AI/ML Subspace-based Parameter Estimation

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ABSTRACT

Subspace based parameter estimation techniques are well known to the signal processing community. The most popular subspace-based parameter estimation techniques include the MUSIC, ESPRIT and Matrix Pencil algorithms. The common thread within this line of techniques is the exploitation of the inherent orthogonality that exists between the noise subspace and signal vectors that span the (signal + noise) subspace. Recently, Model-based AI/ML approaches have been developed that derive surrogate matrices which behave like noise subspace-based projection matrices. These surrogate matrices are learned by the AI/ML algorithms by processing a sufficiently large amount of input/output data through their structure using backpropagation or a similar type of learning approach. A very brief analysis that delved into the characteristics of these surrogate matrices, showed they behave like an interesting cross between beamforming and noise subspace projection matrices, but without the limitations of either group of matrices. Furthermore, it was postulated that these surrogate matrices may serve as a basis for developing some intriguing virtual sensor concepts.

The purposes of this SIAM Challenge problem would be to:

- develop an understanding of the characteristics of these surrogate matrices
- leverage these surrogate matrices to develop virtual sensor concepts that provide enhanced number of degrees-of-freedom for parameter estimation purposes
- develop extensions to this concept of AI/ML subspace-based parameter estimation approaches

The parameter estimation discussed in this brief paper will be DOA estimation.

Brief Motivation for the study of Direction-of-Arrival (DOA) estimation

Over the last few decades, a wide range of applications have spawned interest in advanced (possibly AI/ML based) DOA estimation approaches in the signal processing community. These include radar, sonar, acoustics, astronomy, wireless communications, mobile communications, vehicular communications and various biomedical applications. Our specific interests here stem from radar applications, where various challenges have been shown to impact the precision/accuracy of DOA estimation, such as calibration errors, multipath, nonstationary effects and the need for real time processing. Model-based AI/ML approaches to DOA estimation have shown significant promise in surmounting these challenges and it is one such approach that we are interested in studying here. Our approach started from curiosities about how AI/ML could be used to augment the capabilities of the well-known subspace-based method called Multiple Signal Classification (MUSIC). This ultimately leads us to the Complex Orthogonal Search Network (CosNET) algorithm, our model-based AI/ML approach to DOA estimation, which appears to be an interesting mix/hybrid of the Capon Estimator and MUSIC.

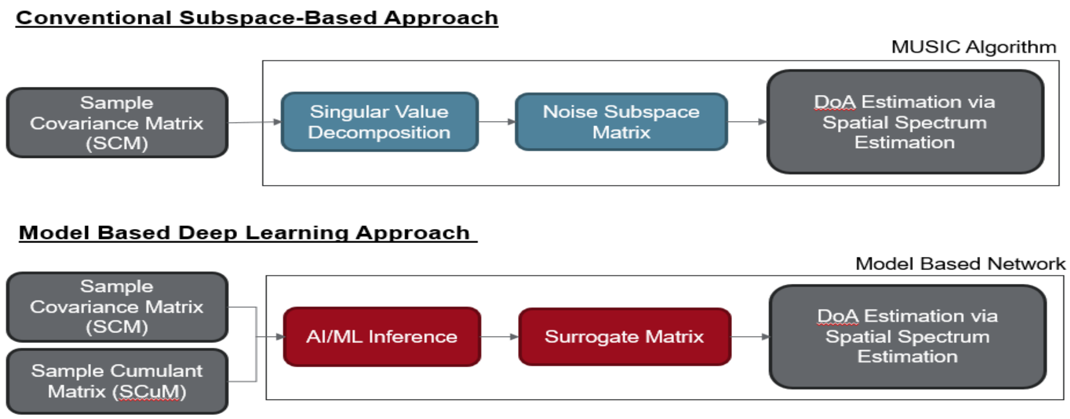
Discussion

The following sections expand on the concepts introduced in the abstract above. The format follows a line of questions posed by the MPI selection committee that sought to provide a more complete picture for the reader. The answers to these questions are shown in blue to highlight the author's response.

The MPI committee believes this could be a particularly interesting problem, especially for those with a statistical focus. However, its current framing is rather generic. Providing a more specific physical context (e.g., a specific example) would be helpful, particularly as a potential basis for proposing surrogate matrices. Sample data and/or codes (MATLAB or Python) would also be appreciated.

