Team #2559

Waste Not, Want Not: Putting Recyclables in Their Place

Summary

The increased usage of plastic, paper, and other recyclable materials, due to convenience and efficiency, has not been matched by available recycling methods. These readily disposable goods have replaced reusable products such as glassware, resulting in landfills inundated by wastes—such as plastic and Styrofoam—that are not biodegradable (Rogers). While the immense consumption of plastics is harsh on the environment, these synthetic polymers are too integrated in modern-day society to be suspended or discontinued. How might we reconcile the use of these goods with cost-efficient recycling methods for every state and township in the United States?

Our team has been asked to predict the production rate of plastic waste over time, and to forecast the amount of plastic waste present in landfills in ten years. To begin, we assumed that while an increase in population over the next ten years will increase plastic waste output, and that there is a limit on the total amount of plastic generated that is discarded. Thus our model for production rate of plastic is sigmoidal in nature, with a carrying capacity (maximum amount of plastic discarded) of 30,000 tons/year. By integrating our sigmoid function, we predicted the amount of plastic waste present in landfills in 2023 to be 1,026,000 tons.

We were also asked to design a mathematical model that could determine which recycling method is most appropriate for a city, and apply it to Fargo, ND; Price, UT; and Wichita, KS. Our approach began with the assumptions that geographic location has a negligible impact on recycling rate for each method of recycling; each city will have at least one recycling facility; the use by citizens of drop-off and curbside pickup recycling is mutually exclusive; people will recycle in the correct manner; every household has recyclable wastes; and cities may be modeled as circles. Thus our first model considered the probability that a person would recycle at a drop-off center based on distance to the center. Our second model then determined the costs of collecting and operating curbside pickup, taking into account area, population density, and total household units of each city. Analysis led to the conclusion that Price, UT should employ drop-off recycling only, while Fargo, ND and Wichita, KS should employ curbside pickup as the most cost-efficient methods.

On a national scale, we must report to the EPA how our model can lead to a municipal recycling guideline policy to govern all states and townships in the United States in an effort to mitigate the problem of recyclables not being recycled. Our model is best applied to cities and townships, as the factors considered—population, area, and household density—are specified on a city and township level. Furthermore, our model should not be used on a state level as states include cities and townships of varying sizes and development, including rural and urban regions. We conducted a cost-benefit analysis of each recycling method based on city population and area. Based on our analysis, we determined that it is more cost-efficient for cities with relatively small populations to adopt drop-off recycling only, while cities with larger populations to adopt curbside pickup recycling.

Therefore we recommend that the EPA allows each municipality to determine their own recycling method based on our mathematical model because the variables involved in costs of recycling are unique to each municipality. However, as a general standard, the EPA should require all cities and townships beginning in 2016 to recycle by the method best for them, in order to put recyclables in their place so that future generations are not left to deal with a world wasted away.

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Introduction

Background

The introduction of plastic in the 20th century was hailed for its economic and social benefits. Plastic is an essential resource in almost all modern-day products, found in everyday kitchen and food supplies to medical instruments. Also, about 50 percent of all synthetic polymers are made for convenient single-use disposable applications (European Commission). However, plastic is not easily biodegraded—it takes about 450 years for plastic to completely decompose (U.S. National Park Service: Mote Marine Lab). In addition to the waste buildup caused by synthetics in landfills, the toxicity of them is also an issue. Plastic continues to be produced using carcinogenic chemicals, generating 100 times more toxic emissions than the manufacturing of glass (Rogers). These along with other environmental effects from the disposal of plastic and man-made wastes have escalated the situation into a dire global dilemma.

The rate at which we consume plastics has grossly overtaken the rate at which they can be decomposed, contributing to the increase in the amount of municipal solid waste, or MSW. Municipal solid waste, more commonly known as trash or garbage, consists of items that are used and disposed of for everyday consumption. In the United States, about 250 million tons of this waste was generated in 2010, which is equal to about 4.43 pounds per person daily (Environmental Protection Agency). While plastic only contributes about 12.4% of the total amount of waste generated in the U.S., it comprises the highest percentage of non-biodegradable waste produced (Environmental Protection Agency). This is a problem because without a change in waste management, plastic use is not environmentally sustainable for future generations.

One method of responding to this excess of MSW is recycling, which involves the collection and processing of discarded materials for remanufacturing (Association of Collegiate Schools of Architecture). The two major types of recycling are drop-off and curbside pickup. Currently, single-stream recycling is becoming one of the most common methods of curbside pickup. Single-stream recycling allows the recycler to throw all the waste away in a single bin, encouraging higher recycling rates and lowering the cost of collection. However, this method also contributes to higher processing costs and contamination rates (Container Recycling Institute). Therefore, careful analysis must be performed prior to deciding which type of recycling to implement in an area.

