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Optimization in Machine Learning and Data Science

By Stephen J. Wright

Machine learning (ML) and artificial intelligence (AI) have burst into public consciousness in the last several years. While large language and multimodal models like GPT-4 have recently taken the excitement to a new level, developments in voice recognition software, novel recommendation systems for online retailers and streaming services, superhuman-level play by computers in Chess and Go, and unfulfilled promises in technologies like self-driving cars have been generating interest for more than a decade. Many research disciplines are feeling the profound effects of AI. For example, scientists can now utilize neural networks (NNs) to predict a protein's structure based on its amino acid sequence [3] — a problem that was identified decades ago as a grand challenge for computational science.

ML, AI, data science, data analysis, data mining, and statistical inference all have different but overlapping meanings; the term "data science" is perhaps the most general. The remainder of this article will refer to ML since the problems that we discuss tend to cluster in that area.

Modern ML rests on multidisciplinary foundations in statistics, mathematics, and computer science. Optimization plays a central role by providing tools that formulate and solve computational problems in ML. Optimization *formulations* encapsulate statistical principles and distill them into computational problems that are solvable with algorithms and software. They also enable tradeoffs between competing goals via the judicious use of constraints or weighting of terms in the objective. Additionally, recent years have seen an outburst of research around optimization *algorithms* that are suited to the structure, scale, and context of ML applications — often building upon long-established foundations with new perspectives and insights.

ML aims to extract meaning from data, learn important features and fundamental structures, and use this knowledge to make predictions about other similar data. For example, an image classification system can learn how to identify an object in an image by processing thousands of sample images, each of which is labeled with the object it contains (see Figure 1).

After preprocessing, the m items of data that are presented to a ML system each consist of a vector of *features* a_j, $j=1,2,\ldots,m$ and a *label* y_j that is associated with each feature. The fundamental task is to learn a function ϕ that (approximately) maps each a_j to its associated y_j. The process of determining ϕ is often known as *training*, and the data (a_j, y_j), $j=1,2,\ldots,m$ are called *training data*. Usually, ϕ is parametrized by some finite vector x and the training problem reduces to finding x such that $\phi(a_j;x)\approx y_j$ for all $j=1,2,\ldots,m$; in short, it becomes a data-fitting problem. In a NN, x can be the vector of weights on all of the connections in the network — a vector that is believed to have more than a trillion components in the GPT-4 NN.

To formulate the data-fitting problem as an optimization problem, we define an objective $f(x)=\frac{1}{m}\sum_{i=1}^{m}\ell(\phi(a_j;x),y_j)$, where the *loss function* ℓ measures the discrepancy between the prediction $\phi(a_j;x)$ and the true label y_j on a single data point.

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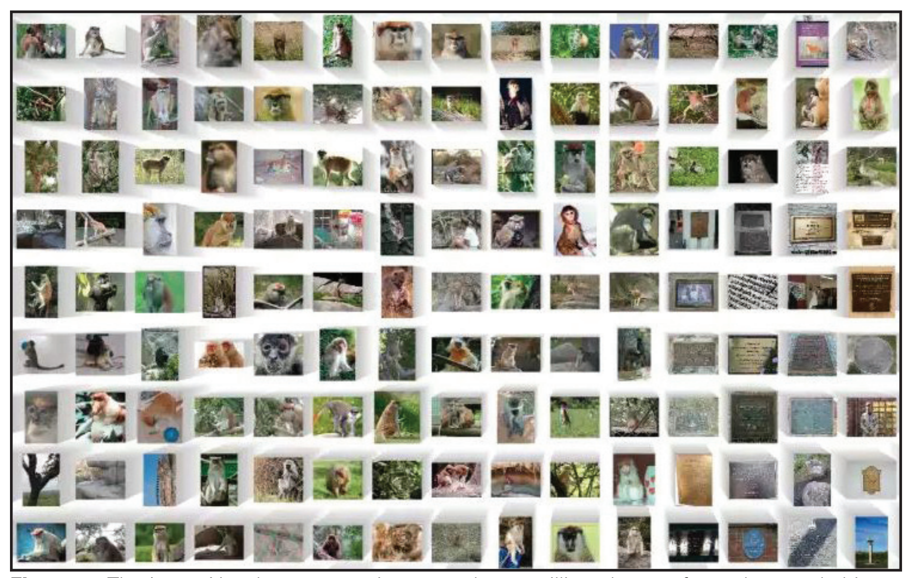

Figure 1. *The ImageNet dataset contains more than a million photos of one thousand objects. Figure courtesy of the ImageNet database at Princeton University and Stanford University.*

Reflections on March *Math*ness

By Tim Chartier

Every March, the U.S. goes mad — for March Madness. For those of you outside of the U.S., March Madness is an annual, highly anticipated single-elimination basketball tournament for National Collegiate Athletic Association Division I teams. WalletHub estimated that corporate losses due to unproductive workers during the 2023 iteration of March Madness totaled \$16.3 billion [1]. They also reported that 37 percent of Americans are willing to call in sick or skip work to watch the games.

The hype begins well before a single ball bounces in the tournament itself. The initial match-ups are announced on Selection Sunday: the Sunday before the first day of the games (this year, it fell on March 12). Over the subsequent days, millions of people complete brackets that predict their proposed winners for the 63 games that comprise the tournament.

While producing a perfect bracket is of course possible, exponential growth gives rise to its improbability. There are 2^{63}—or 9,233,372,036,854,775,808—unique brackets for each tournament. For perspective, note that 2^{63} seconds is almost 300 billion years; picking the perfect bracket out of the entire set of possibilities is therefore akin to me randomly choosing one second in 300 billion years and you accurately selecting the exact second that I chose. This near impossibility provides context to the fact that a perfect bracket has never been submitted to ESPN, CBS, or Yahoo Sports — despite the fact that fans sent more than 15 million brackets to ESPN in 2022 alone.

This period of bracket making encompasses a certain madness for me as well; I'm interviewed on the radio, quoted in print media, and make TV appearances every year. For this year's tournament, *The New York Times*[1] and CNN[2] interviewed me even before Selection Sunday. So how did I end up specializing in brackets and sports analytics? In 2009, I worked with my collaborator Amy Langville of the College of Charleston and our student researchers to apply new ranking research to March Madness. Our efforts adapted the Colley and Massey methods, which officials used during the Bowl Championship Series era of college football to select teams for New Year's Day bowl games. Both methods utilize linear systems to rank teams; their success launched me into the field that is often known as bracketology.

The Massey method derives from a least-squares formulation. We start by defining the point differential for a single game between a winning team W and a losing team L as $d_{WL}=\text{points}_W-\text{points}_L$. The Massey method derives its linear system from the assumption that $r_W-r_L=d_{WL}$, where r_W and r_L are the respective Massey ratings for the winning and losing teams. Applying least squares to this overdetermined system yields the normal equations $M\mathbf{r}=\mathbf{p}$, where $\mathbf{r}$ is the ratings vector. Adding a restriction to specify that all of the ratings sum to zero resolves this singular system, and replacing the last row of $\left[M \mid \mathbf{p}\right]$ with $\left[1\ 1\ \ldots\ 1 \mid 0\right]$ results in the nonsingular system. While infinitely many solutions are still possible (when the graph of the season is disconnected, for instance), a lot of systems now have a unique solution.

Another approach involves forming the linear system $C\mathbf{r}=\mathbf{b}$ from Colley's method, where C is the Colley matrix and $\mathbf{r}$ is the ratings as a vector. Here, $C=M+2I$ and the vector $\mathbf{b}$ contains information about each teams' number of wins and losses: $b_i=1+\frac{w_i-l_i}{2}$. Because the Colley matrix C is strictly diagonally dominant, the Colley method produces a unique rating vector $\mathbf{r}$.

