that of the corresponding link. We will also talk about new invariants of linkoids which are introduced via a surjective map between linkoids and virtual knots. This leads to a new collection of strong invariants of linkoids that are independent of any given virtual closure. We will show that invariants of linkoids give rise to a collection of novel measures of entanglement of open curves in 3-space, which are continuous functions of the curve coordinates and tend to their corresponding classical invariants when the endpoints of the curves tend to coincide.

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MS5

Topological Feature Selection for Time Series Data

I will describe how tools from applied topology may be used to identify components of time series most responsible for observed cyclic dynamics. In this setting, I will show that derivatives of the persistent homology may be computed explicitly and describe a simple algorithm for gradient descent. As an example, we will consider neuronal data from the model organism C. elegans and identify subsets of neurons driving global brain dynamics in the spirit of dimensionality reduction. This is based on joint work with Peter Bubenik.

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MS5

Topological Stability in Extremely High Dimensions: How Can Persistence Distinguish Neu-

roimaging Pipelines?

Evaluating the performance of a dimension-reduction process (e.g., manifold learning) remains a challenge, especially in the regime of very high-dimensional data. When is persistent homology a sensitive tool for comparing dimensionality reductions? In particular, what topological information is persistence robustly sensitive to in a high-dimensional setting with noise? In neuroimaging, extensive post-processing of resting-state functional MRI (rfMRI) data is necessary for its application, especially when investigating neural mechanisms of mental illness. However, widespread variability in these post-processing pipelines has hindered both the reproducibility and accumulation of knowledge in this area of the field; the choice of dimension-reduction algorithm is one of the most variable and impactful ones made in an rfMRI post-processing pipeline. We implement persistence as a comparison tool between different reductions of neuroimaging data in the regime of D ¿¿ d ¿¿ n (where D and d are the embedding dimension of the original and reduced data, resp., and n the number of samples) to investigate persistence stability in a naturally high-dimension, low SNR setting.

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MS5

Circle Bundles: Parametrized Cyclic Processes and Lead-Lag Phenomena

We introduce a framework for using embedded principal circle bundles as models of parametrized cyclic processes in high-dimensional feature space. Previous work by Baryshnikov and Schlafly used the signed area of a multivariate cyclic time series as an indicator of lead-lag behavior across different components. We extend this perspective to circle bundles to describe parametrized lead-lag phenomena. This is joint work with Kelly Maggs and Daniel Tolosa.

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MS6

Toric Multivariate Gaussian Models from Symmetries in a Tree

Given a rooted tree T on n non-root leaves with colored and zeroed nodes, we construct a linear space L_T of $n \times n$ symmetric matrices with constraints determined by the combinatorics of the tree. When L_T represents the covariance matrices of a Gaussian model, it provides natural generalizations of Brownian motion tree (BMT) models in phylogenetics. When L_T represents a space of concentration matrices of a Gaussian model, it gives certain colored Gaussian graphical models, which we refer to as BMT derived models. We investigate conditions under which the reciprocal variety L_T^{-1} is toric. Relying on the birational isomorphism of the inverse matrix map, we show that if the BMT derived graph of T is vertex-regular and a block graph, under the derived Laplacian transformation, L_T^{-1} is the vanishing locus of a toric ideal. This ideal is given by the sum of the toric ideal of the Gaussian graphical model on the block graph, the toric ideal of the original BMT model, and binomial linear conditions coming from vertex-regularity. To this end, we provide monomial parametrizations for these toric models realized through paths among leaves in T.

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MS6

Maximum Likelihood Degree of Brownian Motion Star Tree Models

A Brownian motion tree (BMT) model is a Gaussian model whose associated set of covariance matrices is linearly constrained according to common ancestry in a phylogenetic tree. We study the complexity of inferring the maximum likelihood (ML) estimator for a BMT model by computing its ML-degree. We show that the ML-degree of the BMT model on a star tree with $n+1$ leaves is $2^{n+1} - 2n - 3$. We also prove that the ML-degree of a BMT model is independent of the choice of the root. The proofs rely on the toric geometry of concentration matrices in a BMT model.

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MS6

Equivalence of Graphical Models through Toric Geometry

In the study of (possibly cyclic) directed Gaussian graphical models, one aims to understand how the structure of a directed graph relates to the structure of an associated model. Many graphs yield the same model, a feature called distribution equivalence. In the acyclic case, equivalent graphs are related via a sequence of covered edge flips. We translate these edge flips into generators of a binomial ideal and apply the theory of binomial and toric ideals to study distribution equivalence of cyclic graphical models.

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MS6

Lattice Supported Distributions and Graphical Models

For the distributions of finitely many binary random variables, we study the interaction of restrictions of the supports with conditional independence constraints. We prove a generalization of the Hammersley-Clifford theorem for distributions whose support is a natural distributive lattice: that is, any distribution which has natural lattice support and satisfies the pairwise Markov statements of a graph must factor according to the graph. We also show a connection to the Hibi ideals of lattices.

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MS7

Totally Real Divisors on Algebraic Curves

The study of the convex cone of nonnegative polynomials on a real algebraic curve X (and dually of the moment cone) naturally leads to the investigation of the effective divisors on X whose support is entirely contained in the real locus of the curve. We call such divisors *totally real*. For instance, consider the following question: Does a nonnegative quadric exist on X, which intersects X in only real points? This can be rephrased as follows: Is there an effective totally real divisor in the divisor class of degree 2 hypersurfaces? Thanks to a result of Scheiderer, every divisor class of sufficiently high degree contains an effective totally real divisor. The smallest degree satisfying the previous condition is called the totally real divisor bound, and we denote it by $N(X)$. In this work, we study $N(X)$ and show that it cannot be bounded only using topological properties of the curve. We prove a lower bound for $N(X)$ depending on metric properties of the Abel-Jacobi map, and that $N(X)$ can be arbitrarily large for curves of fixed topological type, unless the real locus of the curve has many connected components.

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MS7

Tropicalization in Extremal Graph Theory

The number of homomorphisms from a graph H to a graph G, denoted by hom(H;G), is the number of maps from V(H) to V(G) that yield a graph homomorphism, i.e., that map every edge of H to an edge of G. Given a fixed collection of finite simple graphs $H_1, \ldots, H_s$, the graph profile is the set of all vectors $(hom(H_1; G), \ldots, hom(H_s; G))$ as G varies over all graphs. We will first discuss graph profiles, which are objects that essentially allow us to understand all polynomial inequalities in homomorphism numbers that are valid on all graphs. Profiles can be extremely complicated; for instance the full profile of any triple of connected graphs is not known. To simplify these objects, we introduce their tropicalization which we show is a closed convex cone that still captures interesting combinatorial information. We explicitly compute these tropicalizations for some sets of graphs, and use those results to answer some questions in extremal graph theory. Different parts are joint with Greg Blekherman, Maria Dascalu, Sasha Razborov, Mohit Singh, Rekha Thomas and Fan Wei.

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MS7

Higher Order Arithmetic-Geometric Inequalities

If $\{\alpha_k\} \subset \mathbb{R}^d$ consists of a simplex and a single interior point, and if you impose the condition that $p(x) = \sum c_k x^{\alpha_k}$ vanishes to the second order at $\mathbf{1} = (1, \ldots, 1)$, then the resulting polynomial is, up to a multiple, a version of the arithmetic-geometric inequality for the monomials $\{x^{\alpha_k}\}$. In this talk, we explore geometric conditions on $\{\alpha_k\}$ so that imposing a higher even-order vanishing at $\mathbf{1}$ leads to an inequality, and present a few preliminary results.

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MS7

Spurious Local Minima in Nonconvex Sum-of-Squares Optimization

We study spurious second-order stationary points and local minima in a nonconvex low-rank formulation of sum-of-squares optimization on a real variety X. We reformulate the problem of finding a spurious local minimum in terms of syzygies of the underlying linear series, and also bring in topological tools to study this problem. When the variety X is of minimal degree, there exist spurious second-order stationary points if and only if both the dimension and the codimension of the variety are greater than one, answering a question by Legat, Yuan, and Parrilo. Moreover, for surfaces of minimal degree, we provide sufficient conditions to exclude points from being spurious local minima. In particular, all second-order stationary points associated with infinite Gram matrices on the Veronese surface, corresponding to ternary quartics, lie on the boundary and can be written as a binary quartic, up to a linear change of coordinates, complementing work by Scheiderer on decompositions of ternary quartics as a sum of three squares. For general varieties of higher degree, we give examples and characterizations of spurious second-order stationary points in the interior, together with a restricted path algorithm that avoids such points with controlled step sizes, and numerical experiment results illustrating the empirical successes on plane cubic curves and Veronese varieties.

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MS8

Topological Behaviour of Regularity of Monomial Ideals and Their Powers

The minimal free resolution of a monomial ideal and many of the algebraic invariants associated to it can be studied using topological methods, where discrete homotopy theory has proven to be a powerful tool. The regularity of the ideal seems to be an exception: it is more difficult to control and more complicated to describe using such tools. This talk will explore instances where the regularity of a monomial ideal (and its powers) are topologically well-behaved.

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MS8

Minisymposium Introduction

Graded and multigraded structures are central in commutative algebra and algebraic geometry, with theoretical applications in fields such as homological and combinatorial algebra, and practical applications in polynomial system solving, tensor decomposition, computer vision, and others. This talk aims to provide an overview of the topic. In particular, we will show how the Castelnuovo-Mumford regularity arises in various problems, based on our own experience.

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MS8

Castelnuovo-Mumford Regularity: Insights and Applications to Binomial Edge Ideals

In this talk, we will discuss applications of Castelnuovo-Mumford regularity to the study of binomial edge ideals. Binomial edge ideals are a class of ideals, introduced by Herzog and others in the last 15 years, that associate a binomial ideal to a graph. The study of the algebraic properties of these ideals, in terms of the combinatorics of the underlying graph, has garnered significant attention. In this talk, we will review past results on the Castelnuovo-Mumford regularity of binomial edge ideals, present improved upper and lower bounds, introduce new families of graphs for which a combinatorial description of the regularity is known, and highlight potential directions for future research in this area.

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MS8

Regularity of Primes

In 1984, Eisenbud and Goto conjectured that the Castelnuovo-Mumford regularity of reduced and irreducible varieties was bounded above by degree or multiplicity of the variety. This conjecture attracted a lot of attention and several special cases were proved, but in 2017 Peeva and the author provided counterexamples that moreover showed that given any polynomial function $f(x)$, there was a variety X such that the regularity of X was larger that $f(deg(X))$. Later Caviglia, Chardin, Peeva, Varbaro and the author proved that there was a function $g(x))$ such that the regularity of X is at most $g(deg(X))$ for all varieties X. We present new counterexamples that show that such a function $g(x)$ must be at least doubly exponential.

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MS9

Geometrizing Synchronization Problems

In synchronization problems, elements of a group are inferred from measurements of their ratios. The derivation of the most popular synchronization methods are dependent largely on the algebraic properties of the underlying group and in particular the algebraic properties of its embeddings in matrix algebras. Here, we propose a geometric interpretation of synchronization problems, which takes into account both geometric and algebraic properties of the underlying group. We characterize the global geometry of the space of all possible observations and relate it to the group action on it, which allows us to derive a characterization of the solution of synchronization problems in terms of a tangent subbundle of the space of observations. We show how our characterization can be used to infer the group elements in practice.

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MS9

Drones, Global Positioning and Algebra

Can a drone "see" the walls of a room from hearing the echoes of a sound? Is this also possible for a ground-based vehicle? How can echoes be used for simultaneous localization and mapping (SLAM)? Such questions can be translated to commutative algebra and ultimately into problems about polynomial ideals. Their solution tends to require massive computer algebra computations. Time permitting, the talk will also delve into recent results on the global positioning problem, whose proofs require tools from algebraic geometry. All work presented is joint with Mireille Boutin.

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MS9

Phase Retrieval for Signals that are Sparse Under a Generic Orthonormal Basis

The crystallographic phase retrieval is the problem of recovering a sparse vector in R^N from its power spectrum. In this talk we prove that (generic) signals which are sparse with respect to a generic orthonormal basis can be recovered from their power spectrum provided the size of the support is $\sim N/2$. This was achieved by proving a more general result about phase retrieval for semi-algebraic sets. Namely that if X is any semi-algebraic set of dimension $M \sim N/2$ then the phase retrieval map is (generically) injective on a generic rotation of X.

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MS10

Positivity of Matrix Moments: (Un-)decidable Cases

Is it possible to algorithmically determine properties such as positivity or non-zeroness of linear recurrence sequences? This fundamental question, known as Skolem's problem, has intrigued researchers for decades. In this talk, we investigate the complexity of a closely related problem: the (generalized) moment membership problem for matrices. We establish that this problem is decidable for unitary matrices, while demonstrating undecidability for matrices over certain commutative and non-commutative polynomial rings. We establish undecidability through a reduction from the positivity problem for tensor networks, a well-studied framework in the simulation of quantum many-body systems. Specifically, we embed the problem of determining whether a given tensor network yields a positive tensor into the problem of deciding the positivity of a linear recurrence sequence with coefficients in the ring of polynomials.

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MS10

Probing Multipartite Entanglement Through Persistent Homology

We propose a study of multipartite entanglement through persistent homology, a tool used in topological data analysis, by interpreting the individual systems as vertices of a simplicial complex. To construct a persistence complex from a given multipartite quantum state, we use a generalization of the bipartite mutual information called the deformed total correlation as the functional defining k-simplices. Computing the persistence barcodes of this complex yields a visualization or topological fingerprint of the multipartite entanglement in the quantum state. The barcodes can also be used to compute a topological summary called the integrated Euler characteristic of a persistence complex. We show that in our case this integrated Euler characteristic is equal to the deformed interaction information, another multipartite version of mutual information. When choosing the linear entropy as the underlying entropy, this deformed interaction information coincides with the n-tangle, a well-known entanglement measure. The persistence barcodes thus provide more fine-grained information about the entanglement structure than its topological summary, the n-tangle, alone, which we illustrate with examples of pairs of states with identical n-tangle but different barcodes. We also comment on a possible generalization of our approach to arbitrary resource theories.

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MS10

A Bound on the Quantum Value of All Compiled Nonlocal Games

Nonlocal games are a foundational tool for understanding entanglement and constructing quantum protocols in settings with spatially separated quantum devices. Recently, Kalai et al. defined a cryptographic compilation procedure for nonlocal games. It translates a (two-player) nonlocal game into a single-player game, using cryptography to simulate the spatial separation between the players. This talk will be about the quantum value of such compiled nonlocal games. We will discuss techniques for bounding the quantum value and see when the value is preserved under the compilation procedure. This talk is based on joint work with Alexander Kulpe, Giulio Malavolta, Connor Paddock and Michael Walter.

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MS10

Four-Qubit Critical States

Many highly entangled or useful four-qubit states that appear in the literature are stationary points of entanglement measures coming from SLOCC invariants. We discuss Vinberg theory and how it can be used to set up equations whose solutions give an extended list of stationary points. Using homotopy continuation to solve, we recover all the critical states in the survey by Enriquez et al (2016).

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MS12

p-Riccati Equation and Applications to Factorisation of Linear Differential Operators in Positive

Characteristic.

In 1995, Marius van der Put published a classification of finite dimensionnal differential modules of positive characteristic and introduced an important nonlinear differential equation whose solutions (or lack thereof) in some precise algebraic function fields hold precious information on the isomorphism class of a given differential module. In this talk we call this equation the p-Riccati equation and focus on differential modules given by a linear differential operator. After explaining how the solutions of the p-Riccati equation can be used to find factorisation of this operator, we present methods to find solutions to the p-Riccati equation, or test their existence, relying on computations of Riemann-Roch spaces, on the divisor class group of a function field as well as some results from class field theory.

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MS12

Combinatorial Structure Behind Sinkhorn Limits

The Sinkhorn limit of a positive square matrix is obtained by scaling the rows so each row sum is 1, then scaling the columns so each column sum is 1, then scaling the rows again, then the columns again, and so on. It has been used for almost 90 years in applications ranging from predicting telephone traffic to machine learning. But until recently, nothing was known about the exact values of its entries. In 2020, Nathanson determined the Sinkhorn limit of a 2 x 2 matrix. Shortly after that, Ekhad and Zeilberger used Grbner bases to determine the Sinkhorn limit of a symmetric 3 x 3 matrix. Recently, Jason Wu and I determined the Sinkhorn limit of a general 3 x 3 matrix. The result suggests the form for n x n matrices. However, the values of the coefficients that arise are not obvious. We used 1.5 years of CPU time to interpolate formulas from numeric examples, leading to a conjecture that the coefficients reflect new combinatorial structure on sets of minor specifications.

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MS13

Galois/monodromy Groups of Parametrized Polynomial Systems and Why we Should Certify Them

Galois/monodromy groups offer us a first glimpse of structure hidden in systems of algebraic equations. Numerical homotopy continuation methods enable heuristic computations of these groups that are orders of magnitude beyond what can be obtained with general Galois-theoretic algorithms. How much can we trust these computations? I will motivate the problem of certified path-tracking with some examples (eg. from computer vision), and discuss ongoing work on this problem, including joint work with Kisun Lee.

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MS13

Real Numerical Algebraic Geometry

Numerical algebraic geometry provides a collection of methods for computing and analyzing the solution set over the complex numbers for a system of polynomial equations. This talk will summarize some recent advances for computing and analyzing the solution set over the real numbers using numerical algebraic geometric techniques. This includes computing smooth sample points and smoothly connected components. Examples will be used to illustrate the new techniques.

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MS13

Finding Tensor Decompositions with Sparse Optimization

For a given tensor $T \in \mathbb{C}^{n_1+1} \otimes \mathbb{C}^{n_2+1} \otimes \cdots \otimes \mathbb{C}^{n_d+1}$, its tensor rank $\mathbf{R}(T)$ is defined as the smallest number of decomposable tensors $T_1, ..., T_r$ required to express the tensor as $T = T_1 + \cdots + T_r$ (called a CP decomposition of T). It is meaningful to calculate the tensor rank in the sense of both "complexity" and "geometry". In particular, $\mathbf{R}(T) = r$ is equivalent to that T is on the r-th but not $(r-1)$-th secant space of the Segre variety $\mathrm{Seg}(\mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_d})$. In most cases, it is extremely hard to compute the explicit value of the tensor rank, one tries to improve its upper and lower bounds. In particular, one can improve upper bound by finding a CP decomposition with less number of decomposable tensors. In this talk, we suggest a numerical method for a given tensor to find CP decompositions with less number of decomposable tensors. By regarding this CP decomposition problem as a sparse optimization problem, our method is based on the Least Absolute Shrinkage and Selection Operator (LASSO). As an application, we design an experiment to find a new CP decomposition of the 4×4 determinant tensor $\det_4$, and we observe that $\mathbf{R}(\det_4) \leq 12$ as a result. If time permits, we also discuss a few more results.

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MS13

A Novel Approach to the Initial Value Problem, with a Complete Validated Algorithm

The Initial Value Problem (IVP) is concerned with finding solutions to a system of ordinary differential equations of the form

$$x' = f(x)$$

with given initial condition $x(0) \in B_0$ for some n-dimensional box $B_0 \subseteq \mathbb{R}^n$. Let $IVP_f(B_0, 1)$ denote this set of solutions. Here $f : \mathbb{R}^n \to \mathbb{R}^n$ and $x : [0, 1] \to \mathbb{R}^n$

where f and x are C^1-continuous. Consider the following **End-Enclosure Problem**:
INPUT: (f, B_0, ε) where $\varepsilon > 0$.
OUTPUT: The output is (A_0, B_1)
where $A_0 \subseteq B_0$ and B_1 are boxes in $\mathbb{R}^n$, such that for all $x \in IVP_f(A_0, 1)$, $x(1) \in B_1$. Moreover, B_1 has diameter at most ε. We give a complete validated algorithm for this problem. This is an important basic problem in many applications. Despite over 60 years of development in validate IVP algorithms, our algorithm appears to be the first complete algorithm. Our formulation allowing a user specified $\varepsilon > 0$ is novel. Our approach exploits the theory of logarithmic norms. Joint work with Bingwei Zhang.

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MS14

Disguised Toric Systems in the General Sense

Toric dynamical systems show simple dynamics: their unique positive equilibrium *attracts every positive solution*. Motivated by the fact that a number of non-toric mass-action systems behave similarly neatly, *disguised toric* systems were introduced a couple of years ago. By definition, the mass-action differential equation of such a system is *dynamically identical to that of a toric mass-action system*. In many instances, this concept allows us to conclude the global stability of reaction networks for a *set of parameter values significantly larger than their toric variety*. In this talk, we take another step forward. We say that a mass-action system is disguised toric in the *general sense* if there *exists a coordinate change* that makes it disguised toric. Even the simplest coordinate transformation, a linear diagonal change of variables, leads to a *powerful method*. We illustrate the strength of this new concept via several examples. Joint work with Gheorghe Craciun, Oskar Henriksson, Jiaxin Jin, and Diego Rojas La Luz.

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MS14

Inferring Genome-scale Metabolic Models Using Genomic and Growth Phenotype Data

Genome-scale metabolic models consist of a network representation of the entire metabolism of a cell. These models are difficult to construct with genomic data alone due to the presence of unidentified sequences in genomes and the possibility of unknown functions for known genes. Once constructed, however, they provide an estimate of cell growth on any given media. A Phenotype Microarray [Bochner et al. 2001] provides relatively high-throughput growth data for microbial strains on a set of different media. This data can be used to test the predictions of a genome-scale metabolic model. We describe a novel method for constructing genome-scale metabolic models using Phenotype Microarray data to constrain an optimization problem that maximizes some measure of agreement between the model and a genome sequence.

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MS14

The Persistence Number Approach for Determining Boundary Stability in Chemical Reaction Systems

We extend the next generation matrix method for computing the basic reproduction number in mathematical epidemiology to computing the persistence number for establishing the stability of boundary faces in biochemistry. In epidemiology, the basic reproduction number indicates whether a disease can invade a population, with values below one leading to disease-free stability and values above one signaling an outbreak. In biochemistry, the persistence number determines whether certain chemical species persist or go extinct, with values greater than one leading to persistence. The persistence method approach is typically significantly more computationally tractable than standard Jacobian or Routh-Hurwitz methods for establishing stability; consequently, the method could have significant applications for understanding metabolic function.

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MS15

Q-Matroids and Secret Sharing

The connection between secret sharing and matroid theory is well-established. In this work, we generalize secret sharing from finite sets to finite complemented modular lattices. Building on the concept of matroid ports introduced by Lehman (1964), we define (L,r)-polymatroid ports and analyze their properties. Lastly, we investigate secret sharing schemes induced by rank-metric codes and study their behavior within this extended framework.

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MS15

Geometry of Codes for Random Access in Dna Storage: Part I

In this talk, we will explore the random access problem in the context of DNA data storage. The focus will be on understanding how many DNA strands need to be read to reliably decode a specific piece of information requested by the user from a large pool of encoded strands. The setup involves information strands encoded into strands using an error-correcting code, with the strands being read uniformly at random during the sequencing process. The key question is: How many reads are needed, on average, to recover a particular information strand of interest? We will introduce new techniques to address this problem for arbitrary codes, capturing its combinatorial and geometric nature. We also introduce recovery balanced codes and show that for this special family of codes, the expected number of reads is always k, their dimension. This approach provides an "easy' way to determine the random access expectation for various families of codes, including MDS codes, Hamming codes, and simplex codes, and it gives insight into what structural properties we need in order to reduce the random access expectation. Finally, we will present results on modified systematic MDS and simplex codes, showing how a specific combination of encoded strands and replicated information strands can reduce the required number of reads for retrieving information.

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MS15

The Service Rate Region Polytope

In distributed data storage, data is stored with redundancy across multiple servers. Redundancy not only protects data from errors, but it also enhances its availability. A distributed storage storage system is defined by a full-rank matrix over a finite field, called the "generator matrix" of the system. The service rate region has recently emerged as an essential measure for the efficiency of a distributed storage system. It is a down-monotone polytope whose points represent the access requests that can be served by the underlying system. The service rate region polytope remembers information about the linear dependencies of the columns of the matrix that defines the storage strategy. This talk is about the properties of the service rate region polytope, with emphasis on its geometric parameters and on the link between a system and its generator matrix (most notably its dimensions and underlying field).

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MS15

Geometry of Codes for Random Access in Dna Storage: Part II

Effective and reliable data retrieval is crucial for the feasibility of DNA-based data storage. A key challenge in this context is achieving efficient random access, which significantly impacts both practicality and reliability. In this talk, we explore the Random Access Problem, which involves computing the expected number of samples required to recover an information strand from DNA storage. We introduce geometric objects that allow us to construct codes with improved performance in reducing the random access expectation. In particular, we present a novel construction for k=3 that outperforms previous constructions aiming to reduce the random access expectation. Furthermore, we confirm a conjecture by Bar-Lev, Sabary, Gabrys, and Yaakobi regarding rate 1/2 codes, extending its validity to any dimension.

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MS16

LLM Steganography

LLM steganography leverages large language models (LLMs) to encode hidden messages within seemingly innocuous text, ensuring covert communication. By carefully manipulating word choice, syntax, or punctuation, these methods embed information in a way that appears natural to human readers while remaining decodable by algorithms, offering applications in secure messaging and watermarking AI-generated content.

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MS16

Schur Products of ℓ-quasi-GRS Codes and Application to Cryptanalysis

An ℓ-quasi-GRS (generalized Reed–Solomon) code is an ℓ-dimensional subspace of $\mathbb{F}_q^n$ that intersects a GRS code in a codimension ℓ subcode. One explicit class of such codes is given by the twisted generalized Reed-Solomon (TGRS) codes. The class of TGRS codes yields families of MDS codes that are non-equivalent to GRS codes. Another significant structural difference from the GRS codes is the large dimension of their Schur squares. Exploiting this property, in 2018, Beelen, Bossert, Puchinger and Rosenkilde proposed a subfamily of Maximum Distance Separable (MDS) Twisted Reed–Solomon (TRS) codes over $\mathbb{F}_q$ with ℓ twists $q \approx n^{2^\ell}$ for McEliece encryption, claiming their resistance to both Sidelnikov–Shestakov attack and Schur products–based attacks. In this talk, I will present some structural properties of Schur squares of ℓ-quasi-GRS codes including the dimension of Schur squares,and probability of the Schur square being a GRS code. As a consequence, we will see how ℓ-quasi-GRS codes can be distinguished from random linear codes. Using the observation, a key recovery attack will be presented on the McEliece cryptosystem instantiated with TGRS codes with single twist (i.e. $\ell = 1$).

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MS16

Decoding from Fragments Using Discrepancy The-

ory

3D printing technology is lauded for democratizing additive manufacturing. However, the prevalence of commodity 3D-printers poses security threats such as the recent assassination of UnitedHealthcare CEO using an untraceable firearm (ghost gun) that can be easily 3D-printed. To ensure traceability and accountability, embedding unique identifiers within printed objects is essential in order to assist forensic investigation of illicit use. The problem of extracting these identifiers from a discarded object is then one of decoding under adversarially introduced noise. While most forms of adversarial noise (e.g., bit erasures or bit flips) have been thoroughly studied in the literature, 3D-printing introduce a new form of adversarial noise—breaking the object apart, and concealing some of the fragments. This talk will focus on combating this new type of noise using concepts from discrepancy theory. Specifically, we study a problem setting in which only one sufficiently large fragment of the object is available for decoding. Our codes operate asymptotically close to the best possible information rate, and involve a concept from discrepancy theory called Van der Corput sets in a novel way.

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MS16

Hypergraphs of Girth 5 and 6 Via Coding Theory

We discuss the relationship between coding theory, extremal graph theory, and combinatorial number theory. Our main results are to use coding theory results to give the best-known lower bound on a hypergraph Turán problem: how many edges may be in an r-uniform hypergraph of girth 5 or 6? and to use extremal graph theory result to improve the sphere-packing bound for linear q-ary codes of distance 6. The relationship between these two fields passes through additive combinatorics, and if there is time we will discuss some related problems in that area. This is joint work with Katie Haymaker, Craig Timmons, and Eric Schmutz.

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MS17

The Algebraic Geometry Viewpoint on the Ranks of Tensors

As tensors are higher-dimensional generalisations of matrices, one may expect that linear algebra would still play an important role in understanding these ranks, and this is indeed the case. Yet tools from algebraic geometry, including its simplest tools, provide extra insight for tensors - compared to linear algebra tools - in a way that they do not for matrices. After briefly recalling some of the most commonly used notions of rank, we will discuss some recent basic results on tensors, the proofs of which were enabled by algebraic geometry.

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MS17

Quasi-Linear Relation Between Partition and Analytic Rank

An important conjecture in additive combinatorics, number theory, and algebraic geometry posits that the partition rank and analytic rank of tensors are equal up to a constant, over any finite field. In this talk I will discuss a recent proof of this conjecture up to a logarithmic factor. Joint work with Daniel Zhu.

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MS17

Decomposing Tensors Via Rank-One Approximations

Matrices can be decomposed via rank-one approximations: the best rank-one approximation is a singular vector pair, and the singular value decomposition writes a matrix as a sum of singular vector pairs. The singular vector tuples of a tensor are the critical points of its best rank-one approximation problem. In this paper, we study tensors that can be decomposed via successive rank-one approximations: compute a singular vector tuple, subtract it off, compute a singular vector tuple of the new deflated tensor, and repeat. The number of terms in such a decomposition may exceed the tensor rank. Moreover, the decomposition may depend on the order in which terms are subtracted. We show that the decomposition is valid independent of order if and only if all singular vectors in the process are orthogonal in at least two factors. We study the variety of such tensors. We lower bound its dimension, showing that it is significantly larger than the variety of odeco tensors.

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MS17

Zariski Closure and Tensor Ranks

One of the difficulties in attacking questions regarding tensor ranks with algebraic methods is that the space of tensors of low rank is, for many notions of rank, not Zariski closed. In this talk, we define the geometric and local ranks of tensors, two notions of rank that obey this Zariski closure property, and discuss how their connections to other tensor ranks can be used to prove various structural results. Based on joint work with Guy Moshkovitz.

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MS18

Tropical Toric Maximum Likelihood Estimation

In algebraic statistics, the maximum likelihood (ML) estimation problem for log-linear statistical models is studied geometrically by intersecting a toric variety (representing the model) with an affine subspace (representing the data). Following [Agostini et al. 2023], we study the asymptotics of the ML problem by considering data in the field of Puiseux series and computing valuations of the resulting ML estimates. For log-linear models, this translates to an intersection problem in tropical geometry, which comes down to intersecting a classical linear subspace X_A with a tropical affine subspace $L_{A,u}$. The latter space is a polyhedral complex whose structure is determined by the matroid underlying the log-linear model. Our results are two-fold. First, we derive new results on the tropical affine subspace $L_{A,u}$. We identify explicit equations for some vertices and cones in this polyhedral complex in terms of the model (A) and data (u). These results and computations are essentially matroid-theoretic and of independent interest. Second, we apply these results to the tropical intersection problem $L_{A,u} \cap_{\text{st}} X_A$. We find that the combinatorics of this intersection are governed by subdivisions of a certain point configuration associated to the model. In particular, this leads to a complete and explicit solution of the tropical toric maximum likelihood problem in certain scenarios, for instance when the data has many independent zeros.

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MS18

ML Degree Monotonicity of Toric Models

In this talk we study toric models in terms of their ML degrees, an algebraic complexity measure for the well-known maximum likelihood estimation. The association of such models with polytopes motivates the following ML degree monotonicity, which was first conjectured by Coons and Sullivant. Given a polytope, we show that the ML degree of its associated model is an upper bound on the ML degrees of the models defined by all its faces. To this end, we first study the maximum likelihood estimation problem in the presence of data zeros. This is joint work with Maximilian Wiesmann.

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MS18

Linear Causal Disentanglement via Higher-order Cumulants

Linear causal disentanglement is a recent method in causal representation learning to describe a collection of observed variables via latent variables with causal dependencies between them. We study its identifiability, assuming access to data under multiple contexts, each given by an intervention on a latent variable. We show that one perfect intervention on each latent variable is sufficient and in the worst case necessary to recover parameters under perfect interventions, generalizing previous work to allow more latent than observed variables. We give a constructive proof that computes parameters via a coupled tensor decomposition. For soft interventions, we find the latent graphs consistent with observed data, via the study of a system of polynomial equations. Based on https://arxiv.org/abs/2407.04605, joint work with Paula Leyes Carreno and Chiara Meroni.

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MS18

Toric Amplitudes and Likelihood Estimation

Toric amplitudes measure dual volumes of convex polytopes. In this talk, I will express the toric amplitude as a sum over the fibres of the algebraic toric moment map. In particular, this provides a trace test for the problem of computing all complex critical points of the log-likelihood function of a discrete log-linear model.

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MS19

Pythagoras Numbers for Ternary Forms

We study the Pythagoras numbers $py(3, 2d)$ of real ternary forms, defined for each degree $2d$ as the minimal number r such that every degree $2d$ ternary form which is a sum of squares can be written as the sum of at most r squares of degree d forms. Scheiderer showed that $d+1 \leq py(3, 2d) \leq d+2$. We show that $py(3, 2d) = d+1$ for $2d = 8, 10, 12$. Our approach is to connect the problem with optimality conditions for a Euclidean distance problem. When paired with Diesel's characterization of height 3 Gorenstein algebras, this allows us to control the syzygies of the forms involved in the decomposition into sums of squares.

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MS19

Stubborn Polynomials

A real polyonomial in several variables is stubborn if it is nonnegative but none of its odd powers is a sum of squares. Stubborn polynomials were introduced, and first studied systematically, in a recent paper by Blekherman, Kozhasov and Reznick. Taking odd powers sometimes allows for more uniform and more geometric statements. For example, there are (many) strictly positive polynomials that are not sums of squares, but no such polynomial is stubborn. In this talk, we will discuss new results on the construction of stubborn polynomials and on stubborn ternary sextic forms.

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MS19

Approximation of starshaped sets using polynomials

We introduce polystar bodies, that is compact starshaped sets whose gauge or radial functions are expressible by polynomials. We prove that polystar bodies are uniformly dense in starshaped sets and obtain quantitative approximation guarantees. We develop tools for the effective computation of polystar approximations and illustrate them via several computational examples.

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MS21

Projective Plane Subdivision Method for Initial Orbit Determination

The Initial Orbit Determination (IOD) is the classical problem of estimating the orbit of a body in the space without any presumed information about the orbit. Given five lines in the space representing the lines of sight (LOS) from the observers to the satellite, one needs to find a conic curve with a given focal point meeting the lines. This is the so-called angle-only IOD problem: the distance between the observer and the satellite is unknown. Five is the minimal number of lines necessary to have a finite number of solutions in a non-special case.A typical application of this method would be to estimate the Keplerian orbit of an observed asteroid orbiting the Sun, a satellite of the Earth, or any negligibly light celestial body orbiting a heavy body.Given a vector normal to the plane containing the orbit, in addition to five LOS, one can easily recover the planar conic. We construct a fast method to search for the normal direction as a point on the real projective plane. The ingredients of our algorithm include a subdivision routine that produces an adaptive triangulation of the sphere which up to the equivalence of the antipodal points represents the projective plane, and fast evaluation routines for the function that, given the normal vector, computes two quantities that vanish if the focal point is at the origin as well as the Jacobian of this function. The subdivision procedure is based on oracles that refine the triangulation while marking some of the triangles as accepted containing a solution or rejected containing no solution. In practice, these certified oracles may be used for post-processing the results obtained by reliable yet approximate oracles that use various relaxations. (This is work in progress joint with Christian, Leykin, and Manchini.)

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MS21

Generic Orbit Recovery from Invariants of Very Low Degree

Motivated by the multi-reference alignment (MRA) problem and questions in equivariant neural networks we study the problem of recovering the generic orbit in a representation of a compact group from invariant polynomials of degree at most 3. We prove that in many cases of interest these low degree invariants are sufficient to recover the orbit of a generic vector. In several cases of interest, we explore efficient algorithms for orbit recovery.