Restatement of the Problem

The United States Environmental Protection Agency has asked our team to:

1. Predict the production rate of plastic waste over time and forecast the amount of plastic waste present in landfills 10 years from today.
2. Develop a mathematical model that serves as a guideline for cities to determine which recycling method they should adopt.
3. Determine the recycling method that Fargo, North Dakota; Price, Utah; and Wichita, Kansas should use based on our mathematical model, taking into consideration the characteristics of the city of interest and the recycling methods.
4. Inform the EPA about the feasibility of recycling guidelines and/or standards to govern all states and townships in the U.S.

Global Assumptions

1. We will assume that no major political, global, or economic crises occur in the 10 year time period. Potential changes in production and waste disposal due to such crises will be ignored.
2. Efficiency and type of technology used for processing recycled waste will remain constant in all cities in which a recycling program is implemented.
3. Recycling programs implemented in each township will be governmentally funded, therefore it is the responsibility of the local government to determine which recycling method is best suited for its township based on the costs of each method.

Part I: Forecasting Production of Plastic Waste

The amount of plastic waste produced has been increasing annually since plastic was first invented. To develop a model for the amount of waste per year, we made the following assumptions:

Assumptions

1. More plastic production leads to more waste generated. This makes sense because plastic is often used for temporary purposes, therefore most of the plastic created would eventually be discarded.
2. There is a limit to the amount of total plastic waste that is generated. This assumption is plausible because although a higher population will demand more plastic production and thus waste generated, there are also factors that limit the amount of the total plastic generated that is discarded. Such factors include: recycling, limited resources to create plastic, and higher-awareness of the importance of recycling.
3. Rate of recycling is proportional to rate of waste discarded, because recycling is a method of disposing of waste. Therefore, the difference in the rates is a constant.

1. Designing the Model

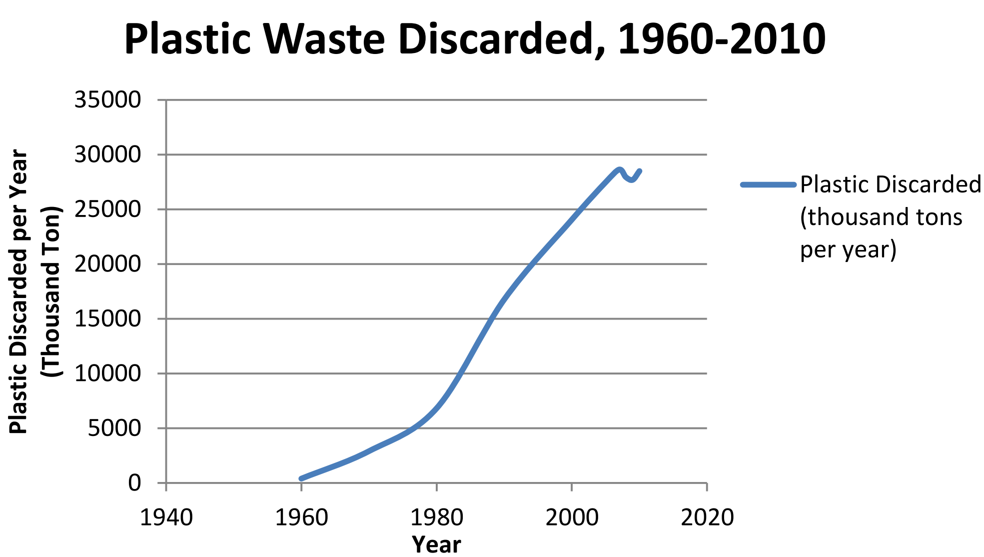

Figure 1: Graph showing the pattern of plastic discarded into landfills over the period 1960-2010.

Year	Plastic discarded(thousand tons per year), P(t)
1960	390
1970	2900
1980	6810
1990	16760
2000	24050
2005	27470
2007	28630
2008	27930
2009	27690
2010	28490

Table 1: Table showing the pattern of plastic discarded into landfills over the period 1960-2010. (Environmental Protection Agency)

By analyzing the pattern of plastic discarded into landfills, we assume that the trend is sigmoidal. This assumption is plausible because even though a higher population will demand more plastic production and thus waste generated, many factors also limit the amount of the total plastic generated that is discarded. Such factors include: recycling, limited resources to create plastic, and higher-awareness of the importance of recycling. We assume that the rate of recycling will be proportional to the rate of waste generated. Thus the difference between waste generated and recycled, the total amount discarded, will remain constant. This is consistent with our sigmoid model.