The Massey and Colley methods generate two different brackets, and my research with Langville and our students allowed us to weight predictive elements. For example, weighting games by time identifies teams

See **March *Math*ness** *on page 4*

[1] https://www.nytimes.com/2023/03/12/sports/ncaabasketball/perfect-bracket-ncaa-tournament.html

[2] https://www.cnn.com/2023/03/13/sport/march-madness-perfect-bracket-odds-spt-intl/index.html

Tim Chartier is the 2022-23 Distinguished Visiting Professor at the National Museum of Mathematics (MoMath) in New York City. Visitors to the museum can smoothly ride a square-wheeled tricycle; extend their arms and become fractal trees; and track data while shooting a basketball at the Hoop Curves exhibit, which is pictured here. Photo courtesy of MoMath.

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Optimization

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This "finite-sum" form of the objective is typical of optimization problems in ML.

Besides fitting the training data, the mapping ϕ often needs to satisfy certain additional properties. Most importantly, it should *generalize* to other data that are similar to the training data. If we present ϕ with a feature vector a_k that is drawn from the same distribution as the training data (but is not identical to any particular training item), the mapping should make a good prediction of the label y_k. We might also require prior knowledge about ϕ to be incorporated into the optimization formulation, alongside the knowledge that is represented by the training data. The addition of *regularization* terms to the objective f can capture these and other desirable properties. For example, we can add a term $\lambda\|x\|_2^2$ for some parameter $\lambda > 0$ to suppress the size of x and (loosely speaking) find a solution that is less sensitive to certain types of noise in the data.

This general framework encompasses a wide range of paradigms in ML. When y_j is a real number, we can point to the special case of classical linear least-squares regression, where $\phi(a_j;x) = a_j^T x$ and $\ell(\phi(a_j;x),y_j) = (a_j^T x - y_j)^2$. Generalizations of least-squares regression abound. The addition of regularization terms such as $\lambda\|x\|_2^2$ or $\lambda\|x\|_1$ induce certain properties on x. We can replace the square loss function ℓ with alternatives like absolute values or other robust losses, or swap the simple linear parametrization of ϕ with a NN — a tactic that typically generates a weight vector x that has many more components and leads to a more powerful class of functions ϕ.

When y_j is a label from a finite set $\{1,2,\ldots,C\}$—as in an image classification scenario—the problem is one of *multi-class classification*. The logistic loss (also known as cross entropy) is the most common objective function for this type of problem, though least-squares losses have also been proposed.

Low-rank matrix problems form another important class of modern ML challenges. In our framework, x parametrizes a low-rank matrix and (a_j, y_j) captures certain observations about the matrix, such as a single element or a random linear combination of elements. Low-rank matrices appear in recommendation systems like the famous Netflix Prize problem,[1] which sought to predict the rating that every user would give to every movie on the streaming platform based on a relatively small sample of ratings from each user. The contest revealed that a linear combination of the preferences of several archetypal movie fans expresses each individual's taste fairly accurately, meaning that the matrix of preferences is approximately low rank.

The label y_j is not present in certain other ML problems, but it still may be interesting to identify structures in the collection of feature vectors. For example, we might find that all a_j lie near a low-dimensional manifold, or that the a_j cluster into a few distinct regions. This structure may reveal some useful information about the data and can even enable prediction. For instance, when a new feature vector a_k lies in a certain cluster, it presumably shares properties with the other feature vectors within that cluster.

What optimization algorithms do researchers use to solve problems that are formulated in the ways described above, particularly to minimize the functions with "finite-sum" form? The most common algorithms for smooth functions are based on gradient descent, which takes steps in a direction that approximates the objective's negative gradient. The most important algorithm of this type is *stochastic gradient*. It is commonly known as SGD where the "D" stands for "descent," though this algorithm may not reduce the function value at each iteration. SGD uses the gradient approximation

$$\frac{1}{|\mathcal{B}|}\sum_{i\in\mathcal{B}} \nabla_x \ell(\phi(a_j;x),y_j) \approx \nabla f(x) = \frac{1}{m}\sum_{i=1}^{m} \nabla_x \ell(\phi(a_j;x),y_j)$$

for some random subset $\mathcal{B} \subset \{1,2,\ldots,m\}$. While the full gradient requires access to the entire training set (which is impractical in most contexts), evaluation of the approximate gradient is much cheaper when $|\mathcal{B}| \ll m$ (as is typically the case).

One can enhance the gradient and stochastic gradient methods in many ways. A proximal-gradient approach explicitly handles any nonsmooth regularization terms in the objective. The use of *momentum*—for which the current search direction depends on the search directions at many previous iterations, rather than just the gradient at the current point—can improve performance in both theory and practice. The idea of momentum harks back to conjugate gradient methods in the 1950s, but the touchstone reference in the convex nonlinear setting is Yurii Nesterov's famous paper [5]. In addition, element-wise scaling of the search direction vector based on accumulated gradient information can enhance performance in practice; NN training employs the Adam approach almost universally [4].

Although NN training is the 800-pound gorilla of ML computations, a plethora of other ML problems utilize optimization formulations and algorithms in novel ways. Techniques from robust and distributionally robust optimization can handle various types of data uncertainty or improve the solutions' resistance to adversarial actions. Conditional gradient or Frank-Wolfe methods are beneficial in many constrained nonlinear problems for which one can cheaply minimize a *linear* function over the constraint set. Mirror descent is applicable when the constraint set is a probability simplex, as is the case in many ML applications. Kernel learning, which was the dominant paradigm before the NN juggernaut arrived, made ingenious use of convex programming duality [1]. Coordinate descent and the alternating direction method of multipliers are relevant to many applications, including those that involve matrices. Quasi-Newton and Newton approaches have not yet found widespread application, but investigations continue. Further details about some of these techniques are available in the literature [2, 6].

ML has influenced optimization in several fundamental ways beyond the challenges that arise from the scale and structure of its applications. For instance, the phenomenon of "benign nonconvexity" has emerged over the past 10 years. Finding the global minimum of a nonconvex function traditionally seemed impossible; the most that one could hope for was a local solution. But in many ML-based nonconvex optimization scenarios, finding a global minimizer is tractable from both a theoretical and practical perspective.[2] In some problems, it is easy to identify an initial point that is close to the global solution. In other problems, all local solutions are in fact global solutions. In still other problems, all stationary points (points with a zero gradient) are either global solutions or strict saddle points that are easily avoided.

Convergence to global minimizers often occurs in the training of overparametrized NNs, wherein the number of weights (length of vector x) far exceeds the number of items m in the training set. Given the nonconvex, nonsmooth nature of the objective function and the relatively naive strategy employed by the SGD algorithm, this development was certainly unexpected. Although researchers have explored several perspectives on this fascinating phenomenon, none have yet provided a complete explanation.

ML has also influenced analysis styles for optimization methods. Complexity analysis—worst-case bounds on the amount of computation that is required to find approximate solutions—has become much more prominent, while the analysis of local convergence properties is less of a focal point. Convexity assumptions were prevalent for many years, but this paradigm has since changed due to the inherent nonconvexity of many applications.

Optimization's engagement with data science and ML has benefited all of these research fields. Fresh perspectives and demands from ML applications have challenged traditional optimization approaches and modes of research. The influx of researchers with ML backgrounds who are collaborating with optimizers or performing their own fundamental work in optimization has revitalized the optimization community. New issues and paradigms continue to arise, and we look forward to many more years of exciting developments.

References

[1] Boser, B.E., Guyon, I.M., & Vapnik, V.N. (1992). A training algorithm for optimal margin classifiers. In *Proceedings of the fifth annual workshop on computational learning theory (COLT'92)* (pp. 144-152). Pittsburgh, PA: Association for Computing Machinery.

[2] Bottou, L., Curtis, F.E., & Nocedal, J. (2018). Optimization methods for large-scale machine learning. *SIAM Rev.*, *60*(2), 223-311.

[3] Jumper, J., Evans, R., Pritzel, A., Green, T., Figurnov, M., Ronneberger, O., … Hassabis, D. (2021). Highly accurate protein structure prediction with AlphaFold. *Nature*, *596*, 583-589.

[4] Kingma, D.P., & Ba, J. (2014). Adam: A method for stochastic optimization. Preprint, *arXiv:1412.6980*.

[5] Nesterov, Y.E. (1983). A method of solving a convex programming problem with convergence rate $O(1/k^2)$. *Soviet Math. Dokl.*, *27*(2), 372-376.