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MS21

Uniqueness Results for the Method-of-moments in Cryo-EM

This talk considers provable methods for cryo-electron microscopy, which is an increasingly popular imaging technique for reconstructing 3-D biological macromolecules from a collection of noisy and randomly oriented projection images, with applications in e.g., drug design. The talk will present uniqueness guarantees for recovering these structures from the second moment of the projection images, as well as associated numerical algorithms. Mathematically, the results boil down to ensuring unique solutions to highly structured non-linear equations.

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MS21

Bilipschitz Invariants

Motivated by problems in data science, we study the following questions: (1) Given a Hilbert space V and a group G of linear isometries, does there exist a bilipschitz embedding of the quotient metric space V/G into a Hilbert space? (2) What are necessary and sufficient conditions for such embeddings? (3) Which embeddings minimally distort the metric? We answer these questions in a variety of settings, and we conclude with several open problems.

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MS22

Entanglement Detection Based on Projective Tensor Norms

In my talk I will present the class of entanglement criteria based on testers, defined as local contractions from the Schatten class S_1 to the Euclidean space l_2. Our criterion detects the state as entangled when, the tester applied on an input state and then compute the projective tensor norm of the output, will give a result larger than 1. We analyze the performance of this type of criteria on bipartite and multipartite systems, for general pure and mixed quantum states, as well as on some important classes of symmetric quantum states. Issues related to the computation of the projective tensor norm will be presented. This new approach of entanglement criteria is of interest as it captures in a unique framework well-known criteria such as realignment and SIC POVM . Moreover, it allows to solve in a positive way conjectures about the power of entanglement criteria, by deriving systematic relations between the performance of these two criteria. This is a joint work with Cecilia Lancien and Ion Nechita (https://arxiv.org/abs/2010.06365).

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MS22

A Quantum Algorithm for MaxLin2: Geometric Extremum Problems in Quantum Circuits

`MaxLin2` is a family of optimization problems over linear equations with solutions over the field of characteristic 2. In general it is NP hard to provide a solution set satisfying all equations, or a set satisfying an optimal number of equations. An example of this is computing the degree of contextuality of an N-qubit quantum system, which is an upper bound of satisfied equations dependent upon measurement outcomes. It is a classical bound analogous to ones given by nonlocal hidden variable models in Bell inequalities. In this work we give a quantum algorithm based on Grover's search algorithm, computing the degree of contextuality with quadratic speedup versus classical computation. Testing on contemporary quantum backends with the IBM quantum experience gives limited results due to hardware restrictions.

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MS22

A Unified Approach to Quantum De Finetti Theorems

The sum-of-squares hierarchy of semidefinite programs has become a common tool for algorithm design in theoretical computer science, including problems in quantum information. This work studied a connection between a Hermitian version of the SoS hierarchy, related to the quantum de Finetti theorem, and geometric quantization of phase spaces in classical mechanics (such as complex projective space $\mathbb{C}P^d$, the set of all pure states in a $(d+1)$-dimensional Hilbert space). We show that previously known HSoS rounding algorithms can be recast as quantizing an objective function to obtain a finite-dimensional matrix, finding the quantum state corresponding to its top eigenvector, and then mapping it back to a classical distribution by using a version of the Husimi quasiprobability distribution. Dually, we can recover several known quantum de Finetti theorems by doing the same steps in the reverse order: a quantum state is first approximated by its Husimi distribution, and then quantized to obtain a separable state. The goals of this talk will be to try to give an exposition of this perspective, and to explain how it can be used to give new error bounds for the HSoS hierarchy over projective varieties in terms of some differential-geometric quantities.

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MS22

On Quantum Birkhoff Polytope

The quantum Birkhoff polytope $\Omega_{n,d}$ is the set of $n \times n$ block matrices $A = [A_{ij}]$ such that each matrix A_{ij} is a $d{\times}d$ positive semidefinite Hermitian matrix ,and each row and column sum is the identity matrix. The quantum Birkhoff polytope corresponds to sets of compatible quantum measurements. A natural generalization of Birkhoff theorem for doubly stochastic matrices holds a convex subset of $\Omega_{n,d}$, which is called a semiclassical Birkhoff polytope. For $n \geq 3$ and $d \geq 2$ it is a strict subset of $\Omega_{n,d}$. The complete characterization of the extreme points of $\Omega_{n,d}$ seems to be a difficult problem even for the case $n = 3, d = 2$. We will discuss some of our results on the extreme points.

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MS24

Linked Partition Ideals and Computer Algebra

The framework of linked partition ideals, which serves as an important tool for integer partition identities, was introduced by George Andrews in the 1970s. One main objective of this framework concerns the construction of Andrews-Gordon type generating functions for partition sets under certain gap conditions. However, while utilizing linked partition ideals, we eventually encounter the problem of solving a system of q-difference equations, making this framework ineffective in the traditional pencil-and-paper mode. In this talk, I will discuss how a computer algebraic procedure works for such q-difference systems, thereby enhancing the power of Andrews linked partition ideals in the modern study of partition identities.

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MS24

Asymptotics of Multivariate Algebraic Generating Functions

The field of analytic combinatorics in several variables (ACSV) develops techniques for studying multivariate generating functions, which encode combinatorial structures with multiple parameters tracked. In this presentation, we describe a collection of methods for computing asymptotics of multivariate algebraic functions, including techniques for embedding the generating function into a higher-dimensional rational function, explicit contour deformations and implicit integration on algebraic varieties. Examples highlighting these techniques on a variety of applications will be shown, as well as implementations of each method in SageMath. This is joint work with Torin Greenwood, Stephen Melczer and Mark Wilson.

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MS25

Identifiability of Tensor Decompositions and Polynomial Neural Networks

Several recent works are focused on studying polynomial neural network models with geometric methods. The key object is the so-called neurovariety (the Zariski closure of the set of polynomial neural networks of fixed format). In the case of single hidden layer, the neurovariety corresponds to the secant variety of the Segre-Veronese variety, and this connection is very helpful for studying various properties such as dimension of the neurovariety. In this work, we look at the identifiability of neurovarieties, which is linked to uniqueness of underlying tensor decompositions. We provide new identifiability results and show that they can be used to provide simpler proofs for some recent results on neurovarieties.

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MS25

Numerical Instability of Algebraic Rootfinders

We demonstrate the numerical instability of several common algebraic methods for global multivariate polynomial rootfinding by considering the conditioning of subproblems that are constructed at intermediate steps. In particular, we give families of polynomial systems for which the most popular global methods must solve a subproblem with a condition number greater than that of the original problem by a factor that grows exponentially with the number of variables.

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MS25

Optimization Landscape of Neural Networks with Algebraic Activations

Recent advances in deep learning have driven remarkable performance gains across domains, but often at the cost of interpretability and computational efficiency. We consider the teacher-student setting of learning shallow neural networks with algebraic activation functions. Leveragin numerical algebraic geometry and other math techniques, we study the optimization landscape associated with the empirical and the population squared risk of the problem, and prove a sufficient condition for the all stationary points to be global optimal with high probability. This implies gradient descent, satisfying given conditions, approximately minimizes the empirical risk and recovers the planted weights in polynomial time.

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MS25

On Computing a Block-Term Decomposition of a Tensor

In this talk, we focus on the decomposition of symmetric tensors. I will discuss a numerical algorithm we proposed for calculating a block-term tensor decomposition, that encompasses the two most common tensor decompositions: the CP decomposition and the Tucker decomposition. I will also show applications where our algorithm performed well when compared with the state-of-the-art.

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MS26

Chemical Reaction Networks: Systematic Design, Limit Cycles and Spatio-temporal Modelling

I will discuss mathematical methods for describing biochemical reaction networks, with applications to modelling of intracellular processes. Several types of mathematical models of chemical reaction systems will be considered, including (i) deterministic models which are written in terms of reaction rate equations (i.e. ordinary differential equations (ODEs) for concentrations of chemical species involved); (ii) stochastic models of reaction networks, given in terms of the Gillespie stochastic simulation algorithm, which provides more detailed information about the simulated system than ODEs; and (iii) spatio-temporal models described by the reaction-diffusion master equation and Brownian dynamics simulations. I will discuss methods for systematic design of relatively simple reaction systems with prescribed dynamical behaviour, including reaction systems with multiple oscillating solutions (limit cycles). I will also present methods for efficient spatio-temporal modelling of intracellular processes.

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MS26

Ubiquitous Asymptotic Robustness in Biochemical Systems

This study explored the implementation of artificial neural networks (NNs) using chemical reaction networks (CRNs). We used the algebraic properties of smooth activation functions to design CRNs where the steady-state concentrations of specific species directly represent the derivative of the activation function. This approach enabled the construction of a single-pot CRN that seamlessly executes both feed-forward and training processes for the NN. Notably, the inherent smoothness of the activation functions in our CRN implementation enhanced computational accuracy by mitigating the impact of noise in reaction rates and species concentrations.

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MS26

Bifunctional Enzyme Action as a Source of Robustness in Biochemical Reaction Networks: A Novel Hypergraph Approach

Substrate modification networks are ubiquitous in living, biochemical systems. A higher-level hypergraph model, called "skeleton" captures only the information about which substrates are modified in the presence of modification-specific enzymes. Many different detailed models (reaction networks) can be associated to the same skeleton. Surprisingly, properties such as existence of positive steady states and concentration robustness depend only on the skeleton and not on the detailed model. We give rigorous hypergraph-based conditions for a variety of robustness properties, including "absolute concentration robustness" in biochemical networks. We prove that bifunctional enzyme action plays an essential role in generating robustness. Additionally, we introduce the notion of "current" on a directed hypergraph which connects remote network components and is the second key notion in combination with bifunctionality that is required for concentration robustness, as well as for existence of positive steady states.

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MS26

Impact of Enzyme Bifunctionality on the Number of Positive Steady States in Substrate Modification Networks

Substrate modification networks such as phosphorylation cycles can exhibit a remarkable suite of steady state and dynamical properties such as multistationarity, oscillations, ultrasensitivity and absolute concentration robustness. Substrate modification networks may contain a bifunctional enzyme, which is an enzyme that has different roles when isolated versus when bound as a substrate-enzyme complex. Previous studies have shown that the presence of a bifunctional enzyme can produce robustness in the steady state concentration of certain substrate. In this talk, we will show that beside facilitating robustness, enzyme bifunctionality also inhibits the number of positive steady states. In particular, our results indicate that phosphorylation cycles with a bifunctional enzyme admit fewer positive steady states than similar ones with non-bifunctional enzymes.

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MS27

A Geometric Invariant of Linear Rank-Metric Codes

In the Hamming metric setting, the dimension of the Schur square of a code distinguishes algebraically structured linear codes, such as ReedSolomon codes, from random ones [Mirandola, Zmor, Critical Pairs for the Product Singleton Bound, IEEE Transactions on Information Theory, 61(9) (2015)]. To extend this result to the rank-metric scenario, simply considering the dimension sequence of the Schur powers of a code is not sufficient. In this talk, we propose a geometric approach to address this challenge. As introduced in [Alfarano, Borello, Neri, Ravagnani. Linear cutting blocking sets and minimal codes in the rank metric. Journal of Combinatorial Theory, Series A, 192, 105658 (2022)], a rank-metric code can be linked to an *associated Hamming-metric code*. This connection relies on a geometric interpretation of rank-metric codes via the concept of *linear sets*, introduced in [Lunardon. Normal spreads. Geometriae Dedicata, 75, 245–261 (1999)]. From this viewpoint, we extend to the rank-metric context the definition of the dimension sequence of the Schur powers of a code, considering instead the dimension sequence of the associated Hamming-metric code. For linear codes defined over

$\mathbb{F}_{q^m}$, our key observation in [Astore, Borello, Calderini, Salizzoni. A geometric invariant of linear rank-metric codes. arXiv:2411.19087 (2024)] is that the $(q+1)$-th term of the dimension sequence marks the onset of distinguishability for Gabidulin codes.

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MS27

Rank-Metric Lattices and Finite Geometry

Rank-Metric Lattices (RML in short) were introduced in 2023 as the q-analogue of Higher-Weight Dowling Lattices. They are special families of geometric lattices whose elements are the $\mathbb{F}_{q^m}$-linear subspaces of $\mathbb{F}_{q^m}^n$ having a basis of vectors with rank weight bounded from above, ordered by inclusion. In this talk, we investigate the properties of RMLs from a finite geometry perspective. We compute the Whitney numbers of the first kind for some of these lattices, providing a recursive formula. In the second part of the talk, we present asymptotic results on the density of some classes of cardinality-optimal rank-metric codes.

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MS27

Decoding Methods for Tensor Codes

Tensor codes are a generalisation of matrix codes. Such codes are defined as subspaces of order-r tensors for which the ambient space is endowed with the tensor rank as a metric. A class of these codes was introduced by Roth [R.M.Roth, Tensor codes for the rank metric, IEEE Transactions on Information Theory] who outlined a decoding algorithm for low tensor-rank errors for particular cases. They may be viewed as a generalisation of the well-known Delsarte-Gabidulin-Roth maximum rank distance codes. We study a class of codes derived from them. We investigate the properties of these codes and outline decoding techniques that leverage their tensor structure, embedding them in different metrics. We first consider a fibre-wise decoding approach, as each fibre of a codeword corresponds to a Gabidulin codeword, we then give a generalisation of Loidreau's decoding method [P. Loidreau, A WelchBerlekamp Like Algorithm for Decod- ing Gabidulin Codes. In: Ytrehus, . (eds) Coding and Cryptography. WCC 2005 Lecture Notes in Computer Science] using the dimension of the slice spaces and fibre spaces, thereby incorporating tensor-specific properties. We show that the number of correctable errors by the fibre-wise decoder can be significantly increased with the same asymptotic complexity. Furthermore, the extension of Loidreau's algorithm can correct errors that are correctable along at least one direction by the previous decoder, though at a higher complexity cost.

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MS27

Schubert Subspace Codes

Schubert subspace codes are constant dimension subspace codes such that all codewords are contained in a given Schubert variety. These codes have a natural geometric description as objects that we name intersecting sets with respect to a fixed subspace. In this talk, we provide a geometric construction of maximum-size Schubert subspace codes with the largest possible value for the minimum subspace distance. Finally, we generalize the problem to different values of the minimum distance.

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MS28

On the Tensor Rank Defect of Rank-Metric Codes

Connections between codes and tensors have been recognized since the 1970s, with algebraic coding theory playing a role in estimating bilinear complexity and rank metric codes naturally described using 3-tensors. More recently, new connections between codes and tensors have been observed. Expressing a generator tensor as a minimal sum of rank-1 tensors gives a compact representation of a code and hence tensor rank is a highly relevant parameter of matrix codes. In this talk, we will explore the tensor rank of rank metric codes. To quantify how far codes deviate from maximum tensor rank, we will introduce the tensor rank defect as a novel invariant. We will present explicit families of rank metric codes derived from algebraic geometry (AG) codes, highlighting their tensor rank properties and exploring potential applications.

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MS28

Linear Sets and MRD Codes Arising from Linearized Quadrinomials

A subset of a finite projective space $PG(r-1, q^n) = PG(V, q^n)$ is called a linear set of rank u if its points are defined by non-zero vectors of a u-dimensional F_q-subspace of V. Any linear set of rank $n(r-1)$ of $PG(r-1, q^n)$ has a canonical form: the coordinates of its points can be written by using $r-1$ linearized polynomials. In this talk, we deal with the case of the projective line $PG(1, q^n)$ and show sufficient conditions for being scattered of a family of linear sets defined by linearized quadrinomials. Finally, we highlight their relations with maximum rank distance codes.

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MS28

Full Weight Spectrum Codes in the Rank Metric

The weight spectrum of a code is the set of all nonzero weights of its codewords. In the Hamming metric, the maximum size of the weight spectrum of a code with dimension k in $\mathbb{F}_q^n$ is noted $L(n, k, q)$. Codes that reach this bound are called full weight spectrum codes In this talk we investigate this quantity in the rank metric. More precisely, we introduce $L(m, n, k, q)$, the maximum number of nonzero weights a rank-metric code of dimension k in $\mathbb{F}_{q^m}^n$ can have. For all admissible values of m, n, k, q, we determine the exact value of $L(m, n, k, q)$ using geometric and combinatorial methods. Furthermore, we provide explicit constructions of rank-metric codes with $L(m, n, k, q)$ nonzero weights in all cases, i.e. codes reaching the bound.

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MS29

Real Subrank of Order-Three Tensors

The subrank of a bilinear map (tensor of order 3) is the maximal number of linearly independent scalar multiplications that can be linearly embedded into the bilinear map. We show that the tensor associated to componentwise complex multiplication in $\mathbb{C}^n$ has real subrank $n$informally, no more than n real scalar multiplications can be carried out using a device that does n complex scalar multiplications. The subrank can increase under field extensions. We present a lower bound of the subrank of a real tensor as a function of its complex subrank. In the space of real $n \times m \times l$ tensors, r is a typical subrank if there exists a nonempty open set (in the Euclidean topology) of tensors with subrank r. In this talk, we will determine the typical subranks of some tensor spaces with small n, m, l. This is joint work with Jan Draisma and Sarah Eggleston.

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MS29

Linear Preservers of Secant Varieties and Other Varieties of Tensors

We study the problem of characterizing linear preserver subgroups of algebraic varieties, with a particular emphasis on secant varieties and other varieties of tensors. We introduce a number of techniques built on different geometric properties of the varieties of interest. Our main result is a simple characterization of the linear preservers of secant varieties of Segre varieties in many cases, including $\sigma_r((\mathbb{P}^{n-1})^{\times k})$ for all $r \leq n\lfloor k/2 \rfloor$. We also characterize the linear preservers of several other sets of tensors, including subspace varieties, the variety of slice rank one tensors, symmetric tensors of bounded Waring rank, the variety of biseparable tensors, and hyperdeterminantal surfaces. Computational techniques and applications in quantum information theory are discussed. We provide geometric proofs for several previously known results on linear preservers.

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MS29

Algebraic Constraints for Linear Acyclic Causal Models

In this work we study the space of second and third order moment tensors of random vectors which satisfy a linear non-Gaussian acyclic model (LinGAM). In such a model each entry of the random vector corresponds to a vertex of a directed acyclic graph and can be expressed as a linear combination of its parents and random noise. We show that a random vector X arises from a LinGAM with a given graph G if and only if certain easy-to-construct moment matrices arising from the graph G drop rank. There is one moment matrix for each vertex and it can be constructed based on the vertexs parents and non-descendants. This model characterization extends previous results proven for polytrees as well as the well-known trek-separation characterization for Gaussian models.

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MS30

Characterizing Single-Cell Transcriptomic Spatial Patterns with Topological Data Analysis

To gain their unique biological function, plant cells regulate protein biosynthesis through gene activation and repression. Additionally, subcellular localization of mRNAs is a complementary regulatory mechanism in fungi and animal cells. However, studies comprehensively reporting the impact of mRNA localization in plant cells are lacking. Here, through the use of single-cell sequencing and molecular cartography technologies, we first produce detailed maps of the spatial location of individual transcripts for different genes, cell types, and organs of the soybean root and nodule. We then use persistent homology to characterize the distribution of these transcripts for each cell. Comparing these topological shape signatures reveals a new perspective on the role of the nuclear and cytoplasmic localization of transcripts as a central mechanism to control protein translation and the biology of plant cells. Our analyses reveal distinct patterns and spatial distributions of plant transcripts between the nucleus and cytoplasm, varying both between and within genes, as well as across different cell types. We believe this differential distribution is an additional, less understood, regulatory mechanism controlling cell identity and cell state.

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MS30

Topological Data Analysis of Molecular Dynamics Simulations of the Nist Monoclonal Antibody

A wide range of medical treatments depend on artificially produced monoclonal antibodies. Similar amino acid sequences can generate biomolecules which adopt different shapes in 3-dimensional space. Molecular dynamics simulations are a valuable tool for revealing potential arrangements of the atoms in these proteins. Topological data analysis detects and quantifies structural features which are not easily measured by classical data analysis techniques. We will discuss our results from using topological data analysis to explore molecular dynamics simulations of monoclonal antibodies, with a focus on the NIST monoclonal antibody (NISTmAb) reference material. We describe matrix summaries which can be used for visualization as well as subsequent classification, clustering, or machine learning tasks.

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MS30

Applying Persistent Homology to Fluorescent Microscopy Data

Microscopy images are rich with complex information that poses significant challenges for analysis and interpretation. Advances in machine learning, particularly convolutional neural networks (CNNs), have significantly improved image segmentation and classification tasks. However, understanding how these pipelines extract and interpret features from training data remains a critical challenge. To address this, we introduce a novel set of image analysis features derived from topological data analysis (TDA), specifically persistent homology and persistence landscapes. TDA provides a mathematical framework to quantify the shape of data and has shown promise in machine learning applications. Building on these concepts, we present TDAExplore, an innovative pipeline for image classification and segmentation. The weakly supervised segmentation model identifies regions whose shape features most strongly characterize image classes, requiring only image labels for training. Across multiple datasets, TDAExplore demonstrates high classification accuracy, robustness to hyperparameter changes, and efficient computational performance. Additionally, the method excels in recognizing diverse subcellular structures, offering biologically meaningful insights from complex image data.

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MS30

Persistent Homology and Zebrafish Neural Dynamics

Using calcium imaging to observe neural dynamics in vivo, neuroscientists witnessed the spontaneous collective activity of assemblies of neurons in the visual system of zebrafish larvae in the absence of visual stimuli. Physically, these assemblies are each localized and together cover the optic tectum along its retinotopic axis. Functionally, less is known - like how these subnetworks of neurons generate and maintain spontaneous dynamics. To better understand the spontaneous dynamics of neural assemblies in zebrafish larvae optic tectum, we recorded calcium imaging of 2000 neurons for 1 hour at 15 Hz for 12 fish, and then identified glutamatergic, GABAergic, and cholinergic neurons using immunostaining. We applied a topological data analysis (TDA) to the correlations of assembly neurons during and outside of spontaneous activity. Using Betti curves to summarize the persistent homology, we identified a plausible low-rank structure to the correlations agnostic to spectral analyses. Incorporating TDA into a sliding window analysis allowed us to cluster spontaneous activations into distinct dynamic profiles, ultimately leading us to hypothesize about the role of cholinergic neurons in recruiting and maintaining assembly activity, and more broadly, visual attention.

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MS31

Bayesian Additive Regression Trees for Network-Linked and Structured Data

We consider regression with network-linked data in which (1) covariate-response pairs are observed at the vertices of a given network but (2) the regression relationship might be different vertex-to-vertex. We describe how to use the popular Bayesian Additive Regression Trees (BART) model for this problem in a way that does not require pre-specifying the functional form of the regression function or how the regression function varies across the network. Key to our proposal are several stochastic processes that randomly partition a network into two, possibly connected, components. We then extend the ideas to more complex structured inputs including images.

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MS31

Efficient and Stable Multidimensional Kolmogorov-Smirnov Distance

We revisit extending the Kolmogorov-Smirnov distance between probability distributions to the multi-dimensional setting and make new arguments about the proper way to approach this generalization. Our proposed formulation maximizes the difference over orthogonal dominating rectangular ranges (d-sided rectangles in R^d) and is an integral probability metric. We also prove that the distance between a distribution and a sample from the distribution converges to 0 as the sample size grows and bound this rate. Moreover, we show that one can, up to this same approximation error, compute the distance efficiently in 4 or fewer dimensions; specifically, the runtime is near-linear in the size of the sample needed for that error. With this, we derive a d-precision two-sample hypothesis test using this distance.

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MS31

Model Selection in Singular Models with Normalizing Flows

Normalizing flows (NF) are a powerful approach to generative modeling in statistics and machine learning. In the context of Bayesian inference, normalizing flows form a highly flexible approach to variational inference. The use of this variational family in the context of singular models remains relatively unexplored. We investigate the use of this variational family to perform model selection for singular models satisfying mild mathematical conditions.

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MS31

Real Birational Implicitization of Statistical Models

Statistical models are often parameterized according to a map from a parameter space, defined by a simple set of interpretable semialgebraic constraints, into the model space. Many of these maps turn out to be birational, with the denominators constrained to be positive on the relevant sets. In this talk, we present a simple method for deriving an implicit description of these models. Statistically, this yields a set of model-defining constraints for globally rationally identifiable models that may be used in tests for model membership, representation learning and the study of model equivalence. The implicit equations recover well-known Markov properties for classical graphical models and provide simple explanations for other well-studied constraints, such as the Verma constraint. They also provide Markov properties for submodels of well-studied models in statistical learning, yielding a mechanism to encode additional modeling assumptions that are easily framed in the parameter space. Of specific relevance to algebraic statistics, we will see that our implicit equations generate the vanishing ideal of the model up to a saturation, under a mild assumption. This generalizes previous results of Geiger, Meek and Sturmfels, Duarte and Grgen and Sullivant. The talk is based on joint work with Tobias Boege.

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MS32

Ranks of Invariant Kernels

Symmetry arises often when learning from high dimensional data. For example, data sets consisting of point clouds, graphs, and unordered sets appear routinely in contemporary applications, and exhibit rich underlying symmetries. Understanding the benefits of symmetry on the statistical and numerical efficiency of learning algorithms is an active area of research. We show that symmetry has a pronounced impact on the rank of kernel matrices by computing the rank of polynomial kernels that are invariant under various groups acting independently on its two arguments. In concrete circumstances, including the three aforementioned examples, symmetry dramatically decreases the rank making it independent of the data dimension. This is joint work with Mateo Daz, Dmitriy Drusvyatskiy, and Rekha Thomas.

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MS32

A Combinatorial Approach to Ramanas Exact Dual for Semidefinite Programming

Ramana's exact dual for Semidefinite Programming (SDP) has no duality gap with the primal and always attains its optimal value. Ramana's dual yields a number of fundamental complexity results in SDP, which to date are still the best known; for example, it proves that SDP feasibility in the Turing model is not NP-complete, unless NP = co-NP. We first connect Ramanas dual to a seemingly very different way of inducing strong duality: reformulating the SDP into a rank revealing form using elementary row operations and rotations. Next we completely characterize the feasible set of Ramana's dual. As a corollary, we obtain a short and transparent derivation of Ramana's dual, which we believe is accessible to both the optimization and the theoretical computer science communities. Our approach is combinatorial in the sense that i) we use a minimum amount of continuous optimization theory ii) we show that feasible solutions in Ramana's dual are identified with regular facial reduction sequences, i.e., essentially discrete structures.

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MS32

Hidden Convexity and the Rotation Group

Many fundamental problems in optimization can be described as minimizing a smooth function over a manifold. In general, efficient techniques used to solve these problems seek out local minima using gradient descent, but such techniques typically do not guarantee global optimality, and do not interact well with additional potentially non-manifold constraints. We show how to leverage convex optimization techniques to give globally optimal results in a number of different settings by making use of 'hidden convexity', in which the image of the manifold of interest under a map to $\mathbb{R}^k$ turns out to be convex, in the vein of the Schur-Horn theorem. We give a simple condition which makes use of some elementary algebraic topology to show that the image of a manifold under some continuous map is convex, allowing us to give efficient algorithms for global constrained optimization over certain manifolds. We will focus primarily on optimization problems involving the rotation group $\mathrm{SO}(n)$, including preliminary work on the unbalanced orthogonal procrustes problem of finding the best rotation and projection mapping a high dimensional point set to a lower dimensional point set.

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MS32

Effective Applications of Second Level Moment Sum-of-Squares Hierarchy

The moment-sum-of-squares hierarchy for polynomial optimization is a powerful tool to obtain solutions of increasing quality, with the advantage of being polynomial size at each fixed level. However for practical purposes even the second level is widely considered to be too computationally costly. There has been a lot of work attempting to mitigate the problem, many of which based on exploiting underlying structure such as sparsity and symmetry. Our work is a new direction in this line of research. We study the second level moment hierarchy with ball constraints, and propose a special version which has drastically reduced size of $n+2$ (instead of $\sim n^2/2$ of the full second level). Another key advantage is that the localizing matrix constraints in this case have a moment cone reformulation that has an efficient second-order cone representation. This moment relaxation turns out to coincide with a RLT-based relaxation already proposed in optimization literature, and known to have very competitive numerical speed and quality. We show that it is strictly stronger than Kronecker RLT inequalities as well as several other inequalities proposed respectively by Jiang, Li and Zhen et al.

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MS33

Gotzmann's Persistence Theorem for Smooth Pro-

jective Toric Varieties

Gotzmann's persistence theorem enables us to confirm the Hilbert polynomial of a subscheme of projective space by checking the Hilbert function in just two points, regardless of the dimension of the ambient space. We generalise this result to products of projective spaces, and then extend our result to any smooth projective toric variety. The number of points to check depends solely on the Picard rank of the ambient space, with no dependence on the dimension.

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MS33

What is the Multigraded Regularity of the Flag Variety?

I will discuss some recent partial results concerning the multigraded regularity of complete flag varieties.

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MS33

Projective Dimension and Regularity over Multigraded Infinite Polynomial Rings in Relation to Stillmans Conjecture

Stillman's conjecture states that there is a bound on the projective dimension of homogeneous ideals in a standard graded polynomial ring which depends only on the number and degrees of the generators of the ideal and importantly not on the number of variables in the polynomial ring. This was proven by Ananyan and Hochster in 2016, and reproven by Erman, Sam, and Snowden and Draisma, Lasn, and Leykin in 2019 using novel techniques. Inspired by recent interest in toric geometry, we give an exact characterization of the gradings which admit Stillman bounds. In the standard graded case, Cavilgia proved that a Stillman bound on projective dimension is equivalent to one on regularity. We investigate this result in the multigraded setting as well. This is joint work with John Cobb (Auburn University) and John Spoerl (UW Madison).

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MS33

Effective Vanishing for Toric Vector Bundles

A toric vector bundle on a toric variety is a vector bundle endowed with a torus action compatible with the one on the underlying space. Almost all familiar examples of bundles on toric varieties are toric, from (co)tangent bundles to kernel bundles associated to complete linear series, which control syzygies of the corresponding embedding. In contrast to the case of line bundles, these are not purely combinatorial objects, as they also depend on linear algebraic information associated to cones known as the Klyachko data. We will discuss a new effective vanishing result for twists of toric vector bundles by line bundles, taking into account both the linear algebra and combinatorics via Batyrev's primitive relations. This vanishing recovers many familiar results, and our techniques lead to new ways to study N_p properties for smooth projective toric varieties.

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MS34

Invariant Theory and Computer Vision

In this talk I will discuss some questions in Invariant Theory that are motivated by applications in computer vision. One direction is the study of perspective invariants, and another is bi-Lipschitz Invariant Theory.

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MS34

PLMP - Point-Line-Minimal-Problems for Uncalibrated Cameras

This talk provides a classification of all minimal problems consisting of arrangements of points and lines completely observed by uncalibrated cameras. We show that there are 8 infinite families and 285 further minimal problems; among the latter ones there is no problem with more than 9 cameras or more than 12 lines and in general there is there is no minimal problem with more than 7 points. Furthermore, we present a novel framework of minimality for problems with few points and lines where only partial reconstruction of the scene is possible. This talk is based on joint work with Albin Ahlbck and Kathln Kohn.

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MS34

Equi-affine Minimal-degree Moving Frames for Polynomial Curves

Classical equivariant differential-geometric moving frames have a long history of applications in the areas of computer vision and image processing. However, such a frame associated with a polynomial curve is, in general, neither polynomial nor even rational. This presents a challenge in applications. We develop a theory and an algorithm for constructing minimal-degree polynomial moving frames for polynomial curves in an affine space. The algorithm is equivariant under volume-preserving affine transformations of the ambient space and the parameter shifts. We show that any matrix-completion algorithm can be turned into an equivariant moving frame algorithm via an equivariantization procedure that we develop. We prove that if a matrix-completion algorithm is of minimal degree, so is the resulting equivariant moving frame algorithm. We propose a novel minimal-degree matrix-completion algorithm, complementing the existing body of literature on this topic.

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MS34

A Problem in Flatland Vision

When is it possible for two sets of labeled points in the projective plane to be projected to the same image on a line? The answer to this question has a surprising connection to traditional 3d reconstruction in computer vision and the loci of the projection centers can be described explicitly using classical invariant theory and algebraic geometry.

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MS35

Quantum Max Flow for Simple (Random) Tensor Networks

We compute the quantum max flow for tensor networks built from simple fixed tensors and show that, in some cases, the value obtained is strictly larger than the quantum min cut of the network. We explore the same phenomenon for tensor networks containing random tensors, relating this problem to the combinatorics of meanders.

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MS35

The Translation-invariant Bell Polytope

Bell's theorem, which states that the predictions of quantum theory cannot be accounted for by any classical theory, is a foundational result in quantum physics. In modern language, it can be formulated as a strict inclusion between two geometric objects: the *Bell polytope* and the *convex body of quantum behaviours*. Describing these objects leads to a deeper understanding of the nonlocality of quantum theory, and has been a central research theme is quantum information theory for several decades. After giving an introduction to the topic, I will focus on the so-called *translation-invariant Bell polytope*. Physically, this object describes Bell inequalities of a translation-invariant system; mathematically it is obtained as a certain projection of the ordinary Bell polytope. Studying the facet inequalities of this polytopes naturally leads into the realm of tensor networks, combinatorics, and tropical algebra. This talk is based on joint work with Jordi Tura, Mengyao Hu, Eloic Valle, and Patrick Emonts.

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MS36

Maximum Likelihood Estimators for Gaussian Distributions with Restricted Covariance, with Connections to Rigidity Theory

A standard problem in statistics is the following: given a multivariate normal distribution with mean $(0, .., 0)$ and unknown covariance matrix that satisfies some known constraints, estimate the covariance matrix by sampling the distribution. A specific example of such a model is a Gaussian graphical model: from a graph with n vertices, we assume that the inverse of the covariance matrix has a zero at entry (i, j) if and only if ij is not an edge of the graph. In this case, the MLE of the covariance matrix can be found by optimising the Gaussian log-likelihood function over the positive definite matrices with zero entries corresponding to non-edges. Recently, Zwiernik (2023) extended this method to other restricted covariance models by introducing entropic covariance models, which are formed by replacing the Gaussian entropy function term in the Gaussian log-likelihood function with a general convex function satisfying certain properties. In this talk I will discuss recent work on the existence of a maximum likelihood estimator for a wide variety of entropic covariance models allowing for non-invertible sample covariance matrix corresponding to a small sample size. Surprisingly, there exists a large class of graphical entropic covariance models for which the existence of a MLE is solely dependent on the structural rigidity properties of the underlying graph. This talk involves joint work with Oliver Clarke, Steven Gortler, Roan Talbut and Louis Theran.

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MS36

Non-Manifold Data Analysis with Hades

High-dimensional data often exhibit low-dimensional structure, and a geometer's first intuition is to model the data using smooth manifolds. However this common assumption, called the "Manifold Hypothesis", is rarely verified to be true. In this talk I will introduce HADES, an unsupervised algorithm to detect data points that show non-manifold behaviour (i.e. singularities in data). This algorithm uses simple statistical methods that makes it much faster than existing topology-based alternatives. Using tools from differential geometry and optimal transport theory, we prove that HADES correctly detects singularities with high probability when the data sample lives on a transverse intersection of equidimensional manifolds. In computational experiments, HADES recovers singularities in synthetically generated data, branching points in road network data, intersection rings in molecular conformation space, and anomalies in image data.

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MS36

K-Fold Circuits in Rigidity Matroids

A k-fold circuit in a matroid $M = (E, r)$ is a set $X \subseteq E$ such that $r(X - e) = r(X) = |X| - k$ for all $e \in X$. Double circuits, I,e, 2-fold circuits, and the so called ‘double circuit property played a fundamental role in Lovaszs celebrated solution to the matroid matching problem in 1980. In this talk we extend results on double circuits to k-fold circuits and then apply the results to the generic d-dimensional rigidity matroid.

