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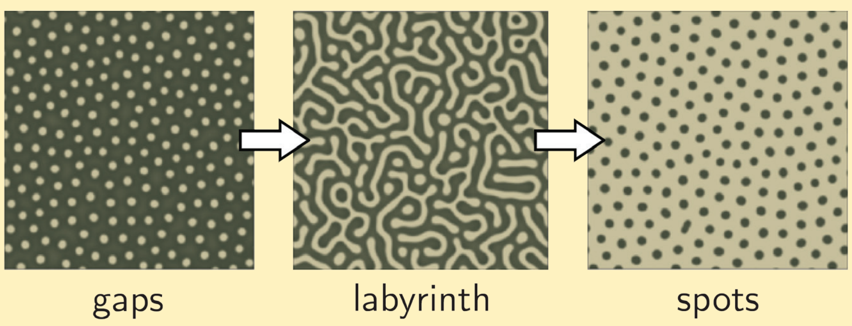
Dynamical Systems Special Issue

Check out articles related to dynamical systems in this ***special issue****, which offers a preview of what to expect at the upcoming SIAM Conference on Applications of Dynamical Systems.*



Vegetation patterns evolve through gaps → labyrinths → spots as ecosystem aridity increases in a vegetation model. Image credit: Karna Gowda.

In the article "Modeling Vegetation Patterns in Vulnerable Ecosystems" on page 6, Lakshmi Chandrasekaran describes work by Mary Silber and her group to mathematically understand repetitive patterns of vegetation in dry and arid regions, such as the Horn of Africa. Silber will present this research at the 2017 SIAM Conference on Applications of Dynamical Systems, to be held in Snowbird, Utah, from May 21-25.

Explaining the East/West Asymmetry of Jet Lag

Dimension Reduction Methods for Analyzing Networks of Circadian Oscillators

By Thomas M. Antonsen, Michelle Girvan, Zhixin Lu, and Edward Ott

Jet lag is a common experience for airplane travelers crossing multiple time zones. Typical symptoms include drowsiness, discomfort, reduced functionality during the local daytime, and difficulty sleeping during the local nighttime. Simply explained, the human body follows a circadian rhythm that synchronizes with the local 24-hour day/night cycle of external natural conditions (particularly the rising and setting of the sun) and social conditions. Upon rapid crossing of several time zones, the body's circadian oscillation needs time to resynchronize to the local oscillation phase of the external conditions; this resynchronization phase manifests as jet lag symptoms in travelers. Since resynchronization of an oscillator is a dynamical process, this phenomenon lends itself to mathematical modeling from a dynamical systems perspective.

While the body produces many signals that help determine its circadian rhythm, one bodily region seems particularly important in this process: the suprachiasmatic nucleus (SCN), a tiny region of the brain's hippocampus. Physiological studies show that the SCN contains of the order of 10^4 neural oscillators, and that it is reasonable to assume that, in isolation, the periods of individual oscillators are distributed with a small dispersion around a mean slightly longer than 24 hours. When coupled together within the SCN, these oscillators are thought to undergo collective synchronization with each other as well as with external stimuli experienced by the individual, e.g., the rising and setting of the sun. This information provides the basis for our model's construction, as well as others that have preceded it. The modeling that we employ [4], however, is different from previous attempts in that we start at the microscopic level of the individual coupled SCN oscillators, but then reduce the high-dimensional microscopic description to a low-dimensional macroscopic description.

See **Jet Lag** *on page 4*

In-Silico Medicine: Multiscale Modeling of Hematological Disorders

By Xuejin Li and George Em Karniadakis

Human red blood cells (RBCs) have remarkable deformability, squeezing through narrow capillaries as small as three microns in diameter without any damage. Several pathological conditions, including malaria, sickle cell disease (SCD), and diabetes can alter the shape and deformability of circulating RBCs. Recent work demonstrates how new computational and analytical models can reveal the ways in which tiny inter-endothelial slits in the spleen prevent old or diseased RBCs from re-entering the systemic circulation. A general computational multiscale framework for RBC modeling is essential in quantifying the altered morphological and biomechanical properties of RBCs in the aforementioned diseases. One can apply this computational framework to other blood pathologies, e.g., in patients with cancer or HIV.