$$P(t) = \frac{KP_0e^{rt}}{K + P_0(e^{rt} - 1)}$$

Logistic curves are of the form shown above. We let $P_0 = e^{\alpha}$ and assume that the -1 is negligible. We shall later demonstrate that this is a reasonable assumption. Thus, we have the sigmoid equation:

$$P(t) = \frac{Ke^{rt+\alpha}}{K + e^{rt+\alpha}}$$

Where $P(t)$ is equal to the rate of plastic discarded per year and where t is time in year. By analyzing the graph, we assume that the carrying capacity (maximum value of plastic discarded) is approximately 30000 thousand tons per year. We shall later show in our sensitivity test that this assumption produces the highest R^2 value and a better regression.

Through algebraic manipulation, we arrive at the conclusion that:

$$\ln\left(\frac{30000P(t)}{30000 - P(t)}\right) = rt + \alpha$$

And so we can perform a linear regression on the left side using the data obtained from the Environmental Protection Agency (Environmental Protection Agency). We obtain a linear regression of $0.14t - 268.13$ and an R^2 value of 0.989. Plugging this back into the initial equation, we get:

$$P(t) = \frac{30000e^{0.14t-268.13}}{30000 + e^{0.14t-268.13}}$$

Therefore, $P_0 = e^{-268.13} < 10^{-114}$ so $K - P_0 \approx K$, justifying our assumption at the beginning that the -1 can be ignored.

2. Validation of the Model

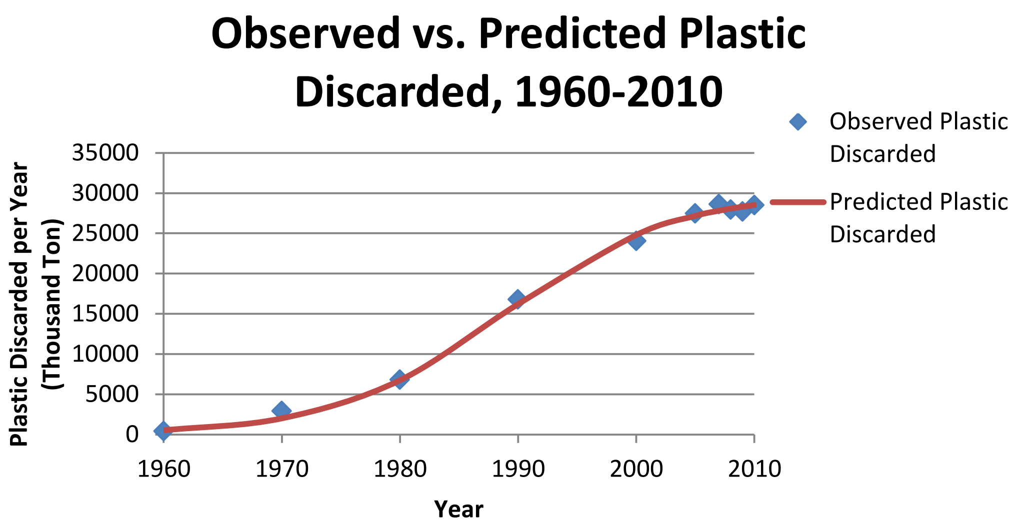

Figure 2: Graph showing observed plastic discarded per year (Environmental Protection Agency) vs. data obtained from our model.

Year	Observed Plastic Discarded	Predicted Plastic Discarded	Percent error
1960	390	519.35	33.17
1970	2900	2000.26	31.03
1980	6810	6738.72	1.05
1990	16760	16205.50	3.31
2000	24050	24795.25	3.10
2005	27470	27168.06	1.10
2007	28630	27809.16	2.87
2008	27930	28077.03	0.53
2009	27690	28314.13	2.25
2010	28490	28523.54	0.12

Table 2: Table showing observed plastic discarded per year vs. data obtained from our model and percent error. (Environmental Protection Agency)

We now aim to prove that our assumption that the "carrying capacity" K is approximately 30000 is correct. We performed the same process for $K = 29000, 29500, 30500, 31000, 31500$ and 32000 as we did for $K = 30000$ and analyzed the percent error obtained from each regression based on a sigmoid function.