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MS37

Computing Characters of Finite Group Actions on Free Resolutions

The Betti numbers of a minimal free resolution of a graded module are invariants that provide a wealth of algebraic and geometric information, including Krull and projective dimension, Castelnuovo-Mumford regularity, multiplicity and Cohen-Macaulay type. When the module carries the action of a finite group, Betti numbers extend to characters of certain representations, thus leading to interesting new interpretations. This talk will discuss novel methods for computing these characters and their implementation in Macaulay2.

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MS37

Calculating Cohomology of Toric Vector Bundles

A toric vector bundle is a vector bundle on a toric variety that is equipped with a compatible action of the underlying torus. Klyachko shows that a toric vector bundle corresponds to a finite-dimensional vector space with a suitable family of filtrations. Building on this classification, we introduce a complex of modules over the Cox ring that simultaneously encodes the cohomology of a toric vector bundle and the cohomology of many of its twists. In addition to offering improved computational efficiency, this method leads to new vanishing theorems. This talk is based on joint work with Michael Perlman.

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MS37

Effective Computation in the Derived Category of Coherent Sheaves in Oscar

Coherent sheaves and their derived category form a backbone of modern Algebraic Geometry. For example, the cohomology groups $H^p(X, \Omega_X^q)$ of the algebraic q-forms on a projective variety X form a fundamental set of invariants for X. Yet, things become surprisingly challenging, once we move on beyond the projection to a point: Already the direct image of a perfect complex along the projection of relative projective space can be computationally difficult. In this talk we will introduce the foundations laid out for Algebraic Geometry and Homological Algebra in the computer algebra system OSCAR, briefly review existing algorithms for the computation of (higher) direct images in various setups, and report on their implementations and performance in particular applications.

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MS37

Higher Direct Images of Toric Morphisms

Higher direct images are relative cohomological invariant generalizing sheaf cohomology. In practice, it is difficult to compute sheaf cohomology, let alone a relative generalization. By specializing to toric varieties, we obtain a combinatorial framework for computing these invariants using the Cox ring. There is a natural choice of module over the Cox ring which yields the higher direct images, but it is almost always infinitely generated for toric varieties, and so we need to find a finitely generated submodule of this. We will discuss these types of challenges and how to overcome them in order to compute higher direct images of line bundles for toric morphisms between smooth projective toric varieties. This is joint work with Mike Roth and Greg Smith.

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MS38

Computing Arrangements of Hypersurfaces

We present a Julia package HypersurfaceRegions.jl for computing all connected components in the complement of an arrangement of real algebraic hypersurfaces in $\mathbb{R}^n$. This is joint work with Bernd Sturmfels and Kexin Wang.

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MS38

Distinguishability of Phylogenetic Networks Under the Multispecies Coalescent Model from Quintet Concordance Factors

Phylogenomics attempts to infer evolutionary relationships between taxa, depicted as a tree or a network, from sequence data for many genes. The *multispecies coalescent model*, describing how gene trees arise from the species network, provides a framework for studying what network features can be inferred from summaries of unrooted topological gene trees. Identifiability of level-1 network features from *quartet concordance factors* (CF_4s) has been well-studied, giving a theoretical basis for practical inference methods, despite the non-identifiability of network roots and 2-cycles from them. Since *quintet concordance factors* identify the root location of a species tree, we began a preliminary investigation of quintet CFs in a network setting. We give several distinguishability results, indicating that quintet CFs indeed contain information on 2-cycles and network rooting.

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MS38

Real Solutions of the Adjoint Newton's Method

Newton's method is a famous iterative root-finding method which quadratically converges locally to a nonsingular solution. However, nuances to the iterative method lead one to consider a continuous version of the method. When such a continuous Newton's method is applied to a bivariate quadratic polynomial system, interesting geometric structures arise relating the system's trajectories and equilibrium points. This talk will explore the geometry of the continuous adjoint Newton's method to a bivariate quadratic polynomial with real solutions through visualizations and eigenanalysis. This is joint work with Jonathan Hauenstein, Hoon Hong, and Francisco Ponce-Carrion.

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MS38

Reality, Perhaps Non-standard

In the real world, it is common to encounter a need to study real algebraic varieties computationally, when the real structure is nonstandard. Such alternate real structures come from an antiholomorphic involution on the ambient complex space. Symbolic methods to handle such alternate reality are either lacking or require a costly reformulation as a standard real object. Numerical algebraic geometry should be able to avoid these pitfalls, as the real points (in any model of reality) are just those fixed under the antiholomorphic involution. I will illustrate some of these ideas with examples I have encountered in my own work.

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MS39

Sign Patterns and Multistationarity in Biochemical Reaction Networks

Biochemical reaction networks give rise to polynomial ODEs in a natural way. Often one is interested in the steady state behavior of these ODEs. Then one studies the zeros set of the corresponding polynomials. And as parameter values are confined to large intervals one effectively studies families of parameterized polynomials. Multistationarity refers to the existence of at least two distinct positive zeros for the same parameter values. It has been recognized as an important feature in biological applications. Because of parameter uncertainties one is often interested in conditions that guarantee multistationarity for some values of the parameters. The focus of this talk are ODEs that admit a monomial parametrization of positive steady states. For such systems multistationarity can be established by analysis fo sign patterns of the linear subspaces occurring in the parametrization. Feasible sign patterns in turn allow for a parametrization of the multistationarity region in parameter and in concentration space.

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MS39

A Path Method for Non-exponential Ergodicity of Markov Chains and its Application for Chemical Reaction Systems

We present criteria for non-exponential ergodicity of continuous-time Markov chains on a countable state space. These criteria can be verified by examining the ratio of transition rates over certain paths. We applied this path method to explore the non-exponential convergence of microscopic biochemical interacting systems. Using reaction network descriptions, we identified special architectures of biochemical systems for non-exponential ergodicity. In essence, we found that reactions forming a cycle in the reaction network can induce non-exponential ergodicity when they significantly dominate other reactions across infinitely many regions of the state space. Interestingly, special architectures allowed us to construct many detailed balanced and complex balanced biochemical systems that are non-exponentially ergodic. Some of these models are low-dimensional bimolecular systems with few reactions. Thus this work suggests the possibility of discovering or synthesizing stochastic systems arising in biochemistry that possess either detailed balancing or complex balancing and slowly converges to their stationary distribution.

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MS39

Weakly Reversible, Deficiency Zero Realizations of Polynomial Systems with Symbolic Coefficients

A polynomial ODE system (S) is said to have a weakly reversible, deficiency zero (short WR0) realization if there is a WR0 mass-action system which gives rise to (S). A lot is known about the dynamics of WR0 networks, and polynomial systems that admit WR0 realizations enjoy the same well-understood properties. We present an algorithm that answers the question: Given a polynomial ODE system (S) whose coefficients depend linearly on some parameters, for what values of those parameters does (S) have a WR0 realization? The algorithm involves computations on Newton polytopes; it is implemented in perl and makes heavy use of polymake.

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MS39

Mapping Dynamical Systems into Chemical Reactions

Polynomial dynamical systems (DSs) can model a wide range of physical processes. A special subset of these DSs that can model chemical reactions under mass-action kinetics is called chemical dynamical systems (CDSs). In this context, a central problem is to map polynomial DSs into dynamically similar CDSs. In this talk, I will introduce the *quasi-chemical map* (QCM) that can systematically solve this problem. The QCM introduces suitable state-dependent perturbations into any given polynomial DS which then becomes a CDS under sufficiently large translations of variables. This map preserves robust features, such as generic equilibria and limit cycles, as well as temporal properties, such as periods of oscillations. Furthermore, the resulting CDSs are at most one degree higher than the original DSs. I will apply the QCM to design relatively simple CDSs with exotic behaviors. In particular, I will address Hilbert's 16th problem in chemistry, and design CDSs with self-excited and hidden chaos.

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MS40

Combinatorial List Decodability of Random Reed-Solomon Codes and Beyond

A code $C \subseteq A^n$ over an alphabet A is said to be (ρ, L)-combinatorially list decodable if any Hamming ball of radius ρn in A^n contains at most L codewords from C. In this talk, I will discuss a recent line of research demonstrating the optimal combinatorial list decodability of random ReedSolomon codes. I will also briefly touch on related progress concerning algebraic geometry, rank-metric, and folded ReedSolomon codes.

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MS40

Normal Binary Codes: A New Approach

Normal codes can be used to construct sparse covering codes with large covering radius via the operation of amalgamated direct sum. Establishing when a code is normal has therefore became a main problem in coding theory. In 1993, Etzion, Greenberg and Honkala constructed the first class non-normal, non-linear binary codes, therefore showing that not all non-linear binary codes are normal. However, to this day, it is still unknown whether there exist non-normal binary linear codes. In this talk we take a new approach to studying normality based on coset leaders and the coset leader ordering.

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MS41

SQIsign: Across the Multiverse

SQIsign is the leading digital signature protocol based on isogenies and the only isogeny-based construction in the NIST standardization process. Its security is tightly linked to the hardness of computing endomorphism rings of supersingular elliptic curves. Over the years, many versions and variants of SQIsign have been proposed. Similarly to most constructions in isogeny-based cryptography, SQIsign has been revolutionized by the introduction of higher-dimensional representations: a new encoding that enables evaluating isogenies at a logarithmic cost in their degree, for any arbitrary degree. This has significantly changed the protocol design, its security, and its algorithmic building blocks, and it has drastically increased its performance. In this talk, we survey the evolution of SQIsign from its first design to its newest iteration, the version recently submitted to the second round of NIST's selection process. We will also discuss recent results that provide a formal and complete proof of security of SQIsign, which fills a long-standing gap in the literature.

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MS41

Isogeny Interpolation in Cryptography

Elliptic curves and arithmetic geometry provide fertile ground for mathematicians and researchers seeking to unearth beautiful mathematics amidst the rich interplay between algebraic and geometric structures. In recent years, cryptographers have gained interest in constructing post-quantum cryptosystems based on isogenies between elliptic curves and abelian varieties. One such cryptosystem, the Supersingular Isogeny Diffie-Hellman (SIDH) scheme, advanced to the fourth round of the NIST post-quantum cryptography standardization process before being broken in 2022. The technique used to break this scheme, based on interpolating between known values of an isogeny, has since been applied to construct new cryptosystems and digital signature schemes from isogenies. In this presentation, we survey the mathematics of isogeny-based cryptography, the construction and break of SIDH, and current proposals for post-quantum isogeny-based cryptosystems.

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MS41

Computing the Trace of a Supersingular Endomorphism

For an elliptic curve E, Elkies' improvement to Schoof's algorithm involves computing the trace of Frobenius modulo primes ell for which E has a rational ell isogeny. In practice, this gives a substantial speedup, but heuristics beyond GRH are required to prove that it yields an asymptotic speedup. Computing the trace of an arbitrary endomorphism can be done with a generalization of Schoof's algorithm. When E is supersingular, computing the trace of an endomorphism is an important subroutine in various algorithms for computing the endomorphism ring of E. And in the supersingular case, assuming E is defined over the finite field of p^2 elements, E always has rational ell-isogenies we leverage this in a generalization of the SEA algorithm for supersingular endomorphisms, yielding an unconditional asymptotic speedup over the generalization of Schoof's algorithm. We gain further speedups in practice: for example, we can compute the trace modulo p (the characteristic) by computing the action on an invariant differential of E, and we can leverage knowledge of the number of points of E since E is supersingular to compute the trace modulo primes dividing $p \pm 1$. This is joint work with Lorenz Panny, Jana Sotakova, and Michael Wills.

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MS41

PEGASIS: Practical Effective Class Group Action using 4-Dimensional Isogenies

We present the first practical algorithm to compute an effective group action of the class group of any imaginary quadratic order $\mathcal{O}$ on a set of supersingular elliptic curves primitively oriented by $\mathcal{O}$. Effective means that we can act with any element of the class group directly, and are not restricted to acting by products of ideals of small norm, as for instance in CSIDH. Such restricted effective group actions often hamper cryptographic constructions, e.g. in signature or MPC protocols. Our algorithm is a refinement of the Clapoti approach by Page and Robert, and uses 4-dimensional isogenies. As such, it runs in polynomial time, does not require the computation of the structure of the class group, nor expensive lattice reductions, and our refinements allow it to be instantiated with the orientation given by the Frobenius endomorphism. This makes the algorithm practical even at security levels as high as CSIDH-4096. Our implementation in SageMath takes 1.5s to compute a group action at the CSIDH-512 security level, 21s at CSIDH-2048 level and around 2 minutes at the CSIDH-4096 level. This marks the first instantiation of an (unrestricted) effective cryptographic group action at such high security levels. For comparison, the recent KLaPoTi approach requires around 200s at the CSIDH-512 level in SageMath and 2s in Rust.

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MS42

A Vector Bundle Approach to Tensor Optimization Problems

This talk explores tensor optimization problems, including finding so-called totally mixed Nash equilibria of noncooperative games. It discusses an approach employing vector bundles for these optimization problems.

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MS42

The Batthacharya-Mesner Product: From Tensor Rank Analysis to Description of Networks

The Bhattacharya-Mesner product is a n-ary operation for tensors of order n. In this talk we present some applications of this product related to the study of Bhattacharya-Mesner rank and to description of networks. First, we give a result about the characterisation of the rank and border rank of the Bhattacharya-Mesner tensor. Next, we show that this product is particularly suitable for the study of networks described by directed acyclic graphs in which each node decides its own activation through tensors of choice. In this way the Bhattacharya-Mesner product becomes an useful algebraic tool for separating various classes of networks.

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MS42

Higher-order Osculating Eigenvectors of Symmetric Tensors

We work in the space of real n-ary symmetric tensors of order d, or of real homogeneous polynomials in n variables of degree d. We equip this space with the Bombieri-Weyl inner product. Let X be the affine cone of real symmetric tensors of rank at most one. For a fixed real symmetric tensor f, we study the distance function from f restricted to X. When f is generic, the number of complex critical points of such distance function is finite and constant, and coincides with the Euclidean Distance degree of X with respect to the Bombieri-Weyl inner product. Furthermore, the critical points correspond to the so-called eigenvectors of f, generalizing the notion of eigenvector of a symmetric matrix. In this talk, we describe the data locus of symmetric tensors f having at least one critical point $x \in X$ of the distance function from f which is k-osculating for some integer $k > 1$, meaning that the vector $f - x$ is orthogonal to the k-th osculating space of X at x. This leads to the notion of k-osculating eigenvector of a symmetric tensor.

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MS42

Border Rank Bounds for $GL(V)$-invariant tensors Arising from Matrices of Constant Rank

We prove border rank bounds for a class of $GL(V)$-invariant tensors in $V^* \otimes U \otimes W$, where U and W are $GL(V)$-modules. These tensors correspond to spaces of matrices of constant rank. In particular we prove lower bounds for tensors in $\mathbb{C}^l \otimes \mathbb{C}^m \otimes \mathbb{C}^n$ that are not 1_A-generic, where no nontrivial bounds were known, and also when $l, m \ll n$, where previously only bounds for unbalanced matrix multiplication tensors were known. We give the first explicit use of Young flattenings for tensors beyond Koszul to obtain lower bounds for border rank, and compare them to previously studied Koszul flattenings when V is of dimension three.

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MS43

Why is Topology a Natural Tool for Studying the Brain?

In the last decade, tools from topology have become increasingly important for quantitative neuroscientists. Many of the links between the two disciplines were surprising at first, but as our understanding has improved it has become clear that many important questions about the brain and our observations of it are naturally addressed from a topological perspective. In this talk, I will provide a brief survey of the fundamental mathematical and neuroscientific ideas that underlie (a few of) these connections, and (a bit of) what we have learned so far.

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MS43

Exploring Neural Responses to Thermal Stimuli in the Gustatory Cortex

The gustatory cortex (GC) is traditionally studied for its role in taste perception, but emerging evidence suggests it may also contribute to encoding other oral sensory modalities. In this study, we investigate the encoding of oral thermal signals by GC neurons recorded simultaneously using multi-electrode probes. Specifically, we recorded the spiking activity of over 400 GC neurons (in groups of 940 simultaneously recorded neurons) from mice allowed to freely lick drops of water at non-nociceptive temperatures. This approach enables us to study the structure of neural population responses across recording sessions to multiple stimuli. We developed an analysis pipeline that applies methods from topological data analysis (TDA) to the spiking activity, using the Victor-Purpura distance to convert spike trains into graphs such that a Vietoris-Rips filtration can be applied. After computing persistence barcodes, we apply leave-one-out classification to quantify the networks ability to encode thermal information, and we further analyze the data using persistence landscapes. Our results show that classification performance based on the topology of neuronal population activity is significantly higher than that obtained from individual neuron spike trains. By leveraging techniques from TDA, spike train distance metrics, and optimal transport, we offer new insights into how neuronal populations encode sensory information.

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MS43

The Role of Graph Partitions in Cluster Synchronization for Weighted Networks

Almost equitable partitions (AEPs) are linked to cluster synchronization in oscillatory systems, providing a mathematical framework for understanding collective behavior. AEPs are of interest in the synchronization process since they have been linked to the study of synchronous clusters, making them the perfect vehicle to study Resting State Networks in the human connectome. These patterns of synchronized activity occur at rest and understanding their formation and relationship to connectome structure is essential for advancing our understanding of the brain's fundamental mechanisms as well as developing treatments for neurological and psychiatric disorders.Spectral properties of AEPs allow us to describe this synchronization behavior in terms of graph Laplacian eigenvectors. Our results shed light on transient hierarchical clustering, multifrequency clustering, and the conditions under which they can occur. Our analysis relates dynamical clustering directly to network symmetry and community detection, relating structure and function in complex networks. This bridges a crucial gap between static network topology and emergent dynamic behavior and allows us to define a relaxation of an AEP called a quasi-equitable partition (QEP). Perfect AEPs are rare in real-world networks since most have some degree of irregularity or noise. While relaxing these strict conditions, QEPs can maintain many of the useful properties allowing for qualitatively similar clustering behavior to AEPs.

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MS43

How to Disentangle Neural Ensembles by Finding Their Local Parts

In studying the spatial properties of neural activity, it is common to construct spaces of neural correlations or spaces of population vectors. However, these approaches carry an often-overlooked limitation: they assume neurons to be point-like entities or, respectively, population activity profiles to be well-represented as points in a space. These assumptions fail, for example, when dealing with simultaneous recordings of multiple neural ensembles exhibiting mixed selectivity. Many modern large-scale neural datasets are of this form. I will present a method for extracting localized combinations of neurons and timepoints from neural datasets. These localized parts serve as more meaningful candidates for points in spatial models of neural data. In particular, this approach provides a principled way to disentangle mixed neural populations and has the potential to reveal previously hidden neural representations in an unsupervised manner. The method is based on theoretical ideas connected to Dowker complexes and Dowker duality. In my talk, I would also like to elaborate further on some of these theoretical underpinnings.

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MS44

Model Invariants for the Equal Input Model on Phylogenetic Trees

This talk focuses on Algebraic Time-Reversible (ATR) models, single-site substitution models used in Markov processes over phylogenetic trees, with appealing algebraic structure. Specifically, our attention is turned to the *Equal Input model* which, despite its simplicity, allows for a non-uniform stationary distribution (as opposed to group-based models). A further advantage is that it can be formulated in any number of *states*. The cases of 4 and 20 states are particularly interesting as they capture nucleotide substitution and amino acid substitution models, respectively. In this work, we present a list of linear phylogenetic invariants for the Equal Input model on trees with three and four leaves, a set of polynomials that cut out the variety on an open set containing the biologically relevant points. Our results provide an alternative to the phylogenetic invariants computed by Steel and Casanellas (2016). However, we are building on the ATR framework recently developed by Casanellas, Homs and Torres (2024), which facilitates finding more comprehensible formulas that have the potential advantage of generalizing to more ATR models.

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MS44

Secant Nondefectivity of Moment Varieties: Curves, Surfaces and Beyond

The method of moments is a classical technique for parameter estimation in statistics, and raises a fundamental question of identifiability: How many moments do we expect to need, in order to determine the parameters? This can be studied from the point of view of algebraic geometry, through the *moment varieties* associated to the distribution, which in the case of mixture distributions are highly structured secant varieties. In this talk, I will discuss some new results on finite and unique identifiability for mixture distributions built from well-known distributions such as the chi-square, exponential, gamma and inverse Gaussian distribution, where the moment varieties turn out to be secant varieties of determinantal curves and surfaces. I will also discuss some recent progress on the threefold case. Most of the talk is based on joint work with Kristian Ranestad, Lisa Seccia and Teresa Yu.

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MS44

Model Selection for Algebraic Time-Reversible Evolutionary Models.

One of the main problems in phylogenetics is to estimate which tree better describes the evolutionary history of some given DNA or protein data. A common approach to reconstruct the phylogenetic tree is to assume a substitution model that explains how characters are substituted at each site of the sequence according to biochemical properties. Classically the selection of a suitable evolutionary model is based on heuristics or relies on the choice of an approximate input tree. In the 90s, several authors suggested that certain linear equations satisfied by the expected probabilities of patterns observed at the leaves of the tree could be used for model selection. In 2012, using techniques from algebraic geometry and group theory, Casanellas et al. obtained necessary and sufficient linear equations for equivariant models of DNA and a method for model selection was successfully implemented by Kedzierska et al. However, the models studied from the algebraic viewpoint up to now are either too general or too restrictive. We provide a framework to study algebraic time-reversible (ATR) models. We showcase the use of these techniques applying them to the Tamura-Nei model and its submodels. We focus on a special type of linear equations that are sufficient to derive the equations of the submodel from the equations of the bigger model. Based in joint work with Marta Casanellas, Anglica Torres, Jennifer Garbet, Annachiara Korchmaros, and Niharika Paul.

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MS45

Moment-Sos and Spectral Hierarchies for Polynomial Optimization on the Sphere and Quantum De Finetti Theorems

In this talk, we discuss hierarchies for polynomial optimization on the sphere. A result of Fang and Fawzi showed a convergence rate of O(1/r) of the r-th approximation level of the sum-of-squares hierarchy towards the minimum. Recent work by Lovitz and Johnston introduced a lightweight spectral hierarchy for polynomial optimization with a convergence rate of O(1/r) in the hierarchy level r. We give a lower bound of O(1/r) for the spectral hierarchy and we connect both hierarchies with real quantum de Finetti representations.

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MS45

Quadratic Determinantal Representations of Positive Polynomials

This is joint work with Mario Kummer. A positive quadratic (determinantal) representation of a nonnegative real polynomial $f \in \mathbb{R}[x_0, x_1, x_2]_{2d}$ is a symmetric matrix of size d with polynomial entries of degree 2 whose determinant is f, and which evaluates to a positive semidefinite matrix in every real point. The study of such representations is a relatively recent development and was initially motivated by the study of extremal rays in the cone of nonnegative biquadratic forms. The (non-)existence of such representations often has to be established on a case-by-case basis. We found a method to show that the Robinson polynomial does not have a positive quadratic representation. Contrarily, a positive result of us shows that at least for smooth positive ternary quartics there always exists a positive quadratic representation. This insight relies on the fact that the combinatorial properties of quartic curves - especially their bitangent lines - are very well understood.

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MS45

Real Stability and Log Concavity Are Conp-Hard

Real-stable, Lorentzian, and log-concave polynomials are well-studied classes of polynomials, and have been powerful tools in resolving several conjectures. We show that the problems of deciding whether a polynomial of fixed degree is real stable or log concave are coNP-hard. On the other hand, while all homogeneous real-stable polynomials are Lorentzian and all Lorentzian polynomials are log concave on the positive orthant, the problem of deciding whether a polynomial of fixed degree is Lorentzian can be solved in polynomial time.

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MS45

A Gap Between Positive Polynomials and Sums of Squares in Various Settings

In 2006, Blekherman established estimates on the sizes of the cones of nonnegative forms and sums of squares forms by comparing the volume radii of compact sections of these cones with a suitably chosen hyperplane. For a fixed degree bigger than 2, as the number of variables goes to infinity, the ratio between the volume radii goes to 0. In the talk we will present estimates for the ratio between the volume radii of nonnegative forms and sums of squares forms in three different vector spaces: 1) the space of biquadratic biforms, 2) the space of biquadratic biforms modulo the ideal of all orthonormal 2-frames, 3) the space of even quartic forms. In 1) and 2), the conclusion is analogous as in the case of all forms, while in 3), the difference between the cones in question does not grow arbitrarily large as the number of variables grows to infinity. The motivation to study these questions is the following: 1) comes from quantum information theory to measure the gap between positive maps and completely positive maps between matrix algebras, 2) comes from financial mathematics to measure the gap between cross-positive maps and completely cross-positive maps between matrix algebras, and 3) comes from matrix theory to measure the gap between copositive matrices and matrices that are a sum of a positive semidefinite matrix and entry-wise nonnegative one.

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MS46

Castelnuovo-Mumford Regularity of Toric Surfaces

In 1996, Lvovsky showed the Castelnuovo-Mumford regularity of the coordinate ring of a monomial curve is bounded by the sum of its semigroups two largest gaps. We explore analogous results for toric surfaces embedded by incomplete linear systems, and show that for certain classes the regularity is controlled by the combinatorics of the associated semigroup.

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MS46

Characterizing KW-Semigroups with Determinantal Defining Ideal

For coprime numbers $p < q$, KW numerical semigroups, introduced by Kunz and Waldi, contain p, q and are contained in $\langle p, q, r\rangle$ where $2r = p, q, p+q$, whichever is even. We characterize KW semigroups whose semigroup ring R/I has determinantal defining ideal, i.e., I is generated by some minors of a matrix. Next, we identify all KW semigroups on the interior of the same face of the Kunz cone, a geometric object in $\mathbb{R}^n$ whose integer points correspond combinatorially to numerical semigroups– with shared properties if they are on the same face. This also gives us the Betti numbers of all such semigroup rings, key invariants of their minimal free resolution.

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MS46

Resolutions Over Numerical Semigroup Rings Using Apry Specialization

Each numerical semigroup S with smallest positive element m corresponds to an integer point in a polyhedral cone C_m, known as the Kunz cone. The faces of C_m form a stratification of numerical semigroups that has been shown to respect a number of algebraic properties of S, including the combinatorial structure of the minimal free resolution of the defining toric ideal I_S. In this work, we prove that the structure of the infinite free resolution of the ground field over the semigroup algebra also respects this stratification, yielding a new combinatorial approach to classifying homological properties like Golodness and rationality of the

Poincar series in this setting.

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MS46
Some Questions on Affine Toric Rings

A numerical semigroup S is a submonoid of the Natural numbers that is minimally generated by n relatively prime positive integers, $A = \{a_1, \ldots, a_n\}$. For any $i, 1 \leq i \leq n$, there is a smallest positive integer $c_i \geq 2$ and non negative integers $p_{ij}, j \neq i$, such that $c_i a_i = \sum_{j \neq i} p_{ij} a_j$. Let $p_{ii} = -c_i$. Then the $n \times n$ matrix P is called the principal matrix of S. We will discuss instances where semigroups can be recognized by their principal matrices. In 2014, Kunz and Waldi considered numerical semigroups beginning with two relatively prime numbers $p = a_1, q = a_2$ and showed that for all such semigroups of embedding dimension n, the first and the last Betti number are respectively $\binom{n+1}{2}$ and $n-1$. In a recent paper, Singh and Srinivasan characterize these semigroups by their principal matrices. The general question of given a P, determining all the semigroups with principal matrix P is interesting. We will discuss some partial results and raise some questions.

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MS47
The Duality of Triangulation and Resectioning

We study the problems of triangulation and resectioning across a variety of contexts and examine the dual nature of these problems.

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MS47
Measuring Spatio-temporal Manifolds with Event Cameras for Structure and Motion Estimation

Event cameras are bio-inspired sensors with a unique sensing paradigm. Unlike standard cameras, which measure synchronous intensity frames at fixed rates, they measure sparse and asynchronous intensity changes (i.e., events) with independent pixels. Their high dynamic range, low motion blur and microsecond-level latency, and temporal resolution have led to a flourishing of event-based research. However, fundamental questions behind the modeling of spatiotemporal event structures generated by moving objects remain open. This talk is aimed at characterizing these structures by studying the spatiotemporal manifolds of events generated by the apparent motion of 3D lines under first-order camera dynamics, which we coin ventails. They satisfy a polynomial constraint derived from geometric relations, and systems of such constraints can be used to derive structure and partial motion information using polynomial solvers. First, I will show how to solve such systems with Grbner bases and how combining solutions from multiple lines yields full 3D linear velocity estimates. Then, I will present a line reparametrization that casts this polynomial system into a linear form that captures both minimal (N=5) and overdetermined systems with a fast linear that is more numerically stable. Finally, this form allows us to derive robust tests for degeneracy, find solution symmetries, and characterize the canonical form of ventails under varying scene and camera parameters.

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MS47
Argus 3D Cameras

There is an emerging class of Dollar Depth Sensors; the term dollar being a metaphor for sensors that operate on an extremely constrained budget not just in terms of cost, but also power, compute resources, size and latency. These minimal sensors often have a single pixel, are based on the time-of-flight principle, and are widely used in robotics to sense proximity (danger) to nearby objects for tasks such as obstacle avoidance. In this talk, we will explore Argus* Depth Cameras a distributed collection of many such sensors (e.g., a robot densely covered with a large number of minimal depth sensors). Such a distributed collection of sensors that are potentially moving with respect to each other can be thought of as a single multi-perspective, deformable camera. Is it possible to capture a consistent representation of the world from such a deformable camera? This is challenging because classical geometric vision techniques (e.g., camera pose estimation, structure-from-motion) for recovering geometric information are largely based on conventional, single-perspective rigid cameras, and do not lend themselves to deformable arrays of these unconventional single-pixel sensors. We will discuss preliminary results on generalizations of geometric vision techniques for such distributed, multi-perspective sensor arrays. * In Greek mythology, Argus Panoptes, is a giant with 100 eyes and a watchman who is often depicted with eyes covering his body.

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MS47
Method of Moments for Estimation of Noisy Curves

In this paper, we study the problem of recovering a ground truth high dimensional piecewise linear curve $C^*(t) : [0,1] \to \mathbb{R}^d$ from a high noise Gaussian point cloud with covariance $\sigma^2 I$ centered around the curve. We establish that the sample complexity of recovering C^* from data scales with order at least σ^6. We then show that recovery of a piecewise linear curve from the third moment is locally well-posed, and hence $O(\sigma^6)$ samples is also sufficient for recovery. We propose methods to recover a curve from data based on a fitting to the third moment tensor with a careful initialization strategy and conduct some numerical experiments verifying the ability of our methods to recover curves. This talk is based on the preprint arXiv:2410.23220.

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MS48

Tropical Subrepresentations and Matroids

In their recent paper, Giansiracusa and Manaker introduced a notion of tropical subrepresentations of linear representations by considering linear actions on tropical linear spaces. In particular, matroids naturally show up in the story. In this talk, I will discuss several elementary results on tropical subrepresentations. This is joint work with Kalina Mincheva and Jeffrey Tolliver.

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MS48

Tropical Subrepresentations of the Boolean Regular Representation in Low Dimension

The study of the tropical regular representation of a finite group over the tropical booleans was initiated by Giansiracusa–Manaker (2018). We study two dimensional and three dimensional tropical subrepresentations of this tropical Boolean regular representation utilizing their theory of group representations over a fixed idempotent semifield. In dimension two we completely classify all two dimensional tropical subrepresentations. In dimension three we make significant progress towards such a classification. These topics and results demonstrate a significant and exciting interaction between matroid theory, group theory, and representation theory.

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MS48

Log-Concavity of Independent Sets of Valuated Matroids and Sampling on Dressians

We show a generalized version of the Mason-Welsh conjecture: logarithmic concavity for independent sets with additional weighting provided by valuations on independent sets of valuated matroids. In the process, we construct free extensions of valuated matroids. These can be seen as the first step towards a Higgs lift of valuated matroids. A strengthening of log-concavity, ultra log-concavity, requires further investigation. To this end, we developed an algorithm automatically generating random testing cases by sampling points in the Dressian, which I will additionally present in this talk. The talk is based on joint work with Jeff Giansiracusa, Felipe Rincn, and Martin Ulirsch.

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MS49

Rigidity and Level Sets of Persistent Homology

Persistent homology (PH) is an operation which, loosely speaking, describes the different holes in a point cloud. How much information is lost when we apply PH to a point cloud? We investigate this question by studying the subspace of point clouds with the same output from PH (via the Cech or Vietoris-Rips filtration). We find that the question of when the persistence map is identifiable has close ties to rigidity theory. For example, we show that a generic point cloud being locally identifiable under Vietoris-Rips persistence is equivalent to a certain graph being rigid on the same point cloud.

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MS49

Paradoxical Flexibility of Reflection Symmetric Frameworks

A reflection symmetric framework is a graph with an involutive automorphism and a reflection symmetric realization, which is a map from the vertex set to the plane such that the vertices in each orbit are reflection symmetric with respect to a fixed line. Such a framework is called reflection symmetric flexible if it can be continuously deformed while maintaining the symmetry and the distances between adjacent vertices, otherwise it is reflection symmetric rigid. Paradoxical situations may occur in the sense that almost all reflection symmetric realizations of a graph are reflection symmetric rigid, but some special ones are reflection symmetric flexible. The existence of a flexible realization has been characterized combinatorially using the valuations of a function field in the non-symmetric case (Grasegger, Legersk, Schicho, 2019) and rotation symmetric one (Dewar, Grasegger, Legersk, 2022). The reflection symmetric case is significantly more complicated. We provide a necessary condition on the existence of a reflection symmetric flexible realization in terms of the existence of an edge coloring by three colors satisfying conditions on the symmetry and the number of occurrences of the colors in each cycle of the graph. We discuss also some sufficient conditions on the existence of a reflection symmetric flexible realization using these edge colorings.

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MS49

Polytope Frchet Means

Fréchet means generalize the classical notion of a mean to arbitrary metric spaces, defined as any point that minimizes the sum of squared distances to the data points. In particular, Fréchet means often coincide with the maximum likelihood estimators of the mean of generalised Gaussian distributions, and are well understood in terms of the Alexandrov curvature of a space. However, when our metric is not strictly convex - as in the case of the taxicab, supremum, or tropical metrics - Fréchet means are not so well behaved. In fact, they will not generally be unique. In this talk, we present the behaviour of Fréchet means on vector spaces with a norm given by some symmetric polytope; we identify the threshold sample size at which our Fréchet means become unique with positive probability, and we prove a central limit theorem for i.i.d. samples. Finally, we demonstrate the statistical applicability of polytope Fréchet means for hierarchical modelling.

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MS49

Jukes-Cantor Identifiability for Phylogenetic Networks

Phylogenetic networks serve as a way to model evolutionary relationships. A common way to model the process of DNA sequence evolution is to view it as a Markov process along a n-leaf directed acyclic graph. We will in particular discuss the Jukes-Cantor model on certain classes of level-2 phylogenetic networks and results obtained for these classes.

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MS50

Certified Algebraic Path Tracking with Algpath

Algpath is a certified homotopy continuation software. We upgrade the previous fixed-precision Rust implementation by incorporating mixed, adaptive precision with minimal overhead. This allows us to tackle problems on which the initial implementation fails due to the inability to increase precision, and where uncertified methods may fail or path jump.

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MS50

An Algorithm for Differential Elimination Via a Support Bound

For a polynomial dynamical system, we study the problem of computing the minimal differential equation satisfied by a chosen coordinate. We give a bound for the Newton polytope of such minimal equation and show that our bound is sharp in “more than half of the cases’. We further use this bound to design an algorithm for computing the minimal equation following the evaluation-interpolation paradigm. We demonstrate that our implementation of the algorithm can tackle problems which are out of reach for the state-of-the-art software for differential elimination.