- Even though the concept is applicable to both Direction-Of-Arrival (DOA) and range estimation, we have focused specifically on DOA estimation.

Diagram of Approaches



- Will provide sample data and test code from a recent model-based AI/ML approach that we developed which is called CosNET. A basic description of the CosNET architecture will be provided (see below).

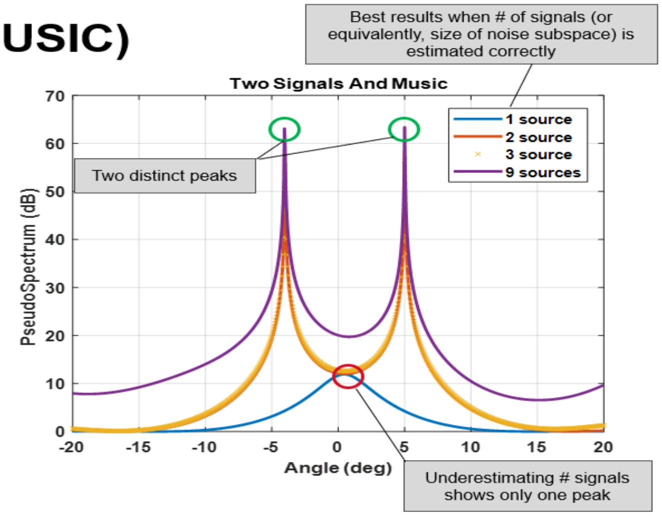
Additionally, could you provide some particularly helpful references to orient problem solvers to the MUSIC, ESPRIT, and Matrix Pencil algorithms in advance of the workshop? Similarly, references on the "recent model-based AI/ML approaches," "beamforming," and "a very brief analysis" would enable problem solvers to make more efficient use of the workshop.

- References for MUSIC, ESPRIT, Matrix Pencil and a basic description of both MUSIC & CosNET architectures are given below.
    1. R. Schmidt, "A signal subspace approach to multiple emitter location and spectral estimation",
       Ph. D. dissertation, Stanford University, 1981.
    2. R. H. Roy, "ESPRIT – Estimation of Signal Parameters via Rotational Invariance Techniques", Ph. D. dissertation, Stanford University, 1987.
    3. Y. Hua and T. K. Sarkar, "Matrix Pencil Method for Estimating Parameters of Exponentially damped/Undamped Sinusoids in Noise", IEEE Transactions on ASSP, Vol. 38, No. 5, May 1990
    4. T. K. Sarkar and O. Pereira," Using the Matrix Pencil Method to Estimate the Parameters of a Sum of Complex Exponentials", IEEE AP Magazine, Vol. 37, No. 1, February 1995.

Conventional Solution:
MUltiple SIgnal Classification (MUSIC)

- **How it Works**
  - Direction finding algorithm based on eigenvalue decomposition of the sample covariance matrix.
  - Performs orthogonal search over noise subspace to determine signals Direction of Arrival.

- **Limitations**
  - Can only detect up to N-1 sources for an N element Uniform Linear Array (ULA)
  - Requires eigen decomposition to determine eigenvectors and a search, which are both computationally intensive
  - Unable to resolve coherent/correlated sources



Conventional Solution: MUSIC Algorithm

– Begin with the sample covariance matrix as defined below:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{i=0}^{N-1} \mathbf{x}_i \mathbf{x}_i^{\ H}$$

1. Perform an eigen decomposition of $\hat{\mathbf{R}}$.
2. Determine the size of the signal and noise subspace by looking at the eigenvalues (AIC/MDL Algorithms)
3. Determine the noise subspace U_N
4. Pseudospectrum: Search over steering vectors to find ones which are orthogonal to the noise subspace (find peaks of $P(\theta)$)

$$\hat{\mathbf{R}} = \mathbf{USU}^{\boldsymbol{H}}$$

$$\{\lambda_i\} = \boxed{\lambda_1, \ldots, \lambda_M}, \boxed{\sigma_1^2, \ldots \sigma_{N-M}^2}$$

Dominant eigenvalues corresponding to number of signals (M)