	29000	29500	30000	30500	31000	31500	32000
Year	% Error	% Error	% Error	% Error	% Error	% Error	% Error
1960	11.05	22.82	33.17	53.13	62.14	54.43	66.43
1970	30.73	31.68	31.03	24.98	23.77	29.50	26.26
1980	13.94	3.54	1.05	2.13	0.46	8.33	6.71
1990	11.08	1.75	3.31	3.21	5.52	12.02	11.89
2000	8.28	5.08	3.10	3.20	2.33	0.71	0.51
2005	0.42	0.57	1.10	0.57	0.59	1.92	1.40
2007	2.35	2.76	2.87	2.19	1.94	2.73	2.09
2008	0.63	0.45	0.53	1.31	1.69	1.10	1.84
2009	1.96	2.00	2.25	3.13	3.64	3.25	4.07
2010	0.51	0.28	0.12	1.04	1.65	1.47	2.34

Table 3: Table of percent errors using different K values in model.

Looking at the table, we see that our initial assumption of having a K value of 30000 proves to produce the least percent errors.

3. Results of the Model

In order to obtain the total amount of plastic in landfills, we take the integral of our function $P(t)$. We need a start year for our integration-when plastic first began to be dumped in landfills. Plastic was invented in the late 1800s although it did not get popularized until the invention of cellophane and polyvinyl chloride (PVC) in the early 1900s (Masterson). Thus, we assume that in 1920, there is no plastic in landfills. The lower limit of integration becomes 1920. To find the amount of plastic in a landfill in 2023, we set that to be our upper limit of integration and integrating gives:

$$\int_{1920}^{2023} \frac{30000e^{0.14t-268.13}}{30000+e^{0.14t-268.13}} = \left[\frac{30000}{0.14}\ln(30000+e^{0.14t-268.13})\right]_{1920}^{2023} \approx 1{,}026{,}000$$

According to our model, in the year 2023, there will be a total of 1,026,000 thousand tons or 1.026 billion tons of plastic in landfills.

<u>Sensitivity Analysis</u>

We examined the sensitivity of our sigmoidal model of the rate of plastic discarded into landfills. Our main assumption was the K constant and so we analyzed how changing our K values would affect our final result. We use the same K values to test as we did to determine if our assumed 30000 was correct: $K = 29000 - 32000$ in intervals of 500. We obtain the following data.

K	2023 Plastic Mass
29000	1061235
29500	1034413
30000	1026293
30500	1033231
31000	1031435
31500	1007263
32000	1015850

Table 4: Table showing how predicted 2023 plastic mass in landfills varies with the K value.

The highest predicted landfill mass was approximately 1,061,000 and the lowest approximately 1,007,000 and are a respective $\frac{|1061000-1026000|}{1026000} * 100\% = 3.41\%$ and $\frac{|1007000-1026000|}{1026000} * 100\% = 1.85\%$ percent difference from our obtained 1,026,000 thousand tons of plastics. This very small difference (<5%) indicates that our model is not very sensitive to changes in our initial parameter (K value). A small error for K would not significantly change the output of our model.

Part II: Assigning Recycling Methods on a Local Scale

The recycling method that a city employs is dependent on population, number of households, area of the city, and consumption of recyclable materials. Locations for dropping off pre-sorted recyclables are free of charge and available in all cities (Association of Collegiate Schools of Architecture). We chose to focus on and analyze only two methods of recycling, drop-off and single-stream curbside pickup, because they are the two main methods of recycling (Robin, Martinez and Palmer).

***Assumptions*:**

1. Geographic location has a negligible impact on recycling rate for each method of recycling (drop-off and single-stream curbside pickup). This is reasonable because society is interconnected, thus products consumed and wastes emitted in each United States city are relatively similar.
2. Every city will have at least one recycling facility that can serve as a drop-off center. This can be assumed because a recycling center is necessary for the implementation of a recycling program. Thus we are only considering operating costs associated with each recycling method in determining which method is best for each city.
3. The use by residents of a city of drop-off and curbside pickup is mutually exclusive. This is because with curbside pickup available, the participation in drop-off is negligible.
4. We will assume that all people who recycle will recycle all recyclables in the correct manner. This assumption is necessary to make the model as simple as needed.
5. We will assume that every household has recyclable wastes so that our model can be dependent upon the number of households in a city.
6. The average recycling truck can hold between 15 and 28 household recycling bin sizes worth of trash before returning to the recycling facility (Resolute Forest Products). For the sake of simplicity, we will assume that any given run of a recycling truck will take the average of these two values, or 22 recycling bins worth of recyclable material.
7. We will assume that all recycling trucks will work every weekday, each day taking care of a different portion of the city. Recycling is typically taken out on a weekly basis, and this is how recycling companies currently work.
8. According to the US National Solid Wastes Management Association, recycled material is on average sold at $30/ton (Murphy, Mueller and Gowda). We will assume that every ton of recycled material can be sold at this price, which is reasonable because it is the national average. Although there may be factors that prevent recycled material being sold at $30/ton, such as regional price discrepancies, they are out of the scope of this model.
9. We will assume that the average recycling truck driver drives for five hours per day (United States Department of Labor).