Why Computational Models?

Blood is a non-Newtonian fluid. The movement of RBCs through and with plasma, which is closely associated with RBC deformability, determines blood's rheological properties. Advances in experimental techniques have enabled accurate measurements of RBC deformability. However, while most of these techniques are suitable for RBC populations, i.e., measuring properties averaged over all RBCs in a blood sample, they do not account for the heterogeneity in shape or size differences within the RBC population. A major challenge for single-cell techniques is the need to obtain a realistic geometry, as experiments on small blood vessels require an especially careful vessel preparation, and in certain conditions the precise determination of RBC membrane properties is difficult to achieve, in part due to resolution limitations. Hence, computational models, such as continuum-based and particle-based RBC models, provide a promising means for tackling a broad range of dynamical and rheological blood-related problems [3].

Continuum-based RBC models treat the RBC membrane and intracellular fluids as homogeneous materials, and describe the modeling system using locally-averaged variables, such as velocity, density, and stress, with ordinary and partial differential equations often governing kinematics and dynamics. While continuum-based RBC models enable simulations of large-scale blood flow, they do not provide the detailed dynamics of local subcellular structures. RBC models based on particle methods, where mesoscopic particle-collision models are employed, fill this gap. Mesoscopic particle-based methods are coarse-grained analogs of the molecular dynamics method, and can be rigorously derived through the Mori-Zwanzig formalism [4]. Such RBC models are increasingly popular as a promising tool for modeling the structural, mechanical, and rheological properties of RBCs. Examples include dynamic deformability for various stages of *Plasmodium falciparum*-infected RBCs (*Pf*-RBCs), and membrane flickering of human RBCs. These studies lead to better understanding of the microvascular transport of RBCs in healthy and diseased states.

Why Two-component RBC models?

A normal RBC is a nucleus-free cell; it adopts a distinctive biconcave shape of about 8.0 μm in diameter and 2.0 μm in thickness. The membrane of an RBC consists of a lipid bilayer supported by an attached spectrin network (cytoskeleton); they are connected by transmembrane proteins, such as band-3. Most RBC models depict the membrane as a single shell with effective properties that represent the combined effects of lipid bilayer and spectrin network, and are referred to as one-component RBC models. Under normal conditions, the cytoskeleton is attached to the lipid bilayer from the cytoplasmic side. However, under certain pathological conditions, such as SCD and other hereditary disorders, the cytoskeleton may dissociate from the lipid bilayer. The biomechanical properties associated with the bilayer-cytoskeleton interactions strongly influence cell function and progression of RBC diseases. One-component RBC models cannot facilitate detailed whole-cell exploration of diverse biophysical and biomechanical problems involved in such cases. There is, hence, a compelling need to develop a more realistic RBC representation, e.g., to