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MS50

The Vacuum Moduli Space Meets Symbolic Computation

The Standard Model of particle physics has been amazingly successful at explaining interactions between three of the four fundamental forces of physics: the strong force, the weak force, and electromagnetism. But it is an incomplete theory, failing to incorporate gravity. The Minimal Supersymmetric Standard Model (MSSM) is an approach to including gravity and supersymmetry in the standard model, by adding new particle states and interactions to the standard model. For example, supersymmetry pairs fermions (such as electrons) with bosons (such as photons). A starting point in the study of the MSSM is the Vacuum Moduli Space, which is a highly complicated algebraic variety introduced by Witten: it is the image of an affine variety $X \subseteq \mathbb{C}^{49}$ under a symplectic quotient map to $\mathbb{C}^{973}$. Even though this is a huge computation, we use computational algebra and Macaulay2 to describe many aspects of the structure of the MSSM Vacuum Moduli Space. This represents joint work with Yang Hui-He, Vishnu Jejjala, Brent Nelson, and Hal Schenck.

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MS51

Robust Numerical Algebraic Geometry

The field of numerical algebraic geometry consists of algorithms for numerically solving systems of polynomial equations. When the system is exact, such as having integer or rational coefficients, the solution set is well-defined. When solving parameterized polynomial systems with parameter values that are not exact due to imprecision in measurement or prior computations, the structure of the solution set can change. This talk will describe methods to robustly recover nearby parameters corresponding to the desired structure for solutions at infinity, positive dimensional components, multiplicity, and irreducible components. These methods will be demonstrated through illustrative examples and problems arising from the kinematics of mechanisms and robots.

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MS51

Positive Steady-State Varieties of Small Chemical Reaction Networks

Reaction systems with complex chemistry give rise to complicated systems of nonlinear equations that dont lend themselves to analytic solution. Chemical Reaction Network Theory aims to tie aspects of reaction network structure in a precise way to the kinds of dynamics the network might admit. Under the assumption of mass action kinetics, the reaction dynamics are given by polynomial equations. We can thus apply methods from algebraic geometry to study the locus of positive steady states. While some (restricted) techniques exist to understand the positive part of an algebraic variety, this computation remains a significant challenge in general. In this talk, we will present a systematic classification of these varieties for small reaction networks with few species and reactions.

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MS51

Implementing Real Polyhedral Homotopy

In this talk, we revisit the Julia implementation of real polyhedral homotopy. The implementation is based on the real polyhedral homotopy algorithm established by Ergr and de Wolff (2023). This algorithm constructs a homotopy with solution paths that do not cross the real discriminant locus obtained by Viro's patchworking to track the optimal number of real solutions. Based on joint work with Lindberg and Rodriguez (2024), we pose open questions and future outlooks for finding real solutions to polynomial systems.

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MS51

Visualizing Parameter Landscapes

Given a parametrized polynomial system, we present an algorithm for visualizing functions over its real parameter space. We focus on functions which can be easily evaluated given the solutions over those parameters, like the function which counts the number of real solutions or Dietmaier's function. Our algorithm works by refining an initial mesh of parameters only in areas which are deemed 'interesting' with respect to the given function, greatly reducing the number of evaluation oracles necessary to produce a detailed picture.

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MS52

Algebraic Geometry As a Window to Machine Learning

An emerging paradigm in machine learning is the study of the algebraic invariants of the functions parameterized by artificial neural networks, as well as their associated fibers in parameter space and the fibers of an estimator in data space. Algebraic geometry can be exploited to study the behavior of neural networks outside simplified linearized regimes and progress towards closing existing gaps between theory and practice. In this talk, I will discuss some of the recent advancements bridging between the two subjects motivated by technical challenges and emerging phenomena in machine learning for which appropriate theoretical frameworks are still missing.

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MS52

Critical Curvature of Algebraic Varieties

We study the curvature of a smooth algebraic hypersurface $X \subset \mathbb{R}^n$ from the point of view of algebraic geometry. We introduce an algebraic variety that encodes the second fundamental form of X. We apply this framework to enumerate umbilical points and points of critical curvature. In particular, we prove that the number of complex critical curvature points of a smooth algebraic surface $X \subset \mathbb{R}^3$ of degree d is of order d^3.

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MS53

Information Theoretic Bounds on the Size of Spheres in the Lee Metric

Back in 1957, the Lee metric has been introduced for channel coding purposes describing conditions under which a channel matches to the Lee metric under maximum likelihood decoding. Designing such a channel model concretely is an interesting task and allows for applications in communications. Further-more, the Lee metric also gained some attention in the past few years in the area of code-based cryptography, where the understanding of an intentionally introduced error of given weight is crucial. To understand a codes error-correction performance, the size of an n-dimensional sphere in the underlying metrics is crucial. While for the Hamming metric this size is described by a wellknown closed formula, in the Lee metric this quantity is more complex to be computed. In this talk, we derive a bound on the n-dimensional sphere in the Lee metric using information theoretic tools, such as typicality. We derive an asymptotically tight bound based on the distribution of a typical sequence. Additionally, we use the bound to derive the error probability of a code over a channel model designed for the Lee metric and give a sphere-packing bound statement in terms of the block error probability.

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MS53

Bounds and Codes for Multiple Phased Burst Errors

Phased burst errors (PBEs) are bursts of errors that occur at one or more known locations and time intervals. The correction of phased bursts of errors is a classical topic in coding theory, with prominent applications to the design of array codes for memories. We propose a new fine-grained approach to this problem, where not only the number of bursts is considered, but also the number of errors in each burst. By modeling PBEs as an error set in an adversarial channel, we investigate bounds on the size of codes that can correct them by studying the related packing problem. We show upper and lower bounds on the size of linear codes correcting PBEs, and compare them with the traditional bounds for block codes in the asymptotic regime. We observe that tensor codes, although they seem designed to tackle this type of errors, fail to achieve the rate given by the lower bound in many instances. We propose an extension of tensor codes via concatenating codes, which closes this gap at the cost of slightly increasing decoding complexity.

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MS53

Bounds on Sphere Sizes in the Sum-Rank Metric and Coordinate-Additive Metrics

In this talk we present new bounds on the size of spheres in any coordinate-additive metric with a particular focus on improving existing bounds in the sum-rank metric. We derive improved upper and lower bounds based on the entropy of a distribution related to the Boltzmann distribution, which work for any coordinate-additive metric. Additionally, we derive new closed-form upper and lower bounds specifically for the sum-rank metric that outperform existing closed-form bounds.

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MS54

Computing the Endomorphism Ring of a Supersingular Elliptic Curve from a Full Rank Suborder

In this talk, we present a polynomial-time quantum algorithm for computing the endomorphism ring of a supersingular elliptic curve, given a full-rank subring of the endomorphism ring. We begin by connecting the Endomorphism Ring Problem to the Isogeny to Endomorphism Ring Problem introduced by [Chen et al, Hidden Stabilizers, the Isogeny To Endomorphism Ring Problem and the Cryptanalysis of pSIDH]. Building on their work, we then generalize their results to fully resolve both problems. Finally, we discuss the implications of our algorithm for isogeny-based cryptography. This is joint work with Christophe

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MS54

Sesquilinear Pairings and Applications to Class Group Action Attacks

Recent work of Castryck, Houben, Merz, Mula, Buuren, and Vercauteren (2023) introduced a class of pairings on oriented supersingular elliptic curves that yield polynomial-time attacks on certain instances of the vectorization problem. In this talk, I'll discuss recent joint work with Katherine Stange that extends the scope of these attacks. The key ingredient is the introduction of a new class of pairings which are sesquilinear with respect to the module structure of an elliptic curve with CM by a given imaginary quadratic order.

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MS54

Computation of the Deuring Correspondence Using 2-Dimensional Isogenies

The Duering correspondence is a correspondence between the set of supersingular elliptic curves over a finite field $\mathbb{F}_{p^2}$ and the set of maximal orders in the quaternion algebra over $\mathbb{Q}$ ramified at p and the infinity place. Under this correspondence, an isogeny between two supersingular elliptic curves corresponds to an ideal of the quaternion algebra. An algorithm that computes the isogeny corresponding to a given ideal of the quaternion algebra is called an ideal-to-isogeny algorithm. The construction of such algorithms has garnered significant attention in algorithmic number theory and cryptography. Recently, the use of 2-dimensional isogenies to represent isogenies between elliptic curves has also been explored. This talk will present recent progress in constructing ideal-to-isogeny algorithms using 2-dimensional isogenies.

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MS54

Smooth Sandwiches: Old and New Techniques

Almost all isogeny-based cryptosystems are instantiated over a finite field $\mathbb{F}_{p^2}$ such that $p + 1$ is smooth. These primes come in abundance and finding them is a straight forward task. Among these cryptosystems some also require (or benefit from) a large smooth factor in $p - 1$. Unfortunately these sorts of primes are non-trivial to find and one needs more sophisticated approaches to get practical parameters. In this talk we explore this challenging problem and take SQIsign1D (the original version of SQIsign) as a case study to compare several parameters found for this application. Additionally we discuss a related problem of finding large smooth twins and discuss significant improvements over the state-of-the-art.

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MS55

Ranks and Classification of Infinite-strength Tensors

A tensor space is a vector space V, typically of countably infinite dimension, equipped with a multilinear form f. Two tensor spaces are isogenous if each embeds into the other. Tensor decomposition yields natural notions of rank which, in the symmetric case, exploit the strength of the form f and its derivatives. We classify isogeny classes of tensor spaces based on rank, and discuss a new class of representations of End(V). We will also touch on the classification of biquadratic tensor spaces with large automorphism groups. Joint work with Andrew Snowden.

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MS55

Groebner Bases, Syzygies and Polynomial Matrix Multiplication

Given elements $f_1, \ldots, f_m$ in a quotient module R^n/N of finite dimension as a K-vector space, where $R = K[x_1, \ldots, x_r]$ and N is an R-submodule of R^n, we consider the problem of computing a Groebner basis of the module of syzygies of $(f_1, \ldots, f_m)$. This is the set of vectors $(p_1, \ldots, p_m)$ in R^m such that $p_1 f_1 + \cdots + p_m f_m = 0$. An iterative algorithm for this problem was given by Marinari, Mller, and Mora (1993) using a dual representation of the quotient as the kernel of a collection of linear functionals. We will present a recent improvement upon this approach, based on the interpretation of syzygy computations via polynomial matrix multiplication. Remarkable applications are Pad approximation (approximating a polynomial by a rational function of low degree) and interpolation (finding a Groebner basis of the vanishing ideal of a zero-dimensional variety).

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MS55

Typical Ranks of Random Order-three Tensors

We introduce typical ranks of tensors highlighting the difference between the real and the complex. We then focus on the case of order-three tensors and give a geometric interpretation of tensor rank through a result by Friedland. Using this result, we will link typical ranks to linear sections of the Segre variety and show some heuristics on typical ranks of a family of tensors exploiting tools coming from integral geometry. If time allows, we will briefly show how the rank of a real random 3x3x5 tensor depends on the number of real lines on a random cubic surface. This is joint work with P. Breiding and S. Eggleston (University of Osnabrck).

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MS55

Asymptotic Tensor Rank is Characterized by Polynomials

Asymptotic tensor rank is notoriously difficult to determine. Indeed, determining its value for the 22 matrix multiplication tensor would determine the matrix multiplication exponent, a long-standing open problem. On the other hand, Strassen's asymptotic rank conjecture makes the bold claim that asymptotic tensor rank equals the largest dimension of the tensor and is thus as easy to compute as matrix rank. Despite tremendous interest, much is still unknown about the structural and computational properties of asymptotic rank. We prove that the sublevel sets of asymptotic rank are Zariski-closed (just like matrix rank). While we do not exhibit these polynomials explicitly, their mere existence has strong implications on the structure of asymptotic rank. As one such implication, we find that the values that asymptotic tensor rank takes, on all tensors, is a well-ordered set. In other words, any non-increasing sequence of asymptotic ranks stabilizes (”discreteness from above”). In particular, for the matrix multiplication exponent (which is an asymptotic rank) there is no sequence of exponents of bilinear maps that approximates it arbitrarily closely from above without being eventually constant. https://arxiv.org/abs/2411.15789

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MS56

Topological Insights into Graphical Motifs in Connectomes

The connectome of an animal's brain is a labeled graph with a vertex for every neuron and directed edges representing synaptic connections between neurons. The field of connectomics has now produced a complete connectome of the fly brain, as well as partial connectomes in several other organisms. Of particular interest are repeated or overrepresented graphical motifs: that is, subgraphs that occur significantly more often in real connectomes than in families of control graphs with similar connectivity statistics. What can motifs tell us about neural dynamics and computation? Ideas from topology provide tools for classifying and interpreting structural and dynamic properties of these graphs.

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MS56

Uncovering Latent Mechanisms of Short-Term Memory in Recurrent Neural Networks

Short-term memory is critical for cognitive processing, yet its mechanistic basis in biological brains remains incompletely understood. A prominent hypothesis proposes that sequential activity patterns, where neurons fire one after another, support memory maintenance. Although recurrent connections are known to generate such dynamics, a deeper mechanistic and geometric understanding has been lacking.In this talk, I will demonstrate that two distinct mechanisms can subserve sequential activations of neurons in both biological and artificial neural networks: (1) slow-point manifolds, which generate exact sequential activity, and (2) limit cycles, which provide temporally localized approximations. Using analytical models, I will derive scaling laws that govern the likelihood of learning these mechanisms, predicting critical learning rates as a function of memory delay duration and identifying thresholds beyond which memory formation fails. I will conclude with a large-scale empirical validation of these theoretical predictions, in which our team trained and analyzed over 35,000 recurrent neural networks (RNNs).Our findings reveal fundamental, low-dimensional, geometric structures supporting memory in high-dimensional neural systems and generate experimentally testable predictions for systems neuroscience.

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MS56

Attractor-Based Models for Sequences and Pattern Generation in Neural Circuits

Neural circuits in the brain perform a variety of essential functions, including input classification, pattern completion, and the generation of rhythms and oscillations that support functions like breathing and locomotion. There is also substantial evidence that the brain encodes memories and processes information via sequences of neural activity. Traditionally, rhythmic activity and pattern generation have been modeled using coupled oscillators, whereas input classification and pattern completion have been modeled using attractor neural networks. In this talk, I will present models for several different neural functions using threshold-linear networks. Our goal is to develop a unified modeling framework around attractor-based models. The models presented include: a counter network that can count the number of external inputs it receives, encoded as a sequence of fixed points; a model for locomotion that encodes five different quadruped gaits as limit cycles; and a model that connects the sequence of fixed points in the counter network with the attractors of the locomotion network to obtain a new network that steps through a sequence of locomotive gaits. I will also introduce a general architecture for layering networks which produces fusion attractors by minimizing interference between the attractors of individual layers.

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MS57

Deriving Phylogenetic Invariants for Equivariant Models: A Simplified Approach

In recent years, algebraic tools have proven effective in phylogenetic reconstruction, relying on equations that describe the algebraic varieties of Markov processes of molecular substitution on phylogenetic trees, known as phylogenetic invariants. Although the theory enables determining these equations for all equivariant modelsincluding several nucleotide substitution modelspractical applications have mostly been limited to the general Markov model, likely due to the reliance on linear representation theory for deriving these equations. To broaden the scope of algebraic phylogenetics, we show that phylogenetic invariants for equivariant models can be derived directly from those of the general Markov model, bypassing the need for representation theory. Our main result establishes that the algebraic variety of an equivariant model on a phylogenetic tree is an irreducible component of the intersection between the variety of the general Markov model and the linear space defined by the equivariant model. Moreover, we demonstrate that for any equivariant model, phylogenetic invariants crucial for practical tasks like tree reconstruction can be deduced from a single rank constraint applied to matrices derived by flattening the joint distribution at the leaves of the tree.

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MS57

Hypothesis Testing of Phylogenetic Models Using Incomplete U-Statistics

Recently, Sturma, Drton, and Leung proposed a general stochastic test of any model defined by polynomial equality and inequality constraints. Even near irregular points, such as singularities or boundaries that create difficulties for traditional testing methodologies, their method should be applicable. In this talk I will discuss recent work implementing this method and investigating its performance in practice on a number of phylogentic models. This talk is based on joint work with Dave Barnhill, Marina Garrote-Lpez, Elizabeth Gross, Bryson Kagy, John Rhodes, and Joy Zhang.

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MS57

Identifiability of Structural Equation Models

The parameter identifiability problem concerns the question of which parameters of a model can be determined from known data. For structural equation models, this question amounts to testing if the parameters of the model can be determined from the covariance matrix, or more precisely, if the mapping from the parameters to the covariance matrix entries is generically finite-to-one. This amounts to checking the rank of the Jacobian of this mapping evaluated at a generic point. However, an interesting question arises: for what parameter values does this Jacobian matrix drop in rank? This can be answered by examining the singular locus, which is given, e.g., by the determinant of the Jacobian matrix when it is square, or more generally by its minors. We consider some families of graphs and find the corresponding formulas for the singular locus equations.

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MS57

Quartet Tree Inference via Site Weights

A common way to infer large phylogenies is to do it one quartet (i.e., four species) at a time. Many methods exist for doing so. In this talk, I will describe a recently introduced approach based on assigning scoring weights to different site patterns or multi-site patterns in an alignment, which has the advantage of being robust to the kind of heterogeneity that is pervasive in phylogenomic datasets. In particular, I will derive algebraic characterizations of valid weight schemes under various models of sequence evolution.

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MS58

Using Symmetry Reduction in Optimization to Investigate Stability of Shear Flows

Determining nonlinear stability of steady states for complex dynamical systems are notoriously difficult problems, even for the seemingly simplest cases. For example, it is expected that the standard steady state shear profile, 2D planar Couette flow, is globally stable for all Reynolds numbers, however the state-of-the-art analysis is decades old and only proves the stability for relatively low Reynolds numbers. In recent years, a promising computational approach uses polynomial sum-of-squares optimization to find Lyapunov functions based on low-mode projections onto an orthogonal basis of $L^2 \cap H^1$. Critically, physical symmetries inherent in the system can be exploited to reduce the complexity of the resulting optimization problem. We will present on rigorous and practical extensions of this work.

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MS58

Global Optimal Control of Semilinear Elliptic PDEs via Polynomial Optimization

I will present a two-step approach, based on polynomial optimization, to solve nonconvex and infinite-dimensional optimal control problems for semilinear elliptic PDEs with monotone polynomial nonlinearities. In the first step, the finite element method is employed to discretize the PDE control problem into a *correlatively sparse* polynomial optimization problem (POP) with a compact feasible set. In the second step, the POP is relaxed into semidefinite programs (SDPs) using a standard sparsity-exploiting version of the moment-Sum-of-Squares hierarchy for polynomial optimization. Optimal SDP solutions approximate *globally optimal* controls for the original PDE control problem and, crucially, converge to the latter (in a precise sense) under suitable assumptions. Gaps between the theory and the practical performance of the proposed computational method will be showcased on examples. Interestingly, these gaps are deeply related to open questions in algebraic geometry concerning the relaxation of sparse POPs.

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MS58

SONC Decompositions in Chemical Reaction Networks - An Empirical Approach

Chemical reaction network theory models the dynamic behavior of biochemical processes through underlying parametrized ordinary differential equation systems. The notions of multi- and monotationarity determine if a network achieves multiple steady states or just one. Identifying which parametric reaction rate constants correspond to which of these notions is a difficult problem linked to the nonnegativity of a specific polynomial. For the dual phosphorylation network, a previous work by Feliu, Kaihnsa, de Wolff and Yrk provides a sufficient condition for monostationarity via a non-unique decomposition of this polynomial into sums nonnegative circuit polynomials (SONC). We extend their work by a systematic approach to classifying the SONC decompositions and comparing them through empirical experiments, thereby improving on previous conditions for the region of monostationarity in dual phosphorylation.

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MS58

Symmetric SAGE and SONC forms, Exactness and Quantitative Gaps

The classes of sums of arithmetic-geometric exponentials (SAGE) and of sums of nonnegative circuit polynomials (SONC) provide nonnegativity certificates which are based on the inequality of the arithmetic and geometric means. We study the cones of symmetric SAGE and SONC forms and their relations to the underlying symmetric nonnegative cone. As main results, we provide several symmetric cases where the SAGE or SONC property coincides with nonnegativity and we present quantitative results on the differences in various situations. The results rely on characterizations of the zeroes and the minimizers for symmetric SAGE and SONC forms, which we develop. Finally, we also study symmetric monomial mean inequalities and apply SONC certificates to establish a generalized version of Muirhead's inequality.

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MS59

Conditional Tangency and Relative Discriminant

Central properties of algebraic models in a space of data points are captured by complex and projective invariants such as the Euclidean Distance Degree, Maximum Likelihood Degree, and Bottleneck Degree. The geometry of tangencies, and consequently discriminants, of the algebraic variety has proven to be a key tool for computing these invariants. In many cases, one may be interested in a specific sublocus of the variety, necessitating the computation of these invariants conditioned on the subvariety. This perspective requires developing a theory of tangencies relative to the given subvariety, which leads to the introduction of relative discriminants and relative polar geometry. The results presented are based on joint work with Lukas Gustafsson and Luca Sodomaco.

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MS59

Towards Learning the Positive and Real Discriminant

Computing the discriminant of a polynomial system is important for many applications across the sciences. It allows us to characterize the number of physically realizable solutions that we expect as we change our parameters. As the dimension of our system grows, computing the discriminant becomes increasingly computationally complex. In this talk, we present a set of software tools for training a polynomial neural network approximating our discriminant and some experimental results.

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MS59

Discriminants in the Sciences: An Overview

In this first introductory talk, we would like to summarise the current results regarding discriminants across a variety of subfields, such as Euclidean distance discriminants, Euler discriminants, maximum likelihood discriminants, Terracini loci, logarithmic discriminants, associated hypersurfaces, Chow-Lam forms, etc. And to shed light on connections between these different notions of discriminants. This is relevant since many problems in algebraic geometry and applications depend on parameters. Discriminants characterize parameter values for which the solution to the problem changes qualitatively or quantitatively.

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MS60

Unique Existence of Positive Solutions to Generalized Polynomial Equations

We consider solutions to parametrized systems of generalized polynomial equations (with real exponents) in n positive variables, involving m monomials with positive parameters; that is, $x \in \mathbb{R}^n_>$ such that $A\,(c \circ x^B) = 0$ with coefficient matrix $A \in \mathbb{R}^{l \times m}$, exponent matrix $B \in \mathbb{R}^{n \times m}$, parameter vector $c \in \mathbb{R}^m_>$, and componentwise product $\circ$. As our main result, we characterize the existence of a unique solution (modulo an exponential manifold) for all parameters in terms of the relevant geometric objects of the polynomial system, namely the *coefficient polytope* and the *monomial dependency subspace*. We show that unique existence is equivalent to the bijectivity of a certain moment/power map, and we characterize the bijectivity of this map using Hadamard's global inversion theorem. Furthermore, we provide sufficient conditions in terms of sign vectors of the geometric objects, thereby obtaining a multivariate Descartes' rule of signs for exactly one solution.

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MS60

Local Stability in Binomial Differential Inclusions

We extend the classical result by Horn and Jackson on the asymptotic stability of complex-balanced equilibria of mass-action systems. As it turns out, all dynamical systems are asymptotically stable that can be embedded in certain *binomial differential inclusions*. For the proof, we use a new decomposition of the Laplacian matrix which allows to write weakly reversible mass-action systems as sums of binomial terms (just like reversible systems). In particular, this suggests to consider regions in the positive orthant with given monomial evaluation orders and corresponding polyhedral cones in logarithmic coordinates (defining the differential inclusion). As another application, the core Laplacian matrix, monomial evaluation orders, and corresponding hyperplane arrangements can be used to study the linear stability of complex-balanced equilibria of *generalized* mass-action systems.

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MS60

Zero Eigenvalues in Networks that Admit a Monomial Parametrization

Reaction networks, whose equilibrium points are defined by binomial ideals are significant and well studied instances of chemical reaction networks. For such networks the monomial parametrization of positive steady states has been used to determine the capacity of multistationarity. In this talk we will discuss networks with smooth positive steady state varieties and link the monomial map to the study of bifurcation points in those systems. A strong emphasis will be placed on applications to well known phosphorylation networks.

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MS61

Computing the Tropical Abel-Jacobi Transform of Metric Graphs

Metric graphs are widely used to model complex real-world data. The problem of how to extract and represent the geometric and topological information from graph data has given rise to various research areas, such as graph representation learning and graph reconstruction. Despite the immense literature in machine learning, the representation of metric graphs as abstract tropical curves has never been studied previously in computational and machine learning contexts. In this talk, I will present an algorithm to compute the tropical Abel–Jacobi transform for metric graphs, and discuss potential applications of the tropical Abel–

Jacobi transform to graph embedding and topological data analysis.

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MS61

Toward Non-Archimedean Machine Learning

In machine learning, the construction and manipulation of models based on hierarchical data, such as trees, genetic data, and word-embeddings, is a well-established problem. For instance, to embed hierarchical data, it has been shown that a low-dimensional hyperbolic space is just as effective as a high-dimensional Euclidean one. In this talk, we consider data that lies in a space over a non-Archimedean valued field and explain how standard methods, such as gradient descent and linear regression, translate to the new setting.

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MS61

Pair-of-pants Decompositions Using Tropical Geometry

The piecewise linear objects found in tropical geometry arise naturally when taking limits of amoebas. In this process, a large amount of the topology of the original algebraic variety is however lost. To recover the remaining information, we can consider the angle sets of the initial varieties, which are often called coamoebas. In this talk, I will discuss recent work with Yassine El Maazouz on recovering the topology of a variety by gluing coamoebas of hyperplane complements, which arise naturally as the initial varieties of smooth tropicalizations. I will focus on the main result in our upcoming paper, which says that a hyperplane complement is homotopy equivalent to its coamoeba. After this, I will discuss how the various initial degenerations of the hyperplane complement give rise to subcomplexes of the coamoeba. Using the theory of Kato-Nakayama spaces, we are then able to generalize the well-known pair-of-pants decompositions for Riemann surfaces to higher-dimensional varieties.

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MS62

Approximating Singular Integrals Using Variational Inference

The marginal likelihood or evidence in Bayesian statistics contains an intrinsic penalty for larger model sizes and is a fundamental quantity in Bayesian model comparison. Over the past two decades, there has been steadily increasing activity to understand the nature of this penalty in singular statistical models, building on pioneering work by Sumio Watanabe. Unlike regular models where the Bayesian information criterion (BIC) encapsulates a first-order expansion of the logarithm of the marginal likelihood, parameter counting gets trickier in singular models where a quantity called the real log canonical threshold (RLCT) summarizes the effective model dimensionality. In this article, we offer a probabilistic treatment to recover non-asymptotic versions of established evidence bounds as well as prove a new result based on the Gibbs variational inequality. In particular, we show that mean-field variational inference correctly recovers the RLCT for any singular model in its canonical or normal form. We additionally exhibit sharpness of our bound by analyzing the dynamics of a general purpose coordinate ascent algorithm (CAVI) popularly employed in variational inference.

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MS63

Pandora.jl - A Julia Package for the Automated Study of Enumerative Problems

We introduce the julia package Pando(RA) [Parallel, Automated, Numerical, Discovery, and Optimization (Research Aid)] for computational enumerative geometry. Relying on the respective symbolic/numerical engines of Oscar.jl and HomotopyContinuation.jl, our hybrid software implements algorithms which explore degrees, Galois/monodromy groups, and reality properties of enumerative problems automatically. It is designed to be used by experts and non-experts alike.

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MS63

Homotopy Continuation for Julia Set Computations

We consider dynamical systems given by the function $z \mapsto z^2 + c$ in the complex plane. For most values of c, the Julia sets defined by these functions are non-algebraic (fractals). Using (algebraic) homotopy continuation, we approximate and study these non-algebraic objects for values of c in the main cardioid of the Mandelbrot set. As a further application, we use this approach to compute Markov partitions of these Julia sets.

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MS63

Solving Bihomogeneous Systems with a Zero-dimensional Projection

We study bihomogeneous systems defining, non-zero dimensional, biprojective varieties for which the projection onto the first group of variables results in a finite set of points. To compute (with) the 0-dimensional projection and the corresponding quotient ring, we introduce linear maps that greatly extend the classical multiplication maps for zero-dimensional systems, but are not those associated to the elimination ideal; we also call them multiplication maps. We construct them using linear algebra on the restriction of the ideal to a carefully chosen bidegree or, if available, from an arbitrary Grbner basis. The multiplication maps allow us to compute the elimination ideal of the projection, by generalizing FGLM algorithm to bihomogenous, non-zero dimensional, varieties. We also study their properties, like their minimal polynomials and the multiplicities of their eigenvalues, and show that we can use the eigenvalues to compute numerical approximations of the zero-dimensional projection. Finally, we establish a single exponential complexity bound for computing multiplication maps and Grbner bases, that we express in terms of the bidegrees of the generators of the corresponding bihomogeneous ideal.

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MS63

Tropical Homotopy Continuation

Given a polynomial system over the complex numbers, we demonstrate two things: First, we explain how to construct homotopies with the optimal number of paths from their tropical points. Second, for some types of polynomial systems, we explain how to obtain said tropical points using homotopies over the tropical numbers. This is joint work with Oliver Daisey, Paul Helminck, and Oskar Henriksson.

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MS64

Certified Curve Tracking

In this talk, we consider the problem of computing a certified approximation to a regular curve in $\mathbb{R}^n$. Specifically, given a regular curve and an approximate point on it, our goal is to construct a tubular neighborhood containing the curve. Our approach uses certified homotopy tracking methods and interval arithmetic, extending the work of Duff-Lee and Guillemot-Lairez. As a key application, we provide a certified algorithm for approximating the image of a generic projection of a curve in $\mathbb{R}^n$ to the plane.

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MS64

Versatile Optimization of Enumerative Problems

In 1998, Dietmaier showed that a certain polynomial system can have 40 real solutions using a hill-climbing algorithm. Since then, his ideas have been used to show other enumerative problems can have many real solutions, although to the best of our knowledge a general implementation does not exist. We will discuss our implementation of a hill-climbing algorithm for enumerative problems which is versatile, in the sense that it accepts objective functions which take values in totally ordered sets other than the real numbers. We showcase our implementation on several examples.

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MS64

Numerically Computing Connected Components of Real Projective Hypersurfaces

A common task in various areas is to compute connected components of a solution set. This talk introduces a computational approach for computing relationships between connected components via Euler characteristics and summarized as a bipartite connectivity graph. This approach uses routing functions to perform computations on both a hypersurface and its complement. Examples in two-dimensional projective space will be visualized on the sphere, utilizing bipartite graphs to demonstrate this new computational approach and describe the nesting structure of the connected components. This is joint work with Jonathan Hauenstein.

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MS65

Applications of Grobner Bases to the Geometry of Value Functions

This talk explores the intersection of algebraic geometry and statistical learning theory through the application of Grbner bases to examining the geometry of the set of value functions for a reinforcement learning problem. After introducing Grbner bases and their construction using Buchberger's algorithm, we connect this algebraic machinery to reinforcement learning by examining the geometry of value functions in Markov Decision Processes (MDPs). Using results from interval matrix theory, we first describe the feasible space of value functions in terms of the parameters of the MDP before demonstrating the use of elimination theory to implicitize our parametric descriptions, providing an intrinsic geometric characterization of the space of value functions. This work bridges modern algebraic geometry with statistical learning theory, offering new tools for analyzing and understanding the geometric structure of value functions in MDPs.

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MS65

Machine Learning to Predict and Enhance Grbner Base Computations

We present a detailed approach and several models aimed at predicting the combinatorial dimension and maximum total degree of a given Grbner basis (GB) from its generating set. This effort supports a broader randomized algorithm for computing minimal GBs efficiently. The dataset, generated using Macaulay2, consists of restricted generating sets of five binomials with five variables and maximum total degrees of 15 or less. Neural network regression models are trained on this dataset to capture structural patterns, outperforming traditional methods in predicting key complexity metrics, but posing potential problems in high variability. The results support a novel way of computing a Gröbner basis, while highlighting the challenges posed by challenges of dataset generation.

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MS65

On the Loss Landscape of Deep Polynomial Networks

Understanding the loss landscape of the deep networks can provide many insights into the theoretical understanding of how the networks learn and why they work so well in practice. Recently, the existence and importance of flat minima within the loss landscapes have been a heavily debated topic in the literature. In this talk, I will present two symmetry breaking methods that provably remove all the flat minima (and flat stationary points) from the loss landscape for any deep feedforward network of any depth and width as long as the activation function is a smooth function. The methods can be essentially viewed as generalized regularizations of the loss function. The proposed methods are applied on the polynomial neural networks and a first result on estimates of the number of isolated solutions is provided.

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MS66

Codes and Designs in Finite General Linear Groups

Permutation codes, which are subsets of the symmetric group endowed with the Hamming distance, have been studied since the 1970s and find applications in areas such as power-line communications. In the context of association schemes, these codes can be interpreted as duals of transitive subsets of the symmetric group, where the latter are designs in the association scheme. In this talk, we give an overview of results known for symmetric groups and discuss a q-analog setting in finite general linear groups. This is joint work with Kai-Uwe Schmidt.

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MS67

Prism: Simple And Compact Identification and Signatures From Large Prime Degree Isogenies

Recent discussions on the security of the SQIsignHD signature scheme raised interest on a relevant gap in our current understanding of isogeny computations. In fact, thanks to recent developments in the use of Quaternion Algebras and Abelian Varieties, we can use information on the Endormorphism Ring of the curve to compute arbitrary degree isogenies. However, without endomorphisms, we still have no clue on how to do that. In this talk we show, instead, how to leverage this gap to get a Hash-and-Sign signature based on the same building blocks of SQIsign, but with a much simpler description. The new scheme preserves the same efficiency and compactness properties of other isogeny based signatures, but it is also suitable for more advanced constructions, on which we briefly touch.

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MS67

Algebraic Pathfinding Between Superspecial Abelian Surfaces

In this talk, we will discuss a polynomial-time algorithm for finding a “connecting matrix” between two given matrices g, g' having entries in the quaternion algebra B_p over $\mathbb{Q}$ ramified at p and ∞, where each matrix encodes a principally polarized superspecial abelian surface in characteristic p. This algorithm should be thought of as a two-dimensional analog of the celebrated KLPT algorithm for finding a connecting ideal between two given maximal orders in B_p, and it has similar applications. One application lies in finding collisions for the two-dimensional analog of the Charles-Goren-Lauter hash function as soon as the matrix g corresponding to the starting surface is known. This is joint work with Thomas Decru, Pter Kutas, Abel Laval, Christophe Petit and Yan Bo Ti.

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MS67

Practical Fault Injection Attacks on Constant Time Csidh and Mitigation Techniques

Commutative Supersingular Isogeny Diffie-Hellman (CSIDH) is an isogeny-based key exchange protocol which is believed to be secure even when parties use long-lived secret keys. To secure CSIDH against side-channel attacks, constant-time implementations with additional dummy isogeny computations are employed. In this study, we demonstrate a fault injection attack on the constant-time real-then-dummy CSIDH to recover the full static secret key. We prototype the attack using voltage glitches on the victim STM32 microcontroller. The attack scheme, which is based on existing research which has yet to be practically implemented, involves getting the faulty output by injecting the fault in a binary search fashion. Our attack reveals many practical factors that were not considered in the previous theoretical fault injection attack analysis, e.g., the probability of a failed fault injection. We bring the practice to theory and developed new complexity analysis of the attack. Further, to mitigate the possible binary search attack on real-then-dummy CSIDH, dynamic random CSIDH was proposed previously to randomize the order of real and dummy isogeny operations. We explore fault injection attacks on dynamic random CSIDH and evaluate the security level of the mitigation. Our analysis and experimental results demonstrate that it is infeasible to attack dynamic random CSIDH in a reasonable amount of time when the success rate of fault injection is not consistent over time.