[6] Sra, S., Nowozin, S., & Wright, S.J. (Eds.) (2011). *Optimization for machine learning*. Cambridge, MA: The MIT Press.

[7] Vapnik, V.N. (1999). *The nature of statistical learning theory*. New York, NY: Springer Science & Business Media.

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[1] https://en.wikipedia.org/wiki/Netflix_Prize

[2] A list of such problems is maintained at https://sunju.org/research/nonconvex.

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President's Column: Musings on SIAM and CSE23

FROM THE SIAM PRESIDENT

By Sven Leyffer

I am writing this article—hopefully the first of many during my two-year tenure as SIAM President—while flying home from the 2023 SIAM Conference on Computational Science and Engineering[1] (CSE23), together with a few dozen bleary-eyed CSE23 attendees (enough for two minisymposia and a small poster session).

CSE23 took place in Amsterdam, the Netherlands, from February 26 to March 3 and was a wonderful and energizing in-person event. With more than 2,000 attendees from across the vast spectrum of computational science and engineering, it was SIAM's largest in-person meeting to date; it was also the first CSE conference outside of the U.S. If holding CSE23 abroad was an experiment, then it was certainly a successful one. The three Organizing Committee co-chairs—Karen Devine (Sandia National Laboratories), Dirk Hartmann (Siemens AG), and Wil Schilders (Eindhoven University of Technology)—easily rose to this historical occasion by planning and executing a fascinating program. Together with the 14 other members of the Organizing Committee, they pioneered innovative conference ideas such as the highly successful two-day SIAM Hackathon[2] that took place just before CSE23. During this fast-paced affair, teams worked together to create cutting-edge mathematical solutions for real-world problems in industry. The meeting also incorporated robust student programming via its "Student Days,"[3] including affinity groups that were led by experts in the field.

In addition, CSE23 hosted a free public event that featured six TED-style talks on "The Role of Mathematics in Solving the World's Main Challenges."[4] These short presentations by Karen Willcox (University of Texas at Austin), Magnus Fontes (Institut Roche), Katherine Evans (Oak Ridge National Laboratory), Luke Bennetts (University of Adelaide), Caoimhe Rooney (Astroscale and Mathematigals), and Bert Zwart (Centrum Wiskunde & Informatica and Eindhoven University of Technology) were also livestreamed on SIAM's Facebook page.[5] The speakers discussed a wide variety of topics that ranged from digital twins, biomedicine, and climate change to ocean dynamics, the power grid, and the way in which blenders chop fruits and vegetables for smoothies. In 15 short minutes, each speaker brought applied mathematics to life with their passion, insights, and individual research experiences. The streaming feed for this public event is available on SIAM's YouTube channel[6] — be sure to check it out!

Prior to attending CSE23, I had the pleasure of representing SIAM at the 2023 Joint Mathematics Meetings[7] in Boston, Mass., in early January (this conference was also held largely in person, though a virtual component was available). Every year, a dedicated team of SIAM volunteers puts together a vibrant program that showcases applied math and computational science to the broader mathematics community. This year's schedule also featured SIAM minisymposium sessions that were organized by Ron Buckmire (SIAM's Vice President for Equity, Diversity, and Inclusion) and Kathleen Kavanagh (SIAM's Vice President for Education).[8] My personal highlights were the three Joint Policy Board for Mathematics prize lectures — and the wild crowd in Cartman's Escape Room after the meeting's conclusion.

One of the main issues that SIAM currently faces is a decline in the number of its student members. This decrease began during the COVID-19 pandemic and has not yet recovered; as of December 2022, student membership was at 76 percent of

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Attendees of the 2023 SIAM Conference on Computational Science and Engineering, which recently took place in Amsterdam, the Netherlands, engage with expert-led affinity groups during "Student Days." Clockwise from left: SIAM President Sven Leyffer (Argonne National Laboratory), Olga Dorabiala (University of Washington), Dev Dabke (Princeton University), Stefan Güttel (University of Manchester), Elizabeth Amankwah (Kwame Nkrumah University of Science and Technology), and Aditi Basu Bal (Florida State University). Photo courtesy of Sven Leyffer.

[1] https://www.siam.org/conferences/cm/conference/cse23

[2] https://www.youtube.com/watch?v=a6RuXmKf4uM

[3] https://www.siam.org/Portals/0/Conferences/CSE23/CSE23_STUDENT_DAYS_A4_color.pdf

[4] https://www.cwi.nl/documents/199407/Flyer_publiek_SIAM2023.pdf

[5] https://www.facebook.com/SocietyforIndustrialandAppliedMath

[6] https://www.youtube.com/watch?v=LtsQAhtk_8w&t=7s

[7] https://www.jointmathematicsmeetings.org/meetings/national/jmm2023/2270_intro

[8] https://sinews.siam.org/Details-Page/siam-session-connects-education-and-research-at-the-2023-joint-mathematics-meetings

Early-career Panel at CSE23 Explores Strategies for Healthy Career Advancement

By Lina Sorg

CAREERS IN MATHEMATICAL SCIENCES

Regardless of whether one is anticipating a future in academia, industry, or government, graduating students and early-career researchers typically focus all of their energy on the immediate next step of their professional journey: getting a job. But securing employment is only a small part of the larger process of navigating career responsibilities, preparing for strategic advancement, and maintaining a healthy work-life balance. During a panel discussion at the 2023 SIAM Conference on Computational Science and Engineering[1]—which recently took place in Amsterdam, the Netherlands—Hartwig Anzt (University of Tennessee), Alicia Klinvex (U.S. Naval Nuclear Laboratory), and Pat Quillen (MathWorks) shared their personal experiences as junior scientists and offered advice to the forthcoming generation of applied mathematicians. Richard Vuduc (Georgia Institute of Technology) moderated the panel and stimulated a lively conversation on career goals and strategies. Attendees were able to actively participate in the session by asking questions firsthand or submitting their queries for the panelists via Slido.[2]

The panelists opened the discussion by reflecting on some of their earliest career experiences. Klinvex commented that her first day in a "real" job at Sandia National Laboratories felt like an extension of her previous internship. Consequently, she initially struggled to differentiate her expectations and responsibilities as a postdoctoral researcher rather than an intern. "You figure those things out over time," she said. "You have conversations; you don't need to have it all figured out on the very first day."

Quillen accepted a job at MathWorks after earning his Ph.D. in mathematics and has remained there for the last 18 years. Prior undergraduate- and graduate-level internships at Sandia, Raytheon Technologies, and Honeywell International had introduced him to numerical linear algebra, provided opportunities to write software, and confirmed his desire to work outside of academia. The mathematical software industry thus seemed like a natural fit. "At MathWorks, I was able to essentially continue what I was doing in my Ph.D. but focus on product development instead of research," Quillen said, adding that he was immediately tasked with fixing system bugs on his first day. "It was a little bit like being dumped into the deep end of the pool, but also pretty entertaining."

When Vuduc asked the panelists how they prioritize when juggling multiple projects, Quillen admitted that he still has not fully mastered this skill. "The way I prioritize now is running from the most urgently burning thing to the next most urgently burning thing," he quipped. He did, however, emphasize the importance of time management in industry settings, which are committed to delivering tangible products on a set timeline. Quillen encouraged listeners to split large projects into smaller, more manageable items. "You need to correctly identify hard deadlines versus softer deadlines and make sure that you're advancing things that need to be advanced," he said.

Anzt agreed that time management and attention to deadlines are critical in all employment areas. "You want to avoid running into the danger of doing all of the small things that are not super urgent," he said. "They always fill up and you'll never get to the important things." In many cases, accurately identifying the most crucial tasks is a trial-and-error process. Klinvex therefore reminded attendees to ask colleagues and mentors for guidance, especially in a new position — doing so is not a failure, but rather an opportunity to grow.

Conversation then turned to mentorship. Anzt noted that one can and should have multiple mentors, each of whom serves a different purpose. For instance, a mentor in Anzt's Ph.D. years helped him navigate the program, another taught him how to write strong papers during his postdoctoral period, and a third offered valuable career perspective. "The best advice I got was from a mentor and friend," Anzt said. "An academic career is a marathon, not a sprint; be wise with your energy."