$N_{chan} - \mathrm{M}$ eigenvalues corresponding to noise

$$\mathbf{U}_N = \mathbf{U}(:, M+1{:}\,N)$$

$$P(\theta) = \frac{1}{\mathbf{a}(\theta)^H(\mathbf{U}_N\mathbf{U}_N^H)\mathbf{a}(\theta)}$$

$$P(\theta,\varphi) = \frac{1}{\mathbf{a}(\theta,\varphi)^H(\mathbf{U}_N\mathbf{U}_N^H)\mathbf{a}(\theta,\varphi)}$$

Complex Orthogonal Search Network (COSnet)

- This network attempts to loosely replicate the orthogonal search performed by MUSIC to find steering vectors which are orthogonal to the noise subspace.

$$P(\theta) = \frac{1}{\boldsymbol{a}(\theta)^H(\boldsymbol{U}_N\boldsymbol{U}_N^H)\boldsymbol{a}(\theta)}$$

- We will refer to the term $(\boldsymbol{U}_N\boldsymbol{U}_N^H)$ as $\boldsymbol{X}_N$, our surrogate matrix

$$= \frac{1}{\boldsymbol{a}(\theta)^H(\boldsymbol{X}_N)\boldsymbol{a}(\theta)}$$

dim: [4 x 4]

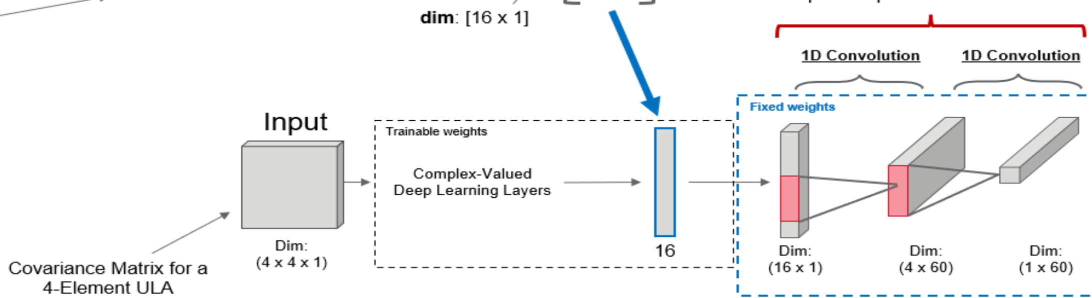
$$vec(\boldsymbol{X}_N) = \begin{bmatrix} \boldsymbol{X}_{N,11} \\ \vdots \\ \boldsymbol{X}_{N,41} \\ \boldsymbol{X}_{N,21} \\ \vdots \\ \boldsymbol{X}_{N,44} \end{bmatrix}$$

dim: [16 x 1]

The vectorized form of $\boldsymbol{X}_N$ is represented by the output of the preceding layer

The spectrum estimate is obtained by taking the inner product of $\boldsymbol{X}_N$ against the array of steering vector

A suitable final layer activation function is chosen to perform the reciprocal operation.



Regarding the development of an understanding of the characteristics of these surrogate matrices, the most likely outcome of the workshop would be some statistical performance characterization on examples, rather than analytical results. Is that the desired outcome?

- No, we simply want to characterize the functionality of the surrogate matrix (as stated above in objective 1). As a result of a very quick characterization that we did previously, it appears to be a hybrid between the surrogate matrices of (1) the Capon Estimator (related to adaptive beamforming), where the surrogate matrix is the inverse correlation matrix and (2) MUSIC, where the surrogate matrix is the orthogonal projection matrix computed from the noise subspace matrix.

It would also be beneficial to explain the goal to "develop virtual sensor concepts that provide an enhanced number of degrees-of-freedom" in more direct language or within the context of a specific test example. Providing a target model problem with the desired improvements over the "popular subspace-based parameter estimation techniques" would offer helpful focus. If it is not feasible to provide this in advance, developing such a test example could be one of the first steps of the workshop.

- Two main objectives:
    1. Characterization of the functionality of the surrogate matrix
    2. Derive alternative surrogate matrices that explicitly leverage virtual sensors inherent to cumulant matrices (such as quadricovariance matrices).