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MS67

Constant-Time Lattice Reduction for SQIsign

SQIsign is the first isogeny scheme considered for practical use that requires computations in a quaternion algebra. These computations operate on secret data, and thus they must be protected against side-channel attacks. However, relatively little research on secure implementations of SQIsign has been reported in the literature, possibly due to the many variants and the high complexity of the scheme. Lattice reduction is a key component of the signing procedure of all SQIsign variants, and it is the most performance-critical basic algorithm on the quaternion side of the scheme. Given that lattice reduction has been used previously in cryptanalysis, but not in constructive cryptography, no known lattice reduction algorithm is resistant against timing side-channels. This talk presents a new, constant-time lattice reduction algorithm, developed in collaboration with Ott Hanyecz, Alexander Karenin, Elena Kirshanova and Pter Kutas. We first explore the use of lattice reduction in SQIsign. Then, we analyze different existing lattice reduction algorithms, and present our constant-time version, which is based on the BKZ-2 algorithm. Finally, we explain some implementation choices and discuss the performance using two sets of parameters: one for provable guarantees on its output, and one for speed with a reasonable success rate.

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MS68

Convolutional Tensors

Convolutional neural networks are a popular deep learning architecture. Each layer of such a network is represented by a convolutional tensor. The composition of layers corresponds to a tensor multiplication that yields another convolutional tensor. We explain how this multiplication can be interpreted as polynomial multiplication, and use this to study the space of convolutional tensors that can be factorized into a fixed number of convolutional tensors. We also explore how optimization on this space of tensors corresponds to training convolutional networks, and how geometric properties of this space affect the optimization. This talk is based on several joint works with El Mehdi Achour, Giovanni Marchetti, Thomas Merkh, Guido Montfar, Holger Rauhut, Vahid Shahverdi, Matthew Trager.

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MS68

Multilinear Quivers and Geometric Invariant Theory

Multilinear quiver representations are generalizations of quiver representations, allowing graphs with directed edges from multiple source vertices to one target vertex, and assigning linear maps on those edges from a tensor product of the vector spaces assigned to the source vertices. They have been introduced to study the geometry of the space of singular vectors of a system of tensors, enabling one to generalize well-known formulae for counting eigenvectors and singular vectors of tensors. In this talk, we study multilinear quivers from the perspective of geometric invariant theory (GIT) and investigate potential applications to (non-reductive) GIT.

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MS68

Computing Generalized Additive Decompositions with Low Regularity

Generalized additive decompositions of degree-d forms (equiv. order-d symmetric tensors) unveil intrinsic information about the considered objects. From a geometric perspective, such decompositions evince 0-dimensional schemes that resolve the membership problem in certain (joints of) osculating varieties to the Veronese variety. For both theoretical and applied purposes, we are interested in computing additive decompositions evincing schemes of minimal length. This minimal integer is usually referred to as the (generalized additive) rank of the considered form. Explicitly computing the rank and corresponding minimal decomposition of a given form is known to be extremely challenging without additional assumptions. In this talk, we discuss this decomposition problem for forms evincing minimal schemes with low regularity. When this regularity does not exceed d/2, we show that such a task can be conveniently performed at the cost of one simultaneous Jordanization of size equal to the scheme length. We also show that by employing the idempotents of the local Artinian Gorenstein algebras defined by the considered 0-dimensional schemes, we can reduce the global decomposition problem to a series of independent local instances, improving the linear algebra part of the decomposition routines. This is joint work with E. Barrilli and B. Mourrain.

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MS68

Detecting Cluster Patterns in Tensor Data

I will discuss fast algorithms to detection continuous clustering in tensor data, and discuss uniqueness claims of the results. The study has roots in Lie theory but the presentation will be accessible to a general audience.

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MS69

Extracting Latent Structures from Neural Data and Insights About How we Perceive Space

High-dimensional neural population recordings pose significant analytical challenges due to their inherent heterogeneity: meaningful neural representations often involve specific subsets of neurons active only during particular time windows. Methods assuming population-wide homogeneous activity typically fail, either missing these localized structures entirely or conflating signals from functionally distinct subgroups. This talk discusses mathematical and computational strategies designed to overcome this limitation by identifying latent structures embedded within neural activity. Our approach focuses on methods that explicitly search for coherent structures within subsets of neurons and time points, thereby assembling a picture of the underlying representation from localized evidence. We demonstrate the effectiveness of these principles from statistical learning and topological data analysis through applications in systems neuroscience, such as uncovering toroidal representations of space in the brains of rat pups. The results also suggest that our biology maintains a priori structure for space, through which we organize the massive data coming from the external world that such representations are innate, and not empirical concepts abstracted from experience.

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MS69

Ensemble Latent Space Geometry Reveals Universals for Reach and Grasp Coding in Motor Cortex

While single neuron computations are subject-specific and embedded in a high-dimensional space, ensemble activity patterns during behavior often exhibit low intrinsic dimensionality. We suggest that equivalent computations occurring on different ensembles across individuals can be identified by comparing features of intrinsic latent space geometry that define the transformations of a networks spike patterns. We compare intrinsic geometries of latent space representations derived from spike train similarity metrics on task-generated activity patterns, and we find neural ensemble activity converges to a common geometry across different non-human primates when performing the same reaching or grasping behaviors. These results highlight the value of geometric approaches in uncovering invariant features of brain computations with applications that may improve the function, speed, and reliability of brain-computer interfaces which rely on accurate and repeatable neural decoding.

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MS69

Learning Circular Coordinate Systems via Spike Timing Dependent Plasticity

Circular coordinate systems are ubiquitous in neural encodings. Important examples include simple cells in primary visual cortex, head-direction cells, and grid cells in entorhinal cortex. In each of these systems, the information encoded by the neural populations is propagated to downstream regions of the brain for further processing. We investigated the following question. How do biological neural populations, with high levels of noise and in which individual neurons do not have access to population-level information, propagate circular coordinate systems? Our primary results are that 1) with biologically plausible levels of inhibition, random architectures fail to propagate circular coordinate systems, 2) in these cases, updating network connections via spike-timing dependent plasticity (STDP) suffices to form networks that propagate circular coordinate systems provided the output population has at least several times as many neurons as the input layer, and 3) STDP forms such networks through a nonlinear processing of learning "local" geometry of the input stimulus space.

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MS70

Phylogenetic Invariants for Algebraic Time-Reversible Models: From TN93 to Its Submodels

Time-reversible models form a class of phylogenetic models widely used by biologists which can also be studied algebraically. These models allow for a non-uniform stationary distribution while remaining amenable to some of the tools used for group-based models. In particular, defining an appropriate inner product allows for diagonalization of the transition matrices with respect to an orthogonal eigenbasis leading to a change of coordinates that generalizes the Fourier transform used for group-based models. We implement this approach to provide model and topology phylogenetic invariants for the Felsenstein 81 and Felsenstein 84 (F81 and F84) submodels of the Tamura-Nei (TN93) nucleotide substitution model on particular phylogenetic trees, describe the phylogenetic varieties for F81 and F84 on open sets containing biologically relevant points, and relate these varieties to those of TN93.

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MS70

The Pfaffian Structure of CFN Phylogenetic Network

This talk will focus on the study of ideals of phylogenetic invariants of the Cavender-FarrisNeyman (CFN) model on a phylogenetic network to describe the invariants, which are useful for network inference. It was previously shown that to characterize the invariants of any level-1 network, it suffices to understand those of all sunlet networks. It is shown that the parameterization of an affine open patch of the CFN sunlet model, which intersects the probability simplex, factors through the space of skew-symmetric matrices via Pfaffians. This affine patch is isomorphic to a determinantal variety, and an explicit Grbner basis is given for the associated ideal. Furthermore, we will see that sunlet networks with at least six leaves are identifiable using only these polynomials and run extensive simulations, which show that these polynomials can be used to accurately infer the correct network from DNA sequence data.

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MS70

Convex Algebraic Geometry of the Allele Frequency Spectrum

Demographic inference aims to use current genetic data to infer historical population dynamics. One technique relies on the sensitivity of the allele frequency spectrum (AFS) to population size history. The AFS is a vector that records how many mutations appear in k out of n individuals, where k ranges from 1 to n-1. The expected AFS, computed for an appropriate random process, can then be thought of as a function of population size history. The set of all expected AFS for any possible population size history forms a semialgebraic set called the coalescence manifold. In this work, joint with G. Scholten and C. Vinzant, we establish a connection between the AFS and the univariate moment problem for piecewise-constant density functions on the interval. We prove that any set of n moments can be represented, up to closure, by a step function with at most n-1 breakpoints. Applying this result, we show that any point in the nth coalescence manifold corresponds to a piecewise-constant population history with at most n-2 changes.

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MS70

Tropical Fermat-Weber Tree Reconstruction

Fermat-Weber (FW) points are solutions to a classical location problem that seeks to minimize the average distance to a finite sample. They have many applications in statistical analysis and optimization, including phylogenetics. In this work, we consider FW points utilizing the tropical metric (also known as the generalized Hilbert projective metric) and, alternatively, the asymmetric tropical distances as proposed by Comaneci and Joswig on simulated sequence data for tree reconstruction. In this talk we discuss an application of these tropical FW points to estimating a phylogenetic tree from a given alignment via the Bayes estimators proposed by Huggins et al.

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MS71

SAGE/SONC Bodies for Polynomial Optimization

A central and challenging problem in optimization and real algebraic geometry is to understand the convex hull of the variety of an ideal. Classical approaches, such as the Laserre hierarchy and theta bodies, obtain a sequence of nested convex relaxations using degree-bounded representations. We generalize this framework, which typically relies on the sums-of-squares (SOS) cone, to arbitrary cones of non-negative polynomials. We then focus on the cone of SAGE (sums of AM-GM exponentials), equivalently SONC (sums of nonnegative circuit), polynomials, where it is more natural to seek a *sparse* representation of a polynomial modulo an ideal, rather than a bounded degree representation. These SAGE/SONC relaxations can be computed efficiently in terms of the relative entropy cone. We consider the following questions:

1. Given an ideal, can we find the sparsest SAGE/SONC representation where this relaxation is exact?
2. Under what circumstances, if any, does a SAGE/SONC approach outperform SOS techniques? One hopes to find varieties for which SAGE/SONC methods are exact, while SOS methods do not converge.
3. Can we provide a more complete understanding of certain families of varieties, such as finite point sets, 0-1 point sets, or something else which is more amenable to SAGE/SONC techniques?

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MS71

Exploiting Symmetry in Optimization over Basis-

dependent Cones: Semisimple *-algebras with Application to Scaled Diagonally Dominant Matrices

Symmetry in semidefinite programs can be exploited by transforming to a symmetry-adapted basis that block diagonalizes the matrix variable. This transformation preserves the optimal value, as the cone of positive semidefinite matrices is basis-independent. For basis-dependent cones, optimization problems with symmetries can similarly use a symmetry-adapted basis to block diagonalize the matrix. However, this change of basis generally alters the constraint cone and may affect the optimum. In this talk, I introduce a framework to establish an a priori relationship between the optima of the original and symmetry-reduced problems, depending on the choice of symmetry-adapted bases. This framework is applied to prove results for optimization problems over the cone of scaled diagonally dominant matrices. I will also discuss the role of the representation theory of (nonsplit) semisimple *-algebras in constructing and analyzing symmetry-adapted bases.

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MS71

Combinatorial Challenges in Polynomial Optimization Problems Arising in Dynamical Systems

We start by recalling two recent results using sums of nonnegative circuit polynomials (SONC) as certificates of nonnegativity tackling polynomial optimization problems emerging from certain dynamical systems: First, deciding mono- vs. multistationarity in 2- and n-site phosphorylation networks in terms of their kinematic parameters (joint with E. Feliu, N. Kaihnsa, and O. Yürük). Second computing Lyapunov functions for polynomial ODE systems (joint with J. Heuer). Both of these results lead to (different) combinatorial follow-up problems, which I will briefly explain. This talk is meant as an invitation to the audience to further explore the intersection of polynomial optimization, dynamical systems, and combinatorics.

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MS72

Matroid Stratification of the ML Degree of Segre Products

We study the maximum likelihood (ML) degree of discrete exponential independence models and models defined by the second hypersimplex. For models with two independent variables, we show that the ML degree is an invariant of a matroid associated to the model. We use this description to explore ML degrees via hyperplane arrangements. For independence models with more variables, we investigate the connection between the vanishing of factors of its principal -determinant and its ML degree. Similarly, for models defined by the second hypersimplex, we determine its principal -determinant and give computational evidence towards a conjectured lower bound of its ML degree.

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MS72

Logarithmic Discriminants of Hyperplane Arrangements

A recurring task in particle physics and statistics is to compute the complex critical points of a product of powers of affine-linear functions. The logarithmic discriminant characterizes exponents for which such a function has a degenerate critical point in the corresponding hyperplane arrangement complement. In this talk I will introduce logarithmic discriminants, discuss interesting examples and present structural results regarding irreducibility, dimension, degree and positivity for (sufficiently general) hyperplane arrangements.

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MS72

The Chow-Lam Form

The classical Chow form encodes any projective variety by a single equation. These were invented by Chow and van der Waerden as coordinates on a moduli space of projective varieties. In this talk we introduce the Chow-Lam form for subvarieties of a general Grassmannian, and show that in "most" cases a variety can be completely recovered from its Chow-Lam form. These forms first arose in physics, as universal projection formulas for positroid varieties, pioneered by Thomas Lam. This is based on joint works with Bernd Sturmfels and Kristian Ranestad.

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MS72

Euler Stratifications of Hypersurface Families

An Euler stratification is a stratification of a family of projective or very affine varieties according to their topological Euler characteristic. This is relevant in particle physics and algebraic statistics: it fully describes the dependence of the number of master integrals, respectively the maximum likelihood degree, on kinematic or model parameters. In this talk new algorithms are presented to compute such stratifications for hypersurface families.

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MS73

The Generic Geometry of Steady State Varieties of Reaction Networks

The mathematical formalism of the study of (bio)chemical reaction networks goes back to at least the works of Feinberg, Horn and Jackson in the 70ies. However, several fundamental questions about the positive part of the algebraic varieties defined by the steady state equations remained unsolved. In this talk, I will describe several properties that these semialgebraic sets display generically with respect to the parameter values. The main message is that the generic geometries of the semialgebraic sets and the complex algebraic variety agree, to a great extent. I will address in particular generic finiteness of steady states, nondegeneracy, toricity, absolute concentration robustness, and the nondegenerate multistationarity conjecture. This is joint work with Oskar Henriksson and Beatriz Pascual Escudero. The talk is based on the results on the three manuscripts with arXiv references: 2304.02302, 2411.15134, 2412.17798.

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MS73

Sampling the Space of Monostationarity in the Dual Phosphorylation Network

The dual phosphorylation network provides an essential component of intracellular signaling, affecting both phenotypic expressions and cellular metabolism. For specific choices of reaction rate constants, this system can either exhibit multistationarity or monostationarity. The switch-like behavior of multistationarity is particularly relevant for cellular decision-making. Precisely classifying the parameter regions corresponding to monostationarity in this network remains an open problem. In "[Feliu, Kaihnsa, de Wolff and Yrk, The kinetic space of multistationarity in dual phosphorylation, 2020]", a sufficient condition for this chemical reaction network's monostationarity has been proposed, linking this property to the nonnegativity of a certain polynomial. Their approach relies on decomposing the polynomial into so-called circuits that cover the associated Newton polytope. This circuit decomposition is not unique. We extend their work by systematically classifying all possible circuit decompositions in the dual phosphorylation network. This classification allows us to find improved sufficient conditions for monostationarity by sampling regions where the circuit decompositions are nonnegative. Still, the resulting criterion is not necessary. As the next step, we consider mixed covers that combine multiple circuit covers to enhance the condition further, though this poses significant computational challenges.

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MS73

Parameter Estimation in ODE Models with Noisy Data

Many biological models are given by systems of ODEs with unknown parameters. We will discuss a new differential algebra based approach to estimating values of these parameters from data when data have noise. The algorithm is based on Lie derivative computation, polynomial approximation, and polynomial system solving.

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MS73

Identifiability of Directed-cycle Linear Compartmental Models

This talk addresses the problem of parameter identifiability that is, the question of whether parameters can be recovered from data for linear compartment models. We focus on an important class of linear compartmental models: those in which the underlying graph is a directed cycle. Our main result complete characterizes the directed-cycle models for which every parameter is (generically, locally) identifiable. Additionally, we investigate the "singular loci' of these models. Here, the singular locus refers to the measure-zero set of parameter vectors that are not identifiable. In other words, this set represents the parameters that can not be estimated from data and therefore ideally should be avoided in applications.

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MS74

Stationary Regimes of Piecewise Linear Dynamical Systems with Priorities

Dynamical systems governed by priority rules appear in the modeling of emergency organizations and road traffic. These systems can be modeled by piecewise linear time-delay dynamics, specifically using Petri nets with priority rules. A central question is to show the existence of stationary regimes (i.e., steady state solutions)—taking the form of invariant half-lines—from which essential performance indicators like the throughput and congestion phases can be derived. Our primary result proves the existence of stationary solutions under structural conditions involving the spectrum of the linear parts within the piecewise linear dynamics. This extends a fundamental theorem due to Kohlberg (1980) from nonexpansive dynamics to a broader class of systems. The proof of our result relies on topological degree theory and the notion of "Blackwell optimality" from the theory of Markov decision processes. Finally, we validate our findings by demonstrating that these structural conditions hold for a wide range of dynamics, especially those stemming from Petri nets with priority rules, and we illustrate on real-world examples from road traffic management and emergency call center operations.

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MS74

Tropical SVMs via Mean-Payoff Games

In 2006, Grtner and Jaggi introduced a tropical analogue of support vector machines, using a single tropical hyperplane in dimension n to separate n classes of points. Efficiently computing tropical separators has remained an open problem. In this work, we develop a scalable algorithmic approach to compute tropical classifiers, based on a reduction to mean-payoff games. We first address the case of separating two sets of points. We show that a separating hyperplane with an optimal margin can be found by computing the eigenvector of a particular Shapley operator, involving projectors onto the tropical convex hulls of the two point sets. The tropical eigenvalue yields the margin; in the case of non-separable instances, it provides a measure of inseparability. We then extend this approach to the multi-class setting, showing how to separate n classes of points via a reduction to a non-linear eigenproblem for a novel Shapley operator. Finally, we combine tropical classifiers with linear feature maps to construct piecewise-linear classifiers.

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MS74

A Tropical Approach to Rigidity: Counting Realisations of Frameworks

A realisation of a graph in the plane as a bar-joint framework is rigid if there are finitely many other realisations, up to isometries, with the same edge lengths. Each of these finitely-many realisations can be seen as a solution to a system of quadratic equations prescribing the distances between pairs of points. For generic realisations, the size of the solution set depends only on the underlying graph so long as we allow for complex solutions. The realisation number of the graph is the cardinality of this solution set. In this talk, we will provide a characterisation of the realisation number of a minimally rigid graph as a tropical intersection product. Explicitly, our characterisation uses tropical geometry to express the realisation number as an intersection of Bergman fans of the cycle matroid. As a consequence, we derive a combinatorial upper bound on the realisation number involving the Tutte polynomial. Moreover, we provide computational evidence that our upper bound is usually an improvement on the mixed volume bound.

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MS74

On the Genus of One Degree of Freedom Planar Linkages via Tropical Geometry

In this talk we focus on studying the configuration spaces of graphs realised in $\mathbb{C}^2$, such that the configuration space is, after normalisation, one dimensional. If this is the case, then the configuration space is, generically, a smooth complex curve, and can be seen as a Riemann surface. The property of interest is the genus of this curve. Using tropical geometry, we give an algorithm to compute this genus. We discuss an implementation in Python and give various examples

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MS75

Hypergeometric Integrals on a Punctured Riemann Surface

We study integrals on a punctured Riemann surface of genus $g > 1$. The twisted cohomology groups associated to these integrals were studied by Watanabe. Here we study the corresponding twisted homology groups of these integrals, and bilinear pairings among homology and cohomology groups. This is joint work with Andrzej Pokraka and Lecheng Ren.

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MS76

Equivalences Between SAGBI and Khovanskii Bases

SAGBI and Khovanskii bases generalize Grbner bases to subalgebras of polynomial rings and valued domains. We will discuss a partial connection between Khovanskii bases and SAGBI bases of quotient algebras. This link lets us apply computational tools of SAGBI bases to the theory of Khovanskii bases. Using this approach, we compute Newton-Okounkov bodies from SAGBI bases data, showing how existing computer algebra tools can be used for computing with Khovanskii bases.

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MS76

Equation-by-equation Solvers and Equidimensional Decomposition

The vanishing set of a sequence of polynomials is a complex variety whose decomposition into subvarieties of pure dimension can be computed in sequential fashion: components in the decomposition of the variety defined by an initial subsequence are intersected with the hypersurface defined by the next polynomial. This can be done either in the classical symbolic framework — relying on Groebner bases — or by methods of numerical algebraic geometry — relying on homotopy continuation. I will highlight the strengths of existing approaches, their bottlenecks, and difficulties arising in implementations. (Part of this is based on work with T.Duff and J.I.Rodriguez.)

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MS76

On the Typical and Atypical Solutions to the Kumamoto Equations

The Kuramoto model is a dynamical system that models the interaction of coupled oscillators. There has been much work to effectively bound the number of equilibria to the Kuramoto model for a given network. By formulating the Kuramoto equations as a system of algebraic equations, we first relate the complex root count of the Kuramoto equations to the combinatorics of the underlying network by showing that the complex root count is generically equal to the normalized volume of the corresponding adjacency polytope of the network. We then give explicit algebraic conditions under which this bound is strict and show that there are conditions where the Kuramoto equations have infinitely many equilibria.

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MS76

A Randomized Computational Framework for Grbner Bases

We present a new approach to compute Grbner bases by blending machine learning techniques with a biased sampling method first used in geometric optimization. The key aspect which we use is an adaptation of a fast sampling algorithm borrowed from computing bases of violator spaces. The violator spaces provide a combinatorial framework that can be leveraged to identify minimal Grbner bases efficiently. We will overview the basics of this construction and randomized algorithm. Authors: Shahrzad Jamshidi and Sonja Petrovic

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MS77

Power Flow on Stars

The power flow equations are at the core of all practical problems in power engineering. They represent the relation between network injected active and reactive powers and nodal (or bus) voltage phasors. The efficient solution of these equations is of considerable interest. For this work we are motivated by prior research results that show that the solution to the power flow equations for networks with a tree graph structure are easily computed, and by recent advances in inverse Kron reduction. Conceptually, if one could use an inverse Kron reduction to transform a general topology to an equivalent, but larger, tree structure, the equations may be easier to solve. Unfortunately, with the exception of very special network parameters, general topological graphs cannot be converted to tree structures using this approach. In this presentation we use the concept of inverse Kron reduction to create a network comprising coupled star networks from a general topology. The individual star network equations are easily solved. We present initial findings in efficiently coupling the star networks to ease calculations on general networks.

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MS77

Nonlinear Modal Decoupling: a New Approach to Real-Time Power System Stability Analysis and Control

With the increasing penetration of inverter-based resources (IBRs), a power system can exhibit more complex dynamics such as nonlinear oscillations, near-resonances and unanticipated instabilities. Conventional small-signal and transient stability analyses will no longer be adequate. For real-time stability analysis and control of a future power system modeled by a network of many nonlinear oscillators, this talk introduces a new methodology named Nonlinear Modal Decoupling that inversely constructs as many decoupled nonlinear oscillators as the systems oscillation modes of interests. Such decoupled oscillators together provide a fairly accurate representation of the systems stability and nonlinear oscillatory behaviors under both small and large disturbances. Each decoupled oscillator, although nonlinear, has only one degree of freedom and can easily be analyzed and controlled using, e.g., a Lyapunovs direct method to ensure the stability of the original system at the corresponding mode of the oscillator. When any oscillator foresees a transition from sustained oscillation to instability in real time from wide-area measurements, a proposed IBR-based distributed controller can quickly stabilize and damp the system.

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MS77

Revisit The Ellipsoidal Formulation of Quadratic Equation Systems New Insights for The Connectivity of All Real Solutions

Solving systems of multivariate quadratic equations, such as the power flow problem, is critical in electric power engineering and other fields. In certain scenarios, evaluating multiple real solutions to these systems is essential for global stability analysis, optimization, and related applications. Exhaustive search methods, such as Grbner basis techniques and homotopy continuation, have been developed to indiscriminately collect all complex solutions. These methods are highly inefficient, as they expend significant computational resources on finding irrelevant complex solutions. To address this issue, the branch tracing method has been proposed to focus exclusively on real solutions. This approach relies on tracking 1D real curves to locate different real solutions, with the design of these curves determining the thoroughness of the search. In prior work, we demonstrated that the power flow problem and the optimal power flow problem admit ellipsoidal formulations, transforming each quadratic equation into a high-dimensional ellipsoid. By tracing carefully designed curves on these ellipsoids, the likelihood of finding all real solutions is enhanced. However, the method still lacks a rigorous theoretical guarantee of its performance. In this talk, we revisit the ellipsoidal formulation of the power flow problem and present new insights into the connectivity of real solutions.

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MS78

Flag Varieties in Linear Algebra, Optimization, and Statistics: An Algebraic Perspective

We discuss the many lives that flag varieties lead in pure and applied mathematics. We present a sample of optimization problems over flag varieties from linear algebra and statistics and we compute their algebraic complexity by describing their critical points

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MS78

Exploring the Landscape of Elliptic Pairs

We explore the applications of machine learning tools to algebraic-geometric constructions, with a focus on elliptic pairs. These pairscomprising a complex surface and a curve satisfying specific geometric constraints are used to determine whether a pseudo-effective cone is polyhedral. Using combinatorial data such as polytopes and the kernels of associated matrices, we construct elliptic pairs and apply diverse neural network architectures to classify and analyze their properties.

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MS78

Open Problems Session

We will explore current open problems at the intersection of algebraic geometry, combinatorics, and machine learning to foster interdisciplinary collaboration among participants. This session will feature problems proposed by the speakers and organizers, highlighting challenges that invite joint research efforts. Attendees will be encouraged to engage in discussions and identify opportunities for future projects.

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MS79

The Linear Reliability Channel: a Framework for Soft Decision Code Construction

In a hard detection setting, the minimum Hamming distance and, more generally, the Hamming weight spectrum of a binary linear error correction code form key indicators of its quality. In the presence of soft information, where each received bit has an associated likelihood of being correct, Hamming statistics are usually regarded as being appropriate proxies for code quality as it is not evident how to incorporate the impact of the continuous side information into considerations. Inspired by the operation of Ordered Reliability Bits Guessing Random Additive Noise, which has recently been proven to be almost capacity achieving, we introduce a simple new model of the soft detection setting called the Linear Reliability Channel. Despite capturing all core facets of soft information decoding, it is a discrete model where where soft detection channel reliabilities are described by the declaration of a random permutation of received bits. This channel model provides a discrete framework for assessing soft detection performance through combinatoric considerations, including the determination of appropriate statistical correlates to high quality soft detection codes.

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MS79

Characterizing Logical Errors of the BP+OSD Decoder

First proposed by Pantaleev and Kalachev, the Belief Propagation + Ordered Statistics Decoding (BP+OSD) decoder has emerged as a promising approach for a broad range of quantum Low-Density Parity-Check (QLDPC) codes. A key step in understanding its performance is characterizing the structural properties of the Tanner graph representations of families of QLDPC codes that lead to logical errors during decoding. In this talk, we will analyze the logical errors that persist after BP+OSD decoding, identifying recurring patterns and structures that contribute to its failures. This is joint work with Christine Kelley and Tefjol Pllaha.

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MS79

Multishot Capacity of Adversarial Networks

Adversarial network coding studies the transmission of data over networks affected by adversarial noise. In this setting, the noise is modeled by an omniscient adversary who is restricted to corrupting a proper subset of the network edges. In 2018, Ravagnani and Kschischang established a combinatorial framework for adversarial networks. The study was recently furthered by Beemer, Kilic and Ravagnani, with particular focus on the one-shot capacity: a measure of the maximum number of symbols that can be transmitted in a single use of the network without errors. In this talk, we present a framework for assessing the potential increase in capacity achieved by utilizing a network multiple times for communication instead of just once. Additionally, we extend cut-set bounds from the one-shot capacity context to the multishot context.

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MS80

Maximum Flag Rank Distance Codes

Flag-rank distance codes are spaces of upper triangular matrices endowed with the flag-rank distance and they have been introduced by Fourier and Nebe in 2021. Such spaces are isometric to spaces of degenerate flag varieties endowed with the componentwise subspace distance (as rank-metric codes are isometric to constant dimensional subspace codes). Hence, they find application in network coding and, in particular, they have been shown to possess some advantageous properties over classical flag codes. In this talk we give a Singleton-like bound for the dimension of flag-rank distance codes for given minimum distance value. Moreover, we provide several constructions of maximum flag-rank distance codes for some small values of the minimum distance.These constructions are all based on different approaches and exploit a wide variety of tools, for instance the theory of rank-metric codes and MRD codes, the theory of linear block codes in the Hamming metric and MDS codes, as well as several geometric objects such as caps in a projective space.

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MS80

A Geometric Perspective on Rank-Metric Codes: Recent Applications

It is well known that a nondegenerate linear code of length n and dimension k can be associated with a set of n points (possibly with multiplicities) in a projective space of dimension $k-1$. Some coding-theoretic properties can be interpreted geometrically. Similarly, in the rank metric setting, the associated object is a q-system (or a linear set in the projective space). The rank of the codewords can be determined by intersections with hyperplanes. In this context, MRD codes correspond to linear sets that are scattered with respect to hyperplanes, provided that the length does not exceed the degree of the field extension. In this talk, we will illustrate some recent results obtained by using this geometrical approach for rank-metric codes, ranging from covering properties, supports' intersection properties and dimension sequences.

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MS80

Hulls of Rank Metric Codes

For linear codes with the Hamming metric it is well known that in the binary and trinary cases monomially equivalent codes have the same hull size; the dimension of the linear hulls are determined by the common Tutte polynomial of the matroids arising from their generator matroids. For codes over larger fields, however, the situation is quite different, one has shown that every linear code is equivalent to an LCD code, that is a code with hull size zero. We study analogous issues for vector rank metric codes, replacing monomial equivalence with a natural equivalence of such codes. We consider various cases for the cardinalities of the fields appearing.

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MS81

Geometry of Special Loci of Symmetric Ternary Tensor Eigenschemes

The geometry of eigenpoints is intimately related with the Waring decomposition of a polynomial, since by a result of J. Draisma, G. Ottaviani and A. Tocino, any best lower rank approximation of a symmetric tensor lies in the so called critical space, spanned by the rank 1 tensors which are powers of eingevectors. In the case of ternary symmetric tensors of order 3, with M. Gallet and A. Logar we developed a discussion of the algebraic and geometric conditions imposed by the existence of an aligned triple of eigenpoints, and the computation of the degree of the hypersurface of ternary cubics with at least one alignment in the eigenscheme. Such a locus contains the ODECO locus properly.

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MS81

Equations for Tensors via Intersection Theory

A number of fundamental problems in mathematics reduce to determine membership of a tensor in some algebraic variety. Therefore, one is interested in studying always new methods to find equations for interesting varieties of tensors. In this talk, I will explain some recent progress in this direction using basic notions from classical intersection theory and resolution of singularities.

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MS81

Extending HOSVD and Applications to Quantum Entanglement Classification

We extended the classical higher-order singular value decomposition (HOSVD), which was discovered and popularized by De Lathauwer and others in the early 2000s. Matrix SVD is a classification of matrices under the left-right action of unitary (or orthogonal) groups and provides normal forms for orbits that are real positive diagonal matrices parametrized by the singular values of the matrix. The HOSVD does something similar, decomposing a given tensor into a product of a tuple of unitary group elements and an all-orthogonal core tensor. The HOSVD is actually a consequence of a much more general lemma on reduction maps. We use these reduction maps to produce a complex orthogonal version of HOSVD, mimicking the complex orthogonal SVD. With these normal-form algorithms for binary tensors we can now classify entanglement types for general qubits. This is joint work with Ian Tan.

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MS81

The Secant Variety and Syzygies of the Hilbert Scheme of Two Points

In this paper, we prove that $\mathrm{Sec}(X^{[2]})$ features the identifiability under the Grothendieck-Plucker embedding $X^{[2]} \hookrightarrow \mathbb{P}^N$ when X is embedded by a 4-very ample line bundle. We also prove that the embedding $X^{[2]} \hookrightarrow \mathbb{P}^N$ satisfies Green's condition (N_p) when the embedding of X is positive enough. Accordingly, the singular locus of $\mathrm{Sec}(X^{[2]})$ is exactly $X^{[2]}$ when the embedding of X is positive enough. As an application, we describe the geometry of a resolution of singularities from the secant bundle to $\mathrm{Sec}(X^{[2]})$ when

X is a surface.

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MS83

Mixtures of Discrete Decomposable Graphical Models

I will talk about mixtures of decomposable graphical models, focusing on their ideals and dimensions. For a special family of graphs, called clique stars, I will characterize these ideals in terms of ideals of mixtures of independence models. In addition, I will present a recursive formula for their maximum likelihood degree. Finally, we will see that second secant varieties of all other decomposable graphical models have the expected dimension. This talk is based on joint work with Jane Ivy Coons and Nils Sturma.

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MS83

Algebraic Aspects of Phylogenetic Networks

Phylogenetic trees are widely used in genetics. However, network structures may better explain biological processes such as hybridization, recombination, and horizontal gene transfer. In this talk, after a survey of fundamental concepts, I will present some critical challenges in estimating phylogenetic networks from sequence data and discuss how algebraic geometry and statistics techniques will be used to address them in the mini-symposium sessions.

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MS83

Detecting Toricness of a Variety via its Symmetry Lie Algebra

The motivation for this talk is to detect when an irreducible projective variety V is not toric. We do this by introducing and analyzing a symmetry Lie group and a Lie algebra associated with the ideal I(V). If the dimension of V is strictly less than the dimension of the above-mentioned objects, then there is no linear transformation that turns I(V) into a toric ideal. We use it to provide examples of non-toric statistical models in algebraic statistics. The talk is based on preprint arxiv: 2309.10741.

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MS84

Higher-order Newton Methods with Polynomial Work per Iteration

We present generalizations of Newton's method that incorporate derivatives of an arbitrary order d but maintain a polynomial dependence on dimension in their cost per iteration. At each step, our dth-order method uses semidefinite programming to construct and minimize a sum of squares-convex approximation to the dth-order Taylor expansion of the function we wish to minimize. We prove that our dth-order method has local convergence of order d. This results in lower oracle complexity compared to the classical Newton method. We show on numerical examples that basins of attraction around local minima can get larger as d increases. Under additional assumptions, we present a modified algorithm, again with polynomial cost per iteration, which is globally convergent and has local convergence of order d.

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MS84

Peak Estimation of Nonnegative Models using Signomial Optimization

Peak estimation involves bounding extreme values of state functions along system trajectories, such as finding the maximum current draw on a transmission line, or the maximum speed obtained by an aircraft. In this talk, we focus on the setting where the dynamics and auxiliary function are chosen to be signomials (linear combinations of exponentials with possibly non-integer exponents). These signomial dynamical models arise in applications such as power-law chemical kinetics, epidemiology, and food web modeling. The nonconvex but finite-dimensional peak estimation problem (in terms of optimizing over the initial condition and stopping time) can be lifted into a primal-dual pair of infinite-dimensional linear programs in auxiliary functions/occupation measures. Under mild assumptions, this lifting provides an exact reformulation. In the signomial-restricted peak estimation problem, we truncate the infinite-dimensional linear constraints into Sum of Arithmetic Geometric Exponential nonnegativity certificates, which can be represented by relative entropy or dual-power-cone expressions. Based on joint work with J. Miller, R. Murray, K. Wallington, and R. S. Smith.

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MS84

Nonnegativity of Signomials with Newton Simplex

over Convex Sets

For certain classes of polynomials or signomials with Newton simplex, sums of nonnegative circuit polynomials (SONC) respectively sums of arithmetic-geometric exponentials (SAGE) provide exact characterizations of global nonnegativity. In this talk, we consider generalizations from the global nonnegativity to the case where the ground set is a convex set.