See **In-Silico Medicine** *on page 3*

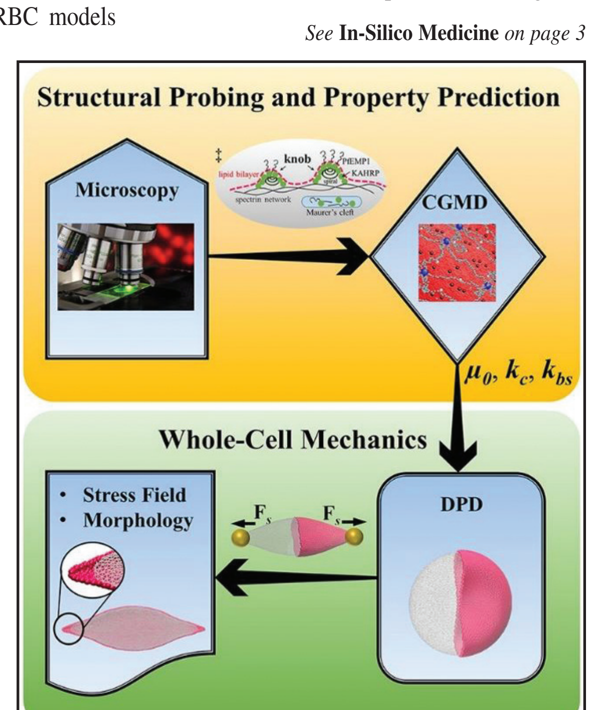


Figure 1. *Two-step multiscale framework for red blood cell (RBC) modeling. Image adapted from [1].*

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Telling a Good Story at Conferences

FROM THE SIAM PRESIDENT
By Nicholas Higham

With peak conference season not far away, many of us will soon turn our attention to writing talks. In doing so, we need to know our audience and put ourselves in the place of a typical listener. I'm sure I am not alone in having two particular wishes when attending a talk. First, I want to hear a good story. We all like to learn why the speaker is working on a given problem, what difficulties were faced, and how they were overcome. And the more personal the story—unique to the speaker—the better. Second, I want to take away a good idea, one that I can perhaps utilize in my own work.

All too often, though, the story and the ideas are obscured by a surfeit of low-level detail and a profusion of equations. Equations are useful only if they can be read and understood, so given the limited time that a slide is displayed and the inadequate legibility caused by imperfect projectors and lighting, the old adage "less is more" applies. And slide text should be as large as possible, for the benefit of those of us with less than perfect eyesight.

It is not necessary to tell the whole story. The narrative needs to be pared down in order to communicate the key ideas and conclusions in the limited time available. Every unnecessary word and symbol should be excised from the slides.

I recently spent some time looking through back issues of *SIAM News* held at the SIAM office in Philadelphia, PA. An article in the October 1996 issue reports on a minisymposium on oral communication held at that summer's Annual Meeting in Kansas City, MO. Margaret Wright, then-president of SIAM and a panelist at the minisymposium, advised, "In planning any talk, ask yourself, 'What do I want to convey? What should the audience remember – later today, next week, next year?'" The article is full of excellent advice. Indeed, the *SIAM News* archive is a real treasure trove, with many articles still pertinent today—years after publication—and others of historical interest. I am hoping that SIAM will be able to digitize the complete archive and make it available online.

Mathematician John de Pillis regularly contributed cartoons to SIAM News from the 1980s onwards. Imbued with de Pillis's unique sense of humor, these cartoons are as relevant today as when they were first published. SIAM News will reprint selected cartoons in future issues, and SIAM hopes to make the complete set available online in due course.

My first official duty as SIAM president was to introduce the SIAM Invited Address by Irene Gamba at the Joint Mathematics Meetings (JMM) in Atlanta, GA, in early January. This was the largest meeting I've ever attended, with over 6,000 delegates. I took the opportunity to attend some sessions on the history of mathematics. These talks have a natural story, but still need to be presented well – and they were. Some speakers read from a script, as is common in history talks. A useful tip for less-experienced presenters is to write down what you want to say for the introduction and conclusion of the talk. Ideally, learn the words and speak them naturally, but if nerves take over or your memory fails, you can always read them out.

An interesting fact I learned from one of these talks is that applied mathematician Richard Bellman, statistician Jon Tukey, and computer scientist John McCarthy all had the same Ph.D. advisor. Can you guess who? The answer is in the footnote.[1]

Scott Bagwell of Swansea University (left) won first prize in the student poster competition at the Annual Meeting of the UK and Republic of Ireland Section of SIAM, held January 12, 2017 at the University of Strathclyde. Professor Des Higham, president of the SIAM UKIE Section, presents Bagwell with his prize. The poster can be viewed online at http://maths.manchester.ac.uk/siam-ukie/annual2017. Image credit: Nicholas Higham.