10. We will assume that 2/3 of people participate in curbside pickup (Robin, Martinez and Palmer).
11. We will assume that the price of diesel fuel remains constant throughout this model. Because fuel prices are very volatile and do not demonstrate any predictable pattern, they are out of the scope of this model. We will use the national average value of Diesel fuel on February 25, 2013, which was \$4.16 / gallon (U.S. Energy Information Administration).
12. Upon observing videos of truck drivers en route, we determined an average value of 15 seconds from the time a truck stops, picks up the recycling bin, empties it, and arrives at the next house (tylertrashtruck). This value is justifiably low because most houses tend to exist close to each other, in neighborhoods. Other isolated houses would optimally be grouped so that they are along existing recycling routes.
13. We will assume recycling trucks average 30 miles per hour on their route to and from the recycling center. This is a reasonable value, as it is the standard speed limit for neighborhoods. Additionally, although the truck will be going faster on major roads, stop lights and other periods of idling will detract from the truck's average speed.
14. Finally, we will assume that population is spread uniformly about the city. Although this is generally not true, it is too complicated to consider other distributions of population. Additionally, although houses will not be uniformly distributed, it is likely that neighborhoods will be approximately uniformly distributed.

Drop-Off Only

Drop-off recycling involves providing locations for citizens to self-haul pre-sorted recyclables to disposal sites (Environmental Protection Agency).

To model drop-off, we calculate the probability that a person a distance d away from a recycling facility will go to the facility and recycle. If $d = 0$ then we will have the probability $p(d) = 1$ and when $d = \infty$, $p(d) = 0$. These are very plausible assumptions because if you are at a recycling facility, you will recycle. And if you are an infinite distance away, you will not recycle. We assume the probability function is of the form:

$$p(d) = \frac{1}{\left(\frac{d}{k}\right)^n + 1}$$

With $n > 0$, this satisfies our conditions at $d = 0$, and $d = \infty$. We choose $n = 4$ because it will ease the calculations in the double integral that we will later do. We must now find what this value k is, which can be thought of as a constant dictating the willingness of a person to recycle. By looking at our function, a larger k value means a higher probability to recycle.

In order to calculate this k value, we look at the effectiveness of a drop-off program. One which provides ample data is the Central Virginia Waste Management Authority. Their drop-off program from July 1, 2011 to June 30, 2012 collected 6,049 tons of recycled materials (Central Virginia Waste Management Authority (CVWMA)). By looking at counties covered and adding their populations and areas, we arrive at the conclusion that the Central Virginia Area encompasses 1,258,251 people and an area of 2532 mi^2. Also, we have that a normal person produces 1.46 pounds of recyclables a day and so 533 pounds a year (Environmental Protection

Agency). We assume that this Central Virginia area can be approximated by a circle for simplicity and by knowing its area, we know its radius is $r = 28.4\ mi$. Then we use the fact that total amount of materials recycled in a year is: Total Population∗Probability to recycle∗Pounds recycled per year and so our double integral is:

$$\int_0^{2\pi}\int_0^r p(l) * \frac{Population}{Area} * \frac{Pounds\ Recycled}{Year} * ldld\theta$$

$$= \frac{1258251}{2532} * 533\int_0^{2\pi} d\theta * \int_0^{28.4} \frac{ldl}{\left(\frac{l}{k}\right)^4 + 1} = 265000 * 2\pi * \left[\frac{1}{2}k^2 \tan^{-1}\frac{l^2}{k^2}\right]_0^{28.4}$$

$$= 832000k^2 \tan^{-1}\frac{28.4^2}{k^2}\ pounds = 6049\ tons * \frac{2000\ pounds}{ton}$$

Solving for the value of k, we get $k = 10$. So our probability function is:

$$p(d) = \frac{1}{\left(\frac{d}{10}\right)^4 + 1}$$

To determine the total amount of tonnage recycled from a drop-off program serving a circular region with radius r and population density ρ, we take the double integral:

$$\int_0^{2\pi}\int_0^r p(l) * \rho * 533 * ldld\theta * \frac{1\ ton}{2000\ pounds} = \frac{533\rho}{2000} * 2\pi\int_0^r \frac{ldl}{\left(\frac{l}{10}\right)^4 + 1}$$

$$= \frac{533\rho\pi}{1000} * \left[\frac{1}{2}(10^2)\tan^{-1}\left(\frac{l^2}{10^2}\right)\right]_0^r = 83.7\rho\tan^{-1}\left(\frac{r^2}{100}\right)$$