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MS84
Computational Complexity of Sum-of-squares Bounds for Copositive Programs

In recent years, copositive programming has received significant attention for its ability to model hard problems in both discrete and continuous optimization. Several relaxations of copositive programs based on semidefinite programming (SDP) have been proposed in the literature, meant to provide tractable bounds. However, while these SDP-based relaxations are amenable to the ellipsoid algorithm and interior point methods, it is not immediately obvious that they can be solved in polynomial time (even approximately). In this paper, we consider the sum-of-squares (SOS) hierarchies of relaxations for copositive programs introduced by Parrilo (2000), de Klerk & Pasechnik (2002) and Pea, Vera & Zuluaga (2006), which can be formulated as SDPs. We establish sufficient conditions that guarantee the polynomial-time computability (up to fixed precision) of these relaxations. These conditions are satisfied by copositive programs that represent standard quadratic programs and their reciprocals. As an application, we show that the SOS bounds for the (weighted) stability number of a graph can be computed efficiently. Additionally, we provide pathological examples of copositive programs (that do not satisfy the sufficient conditions) whose SOS relaxations admit only feasible solutions of doubly-exponential size.

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MS85
Mixed Discriminants and Real Roots

I will discuss the following questions: how to bound the maximal local multiplicity at one point of a sparse system (i.e., with coefficients in the singular strata of the corresponding discriminant) and how to deform the coefficients to get a system with many real solutions.

Alicia Dickenstein
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MS85
Discriminants from the Kuramoto model

The Kuramoto model is one of the most investigated models in the context of coupled oscillators. A primary focus of research is on understanding equilibrium states, which can be expressed as solutions to a system of algebraic equations. In this talk, a novel parametrisation is introduced, leading to algebraic objects which we call Lissajous varieties. In particular, the Lissajous discriminant, which characterizes the singular solutions within these varieties, will be discussed.

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MS85
Bloch Discriminants of Discrete Periodic Operators

Given an operator on a $\mathbb{Z}^d$-periodic graph, its Bloch variety encodes its spectrum with respect to the unitary characters of $\mathbb{Z}^d$. This talk will describe how the Bloch discriminant gives a mapping of the geography of the parameter space that yields a qualitative change in the behavior of the critical points of Bloch varieties for three $\mathbb{Z}^2$-periodic graphs. This includes determining the number and arrangements of critical points and whether or not they are degenerate as the parameters vary. This is joint work with Frank Sottile and Simon Telen.

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MS85
Zeros of 1-Forms and Cohomology of Local Systems

Let X be a closed subvariety of the affine torus T. We study the universal family of translation-invariant one-forms on T and the associated universal degeneration locus on X, known as the likelihood correspondence variety. In parallel, we consider the universal family of rank-one local systems on X and the cohomology jump loci, which consist of those local systems where certain cohomology groups exhibit higher-than-generic rank. We explore connections between the likelihood correspondence variety and the cohomology jump loci, and formulate some conjectures arising from this relationship.

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MS86
Weighted Tropical Fermat-Weber Points

Let $\mathbf{v}_1, \ldots, \mathbf{v}_m$ be points in a metric space with distance d, and let $w_1, \ldots, w_m$ be positive real weights. The weighted

Fermat-Weber points are those points $\mathbf{x}$ which minimize $\sum w_i d(\mathbf{v}_i, \mathbf{x})$. In the unweighted case, Comaneci and Joswig proved that under an asymmetric tropical distance, the Fermat-Weber points form the central cell of the tropical convex hull of $\mathbf{v}_1, \ldots, \mathbf{v}_m$. We extend this result to the weighted setting. We apply the weighted Fermat-Weber points to detect similarities and differences across developmental lineages in mosaic organisms. This is joint work with Maize Curiel.

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MS86

Topological Techniques for Classification of Agent-based Tumour-immune Model

We address the problem of classifying time series of synthetic 2-d spatial data from an agent-based model of tumour growth that includes tumour cells, macrophages, and blood vessels. We implement and compare the predictive power of four topological vectorizations specialized to such cell data: persistence images of Vietoris-Rips and radial filtrations at static time points, and persistence images for zigzag filtrations and persistence vineyards varying in time.

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MS86

A New Graduate Topics Course on Algebraic Systems Biology

In Fall 2024, I designed and taught a graduate topics course on *Algebraic Systems Biology*. It was unique in that only about half of the students were from math. There were students from physics, biomedical physics, bioengineering, chemistry, and plant and environmental science. Topics included biochemical reaction networks, delay differential equation models, Boolean models of molecular networks, algebraic models and finite dynamical systems, network inference, model spaces, statistical mechanics of random Boolean networks (ordered, critical, and chaotic), and biological feedback. One goal was to create an inclusive course where the diversity of backgrounds is a strength. In this talk, I'll tell you how it went, and will show you resources I developed, which I'm happy to share. I'm particularly interested in crowdsourcing your ideas and feedback, which could lead to improvements, new materials, and new modules in future courses, from any of us.

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MS86

Stable Population Dynamics and Combinatorial Structures

We study the univariate moment problem of piecewise-constant density functions on the interval [0,1] and its consequences for an inference problem in population genetics. We show that, up to closure, any collection of n moments is achieved by a step function with at most $n-1$ breakpoints and that this bound is tight. We use this to show that any point in the n-th coalescence manifold in population genetics can be attained by a piecewise constant population history with at most $n-2$ changes. Both the moment cones and the coalescence manifold are projected spectrahedra and we describe the problem of finding a nearest point on them as a semidefinite program.

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MS87

Tropical Combinatorics of Max-Linear Bayesian Networks

A polytrope is a tropical polyhedron that is also classically convex. We study the tropical combinatorial types of polytropes associated to weighted directed acyclic graphs (DAGs). This family of polytropes arises in algebraic statistics when describing the model class of max-linear Bayesian networks. We characterize a minimal non-redundant facet description and the associated graph, resulting in a classification of polytropes from weighted DAGs at different levels of equivalence. This characterization gives rise to a polyhedral cone complex that represents the equivalence classes of weighted DAGs with respect to the associated polytropes.

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MS87

Computations with Tropical Ideals

The tropicalization of the ideal defining a subvariety of projective space remembers more information about the variety than the tropicalization of the variety. In this talk I will indicate some of the ways that this is true, and discuss some challenges with computing with these objects.

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MS87

Shellings of Tropical Hypersurfaces

The shellability of the boundary complex of an unbounded polyhedron is investigated. For this concept to make sense, it is necessary to pass to a suitable compactification, e.g., by one point. This can be exploited to prove that any tropical hypersurface is shellable. Under the hood there is a subtle interplay between the duality of polyhedral complexes and shellability. Translated into discrete Morse theory, that interplay entails that the tight span of an arbitrary regular subdivision is collapsible, not not shellable in general. This talk is based on joint work with George Balla and Michael Joswig.

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MS88

Matroid Lifts and Representability

Given a pair of matroids M and L on the same ground set E, one says that L is a *lift* of M if every flat of L is a flat of M. This happens, for example, when L is the column-matroid of a matrix A, and M is the matroid of a row-submatrix of A. This talk will discuss how a lift of M can be constructed via a matroid whose ground set is the circuit-set of M, and how this construction can be used to derive a combinatorial test for matroid representability.

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MS88

Applications of the 1-Dimensional Group Construction

The 1-dimensional group construction provides a flexible tool for constructing algebraic matroids. I will introduce this construction as well as give some applications.

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MS88

Using Oriented Matroids to Find Low Rank Structure in Presence of Nonlinearity

Estimating the linear dimensionality of a data set in the presence of noise is a common problem. However, data may also be corrupted by monotone nonlinear distortion that preserves the ordering of matrix entries but causes linear methods for estimating rank to fail. In light of this, we consider the problem of computing *underlying rank*, which is the lowest rank consistent with the ordering of matrix entries, and *monotone rank*, which is the lowest rank consistent with the ordering within columns. We show that each matrix of monotone rank d corresponds to a point arrangement and a hyperplane arrangement in $\mathbb{R}^d$, and that the ordering within columns of the matrix can be used to recover information about these arrangements. Using Radon's theorem and the related concept of the VC dimension, we can obtain lower bounds on the monotone rank of a matrix. However, we also show that the monotone rank of a matrix can exceed these bounds. In order to obtain better bounds on monotone rank, we develop the connection between monotone rank estimation and oriented matroid theory. Using this connection, we show that monotone rank is difficult to compute: the problem of deciding whether a matrix has monotone rank two is already NP-hard. However, we introduce an "oriented matroid completion" problem as a combinatorial relaxation of the monotone rank problem and show that checking whether a set of sign vectors has matroid completion rank two is easy.

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MS88

Algebraic Matroids and Secant Varieties

Motivated by the algebraic geometry and combinatorics of Cayley-Menger varieties, we seek to understand the relationship among the algebraic matroids of a variety and its secant varieties. The coordinates of the Cayley-Menger variety $CM(n, d)$ represent pairwise distances among n points in $\mathbb{R}^d$, and its defining ideal consists of the polynomial relations these distances must satisfy. Its algebraic matroid captures the data of which of these distances are independent and which must satisfy a polynomial relation. From the point of view of classical algebraic geometry, $CM(n, 1)$ is the quadratic Veronese embedding of $\mathbb{P}^{n-2}$, and $CM(n, d)$ is its d-secant variety. We seek to understand the combinatorics of algebraic matroids of a variety and its secant varieties more broadly, and will discuss several classes of examples.

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MS89

Exploring the Multi-Manifold Geometry of the Power Flow Solution Space

The Power Flow solution space manifest a convoluted multi-manifold geometries arising from the nonlinear dynamics of power systems. These geometries play a crucial role in defining system stability, operational limits, and feasibility regions, yet their intricate structure poses significant challenges for analysis and computation. We explore the multi-manifold nature of the PF solution space, emphasizing its critical features, including bifurcation points, transition boundaries, and interconnected regions of feasible operation. By leveraging advanced geometric analysis and computational techniques, we develop robust methodologies to characterize and trace these manifolds, providing deeper insights into their influence on grid behavior. The findings enable more effective approaches for ensuring grid reliability, optimizing operations, and addressing challenges associatedwithrenewableintegrationandincreasingoperationaluncertainties. Numerical results from test cases validate the proposed methods, underscoring their potential for advancing power system security and resilience.

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MS89

A Procedure to Assess Nonconvexity via Shortest Feasible Paths

We present a procedure that, given a compact non-convex set $S \subset \mathbb{R}^n$, provides a metric representing how "close" S is to being a convex set. This metric is built upon the ratio of the *shortest feasible path* contained entirely within S to the Euclidean distance between given *endpoints* of such a path. Pairs of endpoints are sampled uniformly from the set S through an efficient mixing algorithm. By computing these ratios over all sampled pairs of points, we obtain an empirical distribution whose average provides a quantitative measure of the degree of convexity of S. We formally show that our sampling-based approach converges to a faithful characterization of the nonconvexity of S as the number of samples increases. As an illustrative application, we consider the non-convex feasible sets of the optimal power flow problem used to determine generator setpoints in electric power grids. We demonstrate the effectiveness of our procedure with numerical experiments using MATPOWER test cases.

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MS89

Global Optimality of Optimal Power Flow Problems

Optimal power flow (OPF) problems are nonconvex and NP-hard. Though hard in theory, they seem to be "easy" in practice in the sense that local algorithms, such as Newton-Raphson or interior methods, tend to yield global optima and convex relaxations tend to be exact. We prove a Lyapunov-like condition for a general nonconvex optimization problem to both have no spurious local optima and have exact convex relaxations. The condition is sufficient, and almost necessary. We illustrate this global optimality condition through application to an OPF problem in the branch flow model.

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MS90

Covering Numbers of Real Algebraic Varieties and Applications to Data Science

In this talk I will discuss covering numbers of real algebraic varieties and applications to data science. Specifically, we control the number of ℓ_2 balls of radius epsilon needed to cover a real variety, image of a polynomial map, and semialgebraic set in Euclidean space, in terms of the degrees of the relevant polynomials and number of variables. The bound significantly improves the best known general bound, and its proof is much more straightforward. On the applications side, I will give consequences for CP tensor decomposition and neural network theory. Joint work with Yifan Zhang at UT Austin (arXiv:2311.05116).

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MS90

An Invitation to Neuroalgebraic Geometry

In this talk, we will discuss the study of function spaces parameterized by machine learning models through the lens of algebraic geometry. To this end, we focus on algebraic models, such as neural networks with polynomial activations, whose associated function spaces are semi-algebraic varieties. We outline a dictionary between algebro-geometric invariants of these varieties, such as dimension, degree, and singularities, and fundamental aspects of machine learning, such as sample complexity, expressivity, training dynamics, and implicit bias. Along the way, we review the literature and discuss ideas beyond the algebraic domain. These connections lay the foundations of a research direction bridging algebraic geometry and deep learning, that we refer to as neuroalgebraic geometry.

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MS90

Activation Degree Thresholds and Expressiveness of Polynomial Neural Networks

Polynomial neural networks are implemented in a range of applications and present an advantageous framework for theoretical machine learning. In this talk, we introduce the notion of the activation degree threshold of a network architecture. This expresses when the dimension of a neurovariety achieves its theoretical maximum. We show that activation degree thresholds of polynomial neural networks exist and provide an upper bound, resolving a conjecture on the dimension of neurovarieties associated to networks with high activation degree. Along the way, we will see several illustrative examples.

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MS91

Dna Data Storage Using Small Finite Fields

Over the past decade, DNA-based data storage systems have emerged as a promising solution to address the ever-increasing volume of information due to their density, durability, and low maintenance costs. Studying the fundamental limits of DNA data storage is therefore essential. The main difference between retrieving data from DNA and traditional storage media is that DNA strands must be read randomly until the desired information is retrieved. In the language of coding theory, this translates into considering the generator matrix G of an $[n,k]_q$ code $\mathcal{C}$ and computing the expected number of columns that one has to draw until the entire space $\mathbb{F}_{q^k}$ is spanned. It is known that MDS codes, for the parameters where they exist, minimize this expectation, when the probability with which the columns are drawn is uniform and the channel is noiseless. Thus, MDS codes are optimal in this context. However, MDS codes do not exist over small fields, raising the question of which alternative codes offer the best performance. This problem is highly combinatorial in nature, connecting coding theory, matroids, and lattice theory. This talk will provide an overview of the problem of retrieving data from DNA and present new results on the performance in this context of codes over small alphabets.

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MS91

Private Linear Computation with Side Information

This talk provides an overview of the Private Linear Computation with Side Information (PLC-SI) problem. In this problem, a client aims to retrieve a linear combination of a subset of data stored on a remote server while keeping the identities of the required data items private. The client's goal is to maximize the download rate by leveraging prior side information, which consists of a subset of data whose identities are initially unknown to the server. We will present lower and upper bounds on the capacity of the PLC-SI problemdefined as the maximum achievable download rateunder two types of side information: reusable and single-use. Additionally, we will discuss the optimality and tightness of these bounds in each scenario.

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MS91

Optimum: Highly Efficient, Scalable, and Secure Atomic Memory Using Random Linear Network Codes

We propose Optimum, a highly-effective (low-latency, high-throughput), scalable, and secure decentralized shared memory solution. It is an atomic read/write memory designed for the Web3 environment, addressing the unique challenges posed by asynchronous communication, high node churn, decentralized decision-making, and the presence of potentially malicious nodes. Achieving a reliable shared memory object in this setting requires careful attention to reducing latency, enhancing fault tolerance, minimizing bandwidth and storage costs, ensuring high throughput, and maintaining non-blocking,high-availability performance. Our approach leverages Random Linear Network Codes (RLNC), selected for their flexible structure, which helps avoid costly distributed synchronization primitives. This flexibility contributes to improved durability, reduced bandwidth usage, and enhanced fault tolerance within a distributed storage framework. Our system is implemented as a network of homogeneous nodes, termed Flexnodes, which collectively provide decentralized storage and communication services. External clients can interact with any Flexnode to perform read/write operations, using this system both for data storage and as a communication socket within the network.

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MS91

Quantum Communications Games

Quantum information science (QIS) promises to revolutionize computing and information processing by exploiting quantum mechanics unique properties. Traditional interest in QIS has revolved around computational speedups using quantum algorithms that utilize phenomena such as superposition and entanglement, which are present in the quantum setting but not in the classical one. The postulates of quantum mechanics also introduce restrictions, such as how quantum information is processed. A famous consequence is the no-cloning theorem (stemming from the unitary processing restriction), which refutes the existence of a universal cloning gate. We exploit the no-cloning theorem to control how classical information encoded in quantum states can be accessed and distributed. This talk considers quantum communications games that involve a referee (Alice) playing against two players (Bob and Charlie) whose goal is to meet a joint condition to win the game. The players can devise a joint strategy in preparation for the game but not communicate once the game starts, which makes the game non-local. Communication during the game is only between the referee and the players. We focus on games whose formulations and strategies involve algebraic geometry.

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MS92

Authenticating in Networks

The goal of authentication is a relaxation of correction over a channel where an adversary may be present but is likely acting infrequently. When authenticating, we allow the receiver to discard any message that has been tampered with, while retaining the correction requirement when the adversary is absent. Feasibility of authentication in the point-to-point case is completely characterized by a channel condition termed (non)overwritability. In this talk, we explore the setting of adversarial multiple access channels, giving channel conditions classifying the feasibility of reliably correcting some fraction of users (and discarding the remainder), as well as coding strategies for achieving positive rate tuples over amenable channels. We conclude by discussing how these ideas extend to more general networks.

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MS92

Optimal Rank-Metric Codes with Rank-Locality from Drinfeld Modules

We introduce a new technique to construct rank-metric codes using the arithmetic theory of Drinfeld modules over global fields, and Dirichlet Theorem on polynomial arithmetic progressions. Using our methods, we obtain a new infinite family of optimal rank-metric codes with rank-locality, i.e. every code in our family achieves the information theoretical bound for rank-metric codes with rank-locality. This is a joint work with Giacomo Micheli and Luca Bastioni.

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MS92

The Alphabet's Impact on Networks: Ideal, Adversarial and Restricted Scenarios

This talk focuses on the (one-shot) capacity of multicast networks where an omniscient adversary has access to a subset of the network's edges. We call the scenarios as ideal, adversarial and restricted, based on whether this subset is empty, a proper subset or the entire edge set, respectively. We revisit, recover and extend previous results on the capacity of single-source networks over small alphabet sizes using a combination of projection, packing and integer programming techniques. In addition, we explore how the known cut-set bounds and linear network coding perform in these three scenarios.

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MS92

Short Rank-metric Codes and Scattered Subspaces

Minimal rank-metric codes have gained attention for their algebraic properties and geometric interpretations. These codes are characterized by their support spaces forming an antichain under inclusion, and correspond to objects in projective geometry called linear cutting blocking sets. In this talk, we investigate an infinite family of such codes with parameters $[m+2,3]_{q^m/q}$, constructed via scattered subspaces in $V(3,q^m)$. This provides the first known explicit constructions for infinitely many values of q and odd m.

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MS93

On Tensor Network varieties

Tensor Network States (TNS) play a central role in the efficient representation of quantum many-body states, especially in the context of condensed matter physics and quantum information theory. Mathematically, the set of TNS of fixed bond and physical dimensions corresponds to a specific algebraic variety, often defined via contractions of tensors constrained by a graph topology. In this talk, I will present recent progress on the problem of computing the dimension of TNS varieties, a fundamental question for understanding both the expressive power and the limitations of tensor network models. I will discuss how tools from algebraic geometry can be used to estimate or exactly determine the dimension of TNS varieties in different scenarios. The talk is based on joint work with Claudia De Lazzari and Fulvio Gesmundo.

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MS93

An Existence Result on Rational Normal Curves

A very classical result, the Castelnuovo Lemma, states that, given $d+3$ points in general position in $\mathbb{P}^d$, there is a unique rational normal curve of degree d passing through them. An easy argument shows that asking for a rational normal curve to pass through a point, imposes, in the parameter spaces of rational normal curves, the same number of conditions imposed by asking for a rational normal curve to be $(d-1)$-secant to a codimension 2 projective subspace of $\mathbb{P}^d$. For this reason, it is quite natural to pose the following problem: given p points in $\mathbb{P}^d$ and l codimension 2 subspaces in $\mathbb{P}^d$ with $p+l=d+3$, is there a rational normal curve passing through the points and being $(d-1)$-secant to the l projective subspaces? A paper of Carlini and Catalisano (2007) solves all the cases except for $(p, l) = (0, d + 3)$. In this talk we show how to solve this missing case by constructing a specific rational map and showing that it is dominant. Moreover, time permitting, we will see some applications of this kind of existence results to the study of secants defectiveness.

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MS93

Ideals of Some k-Secants of Veronese Embeddings

In this talk, we report some results on equations and the ideal of $\sigma_k(v_d(\mathbb{P}^n))$, the k-th secant variety of d-uple Veronese embedding of a projective space, in case of the k-th secant having a relatively small degree. Knowledge on defining equations of higher secant varieties is fundamental in the study of algebraic geometry and in recent years it also has drawn a strong attention in relation to tensor rank problems. First, we recall known results on the equation of a k-th secant variety and introduce key notions for this work, which are 'k-secant variety of minimal degree' and 'del Pezzo k-secant variety', due to Ciliberto-Russo and Choe-Kwak, respectively. Next, we focus on the case of $\sigma_4(v_3(\mathbb{P}^3))$ in $\mathbb{P}^{19}$ as explaining our method and considering its consequences. We present more results which can be obtained by the same method, if time permits. This is a joint work with K. Furukawa (Josai Univ).

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MS93

A Real Generalized Trisecant Trichotomy

The classical trisecant lemma says that a general chord of a non-degenerate space curve is not a trisecant; that is, the chord only meets the curve in two points. The generalized trisecant lemma extends the result to higher-dimensional varieties. It states that the linear space spanned by general points on a projective variety intersects the variety in exactly these points, provided the dimension of the linear space is smaller than the codimension of the variety and that the variety is irreducible, reduced, and nondegenerate. We prove a real analogue of the generalized trisecant lemma, which takes the form of a trichotomy. Along the way, we characterize the possible numbers of real intersection points between a real projective variety and a complimentary dimension real linear space. We show that any integer of correct parity between a minimum and a maximum number can be achieved. We then specialize to Segre-Veronese varieties, where our results apply to the identifiability of independent component analysis, tensor decomposition and to typical tensor ranks.

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MS94

Three Amazing Formulas from Abel's 1826 Paris Memoir

In 1826, Abel submitted a memoir about Abelian integrals to the Paris Academy. After its publication in 1841, many people recognized the importance of Abels Theorem. In this talk, I will focus on some formulas from Abels memoir that have received less attention. We will see that Abel had some interesting insights into symbolic computation and gave a genus formula which (in modern terms) counts interior lattice points in the Newton polygon.

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MS94

Rational "Multi-lump" Solutions of the KP Equation from Cuspidal Algebraic Curves

The Korteweg-deVries (KdV) and Kadomtsev-Petviashvili (KP) equations are nonlinear PDE that model various sorts of surprising wave phenomena. In particular, both equations have "soliton" solutions (self-reinforcing, localized solutions that preserve their shapes while propagating freely). One perhaps unexpected feature of these equations is that they are closely connected with some classical topics in algebraic geometry because they have many exact solutions constructed from algebraic curves. We will give a general overview of this connection by means of explicit examples in the special case of rational function solutions. Our main result is a new construction of "multi-lump" solutions starting from singular rational curves with only cuspidal singularities.

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MS94

Curves, Codes, Computations, Classics: The Mathematical Contributions of John Little

A scenic tour of Johns work.

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MS94

Zeros of Sparse Polynomials and Toric 3-Fold Codes

Given a lattice polytope P, consider the family of polynomials with exponent vectors in P and coefficients in a finite field $\mathbb{F}_q$. Our goal is to provide an upper bound on the number of $\mathbb{F}_q$-zeros of the polynomials in this family in terms of invariants of P. It follows from the Lang-Weil bound that if q is large enough then the polynomials with the most factors have the largest numbers of zeros. Therefore, to obtain the bound one needs to understand the Minkowski length of P, which is defined to be the largest number of nontrivial lattice polytopes whose Minkowski sum is contained in P. I will review the case of dimension 2, whose study was initiated by Little and Schenck, and will then focus on the case of trivariate polynomials, where the combinatorics become significantly richer and new phenomena emerge. I will then explain the application of these results to the problem of estimating the minimum distance of toric codes. This is joint work with Kyle Meyer and Ivan Soprunov.

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MS95

Phylogenetic Networks Evolving under Equivariant Models

One of the prior steps to perform inference on phylogenetic networks is ensuring that the phylogenetic network can be identified from the distributions generated by a Markov process on it. Many different authors have provided equations that allow distinguishing a phylogenetic network from another. These equations have been usually obtained by computational algebra software for small networks evolving under simple models such as CFN, JC69, K80 o K81. In this talk we will explain how to obtain equations for networks evolving under equivariant models (which include the previous but also the general Markov model and other submodels) based on rank conditions from flattening matrices. These equations can be used to prove that certain networks are distinguishable. This is joint work with J. Fernández-Sánchez, E. Gross, B. Hollering, and S. Sullivant

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MS95

The Pfaffian Structure of Level-1 Phylogenetic Networks

In this talk, we will consider the ideal of phylogenetic invariants for level-1 phylogenetic networks under the CFN network. This ideal is the vanishing ideal for the set of joint probability distributions observed at the leaves of the network. Our main result states that for a level-1 network with n leaves, there is a linear change of coordinates for which the ideal is generated by 2×2 and 3×3 minors of a generic $n \times n$ symmetric matrix. We will also show how to list these invariants based off the combinatorics of the network and give a statistical interpretation of the invariants. Finally, we will discuss some partial results for higher level networks.

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MS95

Identifiability of Phylogenetic Level-2 Networks under the Jukes-Cantor Model

We study the identifiability of phylogenetic networks from data generated under Markov models of DNA evolution. Our main result is generic identifiability under the Jukes-Cantor model of binary semi-directed level-2 phylogenetic networks that satisfy two additional conditions called triangle-free and strongly tree-child. We also consider level-1 networks and show stronger identifiability results for this class than what was known previously. Moreover, we prove general identifiability results that do not restrict the network level at all and hold for the Jukes-Cantor as well as for the Kimura-2-Parameter model. We show that any two binary semi-directed phylogenetic networks are distinguishable if they do not display exactly the same 4-leaf subtrees, called quartets. This has direct consequences regarding the blobs of a network, which are its reticulated components. We show that the tree-of-blobs, the high-level branching structure of the network, is always identifiable, as well as the circular ordering of the subnetworks around each blob, for networks in which edges do not cross and taxa are on the outside.

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MS97

Rational SOS Certificates of Any Polynomial Over its Zero-dimensional Gradient Ideal

Given a multivariate polynomial with rational coefficients, we present rational certificates of non-negativity based on sum of squares decomposition, under the assumption that its gradient ideal is zero dimensional.

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MS97

Real Geometry in Reaction Networks

To study the stationary properties, (i.e. properties at the steady states) of reaction networks we are interested in understanding parameterised systems of polynomials. I will highlight two main properties: multistationarity and hopf bifurcation. In the first case, we want to map the parameter space that gives multiple positive real solutions of the said polynomial system and explore their topological properties. In the second case, to preclude Hopf bifurcation in a reaction network, we want to establish positivity of a polynomial over a semi-algebraic set. This polynomial is obtained as a resultant of two polynomials and can be quite large even for small networks. I will explain various techniques from real and computational geometry used to studying these properties and I will conclude the talk with some open problems. The talk will be based on several joint works with: Feliu, Telek, Yrck, Wang, and de Wolff.

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MS97

SONC Exactness and the Positive A-discriminant

A classical approach to certifying the non-negativity of a real polynomial is to express it as a sum of squares. The cone of polynomials that can be represented as a sum of squares is, in general, strictly contained within the cone of non-negative polynomials. The cases when these two cones coincide were characterized by Hilbert. An alternative approach for certifying non-negativity is to express the polynomial as a Sum of Non-negative Circuits (SONC). In this talk, I will discuss necessary and sufficient combinatorial conditions under which the cone of SONC polynomials coincides with the cone of non-negative polynomials. Our approach builds on extending Viros patchworking to singular hypersurfaces and using properties of the positive A-discriminant. This talk is based on joint work with Timo de Wolff.

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MS98

Topological Methods for Molecular Biology

Single-parameter persistent homologythe flagship tool in topological data analysishas witnessed a wide range of applications in the biological sciences in the last decade. Multiparameter persistent homology (MPH) is a natural generalisation allowing for higher-order analysis of more complex phenomena. In this talk, we will explore recent theoretical developments in MPH as well as applications to problems in molecular and structural biology.

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MS98

Indistinguishability of Linear Compartmental Models

An important problem in biological modeling is choosing the right model. Given experimental data, one is supposed to find the best mathematical representation to describe the real-world phenomena. However, there may not be a unique model representing that real-world phenomena, i.e., two distinct models could yield the same exact dynamics. In this case, these models are called indistinguishable. In this talk, we consider the indistinguishability problem for linear compartmental models, which are used in many areas, such as pharmacokinetics, physiology, cell biology, toxicology, and ecology. We exhibit sufficient conditions for indistinguishability for models with a certain graph structure: paths from input to output with detours. These are the first sufficient conditions for indistinguishability of linear compartmental models based on graph structure alone.

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MS98

C-ZIPTF: Stable Tensor Factorization for Zero-inflated Multi-dimensional Genomics Data

In the past two decades, advances in genomics technologies have led to an influx of complex multidimensional data, necessitating robust methods to handle the complexity and multidimensionality of such genomics data. Single-cell RNA-sequencing (scRNA-seq) data, in particular, faces challenges such as sparsity due to low capture efficiency and dropout effects. Tensor factorizations (TF) have emerged as powerful tools to unravel the complex patterns from multidimensional genomics data. However, traditional TF methods, relying on maximum likelihood estimation, struggle with zero-inflated count data, and their inherent stochasticity complicates result interpretation and reproducibility. In this talk, we will introduce Zero Inflated Poisson Tensor Factorization (ZIPTF), a novel approach specifically designed for high-dimensional zero-inflated count data, and Consensus-ZIPTF (C-ZIPTF), which enhances stability of the factorization by integrating a consensusbased framework. We will demonstrate the effectiveness of these methods using synthetic zero-inflated count data, simulated scRNA-seq data, and real multi-sample multi-condition scRNA-seq datasets. Results highlight ZIPTF and C-ZIPTFs superior performance in reconstruction accuracy, robustness to zero-inflated data, and their capacity to uncover biologically meaningful gene expression programs.

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MS98

Steady State Response Curves in Biochemical Systems

A response curve measures the output of a biological system at equilibrium to some input parameter, which could be a rate constant, or the total concentration of a stimulus (poison, drug, ligand, etc.). Of interest is the shape of a response curve: does it have a plateau region (homeostasis), or is it constant? These properties have been studied from the perspective of input-output systems and in chemical reaction networks. For example, absolute concentration robustness (ACR) in reaction networks can be viewed as a constant response curve with respect to total concentrations. We consider generalizations of ACR using techniques from input-output systems.

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MS99

Solving Sparse Tropical Polynomial Systems by Means of Parametric Mean Payoff Games

In a previous work, we showed a Nullstellensatz and a Positivstellensatz for sparse tropical polynomial systems, improving a result of Grigoriev and Podolskii (2018) concerning full polynomial systems. These results rely on a linearization of the systems using the tropical Macaulay matrix with a truncation obtained by a constructed by Canny and Emiris (1993), refined by Sturmfels. Using the equivalence between tropical linear systems and mean-payoff games, the solvability of a tropical polynomial system is then reduced to a winning condition for a mean-payoff game constructed from its Macaulay matrix. In this talk, we shall present an analogue of eigenvalue method for finding the solutions of sparse tropical polynomial systems, which consists in solving parametric mean payoff games. We shall also present a path-following technique to solve these parametric mean payoff games and complexity bounds. This is a joint work with Antoine Bereau and Stephane Gaubert.

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MS99

Tropicalization in Real Algebraic Geometry

I will survey some recent applications of tropicalization in real algebraic geometry and combinatorics. Tropicalization is useful for distinguishing between nonnegative polynomials and sums of squares and capturing a coarse picture of the combined behavior of several quantities in combinatorics.

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MS99

Fewnomial Theory on Average

We present recent results with Telek and Tonelli-Cueto that shows a bound reminiscent of Kushnirenko's conjecture holds true for Gaussian sparse polynomial systems with arbitrary variance.

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MS99

Real and Positive Topicalization of Symmetric Determinantal Varieties

I will present a combinatorial description of real tropicalization of the space of symmetric rank two or corank one matrices and compare two notions of tropical positivity for these varieties. This is based on a joint work with Abeer Al Ahmadieh and May Cai.

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MS100

Internal Zonotopal Algebras and Orlik-Terao Algebras

Given a finite set of points in space, one can ask for a minimal vector space of functions on the ambient space which interpolate all possible functions on the finite set. An elegant answer to this question for the internal lattice points of a zonotope was conjectured by Holtz-Ron and proved by Lenz: a certain space of differential operators applied to the box spline. The space of differential operators is the internal zonotopal space, introduced independently by Holtz-Ron and Ardila-Postnikov. We use the tools they develop to prove a combinatorial conjecture of Mosely-Proudfoot-Young about Orlik-Terao algebras of graphs. The key step is to find new recursive structure on the internal zonotopal space, which we hope will provide new insight into this area and its applications. Contains joint work with Galen Dorpalen-Barry, Andr Henriques, and Nicholas Proudfoot.

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MS100

Covering Relations in the Poset of Neural Codes

A neural code $\mathcal{C}$ is a subset of $2^{[n]}$ for some $n \in \mathbb{N}$. Each $i \in [n] = \{1, \ldots, n\}$ represents a neuron, and each $c \in \mathcal{C}$, called a codeword, records the co-firing event of some neurons. Consider a space $X \subseteq \mathbb{R}^d$, simulating an animal's environment, and a collection $\mathcal{U} = \{U_1, \ldots, U_n\}$. Each subset $U_i \subseteq X$ simulates a receptive field corresponding to a specific region where neuron i is active. Then, $\text{code}(\mathcal{U}, X) = \left\{\sigma \subseteq [n] \middle| \bigcap_{i\in\sigma} U_i \setminus \bigcup_{j\notin\sigma} U_j \neq \right\}$. If a neural code $\mathcal{C} = \text{code}(\mathcal{U}, X)$ for some X and $\mathcal{U}$, we say $\mathcal{C}$ has a realization. Moreover, if $\mathcal{U}$ consists of open convex subsets of X, we say $\mathcal{C}$ is an open convex neural code. A prior study constructed a poset of distinct isomorphism classes of neural codes, partially ordered by the relation "is a minor of" and denoted by $\mathbf{P}_{\mathbf{Code}}$. Recently, a conjecture suggested that any open convex code can be realized as a minor of a neural code derived from a representable oriented matroid. However, the structure of $\mathbf{P}_{\mathbf{Code}}$ remains poorly understood. To investigate this conjecture, we characterize the covering relations in $\mathbf{P}_{\mathbf{Code}}$ and propose an algorithm to enumerate all distinct isomorphism classes of covering codes for a given neural code.

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MS100

The Foundation of a 2-sum of Matroids

The foundation of a matroid is an algebraic invariant that controls representations over any partial field, hyperfield, or more generally, any pasture. We show that the foundation of a 2-sum of two matroids is the absolute tensor product of their foundations. This is joint work with Matt Baker, Oliver Lorscheid, and Tianyi Zhang.

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MS101

Universality Conjecture for Nodal Count Distribution on Discrete Graphs

The eponymous conjecture claims that the distribution of the number of sign changes of a graph's eigenvector (suitably rescaled and averaged) converges to the Gaussian for any sequence of graphs with a growing number of cycles. A promising pathway to this conjecture goes through a characterization of the number of sign changes as a Morse index of a certain critical point in the "dispersion relation" of the graph (more precisely, the dispersion relation of the maximal abelian cover of the graph). To extract this information from Morse inequalities, one needs to understand all critical points of the dispersion relation. We will explain the conjecture and report on the recent progress in classifying critical points in the dispersion relation of the maximal abelian covers. Based on recent work with Lior Alon and Mark Goresky.