Quillen seconded Anzt's comments about mentor variety. MathWorks employs individuals with all types of backgrounds, and Quillen uses that to his advantage. "Mentors come in all shapes and sizes, as does advice," he said. "Look for people who can teach you how to communicate your work." Because industry employees frequently collaborate with individuals whose expertise might not match their own, learning to craft compelling stories and deliver strong presentations is incredibly beneficial.

Klinvex shared one of the most valuable skills that she learned from a mentor:

See **Career Advancement** *on page 5*

From left to right: moderator Richard Vuduc (Georgia Institute of Technology) facilitates a discussion with panelists Alicia Klinvex (U.S. Naval Nuclear Laboratory), Hartwig Anzt (University of Tennessee), and Pat Quillen (MathWorks) about their personal experiences and professional insights during an early-career panel at the 2023 SIAM Conference on Computational Science and Engineering, which recently took place in Amsterdam, the Netherlands. SIAM photo.

[1] https://www.siam.org/conferences/cm/conference/cse23

[2] https://www.slido.com

Conformal Deformation of Conductors

I'd like to describe an observation so simple that it would not be worth mentioning if not for its consequences.

Consider an electrically conducting lamina with constant resistivity $=1$; this means that if we cut a unit square of the material, the resistance between two opposite sides will be 1 unit — say, ohm (see Figure 1a). More precisely, we can coat the opposite sides of the square with a perfect conductor and measure resistance between these conductors. The following property of $\mathbb{R}^2$ is both trivial and fundamental:

All squares, regardless of size, have the same resistance, (1)

where the resistance is measured between opposite sides of the square. Figure 1b explains why: putting m squares in series increases the resistance m-fold, while putting n stacks of squares in parallel decreases the resistance n-fold (in Figure 1b, $m=n=3$).

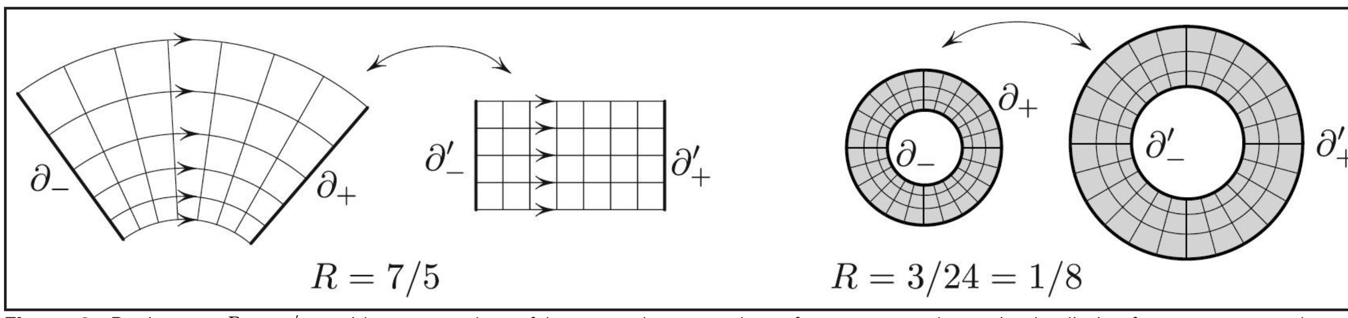

Figure 2. *Resistance $R=m/n$, with m=number of layers and n=number of squares per layer. In the limit of m, n squares that are approaching infinity approximate true squares with increasing accuracy.*

MATHEMATICAL CURIOSITIES
By Mark Levi

This observation has an immediate implication for conformal maps, since they map infinitesimal squares to infinitesimal squares. Thus,

Electrical resistance does not change under conformal deformation, (2)

as Figure 2 illustrates. Indeed, let us divide the domain into infinitesimal squares (to make this rigorous, one has to use equipotential lines and the lines of current). The conformal image is then partitioned into infinitesimal squares as well. The map clearly preserves the ratio m/n, where m is the number of layers and n is the number of squares per layer. But m/n is the resistance, so that the resistance is a conformal invariant (we have to take a limit as the squares become smaller, but I skip those details here).

Observation (1) is at the root of the standard theorem in complex analysis, stating that if two annuli are conformally equivalent (i.e., each is a conformal 1-1 image of the other), then the ratios of their radii are the same. Indeed, if one annulus is a conformal image of another annulus, their resistances must be the same according to (2). But the resistance between the two circles bounding the annulus is the logarithm of the ratio of the radii (again skipping the details), so that the ratios are the same for the two annuli.

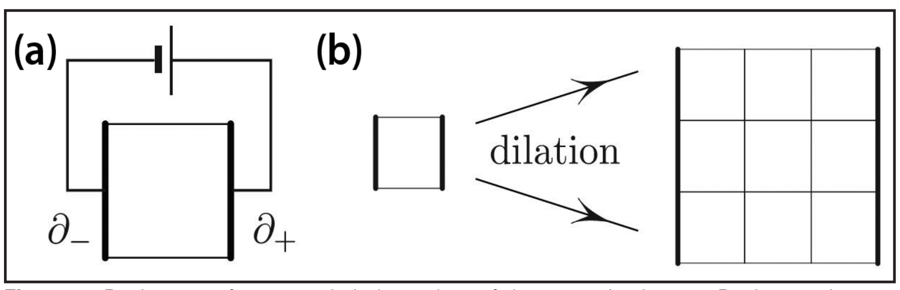

Figure 1. *Resistance of a square is independent of the square's size.* **1a.** *Resistance is measured between boundaries ∂_- and ∂_+.* **1b.** *Dilation increases both the "depth" and "width" of the square; these two opposite effects on resistance cancel each other out.*

The conventional name for the resistance is *modulus*, although "resistance" might be a better term. Another interpretation of the modulus is capacitance, in which case we must replace the current in the conductive lamina with the electrostatic field in the non-conductive plane.

The same electric interpretation of the modulus applies to deformed annuli, i.e., to doubly connected regions. Two such regions are conformally equivalent only if their resistances—measured between the inner and outer boundaries—are equal. And the combinatorial meaning of modulus= resistance=capacitance as the limit of the ratio m/n as $m, n \to \infty$ is the same as for the annuli in Figure 2.

The figures in this article were provided by the author.

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March *Math*ness

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that are playing well as the tournament begins. Our math-based brackets solve these linear systems and assume that a better-ranked team wins any match-up. In 2009, this work produced a bracket that beat over 97 percent of the more than four million brackets that were submitted to ESPN. The following year, I taught a portion of these methods to undergraduate students at Davidson College, where I'm a professor of mathematics and computer science. One of my students created a bracket that beat over 99.9 percent of the more than five million brackets on ESPN that season.

The success of this work quickly captured the media's attention. As a result, the madness to my March now involves participating in interviews and helping people—including the general public—create brackets. Given these responsibilities, I rarely make a full bracket myself. Media personalities typically ask me to make predictions on matters like big upsets, the "Final Four" teams in the tournament, and the national champion. Doing so takes time, analysis, and care, and I usually get help from a group of students. I'm currently serving as the 2022-23 Distinguished Visiting Professor at the National Museum of Mathematics (MoMath), so participants from my eight-week ranking course at MoMath assisted with the 2023 predictions.

How does this work overlap with the interests of the SIAM community? First, there is outreach. I lead March Madness events across the U.S. for students, established researchers, and members of the public. Each year, as many as 600 secondary students come to Davidson College for a math and brackets workshop that prepares them for school-wide competitions. Second, there is education. I teach the bracket methods in various linear algebra courses, from general education to upper-level mathematics. If the course occurs in the spring, I introduce the techniques in time for the students to make brackets for March Madness.

