Quantum Algorithms Teaser (and Connections to Applied Mathematicians)

Jeffrey Larson

Argonne National Laboratory

SIAM Quantum Intersections Convening, 2024

Capabilities of Quantum Algorithms

- Quantum speedup in solving certain classes of problems (e.g., prime factorization, search, linear systems).
- Efficient simulation of quantum systems, which is intractable for classical computers.
- Applications in cryptography, optimization, machine learning, and chemistry.

Key Mathematical Areas Involved

- **Linear Algebra:** Core to quantum mechanics and algorithmic operations (e.g., matrix manipulation).
- **Optimization:** Quantum algorithms for variational methods, quantum annealing.
- **Probability and Statistics:** Understanding measurement, error rates, and noise models.
- **Numerical Analysis:** Algorithms for approximating solutions, stability, and convergence analysis.

Contributions of Applied Mathematicians

- **Algorithm Design:** Creating efficient algorithms by understanding quantum complexity and resource scaling.
- **Error Correction and Mitigation:** Developing methods to mitigate quantum noise and errors.
- **Modeling and Simulation:** Simulating quantum systems for verification and benchmarking.
- **Optimization Techniques:** Enhancing quantum algorithms for practical optimization problems.

Example Quantum Algorithms for Mathematicians

- **Shor's Algorithm:** Efficient integer factorization.
- **Grover's Search Algorithm:** Quadratic speedup in searching unsorted databases.
- **Quantum Approximate Optimization Algorithm (QAOA):** Solving combinatorial optimization problems.
- **HHL Algorithm:** Solves systems of linear equations exponentially faster than classical methods (Harrow, Hassidim, and Lloyd).
- **Quantum Phase Estimation (QPE):** Used to find eigenvalues of unitary operators, crucial in many algorithms like Shor's and quantum simulation.

So many caveats...

- True, HHL can solve $Ax = b$ in time that scales logarithmically with the size of the system (classical methods typically scale polynomially)
- HHL generates a quantum state $|x\rangle$ such that when you measure it, the probability amplitudes correspond to the elements of the solution vector x.
- Extracting the full x requires samples of $|x\rangle$ that scales with the problem size.
- Can your application can leverage the quantum state directly?
- HHL depends heavily on the condition number of A.
- Loading the problem data on the device cost not included.

Conclusion and Future Directions

- Quantum algorithms are still in their early stages, but they hold promise for certain problem classes.
- Applied mathematicians can contribute through expertise in algorithm development, error correction, and optimization.
- Future challenges: Scalability, error correction, algorithmic efficiency, hardware limitations.