While at JMM, I picked up a couple of other tips about giving talks. One is that it is beneficial to provide a shortened URL on the first or second slide from which the audience can download the talk and follow along or investigate links. The other tip is to 3D-print some aspect of the talk's mathematics and pass it around the audience. Finding something suitable to print, however, may not be easy!

It's good to see posters growing in popularity at SIAM conferences. There were 22 posters and six talks at the Annual Meeting of the UK and Republic of Ireland Section of SIAM this January, allowing a wide variety of mathematics to be presented in one day.

A recent innovation at the SIAM Conference on Computational Science and Engineering and the SIAM Annual Meeting has been minisymposteria: minisymposia for posters. A minisymposterium organizer collects poster submissions on a particular topic, which are colocated in the display area. I co-organized a minisymposterium with Françoise Tisseur at the 2016 Annual Meeting (AN16) in Boston, MA, and we managed to attract 22 posters without too much effort. This is a great way to feature a research topic without constraints on the number of presenters and competition from parallel sessions that affect minisymposia.

The option nowadays to print on fabric has made giving poster presentations more attractive. These posters can be carried folded, within a suitcase, without the need for a cardboard tube. Our experience in Manchester with fabric posters, which cost a little more than paper ones, has been very positive, and we have had no problem with creasing. Just don't forget to collect your poster by the after-session deadline, as "abandoned" posters usually get thrown away when poster boards are removed.

At AN16, SIAM experimented with e-posters: large electronic displays that allow interactive material to be displayed from a laptop. E-posters were also featured at the 2017 SIAM Conference on Computational Science and Engineering in Atlanta, GA, in late February. If you attended this meeting, be sure to provide feedback on e-posters (and other aspects) to meetings@siam.org, as this will help us decide whether to continue offering this option.

Nicholas Higham is the Richardson Professor of Applied Mathematics at the University of Manchester. He is the current president of SIAM.

[1] Pure mathematician Solomon Lefschetz

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In-Silico Medicine

Continued from page 1

endow the spectrin-based RBC models with more accurate structure, thus considering the lipid bilayer and cytoskeleton separately but also including the transmembrane proteins.

More recent efforts have focused on this approach, leading to a two-component composite model of the RBC membrane with explicit descriptions of lipid bilayer, cytoskeleton, and transmembrane proteins using coarse-grained molecular dynamics (CGMD) [2]. This CGMD membrane model has been successfully used to study membrane-related problems in RBCs, such as the multiple stiffening effects of nanoscale knobs on *Pf*-RBCs [8]. Recently, a two-component whole-cell model was also developed and implemented using dissipated particle dynamics (DPD) [5]. The DPD-based RBC model also separately accounts for the lipid bilayer and cytoskeleton but implicitly includes transmembrane proteins; thus, it is computationally more efficient than the CGMD model for RBC modeling at the whole-cell level, which has been applied to investigate RBC response and dynamics in various blood flow conditions.