Third, there is research. A number of open questions about March Madness have yet to be explored. Massey and Colley are only two possible ranking methods; we can also apply other techniques like the Elo rating system or Microsoft's TrueSkill ranking system.[3] Furthermore, we might wish to integrate adaptations beyond weighting recency to create more predictive brackets. My students weighted home versus away games in 2010, which proved to be more accurate than our 2009 efforts that only weighted games by date of play. And on Selection Sunday, we can consider how to better predict which methods will be most effective in a tournament. Sometimes, both Massey and Colley do very well; in other years, one method struggles more than the other. Should we employ different ranking methods if we are looking for big upsets versus solely focusing on predicting the national champion? Because March Madness comprises only 63 games per tournament, the sample size of data is quite small. Even if we analyzed every March Madness game ever played, we'd only have a total of several thousand games — not the tens or hundreds of thousands of games that are necessary to hone and train a method. Regardless, bracketology can engage students in data science and help those who are interested in sports analytics create samples of their work.

My efforts in March Madness springboarded my research into the field of sports analytics. I am now a consultant for the National Basketball Association (NBA) League Office, where I explore matters in game integrity. I've fielded analytics queries from ESPN and *The New York Times*, worked with undergraduates on questions posed by National Football League and NBA teams, and aided the U.S. Olympic and Paralympic Committee.

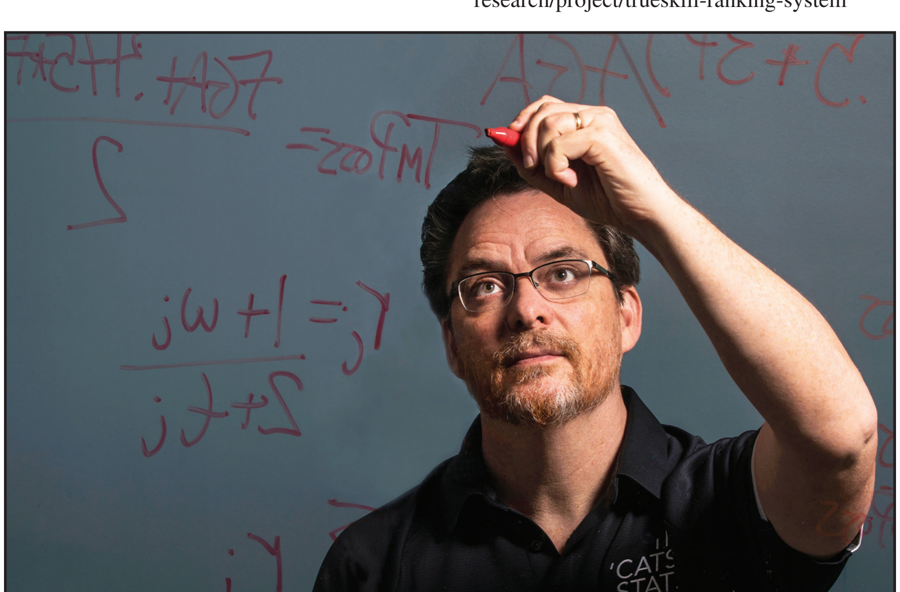
For most people, March Madness crops up annually. For me, it never ends. I'm always looking for novel insights, new ways to develop ranking methods in data science, and fresh opportunities to engage the public in mathematics. This work is an applied context for open questions and offers the chance for individuals to step further into new territories of understanding.

[3] https://www.microsoft.com/en-us/research/project/trueskill-ranking-system

References

[1] Kiernan, J.S. (2023, March 7). 2023 March Madness stats & facts. *WalletHub*. Retrieved from https://wallethub.com/blog/march-madness-statistics/11016.

Tim Chartier is the 2022-23 Distinguished Visiting Professor at the National Museum of Mathematics and the Joseph R. Morton Professor of Mathematics and Computer Science at Davidson College. He received an Alfred P. Sloan Research Fellowship and a national teaching award from the Mathematical Association of America. Chartier's most recent publication is 2022's Get in the Game: An Interactive Introduction to Sports Analytics (published by the University of Chicago Press). He has also worked with Google and Pixar on their K-12 educational initiatives.



Tim Chartier specializes in sports analytics and works with data from a variety of sources, including the National Basketball Association, National Football League, and U.S. Olympic and Paralympic Committee. Photo courtesy of Davidson College.

Career Advancement

Continued from page 3

how to say "no." She cautioned that some people might try to take advantage of new graduates by asking them to undertake additional tasks that might not be appropriate for their level of expertise. "Get comfortable saying 'no' to things that are not of your caliber," Klinvex said, disclosing that she rehearsed doing so before putting it into practice. "The first time I said 'no,' it felt really gross. It surprised the person I said 'no' to, and it surprised me as well." The panelists all agreed that junior scientists can professionally reject a request by explaining that they do not have the time or capacity to sufficiently complete it.

Vuduc then directed the discussion to the challenge of attaining a healthy work-life balance. Quillen encouraged attendees to take control of their personal lives. "One of the things that really helped my work-life balance was removing email from my phone," he said. "That has been extremely important to my personal mental health. If you're at a place where you feel like you can't do that type of thing, maybe it's time to evaluate whether that place is good for you." Quillen mentioned that all companies, laboratories, and employers have their own distinct cultures; what might be suitable for one individual will not necessarily benefit another.

Klinvex knew that she wanted a job that allowed time for family and facilitated a distinct separation between work and home life. She was thus drawn to her current position at the Naval Nuclear Laboratory because she cannot take work home with her. "Different jobs have different expectations, and I purposefully selected a job where the expectation was to work onsite for 40 hours a week," she said.

Anzt recommended that everyone carve out time during the workday to step away and clear their minds, irrespective of workload. For instance, he commits to going for a walk, run, or quick swim during lunch every day. Though it is tempting to make exceptions to this designated break time, Anzt finds that doing so almost guarantees its eventual collapse. "Fighting for work-life balance is an everlasting battle," he said. "It's important not to give in."

Next, an audience member inquired about the feasibility of transitioning between different research subjects. Anzt commented that making this switch is absolutely possible; in fact, some researchers pursue completely new fields after earning their Ph.D.s or completing postdoctoral positions. Quillen agreed and stated that he does not necessarily look for individuals with Ph.D.s in a particular subject during the hiring process. Because he and his MathWorks colleagues often switch applications and contexts depending on the development project at hand, the particular focus areas of prospective employees are less significant than their critical thinking abilities. "The purpose of your education is to teach you how to think and be adaptable to solve the kinds of problems that we have now," Quillen said. "Learn to be flexible; this will make you valuable and fulfilled as well."

When asked about the transition from a research-based role to a managerial position, Quillen—whose official title at MathWorks is "Software Engineering Manager"—urged attendees to refrain from micromanaging tendencies that slow progress. "It's important to learn to let go," he said. "It can be hard to recognize that people may not do exactly what you would have done, but it's still just as effective." New managers must acknowledge that they cannot maintain the same level of research and attention to detail as before.

Another essential aspect of management is coaching. "When exercising the coach muscles, try to understand where the person is and what type of coaching they need," Quillen said. He noted that business books and other professional resources can be surprisingly helpful, especially since coaching and mentoring more junior coworkers is not necessarily an innate skill.

During the 2023 SIAM Conference on Computational Science and Engineering—which recently took place in Amsterdam, the Netherlands—a panel of experienced researchers spoke about various scenarios that applied mathematicians and computational scientists might face in the early stages of their careers. Hartwig Anzt (University of Tennessee), Alicia Klinvex (U.S. Naval Nuclear Laboratory), and Pat Quillen (MathWorks) comprised the panel, which was moderated by Richard Vuduc (Georgia Institute of Technology). SIAM photo.

The panel concluded with several thoughts about the importance of routinely checking in with oneself and evaluating both personal and professional goals. "You need to visualize what you need for your life and decide if your job is compatible with that," Klinvex said, adding that she prefers a structured professional schedule to stay on track. Goal setting influences future trajectories, and Quillen encouraged the audience to establish reasonable goals and take consistent steps towards them. Every job will occasionally include undesirable tasks in the short term, so constant reevaluation and self-reflection is vital to the complete assessment of one's larger career picture. "It's not so much about how much work there is, how much you're doing, or how much is enough," Quillen said. "It depends on how it makes you *feel*. You need to know yourself—or at least learn to know yourself—so that you can do your best in situations that allow you to do your best work."