<u>Sensitivity Test</u>:

We test to see how changing n will affect how accurate our model will be. We do this by varying n and keeping k constant for the case $n = 4$ and see how the predicted tonnage varies. We use the k constant from $n = 4$ and compare predicted tonnage as opposed to seeing how the k values change because it becomes impossible to calculate the k value for values of n other than integers. We obtain the following table:

n	Tonnage	% Error
3.5	6636.4	8.11
3.6	6525.0	6.30
3.7	6419.8	4.59
3.8	6320.5	2.97
3.9	6226.8	1.44
4.0	6138.3	0.00
4.1	6054.7	1.36
4.2	5975.8	2.65
4.3	5901.0	3.87
4.4	5830.5	5.01
4.5	5763.8	6.10

Table 5: Table showing how expected tonnage varies as n varies. It includes the percent error from our assumed n=4 in our model.

We see that varying n by at most 0.5, which is 12.5%, causes the percent error to be 8.11% at maximum. The fact that such a large percentage change in n causes a smaller percent change in the expected tonnage means that our model is stable to changes in initial parameters.

Single-Stream Curbside Pickup and Additional Garbage Fee

Single-stream curbside pickup in addition to a garbage container fee promotes a reduction in the number of garbage containers households put out. This recycling method of unit pricing provides an incentive to reduce waste quantities, but does not necessarily increase proper recycling practice. In fact, an additional garbage fee per container creates artificial value in recycling and may result in people attempting to recycle non-recyclable waste (Robin, Martinez and Palmer). This is problematic as it leads to greater probabilities for contamination in recycle facilities which decreases efficiency and increases costs to remedy the situation. Therefore we do not recommend cities to implement this method and have decided to not include a model for single-stream curbside pickup with additional garbage fee in our report.

Single-Stream Curbside Pickup

Free single-stream curbside pickup provides incentive for households to recycle by making recycling more convenient and less time consuming (Robin, Martinez and Palmer).

Calculation of Required Infrastructure

First, we will attempt to calculate the number of trucks and facilities necessary to institute a curbside pickup program in a given city. Our strategy for this will be as follows: assume that a given facility will provide service to all households within r miles. Thus, the service area is a circle with radius r, and the facility lies at the center.

We can then find the total number of households that a facility must service by finding the area the facility must service, and combining that area with the population density and average people per household. There must also be a factor to account for percent participation of households in curb-side pickup.

$$H(r) = \frac{\pi \chi P_d r^2}{H_a}$$

Where $H(r)$ is the number of households that a recycling facility must service, χ is the percent of total households that would participate in curb-side recycling ($\frac{2}{3}$ by assumption), P_d is the population density, r is the maximal distance that a facility will service, and H_a is the average number of people per household. This can be derived with dimensional analysis and the equation for the area of a circle.

Next, let us analyze how many households an individual recycling truck is able to serve.

By our assumptions, we know a given truck will take care of 22 households every "run" it makes from the recycling center and back. Because the houses are uniformly distributed, it is possible to calculate an equivalent, simpler situation. Consider the following double integral:

$$\frac{\int_0^{2\pi} \int_0^R r^2 \, dr \, d\theta}{\pi R^2} = \frac{2}{3} R$$

This tells us that the average distance from the center of a circle to any point in the circle is $\frac{2}{3} R$. Thus, the average "round-trip" from the center of the circle to a house on the circle and back is $\frac{4}{3}R$. Because of our assumption that it only takes 15 seconds to go from house to house on the recycling route, we can then say that this distance is negligible.

Next, we will calculate the time it takes for a single "run" of a recycling truck, from leaving the recycling center to returning. This is doable with purely dimensional analysis and simple Newtonian mechanics.

$$Truck\ time = \frac{\frac{4}{3}R\ mi}{30\frac{mi}{hr}} + \left(22\frac{households}{run}\right)\left(15\frac{sec}{household}\right)\left(\frac{1\ hr}{3600\ sec}\right) = \frac{4R}{90} + .0917\frac{hr}{run}$$

Next, we find the maximum number of households a truck can service per week.

$$T(r) = \frac{\left(\frac{5\ hours}{day}\right)\left(\frac{5\ days}{week}\right)\left(\frac{22\ households}{run}\right)}{\left(\frac{4R}{90} + .0917\right)\frac{hr}{run}} = \frac{550}{\frac{4R}{90} + .0917}\frac{households}{week}$$

Where $T(r)$is the number of households a truck can service per week, given r. Now we can finally find the two quantities we have been aiming for: the required number of facilities and the required number of trucks for each of these facilities. These quantities are fairly self-explanatory;

$$Trucks\ per\ facility = T_f(r) = \left\lceil\frac{H(r)}{T(r)}\right\rceil$$

This is because every facility must service $H(r)$ households, and every truck can service a maximum of $T(r)$ households.

$$Total\ facilities = \left\lceil\frac{\chi H_t}{T(r)T_f(r)}\right\rceil$$

Where H_t is the total number of households in the city. This equation again comes from unit analysis: if we take the total number of participating households, it must be less than the product of the number of facilities times the number of trucks per facility times the number of households per truck.