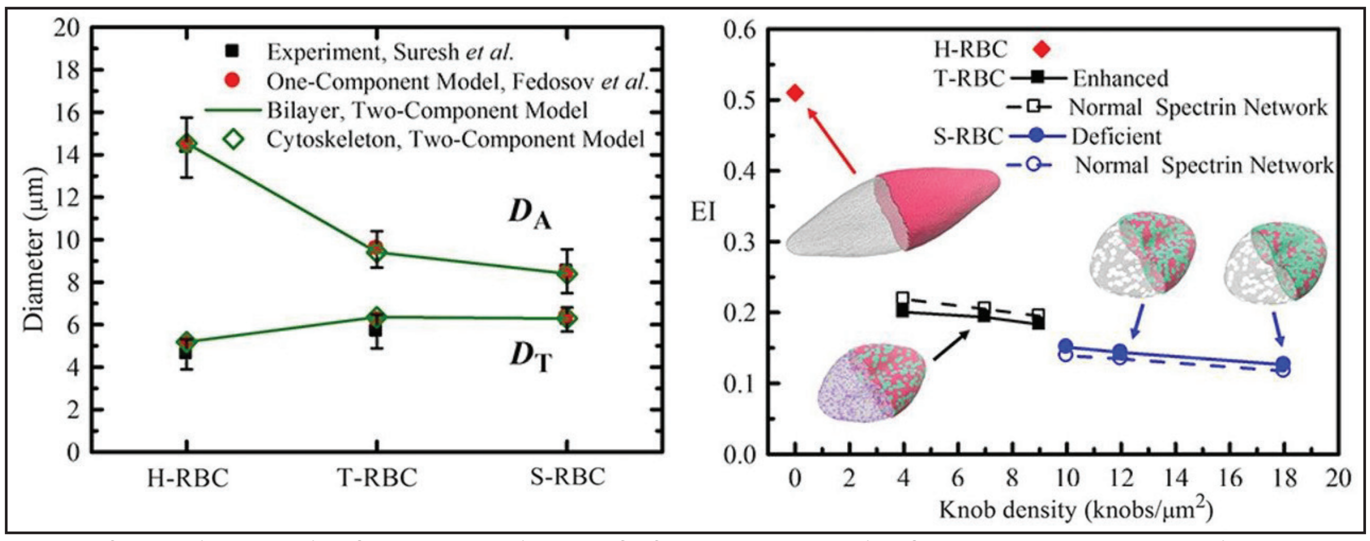

Figure 3. *Shape deformation of RBCs under tensile forcing.* **Left.** *Stretching response of RBCs compared with experiments from [7].* **Right.** *Effect of knob density on cell deformability of T-RBCs and S-RBCs, stages of the malaria parasite. Image adapted from [1].*

Why a Two-step Multiscale Framework for RBC Modeling?

Computational RBC modeling can predict properties beyond available experimental measurements [5, 8]. Modeling a small piece of cell membrane with the two-component composite model can sometimes evaluate modifications of RBC membrane biomechanics, including bending rigidity and shear modulus. However, modeling only a portion of the RBC membrane does not efficiently depict the whole-cell characteristics strongly related to RBC biomechanics and biorheology. On the other hand, the lack of molecular details in the two-component whole-cell model may limit its predictive capacity in identifying key factors that cause the reorganization of the RBC membrane. Incorporating the necessary molecular information from a molecular-detailed composite membrane model into a more coarse-grained whole-cell model effectively addresses this problem. We have recently developed and validated a two-step multiscale framework for RBC modeling by interfacing the two-component CGMD and DPD models (see Figure 1, on page 1). The only experimental input required is information about the structural characteristics of the RBC membrane. Then, we perform CGMD simulations to compute the shear modulus, bending stiffness, and network parameters of a small RBC patch, which we use as input to DPD simulations to predict the stress field and morphology of defective RBCs.

In-silico Predictions

The human spleen acts primarily as a blood filter. By using the simpler one-component whole-cell model based on DPD, we present a recent mesoscopic computational study of physiological and pathological RBCs passing through the spleen that quantifies biophysical limits for splenic slit clearance. A range of possible shapes and sizes allow RBCs to move through the splenic slits (see Figure 2). These closely match the normal ranges observed in healthy human RBCs, with surface areas ranging from 80 to 180 μm^2 and volumes ranging from 60 to 160 μm^3. However, diseases such as malaria can significantly impact the size and shape of affected RBCs, causing them to be filtered by the spleen.

Using the two-step multiscale framework for RBC modeling, we studied the biomechanical characteristics of healthy RBCs (H-RBCs) and *Pf*-RBCs under tensile forcing, and examined the RBC stretching response to large deformation [1]. Our results showed that both the axial and transverse diameters were in agreement with previous experimental measurements (see Figure 3, left). We also investigated the influence of knob density on RBC deformability and found a decrease in elongation index (EI) for *Pf*-RBCs at trophozoite (T-RBC) and schizont (S-RBC) stages of the plasmodium parasite (see Figure 3, right); the increase of knob density indicates that the rigid nanoscale knobs contribute to cell membrane stiffness.