Interested in learning more about possible career paths in the mathematical and computational sciences? SIAM's recently updated "Careers in the Mathematical Sciences" brochure features more than 20 new profiles of applied mathematicians, with a specific focus on industrial careers. The brochure is freely available online[3] and can be purchased at the SIAM Bookstore.[4]

*Additionally, the SIAM Job Board offers job seekers and employers a unique and easy way to connect. Visit **jobs.siam.org** to browse available positions or advertise your organization's openings.*

Lina Sorg is the managing editor of SIAM News.

[3] https://www.siam.org/students-education/programs-initiatives/thinking-of-a-career-in-the-mathematical-sciences
[4] https://my.siam.org/Store/Product/viewproduct/?ProductId=661

President's Column

Continued from page 3

its pre-pandemic levels.[9] During my presidency, I therefore plan to concentrate on attracting more students to the SIAM community. As SIAM members and advocates, you can help too — regular members of SIAM can nominate two students for free membership.[10] Better yet, consider starting a student chapter[11] at your institution if it does not yet have one. I was recently invited to visit the Illinois Institute of Technology SIAM Student Chapter, and I look forward to meeting many more chapters in the coming months.

This short article will hopefully be the first of many such pieces that provide my personal impressions of SIAM. I am always looking for new ideas, volunteers, and suggestions on how SIAM can better serve its membership community. Please contact me at **leyffer@anl.gov** or reach out on Twitter at **@SvenLeyffer** if you have questions, comments, or concerns. I look forward to hearing from you!

Sven Leyffer is a senior computational mathematician at Argonne National Laboratory and the current President of SIAM. He obtained his Ph.D. from the University of Dundee in Scotland and works on nonlinear and mixed-integer optimization.

[9] https://sinews.siam.org/Details-Page/making-up-for-lost-time-in-siam-membership
[10] https://www.siam.org/forms/nominate-a-student
[11] https://www.siam.org/students-education/student-chapters/start-a-chapter

After the 2023 Joint Mathematics Meetings, which took place in Boston, Mass., in January, several attendees tried their luck in an escape room challenge. Top row, left to right: Kathleen Kavanagh (Clarkson University), Karen Bliss (Senior Manager of Education and Outreach at SIAM), Karen Yokley (Elon University), and Nick Luke (North Carolina Agricultural and Technical State University). Bottom row, left to right: Richard Moore (Director of Programs and Services at SIAM), SIAM President Sven Leyffer (Argonne National Laboratory), Ben Galluzzo (Clarkson University), and Adewale Adeolu (Clarkson University). Photo courtesy of Sven Leyffer.

Photos from the 2023 SIAM Conference on Computational Science and Engineering

The 2023 SIAM Conference on Computational Science and Engineering, which was held in Amsterdam, the Netherlands, from February 26 to March 3, included a first-of-its-kind public event titled "The Role of Mathematics in Solving the World's Main Challenges." During this evening program—which was open to conference attendees and the entire Amsterdam community—six expert speakers presented short, broad-interest lectures on a variety of engaging topics, from digital twins and smart grids to biomedicine, ocean dynamics, and the science of making smoothies. Here, Katherine Evans of Oak Ridge National Laboratory discusses climate change and shifting weather patterns. SIAM photo.

Judith Hill of Lawrence Livermore National Laboratory (left)—chair of the SIAM Activity Group on Computational Science and Engineering (SIAG/CSE)—presents the 2023 SIAG/CSE Best Paper Prize to Matthew Colbrook of Cambridge University (middle) and Andrew Horning of the Massachusetts Institute of Technology (right) at the 2023 SIAM Conference on Computational Science and Engineering, which recently took place in Amsterdam, the Netherlands. Colbrook and Horning—along with their coauthor Alex Townsend of Cornell University—were recognized for their 2021 SIAM Review paper on "Computing Spectral Measures of Self-adjoint Operators." Horning then delivered a corresponding talk about this research. SIAM photo.

During the 2023 SIAM Conference on Computational Science and Engineering, which recently took place in Amsterdam, the Netherlands, Devin Matthews of Southern Methodist University (right) accepts the 2023 James H. Wilkinson Prize for Numerical Software from Judith Hill of Lawrence Livermore National Laboratory, chair of the SIAM Activity Group on Computational Science and Engineering. Matthews and Field Van Zee of the University of Texas at Austin received the award for their development of BLIS: a portable, open-source software framework that facilitates rapid instantiation of high-performance BLAS and BLAS-like operations that target modern central processing units. Matthews presented an associated prize talk on "The BLAS-like Library Instantiation Software" at the meeting. SIAM photo.

SIAM president Sven Leyffer (far left) and Association for Computing Machinery (ACM) president Yannis Ioannidis (far right) present the 2023 SIAM/ACM Prize in Computational Science and Engineering to members of Lawrence Livermore National Laboratory's (LLNL) SUNDIALS Core Development Group during the 2023 SIAM Conference on Computational Science and Engineering, which was held last month in Amsterdam, the Netherlands. In the middle, from left to right: Carol Woodward of LLNL, Daniel Reynolds of Southern Methodist University, David Gardner of LLNL, and Radu Serban of the University of Wisconsin-Madison. Cody Balos, Peter Brown, and Alan Hindmarsh (all of LLNL) are also members of the SUNDIALS Core Development Group, which was honored for the innovative development of nonlinear and differential/algebraic equation solvers for high-performance computing that provide unique and critical capabilities in the scientific software ecosystem. Woodward gave a brief presentation about SUNDIALS' efforts at the conference. SIAM photo.

Judith Hill of Lawrence Livermore National Laboratory (left)—chair of the SIAM Activity Group on Computational Science and Engineering (SIAG/CSE)—congratulates Kaibo Hu of the University of Oxford on his receipt of the 2023 SIAG/CSE Early Career Prize at the 2023 SIAM Conference on Computational Science and Engineering, which was recently held in Amsterdam, the Netherlands. Hu was commended for his contributions to finite element exterior calculus, particularly structure-preserving numerical algorithms for magnetohydrodynamics. At the meeting, he spoke about "Structure-preserving Discretization of Incompressible Magnetohydrodynamics Systems." SIAM photo.

An engaging two-hour poster session at the 2023 SIAM Conference on Computational Science and Engineering, which recently took place in Amsterdam, the Netherlands, allowed participants the opportunity to share their research and network with colleagues. SIAM photo.

View more photos from the 2023 SIAM Conference on Computational Science and Engineering (CSE23) at https://go.siam.org/coAznO.

See page 3 of this issue for an article about the early-career panel at CSE23, which includes photos from the event.

A Novel View of an 18th-century Celestial Atlas

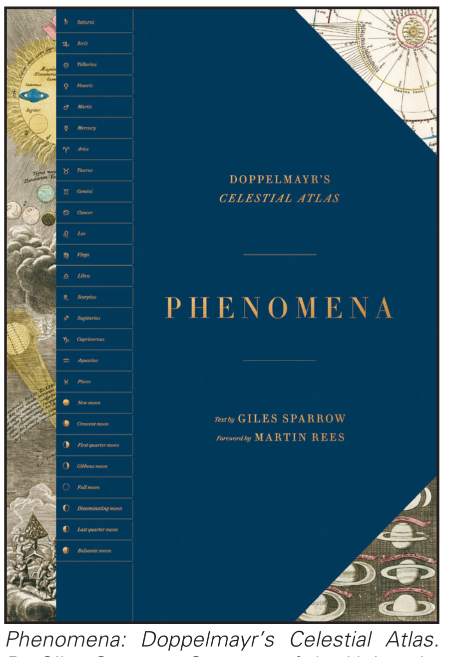
Phenomena: Doppelmayr's Celestial Atlas. *By Giles Sparrow. The University of Chicago Press, Chicago, IL, November 2022. 256 pages, $65.00.*

BOOK REVIEW
By Ernest Davis

In the 17th and 18th centuries, the art of mapmaking and the science of astronomy advanced by leaps and bounds. During this period, art and science came together in celestial atlases: collections of maps and diagrams that depicted the universe as it was understood at the time. Giles Sparrow presents a masterpiece of this form—the 1742 *Atlas Coelestis* by Johann Gabriel Doppelmayr (1677-1750)—in his new book, *Phenomena: Doppelmayr's Celestial Atlas*. Sparrow's extensive tome reproduces Doppelmayr's *Atlas*; enhances it with a wealth of additional illustrations; and explains its science, significance, and preceding and subsequent history.