Calculation of Expenses for Both Models

Fargo is the largest city in North Dakota, consisting of 48.8 square miles with a population of about 107,349 (Cable News Network) and a population density of 2162 persons/mi^2 (U.S. Census Bureau). There are 46,791 residential households within the city, averaging 2.15 persons per house (U.S. Census Bureau).

The city of Price has a population of 8,682 living in 3,045 households, with an average of 2.6 persons per household. It has an area of 4.20 square miles, and a population density of 1980 persons/mi^2 (U.S. Census Bureau).

Wichita, Kansas is 163.6 square miles and has a population of 384,445. Its population density is 2305 persons/mi^2. The number of households is 139,027, with an average of 2.44 persons/household (U.S. Census Bureau).

Drop-off Only Cost

By analyzing a table of operating and maintenance costs for various recycling facilities in 1990, calculating the average, and adjusting for inflation, we found that the typical facility spends $258,750 per year (Environmental Protection Agency). This yields the simple equation:

$$Cost = \$258750 * (Total\ Facilities)$$

Using this equation yields the following table:

City	Cost per Year
Wichita, KS	$5,950,000
Fargo, ND	$2,070,000
Price, UT	$259,000

Table 6: The cost in dollars per year of running a drop-off only recycling system.

Drop-off Only Revenue

From our assumption of the selling of recyclable material at $30/ton and the results in tonnage of our model for drop-off, the revenue can be calculated as

$$Revenue = \frac{\$30}{ton} * \ 83.72\rho \tan^{-1}\left(\frac{r^2}{100}\right)$$

This yields the following table:

City	Revenue per Year
Wichita, KS	$1,330,000
Fargo, ND	$434,000
Price, UT	$49,700

Table 7: The revenue in dollars generated per year by running a drop-off only recycling system.

Curb-side Pickup Cost

The expenses associated with curb-side pickup can be divided into three large categories: operation & maintenance for facilities, gas for the trucks, and salary for the drivers. We will ignore the initial capital required to fund a recycling system, as we are only looking for cost once a system is established. Looking up the average mileage per gallon of a recycling truck, we found 3 mi/gal (Inform Inc.).

$$\frac{\left(\frac{\$4.15}{gal}\right)\left(30\frac{mi}{hr}\right)\left(5\frac{hr}{day}\right)\left(5\frac{day}{week}\right)\left(52\frac{weeks}{year}\right)}{3\frac{mi}{gal}} = \$53950$$

Thus there is $53,950 spent on gas per truck, per year. Looking up data on salary of recycling workers per year, we found a value of $34,420 (United States Department of Labor). Finally, we use our previous figure for the cost of operating and maintaining facilities. Combining this into one equation yields that the overall cost of curb-side pickup is

$$Cost = \$88370T(r)(Total\ facilities) + \$258750(Total\ Facilities)$$

Using this equation, it is possible to find the minimal cost for varying values of r. Doing so yields the following table:

City	Total Cost per Year
Wichita, KS	$4,828,550
Fargo, ND	$1,584,300
Price, UT	$347,120

Table 8: The cost in dollars per year of running a curb-side pickup recycling system.

Curbside Pickup Revenue

According to the EPA, the average person recycles 1.51 pounds per day. Using unit analysis, it is then possible to find total tonnage of recycled material, and then using our assumption of fixed value for tonnage, the overall revenue for each city.

$$Revenue = \left(1.51\frac{lbs}{day * person}\right)\left(365\frac{days}{year}\right)\left(\frac{1\ ton}{2000\ lbs}\right)\left(\frac{\$30}{ton}\right)P$$

Where P is the population of the city. Using this equation, we obtain the following results:

City	Projected Yearly Revenue
Wichita, KS	$2,810,000
Fargo, ND	$832,000
Price, UT	$65,500

Table 9: The revenue in dollars generated per year by running a curb-side pickup recycling system.