Outlook

Our aforementioned simulation highlights demonstrate that stochastic multiscale modeling, based on particle methods to simulate RBCs at the protein level, can facilitate the effective study of longstanding biophysical questions not possible by other computational methods or experimental techniques. One can further extend the two-step computational framework to investigate the following problems related to pathological blood flow: **(i)** Development of hybrid models, which encompass all scales by combining continuum description for blood plasma with particle description for RBCs, for cost-effective simulations. Such simulations could shed light on the coupling of biology, chemistry, and mechanics (the "triple-point"). **(ii)** Development of predictive patient-specific models to describe heterogeneity-related issues in hematological disorders such as malaria, diabetes, or SCD. Perhaps the most important extension is to connect such multiscale models to all the "omics" technologies (genomics, proteomics, metabolomics, etc.) to implement the vision of *precision medicine* advocated both in the U.S. and around the world.

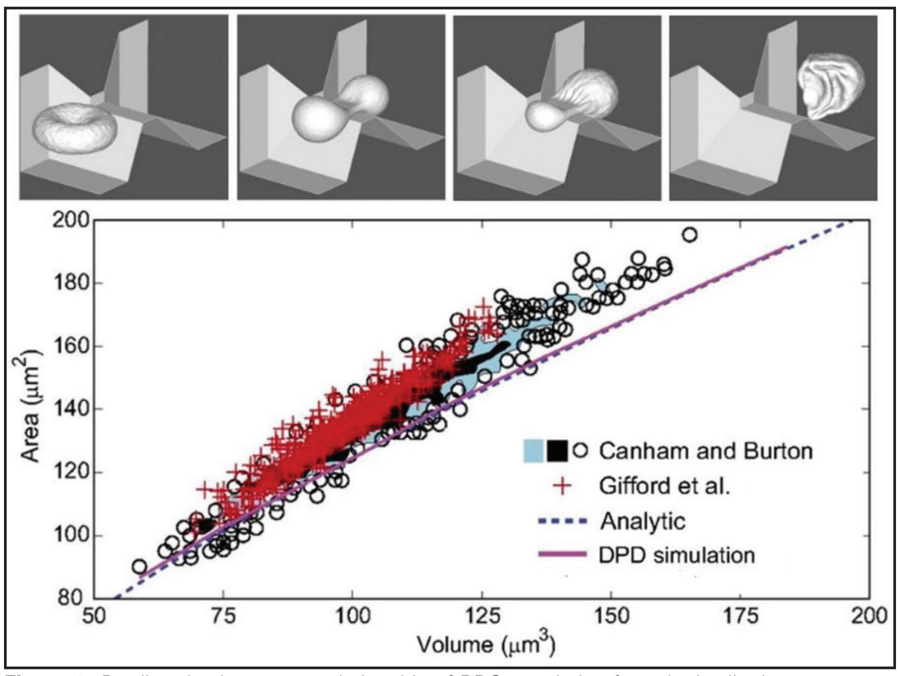

Figure 2. *Predicted volume-area relationship of RBC population for splenic slit clearance, compared with experiments. Healthy RBCs with volumes and areas to the left of the curves would cross the splenic slits, whereas RBCs located to the right of the curves would be filtered out. Image adapted from [6].*

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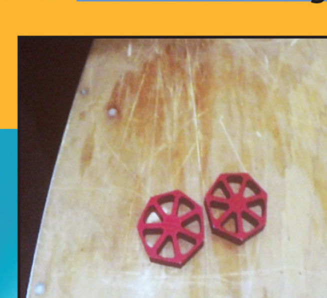
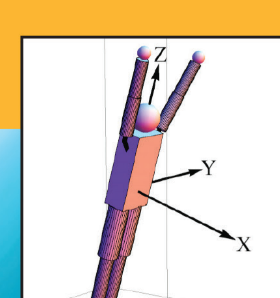
Xuejin Li is a research assistant professor in the Division of Applied Mathematics at Brown University. George Em Karniadakis is the Charles Pitts Robinson and John Palmer Barstow Professor of Applied Mathematics at Brown University. He is also a SIAM Fellow.