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MS101

Opening Gaps in the Spectra of Periodic Word Models

We describe a new approach to opening spectral gaps for periodic word models via a simple algebraic criterion. This streamlines existing gap opening constructions and paves the way for new examples of ergodic Schrdinger operators with exotic properties.

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MS101

Is a Matrix Factorization Approach Useful for Proving Irreducibility of the Bloch and Floquet Varieties?

The talk is devoted to the formulation of a problem, or rather of a new approach to a well-known problem. Namely, all known to the author approaches rely upon proving irreducibility of the (in the continuous case, regularized) determinant of the (operator) matrix polynomial. This determinant is a high dimensional polynomial (and even an entire function in the continuous case). On the other hand, one can prove that in fact in the reducible case the quadratic matrix (operator) polynomial should itself be decomposable. At first glance, this seems to make the problem much easier, due to the low degree and simple structure of the (operator-) matrix-valued polynomial. However, in the matrix (operator) case, factorization is not unique, which seems to erase the advantage. The problem is in the spirit of matrix factorization of polynomials, and thus hard. Any progress would be welcome.

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MS101

Some Algebraic Problems from Periodic Operators

I will pose some problems in commutative algebra that emanate from periodic graph operators in physics. Particularly, the localization of defect states in the continuum for tight-binding models without symmetry leads to a problem in factorization of polynomial matrices. I will illustrate the significance of this problem by the example of AB-stacked graphene.

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MS102

Totally Positive Skew-Symmetric Matrices

A matrix is totally positive if all of its minors are positive. This notion of positivity coincides with the type A version of Lusztig's more general total positivity in reductive real-split algebraic groups. Since skew-symmetric matrices always have nonpositive entries, they are not totally positive in the classical sense. The space of skew-symmetric matrices is an affine chart of the orthogonal Grassmannian OGr(n,2n). Thus, we define a skew-symmetric matrix to be totally positive if it lies in the totally positive orthogonal Grassmannian. We provide a positivity criterion for these matrices in terms of a fixed collection of minors, and show that their Pfaffians have a remarkable sign pattern. The totally positive orthogonal Grassmannian is a CW cell complex and is subdivided into Richardson cells. We introduce a method to determine which cell a given point belongs to in terms of its associated matroid. This is joint work with Jonathan Boretsky and Veronica Calvo Cortes.

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MS102

Canonical Forms and Dual Volumes

It is well-knwon that the canonical function of a projective polytope has an integral representation as the volume of the dual polytope. I will comment on the recent exploration of such dual volume representations for canonical forms of positive geometries with non-linear boundaries. This pictures nicely connectes to positivity, and more strongly, to complete monotonicity: an analytic property of real-valued functions defined on real cones, that has been recently discovered for various quantites arising in high-energy physics. This perspective drawes intriguing connections to algebraic statistics, partial differential equations and to convex optimization. It is also relevant for scattering amplitudes, where it offers a different perspective on the implications of an underlying positive geometry description. This point of view may also be suited for extending the notion of positive geometries to more realistic physical theories.

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MS102

Adjoints in Positive Geometry

I will present results of joint work with Kristian Ranestad and Simon Telen on adjoints (and canonical forms) in the context of amplituhedra. I might also report on currently ongoing work on these questions.

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MS102

Wondertopes

Polytopes in projective space are well-studied, prototypical examples of positive geometries. Considering a hyperplane arrangement in projective space, a region in the complement of the arrangement is a polytope, hence a positive geometry. We define a *wondertope* as the transform of such a polytopal region under a DeConcini-Procesi wonderful compactification of the hyperplane arrangement. We prove that wondetopes are also positive geometries. We also provide a combinatorial description of the product structure of the facets of a wondertope, in terms of nested set complexes in the intersection lattice of the underlying hyperplane arrangement.

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MS103

The Expressive Strength of Relu Transformers

In this talk, we will explain two works on the algebraic structure of ReLU transformers. We first explain the closed connection of ReLU-transformers and piecewise continuous functions. Viewed in this manner, this mysterious but critical component of a transformer becomes a natural development of an old notion deeply entrenched in classical approximation theory. We will further give quantitative bounds on the parameters of transformers and its connections with multivariate splines and wavelets.

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MS103

Algebraic Geometry Learns Machines and Machines Learn Algebraic Geometry

In this talk I will overview some existing results and ongoing work at the intersection of algebraic geometry and machine learning. I will present how a piecewise linear and combinatorial variant of algebraic geometry-known as tropical geometry-has been shown to be relevant in defining neural networks and talk about some recent and current work that our group is doing that adapts tropical geometry theory in numerical studies towards a better understanding of neural network behavior during training. While algebraic geometry holds much potential for better understanding machine learning, it turns out that machine learning is also a powerful tool that can help develop algebraic geometry theory. I will also overview some recent and ongoing work by researchers in my group where we use neural networks for theorem discovery in algebraic geometry.

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MS104

Growing Sparse Codes from Quantum Lego

A central challenge in quantum computing is the design of effective quantum error-correcting codes. Traditionally, quantum codes can be built from classical codes. However, a recently introduced Quantum Lego framework also allows one to construct quantum codes directly from quantum codes. This modular construction generalizes code concatenation, provides substantial computational speedups for the computation of weight enumerator polynomials, and enables a number of novel applications in code design. Although an existence proof has been given earlier showing that one can build any quantum code from a few types of atomic lego blocks, it is generally unclear how they should be combined to grow quantum LDPC codes such that the degree of the Tanner graph can remain bounded while increasing the code distance. In this talk, we introduce a novel method that uses non-isometric code concatenation involving quantum repetition codes. We show that this method can build up quantum LDPC codes and sparse subsystem codes. We also show that the (non-isometric) concatenations of two-qubit quantum bit flip and phase flip repetition codes are sufficient to create any CSS code.

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MS104

An Introduction to Quantum Anticodes

We introduce a notion of anticode tailored to quantum codes, and specifically stabilizer codes. In this context, a stabilizer code can be treated as a type of code over an Abelian group with symplectic form and their anticodes then blend the features of the Hamming metric and rank metric. We provide several notions of dually between quantum analogues of higher weights and binomial moments, including the quantum MacWilliams identities.

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MS104

Hulls of Linear Codes and Applications

The hull of a linear code is the intersection of the code with its dual. In this talk, we share recent results on hulls and their applications.

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MS104

Quantum Fault-Tolerance Via Classical Codes

Implementing non-Clifford gates, particularly the T gate, is one of the main challenges in fault-tolerant quantum computing. CSS-T codes are quantum codes specifically designed to support the transversal T gate. We provide a condition for CSS codes to qualify as CSS-T codes in terms of the Schur product of the constituent classical codes. A subfamily of CSS-T codes is formed by triorthogonal codes, where applying the transversal T gate results in a logical T gate over the logical qubits, up to a Clifford correction. We investigate some properties of these codes and derive rules to generate new triorthogonal codes from existing ones. Moreover, we construct CSS-T codes and triorthogonal codes from cyclic and extended cyclic codes. This is a joint work with Eduardo Camps-Moreno, Hiram H. Lpez, Gretchen L. Matthews, Rodrigo San-Jos, and Ivan Soprunov.

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MS105

Matroid Connectivity in Hamming and Rank Metric Codes

Connectivity is a fundamental concept in graph theory that was later extended to matroid theory. Several notions of matroid connectivity exist, most notably Tutte connectivity and vertical connectivity. In this talk we will characterize linear codes that have full vertical connectivity. Furthermore, we will propose a definition of (higher) connectivity for a q-matroid, explore some of its properties, its connections to rank metric codes and how it relates to connectivity for classical matroids.

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MS105

Nested q-Matroids

q-Matroids generalise both matroids and vector-rank-metric codes. Matroids, themselves generalising Hamming-metric linear codes, have seen a vast amount of study and development since the 1930s, whereas q-matroids have only seen a surge in popularity within the last decade. Naturally, a lot of q-matroid research involves developing q-analogues of concepts from matroid theory. Some of these concepts, such as graphic matroids, matroid polytopes, and many others, don't seem to have an intuitive q-analogue. One concept that extends remarkably well to the q-analogue is cyclic flats. Furthermore, cyclic flats have shown to have powerful proving capabilities in both matroid and q-matroid theory. A subclass of matroids, called nested (or Schubert) matroids, are characterised via chains of cyclic flats, and have a rich collection of properties. In this talk, I will present a q-analogue of nested matroids, and discuss their capacity for furthering q-matroid research.

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MS105

Higher Weight Spectra of Linear Codes: A Matroid Approach

Following [Johnsen and Verdure, Hamming weights and Betti numbers of Stanley-Reisner rings associated to matroids, *Appl. Algebra Eng. Commun. Comput.*, 2013], one can associate a vector matroid to a linear code C, and hence, a Stanley-Reisner ring R_C. The ring R_C admits a graded minimal free resolution, and the resulting graded Betti numbers, referred to as the Betti numbers of C, are known to determine the generalized Hamming weights of C. Further, the works of [Johnsen, Roksvold and Verdure, A generalization of weight polynomials to matroids, *Discrete Math.*, 2016], and [Helleseth, Klve and Mykkeltveit, The weight distribution of irreducible cyclic codes with block lengths $n_1((q^l-1)/N)$, *Discrete Math.*, 1977] show that the Betti numbers of C and its matroid elongations determine the higher weight spectra of C. In this talk, we explore this connection and use it to determine the complete higher weight spectra of first non-trivial class of generalized Reed-Muller codes.

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MS105

Latroids in the Rank Metric

It is widely known that many properties of a rank-metric code can be determined by the associated q-polymatroid. In this talk we will present a generalization of this combinatorial object, the so-called latroid. We will discuss the basic properties and we will show how we recover the classical definition of q-polymatroid. Working in this generality will allow us to extend some results to the case of rank-metric codes over certain classes of finite commutative rings.

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MS106

Forms of Low Slice Rank and Strength

We analyze slice rank and strength of polynomials and we study geometric and set-theoretic properties of varieties parameterizing forms with low strength and slice rank. In particular, we focus on the related properties of cubic forms. This is a joint work with Fulvio Gesmundo, Alessandro Oneto, and Emanuele Ventura.

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MS106

How Many Minimal Decompositions Exist of an Even Degree Complex Polynomial as a Sum of Squares ?

Homogeneous polynomials of even degree d can be written as a sum of squares of homogeneous polynomials of degree $d/2$. Such decompositions are never unique and they are divided into$O(r)$-orbits, where r is the number of summands and $O(r)$ is the orthogonal group in r variables. The problem becomes counting how many different$O(r)$-orbits exist. We say that$f$is$O(r)$-identifiable if there is a unique$O(r)$-orbit. We give sufficient conditions for generic and specific$O(r)$-identifiability. Moreover, we show the generic$O(r)$-identifiability of ternary forms.Joint work with Ettore Turatti.

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MS106

Spans of Derivatives and Catalecticant Enveloping Varieties

Given a degree d form, its k–th order derivatives span a linear subspace in the space of forms of degree $d-k$. This linear subspace is dual to the degree k–th component of the apolar ideal of the form and it is closely related with the k–th gradient rank. In this work in progress with Leonie Kayser, we study the geometry of the variety of linear subspaces of a fixed dimension of the space of degree $d-k$ forms that are spanned by all the k–th derivatives of a degree d form. To do so, we introduce the catalecticant enveloping variety and study its irreducible components.

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MS106

Configurations of Points in Projective Space and

Their Projections

We call a set of points $Z \subset \mathbb{P}^3_{\mathbb{C}}$ an (a,b)-geproci set (for GEneral PROjection is a Complete Intersection) if its projection from a general point P to a plane is a complete intersection of curves of degrees a and b. Examples which we call grids have been known since 2011. During the talk I will show a construction of nongrid nondegenerate (a,b)-geproci sets, for any $4 \leq a \leq b$. I will also show that the only such example with $a = 3$ is a $(3,4)$-geproci set coming from the D_4 root system. This is the joint project with Luca Chiantini, Łucja Farnik, Giuseppe Favacchio, Brian Harbourne, Juan Migliore and Tomasz Szemberg.

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MS107

An Algorithmic Approach to Codimension Theorems for Complete Toric Varieties

Let X be a complete toric variety with homogeneous coordinate ring S. Computing dimension and bases of the critical degree of ideals of S generated by dim(X) + 1 homogeneous polynomials which do not vanish simultaneously on X is still an area of active research, some bounds have been produced combinatorially but still we do not have exact results and "canonical" bases representing these vector spaces. We will report on the state of the art of the problem, and joint work with Alicia Dickenstein and Ivan Soprunov to shed some light in this topics.

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MS107

Kp Solitons: Tropical Curves Meet Grasmannians

The KP equation is a partial differential equation describing the motion of shallow water waves. A key characteristic of this equation is its remarkable algebraic integrability, with its solutions expressed in terms of algebraic-geometric structures. In this talk, we will give a brief overview of past approaches toward obtaining KP solitons from combinatorial and algebro-geometric techniques. We will then discuss some work in progress towards unifying them.

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MS107

A Problem Coming from the Evaluation of a Quartic Integral

Let $q(x) = x^4 + 2ax^2 + 1$ be an even quartic polynomial. The integral of $q(x)$ raised to a negative integer power over the real line can be expressed in terms of a polynomial in the parameter a. It turns out that this is a Jacobi polynomial of a non-classical type. The limiting behavior of the roots of q were conjectured to lie on a lemniscate. A solution to this conjecture, based on a Riemann-Hilbert method, will be outlined. This is joint work with John Lopez and Ken McLaughlin.

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MS107

The Volume Polynomial and Absolute Value of the Grassmannian

The volume polynomial is an n-variate homogeneous polynomial of degree d associated with a collection of n convex bodies in $\mathbb{R}^d$. It originates in Brunn-Minkowski theory and has appeared in various fields including convex and algebraic geometry, combinatorics, and statistics. I will present a computational framework that connects the volume polynomial of zonoids with combinatorics of the image of the real Grassmannian under the coordinate-wise absolute value map. This approach allows us to derive new coefficient inequalities for the volume polynomial of n zonoids in dimension d when $(n,d) = (4,2)$, $(6,2)$, and $(6,3)$. This is joint work with Gennadiy Averkov, Katherina von Dichter, and Simon Richard.

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MS108

Hypothesis Testing for Phylogenetic Networks Using Composite Likelihood Ratio Tests

Statistical inference for species-level phylogenetic networks from genome-scale data is complicated by the computational intractability of likelihood calculations under the multispecies coalescent model. Composite likelihood provides an alternative approach for which theoretical properties can be derived to allow standard statistical methods to apply. In this talk, the utility of the composite likelihood approach is demonstrated by deriving conditions under which likelihood ratio tests can be applied with composite likelihood in place of the full likelihood for testing nested phylogenetic network hypotheses. The performance of the approach is demonstrated with both simulated and empirical data.

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MS108

Species Delimitation Under the Coalescent Model

We introduce a new distance-based method for delimiting species under the multispecies coalescent model. The method is developed under the hypothesis that pairs of individuals whose species tree distance is zero belong to the same species. The species tree distance is estimated using a formula derived from the multispecies coalescent using a Jukes-Cantor distance correction term. We introduce two algorithms for sorting individuals into species. The first estimates the number of species using a rank condition on the matrix of pairwise distances. The second method uses an error threshold computed in terms of an estimated effective population size.

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MS108

Ultrafast Learning of 4-Node Hybridization Cycles Using Phylogenetic Invariants

The abundance of gene flow in the Tree of Life challenges the notion that evolution can be represented with a fully bifurcating process which cannot capture important biological realities like hybridization, introgression, or horizontal gene transfer. Coalescent-based network methods are increasingly popular, yet not scalable for big data, because they need to perform a heuristic search in the space of networks as well as numerical optimization that can be NP-hard. Here, we introduce a novel method to reconstruct phylogenetic networks based on algebraic invariants. While there is a long tradition of using algebraic invariants in phylogenetics, our work is the first to define phylogenetic invariants on concordance factors (frequencies of four-taxon splits in the input gene trees) to identify level-1 phylogenetic networks under the multispecies coalescent model. Our novel hybrid detection methodology is optimization-free as it only requires the evaluation of polynomial equations, and as such, it bypasses the traversal of network space, yielding a computational speed at least 10 times faster than the fastest-to-date network methods. We illustrate our methods performance on simulated and real data from the genus Canis. We present an open-source publicly available Julia package PhyloDiamond.jl with broad applicability within the evolutionary community.

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MS109

Computing a Loss Function to Bound the Interleaving Distance for Mapper Graphs

Data consisting of a graph with a function mapping into R^d arise in many data applications, encompassing structures such as Reeb graphs, geometric graphs, and knot embeddings. As such, the ability to compare and cluster such objects is required in a data analysis pipeline, leading to a need for distances between them. In this work, we study the interleaving distance on discretization of these objects, called mapper graphs when $d = 1$, where functor representations of the data can be compared by finding pairs of natural transformations between them. However, in many cases, computation of the interleaving distance is NP-hard. For this reason, we take inspiration from recent work by Robinson to find quality measures for families of maps that do not rise to the level of a natural transformation, called assignments. We then endow the functor images with the extra structure of a metric space and define a loss function which measures how far an assignment is from making the required diagrams of an interleaving commute. Finally, we use integer linear programming to obtain an assignment with a minimal loss function value. This work is joint with Erin Chambers, Ishika Ghosh, Elizabeth Munch, and Bei Wang.

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MS109

Solving Parametric Linear Matrix Inequalities and Convergence Analysis in Optimization

We consider linear matrix inequalities (LMIs) with coefficients in a ring R. When R consists of real numbers, the feasibility problem for generic LMIs is known to be solvable by algebraic methods. When R is a polynomial ring involving some parameters, the problem then asks for a formula Phi on the parameters, which describes the values of the parameters for which the specialized LMI is feasible. Such a formula Phi can be computed using classical quantifier elimination (QE) algorithms, but with a prohibitive complexity. In the case where only a dense subset of the region defined by Phi is needed, we design a dedicated algorithm, leveraging the LMI structure, to compute much more efficiently such a formula. We demonstrate the general results, with applications to convergence analyses of first-order optimization algorithms, known to be equivalent to the feasibility problems of certain LMIs. This is joint work with Simone Naldi, Mohab Safey El Din and Adrien Taylor.

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MS109

New Approach to Streaming Data Tensors

In this talk, I will present a new randomized optimization method for maintaining low-rank CP decompositions of tensorial data streams. Numerical results suggest the approach has acceptable computational costs at scale, while improving accuracy and adaptivity to changes in the data stream over existing methods. Theoretically, the approach relies on algebro-geometric properties of the Segre variety. Joint work with Joe Kileel (UT Austin).

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MS109

A Disjunctive Approach for Polynomial Bilevel Optimization

In this talk, we introduce a new approach for bilevel polynomial optimization in which lower-level constraining functions depend linearly on the lower-level variables. We show that such bilevel program can be reformulated as a disjunctive program by using Karush-Kuhn-Tucker (KKT) conditions with a sparse type of Lagrange multipliers. This kind of Lagrange multipliers can be conveniently represented by polynomials, for which we call partial Lagrange multiplier expressions (PLMEs)

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MS110

New Results on Computing Roadmaps of Semi-algebraic Sets

I will discuss two new results on computing roadmaps of semi-algebraic sets. The first result (with Sarah Percival) is a singly exponential algorithm for computing a description of a one-dimensional subset of a given semi-algebraic set S that inherits not just zero-dimensional but also one-dimensional connectivity information of S. The second result (with Marie-Francoise Roy) is an efficient divide-and-conquer algorithm for computing roadmaps of semi-algebraic sets. Such an algorithm was known before only for algebraic sets.

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MS110

On the Convergence of Critical Points on Varieties and Applications

Let $F \in \mathrm{R}[X_1, \ldots, X_n]$ and the zero set $V = \mathrm{zero}(\{P_1, \ldots, P_s\}, \mathrm{R}^n)$ be given with the canonical Whitney stratification, where $\{P_1, \ldots, P_s\} \subset \mathrm{R}[X_1, \ldots, X_n]$ and R is a real closed field. We explore isolated trajectories that result from critical points of F on $V_\xi = \mathrm{zero}(\{P_1 - \xi_1, \ldots, P_s - \xi_s\}, \mathrm{R}^n)$ when $\xi \downarrow 0$, in the sense of stratified Morse theory. Our main motivation is the limiting behavior of log-barrier functions in polynomial optimization which leads to a critical path, an underlying notion behind the theory of interior point methods. We prove conditions for the existence, convergence, and smoothness of a critical path. We also consider the cases where F and P_i are definable functions in a (polynomially bounded) o-minimal expansion of $\mathbb{R}^n$.

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MS110

Diophantine Approximation Speeds Up Real Root Counting

The number of bits needed to describe a system F of n polynomials in n variables, having a total of $n + k$ distinct exponent vectors with coordinates of absolute value at most d, and integer coefficients of absolute value at most 2^h is $O(n^2(n + k)h \log d)$. However, all general algorithms for counting the roots of F in the positive orthant currently have complexity exponential in this input size. We show that for $k \leq 2$, the complexity is in fact polynomial in the input size, outside of a small set of exceptions. This expands an earlier speed-up for counting the number of connected components of the zero set of a single $(n + 2)$-nomial in the positive orthant. This is joint work with Weixun Deng, Alperen Ergur, and Grigoris Paouris.

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MS110

Sharper Bit Complexity Estimates for Computing One Point per Connected Component of a Semi-algebraic Set Defined by a Single Inequation

We analyze the bit complexity of a randomized algorithm for computing at least one point per connected component of a semi-algebraic set defined by a single inequality, as well as its error probability. When the input n-variate polynomial has degree d and its coefficients have bit size bounded by t, we prove that under a regularity assumption (satisfied generically), the bit complexity estimate is cubic in $d(d-1)^n$ and quasi-linear in t. This bit complexity estimate is a bit larger when the regularity assumption is removed (quartic in $d(d-1)^n$). We also discuss the practical performances of this algorithm.

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MS111

Weakly Reversible Chemical Reaction Networks Are Recurrent in 2d

We prove that the continuous time Markov chains modeling the evolution of weakly reversible chemical reaction networks equipped with mass action kinetics are positive recurrent.

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MS111

Stochastic Ordering Tools for Reaction Network Models

When studying stochastic reaction networks, one is typically interested in understanding their dynamical and asymptotic behaviors. Except for a few cases where it is possible to directly compute the quantities of interest, it is often difficult to predict how the abundances of each species will evolve once the reactions are initiated. One approach to address this is to compare the reaction network under study with a similar one whose behavior is better understood. A first step in this direction was taken by Campos et al. in their 2023 paper *Comparison Theorems for Stochastic Chemical Reaction Networks*, in a framework and language which are slightly more general than just stochastic reaction networks. Our main contribution is to provide more direct and computable conditions that can be checked to guarantee the validity of the hypotheses in their theorems. Thanks to the algorithmic nature of our method, it is now possible not only to quickly check whether two models are comparable with respect to a chosen preorder in their common state space, but also to identify from scratch which are the preorders enabling the application of their theorems.

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MS112

Improved Fewnomial Upper Bounds from Wronskians and Dessins Denfant

We consider the problem of counting the number of real solutions to a real square polynomial system $f = g = 0$, where f has three monomials terms, and g has t monomial terms. Currently, the best-known upper bound on the maximal number of positive solutions, that any such a system can produce, is a polynomial in t of degree three. The collective effort to tackle this fewnomial problem produced a sequence of diverse methods that incrementally improved the upper bound thoughout the last 25 years. I will highlight some of the methods mentioned above, and focus on the latest iteration that uses a combination of Wronskians and Grothendieck's dessins d'enfant. I will also present some conjectural necessary conditions inspired from Viro's patchworking constructions. This is a joint work with Sbastien Tavenas.

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MS112

Combinatorial Patchwork: Now and Then

In the 1970s, Viro's method opened an important path in the study of the topology of real algebraic varieties and constituted one of the foundations of tropical geometry. This method is based on subdividing an integer polytope and using the information from each of the pieces to glue them together and obtain information about the union. A key condition of this method is that the subdivision needs to be convex or regular. A special case of Viro's method is known as combinatorial patchwork. In this talk, we will talk about a joint work with Erwan Brugall and Johannes Rau where we translate certain tropical geometry results into the original language of combinatorial patchwork and study the combinatorial objects obtained, without taking into account the convexity condition. In this new setting, we generalise some results of tropical homologie obtained by K. Shaw, J. Rau and A. Renaudineau.

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MS112

Real Algebraic Varieties Close to Non-singular Tropical Limits

In order to study the topology of real algebraic varieties, a combinatorial description of these spaces can be extremely useful. Such a description appears for example in Viro's combinatorial patchworking which is a powerful technique for studying the possible topological types of real algebraic hypersurfaces. I want to present a generalization of this description to higher codimensions which uses the idea of tropical limits. More precisely, given a family of real algebraic varieties with "non-singular tropical limit", we give a description of the topology a generic fiber via a "real structure" associated to the tropical limit. In my talk, I will explain tropical limits and how to use them to recover the topology. (joint work with Kris Shaw and Arthur Renaudineau)

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MS113

Higher Order Cumulants for Identifying Discrete Lyapunov Models

Discrete Lyapunov models are used for the study of multivariate time series data. The process is defined by a directed weighted graph, that depicts dependencies of variables as edges. Assuming that variables are non-gaussian, we use higher order cumulants to study the model identifiability question. Our approach is to use algebraic methods. In this talk we will discuss the parameter identifiability of certain classes of graphs, and the graph identifiability using characterization of the second, third, and higher order cumulants.

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MS113

Conditional Independence in Stationary Diffusions

Stationary distributions of multivariate diffusion processes have recently been proposed as probabilistic models of causal systems in statistics and machine learning. Taking up this theme, I will present a characterization of the conditional independence relations that hold in a stationary distribution of a diffusion process with a sparsely structured drift. The result draws on a graphical representation of the drift structure and clarifies that marginal independencies are the only source of independence relations. Central to the proof is an algebraic analysis of Gaussian stationary distributions obtained from multivariate Ornstein-Uhlenbeck processes.

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MS113

Identifiability in Continuous Lyapunov Models

We study causality in systems that allow for feedback loops among the variables via models of cross-sectional data from a dynamical system. Specifically, we consider the set of distributions which appears as the steady-state distributions of stochastic differential equations (SDEs) where the drift matrix is parametrised by a directed graph. The nth order cumulant of the steady state distribution satisfies the corresponding nth order continuous Lyapunov equation. Under the assumption that the driving Lvy process of the SDE is not a Brownian motion (so the steady state distribution is non-Gaussian) and the coordinates are independent we are able to prove generic identifiability for any connected graph from the second and third order Lyapunov equations while allowing the cumulants of the driving process to be unknown diagonal.

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MS113

Causal Discovery in Time-Series Data Using Signature Tensors

In this work, we model causal relations among time-series data using a path-dependent stochastic differential equation (SDE). Any function of a path can be approximated by some linear functional of its signatures i.e. some linear functional of the iterated integrals of the path. We leverage this fact to model the causal relations among the time-series data using an SDE with a finite number of parameters. We then propose a causal discovery method, where these parameters are estimated by solving a polynomial system of equations. We show that this parameter estimation method is consistent under certain conditions on the driving noise.

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MS114

Solving Polynomials for Collaborative Navigation

Statistical inference for vehicle navigation typically involves minimizing the negative log likelihood function, which in the case of Gaussian distributed measurement noise is a weighted sum of squared errors between algebraic functions of the states (i.e. position and attitude) and measurements of angles and distances. These errors may be formulated as polynomial functions of the states, and the associated minimization problem may have multiple local minima. For small systems of collaboratively navigating vehicles using odometry and inter-vehicle range measurements, all local minima can be determined using computational nonlinear algebra tools such as homotopy continuation. This talk will present an analysis of such systems and the complications that can arise from the presence of multiple local minima in larger systems.

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MS114

Algebraic Geometry Cooperative Localization for Gps-Denied Environments

In GPS-denied environments, cooperative localization and navigation of autonomous aerial vehicles (AAVs) face challenges in connectivity and estimation accuracy. This talk explores the intersection of algebraic geometry and network rigidity theory to address these issues. AAV interactions are modeled as dynamic relative position measurement graphs, emphasizing connectivity between vehicles and landmarks to maintain localization accuracy. Using algebraic geometry, we derive localization error bounds and their dependence on graph connectivity. We propose a joint localization and path-planning framework leveraging nonlinear model predictive control (NMPC) and moving horizon estimation (MHE), demonstrating the impact of network structure and landmark configurations on trajectory optimization and covariance estimation. By aligning with rigidity theory, the framework ensures robust localization in complex environments. Numerical simulations validate its efficacy, highlighting computationally efficient strategies for optimizing connectivity and navigation in contested or urban airspaces. This talk bridges theory and application, appealing to researchers in mathematical and geometric approaches to networked navigation.

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MS114

Analyzing Distributed Formation Control Strategies Using Rigidity Theory

The formation control problem is to find a strategy that allows a set of autonomously acting agents to arrange themselves in a pre-specified spatial configuration, under a constraint of limited communication and local sensing. In a well-studied setup, the agents can sense the pairwise distance to a fixed set of other agents. Natural strategies for the variant of the problem in which the agents sense each other symmetrically and without error have been proven to locally converge, but the more realistic setting, in which sensing is asymmetric, is not well understood. I will describe how to view these strategies as members of a family of dynamical systems on the measurement variety of the sensing graph, and derive combinatorial necessary conditions using techniques from rigidity theory and algebraic geometry.

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MS115

One-Dimensional Fourier Quasicrystals, Quantum Graphs, and Lee–Yang Polynomials

Kottos and Smilansky (1997) proposed the model of a quantum graph as a one-dimensional model for quantum chaos. In particular, they introduced the secular polynomial of a graph as an analytic tool for computing eigenvalues, which they used to derive the trace formula for quantum graphs. The trace formula states that the counting measure of the spectrum of a quantum graph has a Fourier transform supported only on the possible lengths of periodic orbits of the graph. In 2020, Kurasov and Sarnak demonstrated that the trace formula of a quantum graph forms a one-dimensional Fourier quasicrystal (1D FQ): a counting measure of a discrete set whose Fourier transform is supported on a discrete set. Moreover, they observed that the secular polynomial of a quantum graph is a LeeYang polynomial, and that any LeeYang polynomial gives rise to a family of one-dimensional Fourier quasicrystals. I will present recent work, joint with Cynthia Vinzant and Alex Cohen, in which we show that all one-dimensional Fourier quasicrystals arise from this KurasovSarnak construction. If time permits, I will sketch a proof of the KurasovSarnak theorem based on a concept called lighthouse manifolds, introduced by Yves Meyer, and explain why the zero set of a LeeYang polynomial is such a manifold.

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MS115

On Spectra of (finite) Metric Tree Graphs

The secular manifold Σ_G and its singularities are intimately related to the spectra of metric graphs (G, ℓ) for some fixed graph G. I present a complete description of the singular locus for finite tree graphs and confirm that it agrees with a conjecture of Yves Colin de Verdire [Colin de Verdire, Semi-Classical Measures on Quantum Graphs and the Gau Map of the Determinant Manifold, 2015]. I also discuss numerous applications of this result toward the study of generic properties of eigenfunctions on finite tree graphs.

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MS115

High-dimensional Fourier Quasicrystals, Lighthouse Manifolds and Lee–Yang Varieties

In this talk we will explain how to construct Fourier quasicrystals in any dimension from a certain class of real algebraic varieties called Lee–Yang varieties. These are higher codimension analogues of stable polynomials. We will focus on a crucial spectral property of Lee–Yang varieties that makes the construction work, namely being a lighthouse manifold. This is joint work with Lior Alon, Pavel Kurasov and Cynthia Vinzant.

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MS115

Morse Inequalities for Ordered Eigenvalues of Generic Families of Self-adjoint Matrices

The topic of the talk was originally motivated by the Floquet-Bloch theory of Schrdinger equations with periodic potential and other problems in Mathematical Physics. The eigenvalue branches of families of self-adjoint matrices are not smooth at points corresponding to repeated eigenvalues (called diabolic points or Dirac points). Generalizing the notion of critical points as points for which the homotopical type of (local) sub-level set changes after the passage through the corresponding value, in the case of the generic (i.e., transverse to the discriminant set) family we give an effective criterion for a diabolic point to be critical for those branches and compute the contribution of each such critical point to the Morse polynomial of each branch, getting the appropriate Morse inequalities as a byproduct of the theory. These contributions are expressed in terms of the homologies of Grassmannians. If the time permits, we also discuss the case of non-transverse families. The talk is based on the joint work with Gregory Berkolaiko.

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MS116

Taking the Amplituhedron to the Limit

The amplituhedron is a semi-algebraic set given as the image of the positive Grassmannian under a linear map subject to a choice of additional parameters. We define the limit amplituhedron as the limit of amplituhedra by sending one of the parameters, namely the number of particles n, to infinity. We study this limit amplituhedron for $m = 2$ and any k. We determine its algebraic boundary in terms of Chow hypersurfaces and stratify this hypersurface in the Grassmannian by singularities in terms of higher order secants of the rational normal curve. In conclusion, we show that the limit amplituhedron is a positive geometry with a unique differential form and a residual arrangement that is empty.

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MS116

Kinematic Varieties for Massless Particles

We study algebraic varieties that encode the kinematic data for n massless particles in d-dimensional spacetime subject to momentum conservation. Their coordinates are spinor brackets, which we derive from the Clifford algebra associated to the Lorentz group. This was proposed for $d = 5$ in the recent physics literature. Our kinematic varieties are given by polynomial constraints on tensors with both symmetric and skew symmetric slices.

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MS118

Properties of Norm-Trace Codes from Its Numerical Semigroup.

We compute some invariants of the Weierstrass semigroup of certain divisors to compute properties of the associated Norm-Trace codes, such as the Clifford Defect or certain non-special divisors.

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MS118

Optimal Computational Secret Sharing

In (t, n)-threshold secret sharing, a secret S is distributed among n participants such that any subset of size t can recover S, while any subset of size $t-1$ or fewer learns nothing about it. For information-theoretic secret sharing, it is known that the share size must be at least as large as the secret, i.e., $|S|$. When computational security is employed using cryptographic encryption with a secret key K, previous work has shown that the share size can be reduced to $\frac{|S|}{t} + |K|$. In this paper, we present a construction achieving a share size of $\frac{|S|+|K|}{t}$. Furthermore, we prove that, under reasonable assumptions on the encryption scheme — namely, the non-compressibility of pseudorandom encryption and the non-redundancy of the secret key — this share size is optimal.

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MS118

About the Generalized Hamming Weights of Decreasing Norm-Trace Codes.

The generalized Hamming weights of a linear code determine its performance on the wire-tap channel of type II, they have applications to t-resilient functions and they can be used to obtain improved bounds on the list size in list decoding algorithms. This has motivated recent work on the generalized Hamming weights of many well-known families of codes, such as affine Cartesian codes, hyperbolic codes, projective Reed-Muller codes, or matrix-product codes. We determine the generalized Hamming weights of decreasing norm-trace codes, which are linear codes defined by evaluating sets of monomials that are closed under divisibility on the rational points of the extended norm-trace curve. A particular case of these codes are one-point algebraic geometry codes over the norm-trace curve and, in particular, the Hermitian curve. We achieve this using an improved footprint bound for these codes, and showing that this bound is sharp. We also study the relative generalized Hamming weights of these codes, and we use them to construct impure quantum codes with excellent parameters.