Analysis of the Model

$$Net\ cost\ per\ tonnage = \frac{Cost - Revenue}{Tonnage}$$

City	Curb-side pickup cost per tonnage ($/ton)	Drop-off only cost per tonnage ($/ton)
Wichita, KS	$55.37	$104.11
Fargo, ND	$70.17	$112.98
Price, UT	$129.10	$126.13

Table 10: The cost in dollars per tonnage of recycling for both the curb-side pickup and drop-off only recycling systems.

Lastly, we will analyze the two options to determine which one is a better fit for each city. By calculating net cost per tonnage as follows, we obtain the above equation and table. Because we want to minimize the cost required per ton of material recycled, we are looking for the minimum cost per tonnage. We can then conclude that it is best for Wichita and Fargo to use a curb-side pickup method, whereas Price should use a drop-off only recycling method. This makes sense! A smaller town would be expected to have a more successful drop-off only recycling program, because it would only require one large facility and the distance from any citizen to that facility is relatively small. Therefore, for a small town, costs are lower than they would be with curbside, while incentive to recycle is still relatively high.

<u>Part III: National Recycling Policy Recommendation</u>

Recycling is vital to sustaining our environment, thus it is pertinent that all states and townships in the United States take part in some method of recycling. However, cities and townships should be allowed to self-determine their form of recycling based on our model. This is due to the fact that the variables in our model involved in determining whether drop-off only or curbside pickup recycling is most appropriate for a certain municipality—population, area, and household density—are specific to each municipality. Hence, we recommend that the EPA permits each municipality to implement the most appropriate method of recycling for its city after applying its city statistics into our model.

From our model, we conclude that cities with relatively small populations and household densities would economically benefit more from adopting drop-off recycling only (Table 10). Under these population conditions, the cost of recycling using the drop-off only method will be less than the costs associated with curbside recycling pickup, as the number of households participating in recycling is not enough to compensate for the cost of operating curbside pickup. Therefore the city should choose to have drop-off recycling only. On the other hand, cities with high populations and household densities would be better off economically if they implemented the curbside pickup method of recycling. In this case, the cost of curbside pickup is more cost-efficient than the cost of drop-off only, due to a lower net loss (Table 10). With curbside pickup, participation in recycling by household increases, but the increase is only impactful in cities with higher populations, as the ratio of households participating to total household units is greater than the corresponding ratio in small populations (Robin, Martinez and Palmer).

Although the EPA should allow each municipality to determine their own recycling method, it should also legislate that all cities and townships recycle by the method best for them beginning in 2016. This is because recycling is necessary to sustain our environment. The deadline of 2016 provides time for governments to decide which recycling method is most appropriate, and allows time for construction of recycling facilities and implementation of the recycling method.

Part IV: Strengths and Weaknesses

Strengths of our model in forecasting the amount of plastic waste present in landfills 10 years from now include insensitivity to minor changes, and that our assumption of a carrying capacity *K* of 30,000 tons has the least percent error when a linear regression of data obtained from the EPA is performed alongside our model. Furthermore, the linear regression resulted in a R^2 value of 0.989, which indicates a strong correlation between our model and past data, demonstrating that extrapolation to 10 years from now is appropriate. A possible weakness is that we assumed the rate of waste discarded is proportional to the rate of waste recycled, which may not necessarily apply to all households.

Strengths of our model in determining the most cost-efficient method of recycling per city include its consideration of many characteristics of each city, as well as costs associated with each recycling method. This ensures that the recycling method determining model produces a result that is accurately specific to each city. Possible weaknesses include that our model uses average values for operation and maintenance costs of recycling facilities, household size, and recycling truck procedures.

Part V: Conclusion

In this report, we found a mathematical model to predict the amount of plastic waste in the year 2023. By looking at past amounts of waste and performing a regression, we determined that a sigmoidal equation with a carrying capacity of 30,000 tons per year was the best fit for the past data with an R^2 value of 0.989. Using this model, we predict the waste in ten years to be 1,026,000 tons of plastic.

This report also created a model to analyze whether a city in the United States should adopt a drop-off or curbside pickup recycling method. Although it was found that recycling is unprofitable for cities, the environmental necessity for sustainability has demanded that some form of the practice to be adopted.

In addition, utilizing landfills for recyclables involve monetary costs and is much more detrimental to the health of our environment. Therefore, based on our models, we determined that it is more economically beneficial for a large township, such as Wichita, KS or Fargo, ND, to implement a curbside pickup recycling method, while a small township, such as Price, UT ought to use the drop-off method of recycling.

Also, these recycling methods should be determined individually by each township because the decision of which type of recycling to implement depends upon factors specific to each municipal area, such as population. We must take action on both governmental and individual levels, before this swamp of waste trashes our environment, our lives, and eventually—our future.

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