Doppelmayr lived in Nuremberg, Germany, and worked as a mathematics professor at the Aegidien Gymnasium. He pursued a variety of scientific interests by making maps and globes, studying astronomy, grinding lenses and building telescopes, and conducting experiments with electricity. Doppelmayr wrote a number of books on topics in mathematics and astronomy (including sundials and spherical trigonometry) and penned a collection of short biographies for the mathematicians and scientists of Nuremberg. Though he is not known for making any original discoveries, he was nevertheless well respected in the research community of his day.

Doppelmayr's original atlas consists of 30 large plates that use color in a moderate, unlavish way. Each plate contains one or two large maps in the center. Plate 1 provides an abstract presentation of Earth's spherical geometry. Plates 2 through 10 describe the motion of the planets (see Figure 1, on page 8) and offer a detailed presentation of Johannes Kepler's heliocentric theory—in which the planets revolve around the Sun in elliptical paths—as well as Tycho Brahe's geocentric theory, in which the Sun orbits the Earth and the planets revolve around the Sun. Plates 11 and 12 focus on the Moon, plate 13 explains eclipses, plate 14 describes Saturn's rings and Jupiter and Saturn's moons, and plate 15 serves as a map of the Earth that is divided into two hemispheres (each of which is presented as a circle). Plates 16 through 25 are star maps that portray the stars and constellations in a variety of projections; aesthetically, these are the richest plates (see Figure 2, on page 8). Although the maps mostly lay out the same stars and constellations that were listed in Ptolemy's *Almagest* and evident in many star maps over the intervening millennia, Doppelmayr does add some stars in the Southern Hemisphere that were not known to Ptolemy; his chart also includes six novas that were recorded over the preceding two centuries. Plates 26 through 28 address comets, and plates 29 and 30 depict the solar system from the perspective of the other planets in both Kepler's and Brahe's theories. Doppelmayr created the different plates at various points between 1707 and 1742.



Phenomena: Doppelmayr's Celestial Atlas. By Giles Sparrow. Courtesy of the University of Chicago Press.

Many types of additional materials encircle the large central images on the plates. All plates have hand-written explanations of the images in Latin, some of which are quite long. They also contain sizeable tables of data; for instance, the world map lists the latitudes and longitudes of roughly 100 European cities and 40 non-European cities, each of which is labeled with the name of the geographer who provided the information (Doppelmayr was generally very careful to appropriately credit his sources). The star maps include tables that enumerate the stars in every constellation, along with the stars' celestial coordinates and a letter key that identifies them on the map. Meticulously drawn schematic diagrams illustrate various technical astronomical points. Doppelmayr crafted hand-drawn images of astronomical objects such as Montes Alpes (the lunar Alps), the rings of Saturn, and the Orion Nebula, many of which were copied from other astronomers (and again diligently credited). Two of the plates feature pictures of prominent observatories in their corners (see Figure 2, on page 8), and several others incorporate illustrations of astronomical or scientific instruments—telescopes, sextants, pendulum clocks, and so forth—that are carried by putti.

Doppelmayr was clearly deeply interested in both the science and *history* of astronomy. He often utilizes multiple diagrams to explore alternative viewpoints; in addition to Kepler and Brahe's theories of the solar system, other representations overview the theories of Ptolemy, Plutarch, Eudoxus of Cnidus, Porphyry, Johannes Cocceius, William Gilbert, and Sébastien Le Clerc. Doppelmayr frequently provides multiple comparative images of the same subject that are drawn by different astronomers at different times. For example, his *Atlas* includes four pictures of the markings of Venus, 11 of the markings of Mars, 10 of the markings of Jupiter, eight of the rings of Saturn, and two of the Orion Nebula. The frontispiece highlights Doppelmayr's four astronomical heroes: Ptolemy, Nicolaus Copernicus, Brahe, and Kepler.

Sparrow's *Phenomena* is both large (14 inches by 10.5 inches) and beautifully made. It reproduces each of Doppelmayr's plates to fill a pair of facing pages. The subsequent pages also depict somewhat enlarged renderings of the plates' individual, non-text components with brief explanations. Interested readers can view the plates of Doppelmayr's *Atlas* online at the David Rumsey Map Collection,[1] which allows users to enlarge details and better interpret Doppelmayr's text (much of the text is in the horizontal center of the plate, which is difficult to read in *Phenomena* because it falls where the two facing pages curve in toward the binding). Sparrow offers an extended discussion of the astronomical and

See **Celestial Atlas** *on page 8*

[1] https://www.davidrumsey.com

SIAM Establishes 19 New Student Chapters in 2022

By Susanne C. Brenner and Kathleen Kavanagh

The SIAM student chapter program[1] has experienced steady growth in recent years. 2022 was no exception, as students started 19 new chapters at institutions all over the world. Last year saw the noteworthy formation of the first three SIAM student chapters in Japan, a new chapter in China, a twofold increase (from two to four) in the number of chapters in Hong Kong, and three new chapters in South Asia: two in India and one in Pakistan. Nine additional chapters also formed in the U.S., as did one in the U.K.

In November 2022, we once again hosted a series of "Meet and Greet" events with the assistance of Maggie Hohenadel, SIAM's Student Chapter and Fellows Coordinator. Three separate virtual sessions on Zoom welcomed the newly formed chapters. During the first session, we met with faculty advisors, officers, and members from the Waseda University SIAM Student Chapter in Japan and the Hong Kong University of Science and Technology SIAM Student Chapter. Representatives from chapters at St. Olaf College, the United States Military Academy West Point, and West Texas A&M University joined the second meeting. And the final meeting included participants from Durham University in the U.K., Jaypee University of Information Technology in India, and Vellore Institute of Technology, Bhopal in India.

During these get-togethers, we discussed opportunities and procedures for SIAM student chapters within the Society. For example, SIAM conferences are a particularly effective way for students to learn about current trends in applied mathematics and computational science, expand their professional networks, and engage with the broader scientific community. Chapter members were especially interested in SIAM Student Travel Awards[2] and conference "Student Days," which sometimes include undergraduate research minisymposia. We also highlighted the SIAM Visiting Lecturer Program[3] as an excellent source of speakers for both in-person and virtual chapter events. Finally, each participating chapter shared some of the exciting plans that they intend to implement in the future.

We look forward to reviewing the annual reports for all of the new and existing SIAM student chapter activities, and learning more about each chapter's endeavors in the coming year. Here's to a successful and prosperous 2023!

Top row, left to right: Maggie Hohenadel, Susanne Brenner, and Kathleen Kavanagh meet with representatives from the new SIAM student chapters at Waseda University and Hong Kong University of Science and Technology.

Susanne C. Brenner is a professor of mathematics at Louisiana State University and a Past President of SIAM. Kathleen Kavanagh is a professor of mathematics at Clarkson University and the Vice President for Education at SIAM.

[1] https://www.siam.org/students-education/student-chapters

[2] https://www.siam.org/conferences/conference-support/siam-student-travel-awards

[3] https://www.siam.org/students-education/programs-initiatives/siam-visiting-lecturer-program

Celestial Atlas

Continued from page 7

historical significance of each plate's contents; these sections are clear, carefully researched, and very readable. *Phenomena* also contains many other illustrations from multiple contemporary and pre- and post-Doppelmayr sources. In addition, Sparrow supplies a short history of astronomy before and after Doppelmayr, a short biography of Doppelmayr himself, a timeline, a glossary, and an illustrated list of 18 major astronomers — from Aristotle to Edwin Hubble. Martin Rees, the U.K.'s Astronomer Royal, wrote the book's forward.

Phenomena does not provide a translation, transcription, or detailed summary of Doppelmayr's texts, and Sparrow's general discussion does not clearly distinguish between information from Doppelmayr's original *Atlas* and external matter. It would hence be quite laborious for readers like myself—who do not have a fluent knowledge of 18th-century astronomical Latin—to determine Doppelmayr's exact thoughts on any topic; I have certainly not attempted it.