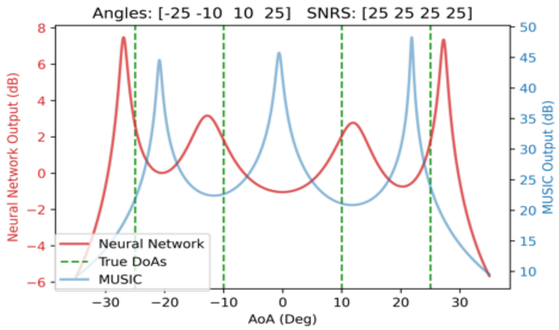
- Here are some useful references about the virtual sensors that are inherent to cumulant matrices:
    1. B. Porat and B. Friedlander, "Direction finding algorithms based on high-order statistics," *IEEE Trans. Signal Processing*, vol. 39, 2016-2024, Sep. 1991.
    2. M. C. Dogan and J. M. Mendel, "Applications of cumulants to array processing: I. Aperture extension and array calibration." *IEEE Trans. Signal Proc.*, vol. 43, pp. 1200-16, May 1995.
    3. P. Chevalier, L. Albera, A. Ferreol, and P. Comon, "On the virtual array concept for higher order array processing," *IEEE Trans Signal Processing,* vol. 53, pp. 1254-1271, Apr. 2005.
    4. P. Chevalier and A. Ferreol, "On the virtual array concept for the fourth-order direction finding problem," *IEEE Trans. Signal Processing*, vol. 47, pp. 2592-2595, Sep. 1999.

From the description, it is not immediately clear where the need for ML (i.e., nonlinear) tools is necessary if we are estimating a subspace or a linear operator. Focusing on the bullet points, it should be clarified why traditional linear algebraic theory (e.g., analyzing the spectrum, singular values, invariant subspaces, etc.) is insufficient for achieving the objectives.

- The non-linear inference provided by model-based AI/ML allows us to estimate a surrogate matrix that appears to exceed the inherent capabilities of surrogate matrices derived through traditional linear algebraic methods.

- Preliminary work that we did seem to indicate that CosNET may provide the capability to resolve more sources than allowed with conventional algorithms which employ traditional linear algebraic theory. However, note that we did not explicitly leverage the virtual sensors Inherent to cumulant matrices, which may be why we were only able to show a vestigial capability to resolve more sources than allowed with conventional algorithms.

Resolving More Sources than Conventional Limit and Alternative Input Data

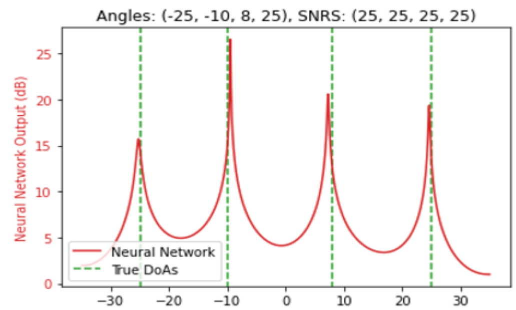


Resolving up to N Sources

- MUSIC can theoretically predict up to (N-1) sources for N spatial degrees of freedom(# of antenna channels)
- *(Left)* The learned X_n surrogate matrix produced by COSnet shows the ability to resolve up to N sources.

Utilizing Alternative Input Data

- Additional information can be provided to the neural network as input to improve predictive ability
- (*Right*) Providing a Cumulant matrix as input to the COSnet showed enhanced resolution of N sources, but did not expand the degrees-of-freedom (DoFs).



Additional information about the data that we plan to share with SIAM

- Given that we don't expect all participants to have access to the Deep Learning Framework used to train and test the neural network, we will be providing an extensive dataset for analysis. Each entry in the dataset will contain:
  - the sample covariance matrix provided to the network as input,
  - the surrogate matrix estimated by the network and
  - the truth information of the noise sources/signals modelled when forming the input data (that is, the Source Number, Directions- of-Arrival (DOAs), SNR).

- The dataset will be representative of the noise source/signal parameter distributions used during network training. Data will be stored in MATLAB *.mat files.

- Additional scripts will be provided to easily read in the provided data and to apply the MUSIC algorithm to the various matrices.