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MS119

Distributed Matrix Multiplication with Straggler Tolerance Using One Point Algebraic Geometry Codes

The tendency for speeding up the majority of algorithms is to parallelize them. An example is matrix multiplication, which can be performed faster if we split the original multiplication into smaller ones that can be computed simultaneously. When implemented, the execution time of the algorithm will be determined by the computation of the slowest subproduct. This is a problem when the concurrency of the algorithm grows since it causes a severe bottleneck in the execution time, i.e., the greater the number of subproducts the greater the probabilities of achieving a worse execution time in one of them. To mitigate this effect, methods using erasure-correction codes, more specifically Reed-Solomon codes, have been proposed. We extend these ideas using one-point algebraic geometry codes in order to design similar algorithms of distributed matrix multiplication that tolerate the bottleneck of sparse execution times, but without the alphabet size restriction that Reed-Solomon codes suffer from. We focus on studying the parameters of the obtained code through the associated Weierstrass semigroup.

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MS119

A New General Construction of Semifields, and their Relations to Rank-metric Codes

A finite semifield is an algebraic structure similar to a finite field, except that multiplication is not assumed to be commutative or associative. Semifields are in 1-1 correlation with certain optimal rank-metric codes. In this talk, I shall present two new constructions for semifields of order p^2m. Together, the constructions unify and generalize around a dozen distinct semifield constructions, including both the oldest known construction by Dickson and the largest known constructions in odd characteristic by Taniguchi. The constructions also provably yield many new semifields. We give precise conditions when the new semifields we find are equivalent and count precisely how many new inequivalent semifields we construct, and discuss the meaning of these results to rank-metric codes.

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MS119

A Zero-Knowledge Protocol for Bounded Hamming and Rank Vectors

Zero-knowledge (ZK) protocols have emerged as a crucial technology in building advanced digital signature schemes, ensuring privacy and security without revealing sensitive information. In ReSolveD, the authors make use of the ability to ZK prove that a vector is a unit vector with entries only 0/1. This talk introduces a ZK protocol for vectors over an extension field with bounded hamming and rank weight, resulting in more general ZK schemes.

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MS119

Algebraic and Combinatorial Algorithms for the Matrix Code Equivalence Problem

Broadly, an equivalence problem considers two instances of the same mathematical object and asks if there exists a map between them that preserves some defined property. Specifically, the matrix code equivalence problem takes as input two error-correcting codes in the rank metric and the map we are tasked to find is an isometry that preserves the rank of codewords. In this talk, we will additionally look into a related problem called the alternating trilinear form equivalence, where we are given two alternating trilinear forms and the goal is to find an isomorphism between them. We first show how these two problems are similar, namely that an alternating trilinear form can be viewed as a matrix code with special properties, or that a matrix code can be viewed as a trilinear form without the alternating property. We then present a survey of recent advances in solving these two problems both with purely algebraic algorithms and with combinatorial algorithms that have algebraic system solving as subroutines. The rising interest in these problems is due to their aptness for building a zero-knowledge-based identification scheme. As such, these two problems, alongside the code equivalence problem in the Hamming metric, have been used as a hardness assumption in the design of the Fiat-Shamir-based digital signature schemes MEDS, ALTEQ, and LESS.

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MS120

Hadamard Ranks of Algebraic Varieties

Inspired by X-ranks and secant varieties, we introduce Hadamard-X-ranks. Given a subvariety $X \subset \mathbb{P}^n$, the task is to write a point $p \in \mathbb{P}^n$ as a coordinate-wise product of points in X (Hadamard product) with as few factors as possible. In this context, the r-th Hadamard power of X plays the role of its r-th secant variety. Hadamard powers of secant varieties of Segre embeddings appear in Algebraic Statistics as they correspond to algebraic statistical models known as Restricted Boltzmann Machines. We discuss finiteness of some Hadamard ranks of tensors. Then, we focus on generic Hadamard-X-ranks. If the Hadamard powers of X do not eventually fill the ambient space, then X is contained, up to a diagonal action, in a proper Hadamard-idempotent subvariety. An example of such varieties are toric varieties. We investigate Hadamard-idempotent varieties by using Tropical Geometry.

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MS120

Tensor Varieties: Uniformity for Limits and Singularities

A *tensor variety* is a closed subvariety $X(V)$ of a space $T(V)$ of tensors, such that both depend functorially on the choice of a finite-dimensional vector space V. In earlier work, I showed that all $X(V)$ are defined by equations inherited from those for a fixed $X(U)$. My talk will report on two new extensions of this uniformity result. First, we established that the singular locus $X^{\mathrm{sing}}(V)$ of $X(V)$ is again a tensor variety, and hence defined by inheritance from some fixed $X^{\mathrm{sing}}(U)$. And second, we showed that if $\alpha : X \to Y$ is a dominant morphism of tensor varieties, then any point in $Y(V)$ equals $\lim_{\epsilon\to 0} \alpha(x(\epsilon))$ for some formal curve $x(\epsilon) \in X(V)$ whose negative exponents are bounded independently of V. Joint work with Chiu-Danelon and with Bik-Eggermont-Snowden.

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MS120

Hilbert Scheme of 9 and 10 Points and the Secant Varieties of Pencils

We describe the irreducible components of the Hilbert scheme of d points on affine space for $d = 9, 10$. The main techniques we use are the variety of commuting matrices and analyzing loci of local algebras with a specific Hilbert function. As the main consequence, we establish the equality of cactus Grassmann and the secant Grassmann variety in the corresponding cases. This is a joint project with Hanieh Keneshlou and Klemen Sivic.

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MS120

Matrix-pencil Invariants from Subvarieties of Grassmannians

In recent years new approaches to detect invariants of tensors have been proposed, e.g. characteristic polynomial (Conner-Michalek, 2021) and collineation varieties (Gesmundo-Keneshlou, 2024) of tensors. However, these approaches do not recover all invariants, not even for matrix pencils for which the Kronecker-Weierstarss form provides a complete classification. In this work, to a regular matrix pencil L with distinct eigenvalues we associate a subvariety Y_L of a Grassmannian endowed with a $\mathbb{C}^{\times}$-action. We identify Y_L with a blow-up of $\mathbb{P}^2$ along a collinear subscheme of points, for which we exhibit Groebner equations. This allows us to recover all the invariants of L from Y_L. This is an on-going joint work with Fulvio Gesmundo and Hanieh Keneshlou.

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MS122

Identifying Features of Phylogenetic Networks from Various Data Types

Phylogenetic networks and admixture graphs allow for the representation and modelling of gene flow, introgression and hybridization. Under the assumption that the network is of level-1, most of its topology and edge parameters are known to be identifiable, from various data types and under various models. But little is known about what network features are identifiable if the level-1 constraint is removed. I will show some features that are identifiable, and give examples of networks and substructures that cannot be distinguished, under models with and without incomplete lineage sorting.

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MS122

Identifiable Network Features from Genomic Sequences Using the Logdet Distance

Species network inference faces significant challenges due to the intricate interplay of hybridization and incomplete lineage sorting. These challenges are further compounded when inference is conducted directly from genomic data, as differences in gene content and rates of evolution must also be accounted for. The Network Multispecies Coalescent model, combined with a mixture of General Time-Reversible (GTR) sequence evolution models across individual gene trees, provides a stochastic framework to describe this complex process. In this talk, we present identifiable network features for different classes of networks within this model, using logDet distances computed from genomic-scale sequences.

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MS122

Identifiability of Level-1 Species Networks from Gene Tree Quartets

Phylogenetic networks offer a more realistic representation of evolutionary relationships when events such as hybridization and horizontal gene transfer are involved. Methods employing quartet concordance factors, the probabilities of four-taxon gene tree topologies, have emerged as powerful tools for inferring these complex networks. While earlier studies have provided initial insights into the identifiability of certain network aspects, a complete picture of what can and cannot be uniquely determined in level-1 phylogenetic networks under the network multispecies coalescent model has remained incomplete. In this talk, we address this gap by presenting a comprehensive analysis of the identifiability of level-1 networks. By investigating the algebraic varieties defined by quartet concordance factors, we achieve a complete understanding of which topological structures and numerical parameters of level-1 networks are identifiable and, crucially, which are not. In particular, we will provide answers to several open questions concerning 3-cycles. This definitive characterization of identifiability is fundamentally important for developing and applying rigorous inference methods.

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MS123

A Generating Set for Rational O(n) Invariants

The aim of the talk is to present a concrete and workable generating set for rational invariants for the action of the orthogonal group on polynomials of even degree. Such invariants are among other things useful in determining whether certain polynomial surfaces can be converted into one another by rotation, thus starting with such a given set of invariants can be used to rescue complexity of algebraic computations arising for example in optimization. Working with the field of rational invariants instead of the ring of invariants allows for a greater flexibility which is illustrated by a surprising connection to graph theory and the graph isomorphism problem.

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MS123

Computing Isotopy Type of Positive Zero Sets of Near-Circuit Polynomials

We present initial results on the algorithmic classification of the isotopy types of positive zero sets of polynomials supported on near-circuits. A near-circuit is a configuration of $n+3$ points in Z^n that do not lie in any affine hyperplane. Our main contribution is a significant improvement to the upper bound on the number of connected components of these zero sets, reducing it from O(n) to a constant value of at most 3. Additionally, we establish a criterion for identifying cases where the number of connected components is at most 2. A key tool in our approach is the computation of cusps on the associated signed A-discriminant contour.

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MS124

Learning Equivariant Tensor Functions with Applications to Sparse Vector Recovery

This work characterizes equivariant polynomial functions from tuples of tensor inputs to tensor outputs. Loosely motivated by physics, we focus on equivariant functions with respect to the diagonal action of the orthogonal group on tensors. Our goal behind these characterizations is to define equivariant machine learning models. In particular, we focus on the sparse vector estimation problem. This problem has been broadly studied in the theoretical computer science literature, and explicit spectral methods, derived by techniques from sum-of-squares, can be shown to recover sparse vectors under certain assumptions. Our numerical results show that the proposed equivariant machine learning models can learn spectral methods that outperform the best theoretically known spectral methods in some regimes. The experiments also suggest that learned spectral methods can solve the problem in settings that have yet to be theoretically analyzed. This is an example of a promising direction in which theory can inform machine learning models and machine learning models can inform theory.

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MS124

New Algorithms for Whitney Stratification of Real Algebraic Varieties and Applications

We describe several new algorithms to compute Whitney stratifications of real algebraic varieties, and of polynomial maps between them. One of the map stratification algorithms described here yields a new method for solving the real root classification problem. We also explore applications of this new map stratification algorithm to the study of the singularities of Feynman integrals; understanding and evaluating these integrals is a fundamental component in a wide variety of problems arising in quantum field theory.

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MS124

Is Uniform Expressivity Too Restrictive? Towards Efficient Expressivity of Graph Neural Networks

Uniform expressivity guarantees that a Graph Neural Network (GNN) can express a query without the parameters depending on the size of the input graphs. This property is desirable in applications in order to have number of trainable parameters that is independent of the size of the input graphs. Uniform expressivity of the two variable guarded fragment (GC2) of first order logic is a well-celebrated result for Rectified Linear Unit (ReLU) GNNs [Barcelo & al., 2020]. In this article, we prove that uniform expressivity of GC2 queries is not possible for GNNs with a wide class of Pfaffian activation functions (including the sigmoid and tanh), answering a question formulated by [Grohe, 2021]. We also show that despite these limitations, many of those GNNs can still efficiently express GC2 queries in a way that the number of parameters remains logarithmic on the maximal degree of the input graphs. Furthermore, we demonstrate that a log-log dependency on the degree is achievable for a certain choice of activation function. This shows that uniform expressivity can be successfully relaxed by covering large graphs appearing in practical applications. Our experiments illustrates that our theoretical estimates hold in practice.

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MS124

Condition-based Low-Degree Approximation of Real Polynomial Systems. I: The Zero-Dimensional Case

The most fundamental fact of real algebraic geometry is that the number of real zeros of a generic real polynomial system is not constant. However, what does influence the size of the zero set? In this talk, we show how the condition number of a real polynomial system restrict the possible number of real zeros. As a consequence, we derive important relations between the well-posedness of a real polynomial system and its number of real zeros, and derive new bounds for the number of real zeros of a random real polynomial system.

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MS125

Identifiability of SDEs for Reaction Networks

Biochemical reaction networks are widely used across fields, typically analyzed using one of three mathematical frameworks. In this talk, we focus on the diffusion approximation of networks with mass-action kinetics and explore the identifiability of the generator of the associated stochastic differential equations. We provide theorems and practical algorithms for assessing identifiability in different cases. Notably, we demonstrate that some networks have unidentifiable reaction rates, even with full knowledge of the stochastic process, and that distinct networks can generate the same diffusion law under specific conditions. Finally, we compare our results with those from deterministic models and Markov chain frameworks.

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MS125

Stability of Reaction Networks with Randomly Switching Parameters

Since the dawn of stochastic chemical reaction network theory over 50 years ago, there have been many general results about (positive) recurrence, especially in the case of mass-action kinetics. One less-explored area is that of mass-action models whose rate constants, rather than being static, are themselves stochastic. Such models have relevance in applications, since biomolecular systems rarely exist in isolation and their rates often depend on time-changing quantities. In this series of two talks, we investigate the positive recurrence of one of the simplest cases of these stochastic rate constants models, with two species, at most monomolecular source complexes, and rate-constants that switch stochastically between two possible sets of options. Already in this special case, it will turn out that the possible overall behaviors are rich. Specifically, this second talk will present a Hurwitz-matrix-type sufficient condition for a parameter-switching model to be transient even when the two individual networks that it switches between are positive recurrent, provided that the switching rate is fast enough. We will discuss some of the high-level ideas behind the proof, which will hopefully be applicable for proving transience in other settings. This talk is based on joint work with Daniele Cappelletti and Chuang Xu.

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MS125

Diffusion Limits for Measure-Valued Process Models of Stochastic Networks

In this talk, we obtain stochastic differential equations that will be satisfied by the diffusion limit of a measure-valued state descriptor for a multiclass, multi-server, random order of service queue with reneging and general distributional requirements. We develop a methodology to represent queueing systems similar to this one in terms of time-changed renewal processes and pure jump martingales. Then, in a general setting, we give conditions for tightness and the form of the SDE satisfied by the subsequential diffusion limits of systems represented in this manner. Finally, we use this methodology on our particular model in order to obtain tightness and an SDE satisfied by its subsequential diffusion limits.

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MS125

Scaled and Conditional Stationary Distributions for Complex Balanced Reaction Networks

The talk is about complex balanced reaction networks (RNs) and their stationary distributions. We assume the intensities of the reactions take a general form and let (N, λ) denote the RN with intensities $\lambda = (\lambda_1, ..., \lambda_m)$ where λ_r is the intensity of the rth reaction. In this framework, we study the reduction of (N, λ) by graphical elimination of non-interacting species along reaction paths.

Specifically, let U be a subset of non-interacting species of (N, λ) and let (N_U, λ_U) be the reduced RN. Moreover, let Γ be an irreducible component of $\mathbb{N}_0^n$, and let $\Gamma_0 \subseteq \Gamma$ be the states for which the species in U have zero counts. Let π be a complex balanced stationary distribution of (N, λ) on Γ. Under regularity conditions, we show that **1.** The conditional distribution $\pi(\cdot|\Gamma_0)$ of π conditioned to Γ_0 is complex balanced for (N_U, λ_U). **2.** Let λ^K be a sequence of scaled intensities on N, in particular $\lambda_r^K(x) \to 0$ as $K \to \infty$. Then, the stationary distribution on Γ of (N, λ^K) exists, provided it exists for (N, λ^1). Furthermore, the stationary distribution of (N, λ^K) converges to that of (N_U, λ_U) as $N \to \infty$.

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MS126

The Tropical Amplituhedron

The Amplituhedron is a geometric object discovered recently by Arkani-Hamed and Trnka, that provides a completely new direction for calculating scattering amplitudes in quantum field theory (QFT). We define a tropical analogue of this object, the tropicial amplituhedron and study its structure and boundaries. It can be considered as both the tropical limit of the amplituhedron and a generalization of the tropical positive Grassmannian.

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MS126

Scattering Amplitudes and the Chirotropical Grassmannian

In this talk, we introduce and study the chirotropical Grassmannian $\mathrm{Trop}^\chi G(k,n)$ and the chirotropical Dressian $\mathrm{Dr}^\chi(k,n)$, arising at the intersection of tropical geometry, oriented matroids, and particle physics. Focusing on the rank-three case, we present an equality of these two polyhedral fans for $n = 6, 7, 8$ and outline how to compute their maximal cones via novel combinatorial algorithms based on χ-compatibility. Using these methods, we classify the resulting fans for all isomorphism classes of uniform realizable chirotopes, producing a complete census of chirotropical Grassmannians in these dimensions. We also show that each chirotopal configuration space $X^\chi(3,6)$ is homeomorphic to a polytope and propose a canonical differential form associated with it. Finally, we demonstrate that the equality $Trop^\chi G(k,n) = Dr^\chi(k,n)$ does not extend to certain higher-rank cases, offering insight into the subtleties of realizability beyond rank three.

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MS126

Tropical Geometry, Divergent Integrals and Particle Physics

Physical phenomena at very high energies are described through the celebrated Feynman integrals. The computation of these integrals is notoriously plagued by divergences. Rather than being an unpleasant feature of the formalism, as they are often thought of, these divergences actually reflect the vast hierarchies in the energy scales of the physical processes under consideration, and thus encode important physical knowledge. It is therefore crucial to understand the mathematical structures that better describe them. In this talk, I will explain how concepts from combinatorics and tropical geometry illuminate our understanding of the singularities of Feynman integrals, while concretely providing new algorithmic tools for their computation. I will also discuss various open problems, for which we can anticipate significant breakthroughs in the future by further deepening the synergy between physics and discrete mathematics.

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MS127

Noetherian Operators: Foundations and Perspectives for Machine Learning

The study of solutions to partial differential equations (PDEs) using algebraic methods has experienced a resurgence, driven by recent computational and algorithmic advancements. In this talk, we begin by exploring the deep connections between PDEs and algebra through the Ehrenpreis-Palamodov fundamental principle and Noetherian operators, providing a comprehensive overview of these foundational ideas. We then transition to applications in statistics and machine learning, focusing on Gaussian processes and state-space models an important class of machine learning frameworks based on ordinary differential equations. Finally, we present preliminary results on extending these models to partial differential equations, leveraging algebraic tools to unlock new possibilities and insights.

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MS127

Towards a Polynomial-by-Polynomial Method from Noetherian Operators for Non-Primary Ideals

The advances in using differential operators to characterize membership in arbitrary polynomial ideals allow us to solve arbitrary homogeneous linear PDE systems with constant coefficients. Manssour, Harkonen, and Sturmfels's approach relies on computing Noetherian operators for the associated ideals via primary ideal decomposition. The work rests on the Fundamental Principle of Ehrenpreis-Palamodov from the 1960s, which bridges between analysis and algebra. We propose an efficient method for constructing and expressing the solutions in exact form by the properties of Groebner bases. In the second part, we focus on improving the efficiency of solving polynomial systems adjoining their derivatives, building on ideas from Noetherian operators.

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MS127

Gaussian Process Methods for Linear Pde Systems

Partial differential equations (PDEs) are important tools to model physical systems and including them into machine learning models is an important way of incorporating physical knowledge. Given any system of linear PDEs with constant coefficients, we propose a family of Gaussian process (GP) priors, which we call EPGP, such that all realizations are exact solutions of this system. We apply the Ehrenpreis-Palamodov fundamental principle, which works as a non-linear Fourier transform, to construct GP kernels mirroring standard spectral methods for GPs. Our approach can infer probable solutions of linear PDE systems from any data such as noisy measurements, or pointwise defined initial and boundary conditions. Constructing EPGP-priors is algorithmic, generally applicable, and comes with a sparse version (S-EPGP) that learns the relevant spectral frequencies and works better for big data sets. We demonstrate our approach on three families of systems of PDEs, the heat equation, wave equation, and Maxwell's equations, where we improve upon the state of the art in computation time and precision, in some experiments by several orders of magnitude.

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MS127

Multivariate D-Algebraic Functions

We discuss the arithmetic of solutions to partial nonlinear differential equations, which are algebraic in the indeterminates and their derivatives. These solutions are multivariate D-algebraic functions, and their equations are algebraic partial differential equations (APDEs). We present a general algorithm for the arithmetic of multivariate D-algebraic functions and conclusions on the order bounds of corresponding APDEs. We will also use the accompanying Maple software to discuss potential applications.

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MS128

Spectral Graph-Theoretic Methods for (Near-)Optimal Robotic Perception and Control

Many problems in robotics and computer vision naturally take the form of optimization over graphs; this class includes (in particular) the fundamental problems of mapping and navigation, formation control, and communication network design (among many others). In these applications the relevant figures of merit characterizing system performance (e.g. accuracy of a statistical estimate, robustness, etc.) very often turn out to be spectral graph-theoretic quantities associated with the underlying graph over which the problem is defined. Spectral graph theory thus furnishes an elegant mathematical framework for both analyzing the performance of many autonomous systems, as well as natural criteria for optimizing their performance. In this talk, we describe several recent lines of work in which spectral graph-theoretic techniques have enabled the efficient recovery of provably (near-)optimal solutions to generally-intractable problems in robotic perception, including spatial estimation, map compression, and optimal sensor package design. A primary focus of our discussion is the design of semidefinite and eigenvalue optimization methods that enable large-scale instances of these estimation and design problems to be solved efficiently in practice.

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MS128

Symmetry-Forced Formation Control

This work considers the distance constrained formation control problem with an additional constraint requiring that the formation exhibits a specified spatial symmetry. We employ recent results from the theory of symmetry-forced rigidity to construct an appropriate potential function that leads to a gradient dynamical system driving the agents to the desired formation. We show that only $(1 + 1/|G|)n$ edges are sufficient to implement the control strategy when there are n agents and the underlying symmetry group is G. This number is considerably smaller than what is typically required from classic rigidity-theory based strategies ($2n - 3$ edges). Numerical examples are provided to illustrate the main results. This is joint work with Daniel Zelazo and Shin-ichi Tanigawa.

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MS128

Utilizing Spline-based States in Factor Graphs

The "factor graph" has become the de-facto standard for optimization when performing highly non-linear navigation tasks – e.g., vision-aided navigation. Traditionally, states at discrete times are optimized over, balancing dynamic models, measurements from different sensors, and other parameters as defined by the graph. In this talk we discuss a new method for expressing the state over time: splines. Using a spline enables the factor graph to optimize over a continuous time state space as opposed to a set of states at discrete time steps. We will discuss three specific examples where this is beneficial. First, we will briefly discuss navigation with neuromorphic camera sensors. Because this sensor detects when events occur, at a very high time resolution, the concept of discrete time steps is a mismatch for this sensor. Second, we discuss how high-rate inertial sensors can be used with a spline representation of the state. This application is particularly interesting as the spline lies on a Lie manifold so that rotation matrices can be properly optimized over. Third, we discuss an application where the timing offset of different sensors needs to be estimated. Because the spline provides state information as a function of time, the time offset of specific sensors can be estimated. This estimation enables a factor graph that is robust to timing problems with specific sensors, enabling real-world navigation applications with lower cost sensor design.

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MS129

(Ir)Reducibility For Periodic Graph Operators

Given a $\mathbb{Z}^d$-periodic graph, a discrete periodic graph operator is a Laplacian together with a discrete periodic potential. Through Floquet theory, one can realize the spectrum of such an operatoracting on the Hilbert space of square-summable functions on the periodic graphas the projection of a real algebraic variety, called the Bloch variety (or dispersion relation). The fibers of this projection, taken at particular spectral values, yield the so-called Fermi varieties. In this (mostly expository) talk, we will provide a brief overview of both past and (some) recent developments concerning the reducibility of these varieties. In particular, we will discuss the spectral consequences of reducibility and summarize what is currently known.

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MS129

Spectral Properties of Periodic Graphs with Discontinuities

Peter Kuchment and Olaf Post explicitly derived dispersion relations and spectra for graphene Hamiltonian in their "On the Spectra of Carbon Nano-Structures" paper. In this talk I will consider a generalization of this Hamiltonian. I will discuss spectral properties of periodic Schrdinger operators with self-adjoint vertex conditions that allow discontinuities. Specifically, jumps at the vertices that reflect existence of controlled discontinuity and mass at the vertices. This is a joint work with Mahmood Ettehad.

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MS129

Toric Compactifications of Periodic Graph Operators

A periodic graph operator is a weighted Laplacian plus potential acting on functions on the vertices of a periodic graph. It is well-known that the spectrum of a periodic graph operator is the projection of an algebraic variety known as the Bloch variety. Motivated by Bttig, we compactify the Bloch variety of a periodic graph operator inside the toric variety associated to its Newton polytope. For a family of periodic graphs, we extend this operator to the toric variety by expressing the compactification as the support of a kernel sheaf. We outline a few spectral-theoretic consequences of this compactification. This is joint work with Matt Faust, Stephen Shipman, and Frank Sottile.

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MS129

The Critical Point Degree of a Periodic Graph

Given an operator on a $\mathbb{Z}^d$-periodic graph, its Bloch variety encodes its spectrum with respect to the unitary characters of $\mathbb{Z}^d$. Finer questions about the spectrum involve understanding the critical points of the projection to $\mathbb{R}$. Previous work of Faust and Sottile gave a bound for the number of complex critical points in terms of the volume of the Newton polytope of the dispersion polynomial. This talk will present background and then describe refined bounds on the number of critical points that are combinatorial in nature and involve an analysis of asymptotic behavior of the Bloch variety. This is joint work with Faust and Sottile.

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MS130

Cutoff for Contingency Table and Torus Random Walks with Low Incremental Correlations

We use the correlation matrix of the generating distribution to determine the mixing time for random walks on the torus $(\mathbb{Z}/q\mathbb{Z})^n$. We present our method in the context of the Diaconis-Gangolli random walk on both $1 \times n$ and $m \times n$ contingency tables over $\mathbb{Z}/q\mathbb{Z}$. In the $1 \times n$ case, we prove that the random walk exhibits cutoff at time $\frac{nq^2 \log(n)}{8\pi^2}$ when $q \gg n$; in the $m \times n$ case, where m, n are of the same order, we establish cutoff for the random walk at time $\frac{mnq^2 \log(mn)}{16\pi^2}$ when $q \gg n^2$. Our method reveals that a general class of random walks on $(\mathbb{Z}/q\mathbb{Z})^n$ with a marginal incremental variance $\frac{\sigma^2}{n}$ (when mapped to $(\mathbb{Z} \cap [-q/2, q/2))^n$) and sufficiently low incremental correlations between coordinates has cutoff at time $\frac{nq^2 \log(n)}{4\pi^2\sigma^2}$. This is a joint work with Zihao Fang.

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MS130

Spectral Estimators in General Linear Models

General linear model concerns the statistical problem of estimating a vector x from the vector of measurements y=Ax+e, where A is a given design matrix whose rows correspond to individual measurements and e represents errors in measurements. Popular iterative algorithms, e.g. message passing, used in this context requires a "warm start", meaning they must be initialized better than a random guess. In practice, it is often the case that a spectral estimator, i.e. a principal component of certain matrix built from Y, serves as such an initialization. In this talk, we discuss the theoretical aspect of the spectral estimator and present a theorem on its performance guarantee. The theorem gives a threshold for the sample complexity, that is, how many measurements are needed for a warm start to be obtainable.

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MS130

Large Random Matrices with Given Margins

We study large random matrices with i.i.d. entries conditioned to have prescribed row and column sums (margins), a problem connected to relative entropy minimization, Schrdinger bridges, contingency tables, and random graphs. Our main result is a *transference principle*, which states that the margin-conditioned random matrix can be approximated by a random matrix with independent entries drawn from an exponentially tilting of the base i.i.d. model. We show that the tilting parameters are given by the sum of two potential functions for the row and column constraints, which can be computed by a generalized Sinkhorn algorithm at an exponential rate. We also show that, for a convergent sequence of "tame" margins, the potential functions converge in L^2 as fast as the margins converge in L^1. Utilizing our general framework, we establish several results for the conditioned matrix. The marginal distribution of an entry is shown to be asymptotically an exponential tilting of the base measure, resolving Barvinok's 2010 conjecture on contingency tables. Also, the conditioned matrix concentrates in cut norm around the *typical table*, the expectation of the tilted model, which serves as a static Schrdinger bridge between the margins. Moreover, the empirical singular value distribution of the rescaled and centered conditioned matrix has an explicit limiting law, depending only on the variance profile of the tilted model. For constant margins, this confirms the universality of the Marchenko-Pastur quarter-circle law under mild conditions, while for non-constant margins, it establishes the non-universality of the limiting spectral distribution. We also address a conjecture of Chatterjee, Diaconis, and Sly in 2011 on δ-tame degree sequences.

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MT1

Metric Algebraic Geometry

A central class of problems in metric algebraic geometry is algebraic optimization, where the goal is to optimize a loss function with algebraic derivatives (Euclidean distance, cross entropy etc.) on a semi-algebraic model. This includes training neural networks, which parametrize functions in an algebraic way. For instance, a popular network architecture is to build network layers from self-attention mechanisms (which is the central building block in transformer models such as ChatGPT). Each layer of unnormalized attention mechanisms parameterizes certain cubic matrix polynomials, forming a semi-algebraic set. In this tutorial, we discuss various concepts on the intersection of metric geometry and algebraic geometry that enable the theoretical analysis of algebraic optimization problems, such as Voronoi cells, medial axes (i.e., the union of the boundaries of all Voronoi cells), offset hypersurfaces, volumes of tubular neighborhoods. We will explain the practical importance of these concepts at the example of neural network training. For instance, large-dimensional Voronoi cells can explain implicit bias of machine learning models, while the volume of the tubular neighborhood around a machine learning model measures its approximative expressivity. All concepts will be discussed with hands-on computational examples.

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MT1

Metric Algebraic Geometry

A central class of problems in metric algebraic geometry is algebraic optimization, where the goal is to optimize a loss function with algebraic derivatives (Euclidean distance, cross entropy etc.) on a semi-algebraic model. This includes training neural networks, which parametrize functions in an algebraic way. For instance, a popular network architecture is to build network layers from self-attention mechanisms (which is the central building block in transformer models such as ChatGPT). Each layer of unnormalized attention mechanisms parameterizes certain cubic matrix polynomials, forming a semi-algebraic set. In this tutorial, we discuss various concepts on the intersection of metric geometry and algebraic geometry that enable the theoretical analysis of algebraic optimization problems, such as Voronoi cells, medial axes (i.e., the union of the boundaries of all Voronoi cells), offset hypersurfaces, volumes of tubular neighborhoods. We will explain the practical importance of these concepts at the example of neural network training. For instance, large-dimensional Voronoi cells can explain implicit bias of machine learning models, while the volume of the tubular neighborhood around a machine learning model measures its approximative expressivity. All concepts will be discussed with hands-on computational examples.

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MT2

Topological and Geometric Methods in Neuroscience

Recent technological advancements in neuroscience have led to an unprecedented view into the functioning of neural circuits, from single-neuron dynamics to the mapping of entire connectomes. This explosion of data brings with it profound challenges: How do these data sets reveal insights into brain function? What mathematical frameworks can reconcile these findings with theoretical models? Can we unify these approaches to better understand neural circuits and dynamics? This minitutorial will introduce the powerful role of topological and geometric methods in addressing these questions, focusing on their application to questions in modern theoretical neuroscience and data science. Designed for participants with a minimal background in topology and graph theory, the course will begin with an introduction to theoretical neuroscience with a focus on applications to real-world data. What do typical neuroscientific data sets generally look like? What sorts of questions are we well-equipped to answer? What sorts of questions remain out of reach? Next, we will introduce a collection of central methods in geometric/topological neuroscience, including the use of directed graphs for modeling neuronal networks and persistent homology with associated Betti curves for handling large-scale neural recordings. Finally, we will see these methods in action on real-world data sets.

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PP1

Relative Homology of Neural Networks Through the Overlap Decomposition

It is known that the set of maps implemented by networks with a ReLU activation is equal to the set of piecewise-linear continuous maps [Arora et al., 2016]. Furthermore, these networks induce a hyperplane arrangement splitting the input space into regions where such maps are affine. In this work we leverage these features to describe the set of inputs on which the network acts non-injectively. We show there are two sources of non-injectivity related to the rank of a map at a region and the intersections between images of regions. Assuming the input space is closed and convex, only the latter can change the homology groups of a neural representation. Additionally, we show that the intermediate representations of deep neural networks $\Phi : X \to Y$ are homeomorphic to the canonical map, induced by the equivalence classes of each $x \in X$ given by $[x]_\Phi = \{z \in X | \Phi(z) = \Phi(x)\}$. We call the set of these equivalence classes the Overlap Decomposition $O_\Phi = \{[x]_\Phi | x \in X\}$ and use it to compute homology groups $H_k(\Phi(X)) = H_k(X, O_\Phi)$. Thus we avoid issues arising from the choice of a good metric for persistent homology on the output. Using this framework, we show that previous calculations using persistent homology [Naitzat et al, 2020] have described geometric rather than topological features of neural networks. Our results indicate that topological simplification occurs gradually along the depth of a network, rather than sharply at the start.

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PP1

Determining Nonnegativity of Real Polynomials

The question of determining the nonnegativity of real polynomials has a rich history, tracing back at least to the work of Hilbert. This problem is relevant in areas like optimization or reaction network theory, where many questions can be reduced to verifying polynomial nonnegativity. In this poster, I present a new criterion for checking nonnegativity on the positive orthant, based on solving a specific polynomial system and analyzing its positive solutions. Under certain conditions on the support of the polynomial, this system has a single positive solution which can be tracked using homotopy continuation methods, achieving a higher efficiency. Beyond its practical applications, the criterion has theoretical value. For instance, it can be used to show that, for some supports, the boundary of the nonnegativity cone coincides with the positive A-discriminant. This poster is based on joint work with Elisenda Feliu and Mt L. Telek.

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PP1

Computational Aspects of the Short Resolution

Let $I \subset R = k[x_1, \dots, x_n]$ be a homogeneous ideal such that $\dim(R/I) = d$, and assume that $A = k[x_{n-d+1}, \dots, x_n]$ is a Noether normalization of R/I. The minimal graded free resolution of R/I as an A-module is called its short resolution. In this poster, we show how to compute the short resolution using Grbner bases. Moreover, we present some of its advantages over the usual resolution for computing some geometric invariants of R/I. These include, for example, the Hilbert series and the Castelnuovo-Mumford regularity of R/I. Supported in part by the grant PID2022-137283NB-C22 funded by MICIU/AEI/10.13039/501100011033 and by ERDF/EU.

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PP1

Tensor-Based Methods for Omics Data: Applications in Multi-Omics, Cell-Cell Interactions, and Data Imputation

The advancement of technology has granted us access to extensive data across various domains. This expansion yields a multidimensional data set, and the analysis of this type of data is highly challenging. Tensor decomposition methods provide powerful tools for dimensionality reduction and latent factor extraction, which outperform traditional matrix-based techniques in capturing complex multi-way relationships. Omics technologies, including genomics, transcriptomics, and proteomics, have transformed biological research by enabling high-throughput molecular analyses. Biological data are inherently multi-dimensional, for which tensors can serve as efficient representations that capture complex relationships across different omics layers and experimental conditions. This poster will overview some tensor decomposition techniques and their applications to impute missing values, infer cell-cell interactions, and integrate multi-omics data. We discussed how tensor methods outperform matrix-based approaches in each application. We will also show that sometimes turning matrix data into a tensor (tensorizing the data) will reveal connections between different samples, which is very helpful in omics tasks like imputing missing values.

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PP1

Classification of Arrangements Formed by Three Ellipsoids

We classify arrangements of three ellipsoids in space up to rigid isotopy classes, focusing on nondegenerate configurations that avoid singular intersections. Our approach begins with a combinatorial description of differentiable closed curves on the projective plane that intersect a given arrangement of lines transversally. This framework allows us to label classes of spectral curves associated with ellipsoid configurations, which are real plane quartic curves. We determine necessary and sufficient conditions for these classes to be inhabited through arguments coming from geometry and combinatorics, and by computations in Mathematica and Macaulay2.

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