Three points about Doppelmayr's book struck me as particularly remarkable. The first is the very fine detail and wealth of scientific issues that he engages, some of which I have already mentioned. Additionally, Doppelmayr included diagrams that show the effect of the Sun's gravity on the Moon's orbit, explain Ole Rømer's 1676 measurement of the speed of light based on observations of Jupiter's moons, explore various conjectures about the sizes of the planets and the Sun, describe the Moon's libration, exhibit zodiacal light, and much more.

The second noteworthy point is Doppelmayr's deep interest in Brahe's alternative theory of the solar system; two full plates and multiple small diagrams illustrate various astronomical phenomena from Brahe's theory. From a modern-day standpoint, this interest feels surprising in an atlas that was published nearly 200 years after Copernicus' *On the Revolutions of the Heavenly Spheres*, 120 years after Kepler's *Epitome Astronomiae Copernicianae*, 110 years after Galileo's *Dialogue Concerning the Two Chief World Systems*, and 55 years after Newton's *Philosophiæ Naturalis Principia Mathematica*. Doppelmayr was committed to the heliocentric theory of Copernicus and Kepler, but he clearly considered Brahe's theory to be an important alternative — at least at the beginning of his career.

Third, there appears to be an extraordinary omission; as far as I can tell, Doppelmayr never mentions Galileo in any form. While Galileo did not make any scientific contributions to the theory of planetary motion, his other contributions—including his world-famous, brilliantly written, heroic advocacy of heliocentric theory; pioneering use of the telescope in astronomy; and discovery of the moons of Jupiter, phases of Venus, craters of the Moon, and sunspots—unquestionably should have justified a large, prominent place in a book like this. Doppelmayr does not seem to have been biased against Italians; Giovanni Domenico Cassini, Francesco Bianchini, and others are featured prominently. The Catholic Church had completely dropped its opposition to Galileo by 1742—and Doppelmayr was likely a Protestant in any case—so that was not the issue. All I can suppose is that he had some personal animus against Galileo, who was indeed an extremely disputatious and unpleasant person and therefore easy to dislike. Nevertheless, it seems surprising for these traits to have been a consideration a century after Galileo's death. Sparrow does not discuss this strange gap.

I must confess that I am a sucker for atlases, especially atlases with historical and scientific content. I spent many happy hours poring over Sparrow's book and have enjoyed my time immensely. I learned a lot about the history of astronomy, and even a bit about its science as well. I am thus extremely grateful to Sparrow for putting this delightful book together.

Doppelmayr ends his *Atlas* with a haunting quotation from Seneca the Younger's mystical scientific work, *Investigations into Nature*, and I will likewise do so:

> Nature does not reveal all her secrets at once; her arcana are not shared with all indiscriminately, they are withdrawn and shut up within the inner shrine. For many ages to come, when our memory will have faded, are reserved those things for which those of our age will see one thing, and those who follow us, another. When, then, will these things be brought to our full knowledge? Great results proceed slowly, especially when labor ceases.

Ernest Davis is a professor of computer science at New York University's Courant Institute of Mathematical Sciences.

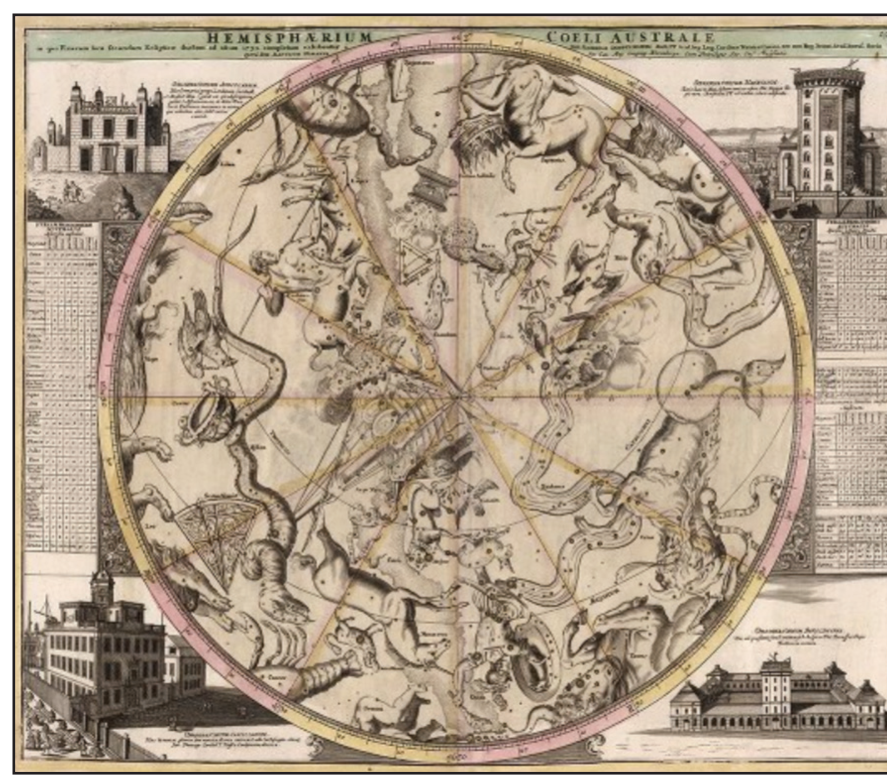


Figure 2. *Plate 19 of Doppelmayr's Atlas Coelestis, "The Southern Hemisphere of the Heavens, Determined in Relation to the Ecliptic." Note the corner illustrations of the observatories in Greenwich, Copenhagen, Kassel, and Berlin. Figure courtesy of davidrumsey.com.*

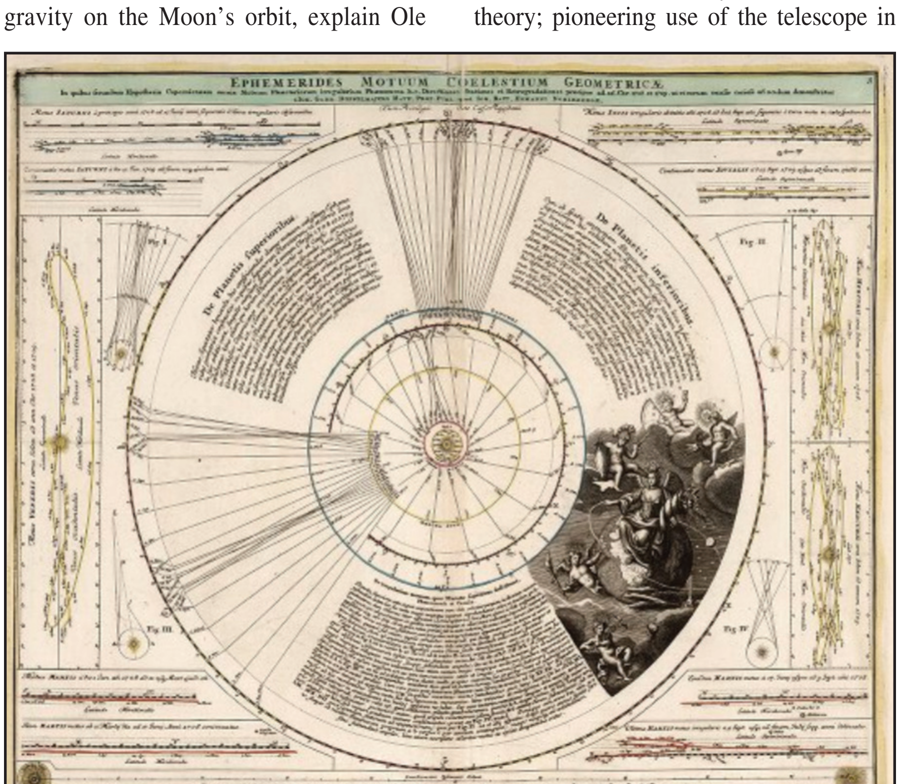


Figure 1. *Plate 8 of Doppelmayr's Atlas Coelestis, "Ephermerides of Geometric Celestial Motion." Figure courtesy of davidrumsey.com.*