Jet Lag

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Our model's primary purpose is to address the empirical observation that jet lag is more severe (i.e., requires a longer recovery time) for eastward travel than for westward travel across the same number of time zones. Specifically, we explore to what extent this east/west jet lag asymmetry may be explained by the small amount by which the average SCN period exceeds 24 hours.

Since we wish to understand the interplay between global travel and resynchronization in a large collection of *N* neuronal oscillators, we use a very simplified model for the neuronal oscillators, perhaps at some expense of realism. As in the well-known Kuramoto model [1], the complicated dynamics of each oscillator are reduced to a time evolution equation for a phase $\theta_i(t)$, representing the state of the i^{th} oscillator where $i = 1,2,\ldots,N$. The phases of the oscillators advance in time, according to the model

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{i=1}^{N} sin(\theta_j - \theta_i) + Fsin(\sigma t - \theta_i + p). \qquad (1)$$

Here, ω_i is the natural frequency for each oscillator which, in our model, is drawn from a distribution $g(\omega_i)$ that peaks at a value of ω, corresponding to a period just over 24 hours. The second term on the right describes the cells' tendency to synchronize with each other, and has coupling strength *K*. The third term represents interaction with the outside world, particularly the effect of sunlight. It attracts, with strength *F*, each cell to a phase $\sigma t + p$, where $\sigma = 2\pi \,/\, 24\,\mathrm{hrs}^{-1}$ is the daily frequency and *p* is a phase that depends on time zone. If *t* is Greenwich Mean Time, positive *p*, $0 < p < \pi$, corresponds to east of Greenwich and negative *p*, $0 > p > -\pi$, to west of Greenwich.

As simple as this model is, it still requires solving a large number, $N{\sim}10^4$, of coupled equations. We address this requirement with two further simplifications. First, we pass to the continuum limit, $N \to \infty$, where the state of the SCN is characterized by a time-dependent distribution of oscillator phases and frequencies, $f(\theta,\omega,t)$. This $N \to \infty$ limit should be a good approximation for $N \gg 1$. Second, we use the so-called Ott-Antonsen ansatz [5], which represents an exact solution for the distribution *f*. This ansatz postulates a particular form for $f(\theta,\omega,t)$, which yields a reduced system when substituted into the continuum system. Furthermore, [6] shows that under weak conditions and at large time, *f* converges in probability to the solution of the reduced description. Thus, we capture all attractors and bifurcations. In the case of a Lorenzian distribution of natural oscillator frequencies, $g(\omega) = (\triangle/\pi)[(\omega-\omega_0)^2 + \triangle^2]^{-1}$, the macroscopic state of the SCN is described by a single complex variable $z = N^{-1}\sum_j \exp[i(\theta_j - \sigma t - p)]$, which evolves according to the equation

$$\frac{dz(t)}{dt} = \frac{1}{2}[(Kz+F) - (Kz+F)^* z^2] - \Big(\triangle + i(\omega_0 - \sigma)\Big)z, \qquad (2)$$

where the polar angle of the complex variable *z* represents the collective global oscillation phase of the SCN. Thus, the ansatz reduces an *N*-dimensional system to this single, first-order, complex ordinary differential equation, enabling rapid scanning in parameter space and enhanced understanding of the dynamics.

Although the ansatz owes its discovery to an investigation [2] into the macroscopic behavior of solutions of (1), it turns out that this dimension reduction result applies, not only to (1) [3] but to a very large class of interesting situations. These include, for example, a model of pedestrian-induced wobbling of London's Millennium Bridge, Josephson junction circuits, a model of birdsong, and networks of pulse-coupled neurons. One can also generalize it to include additional dynamical features, like time delays in the effect of one oscillator upon another, the effects of different types of network topology, spatial coupling, and feedback control.

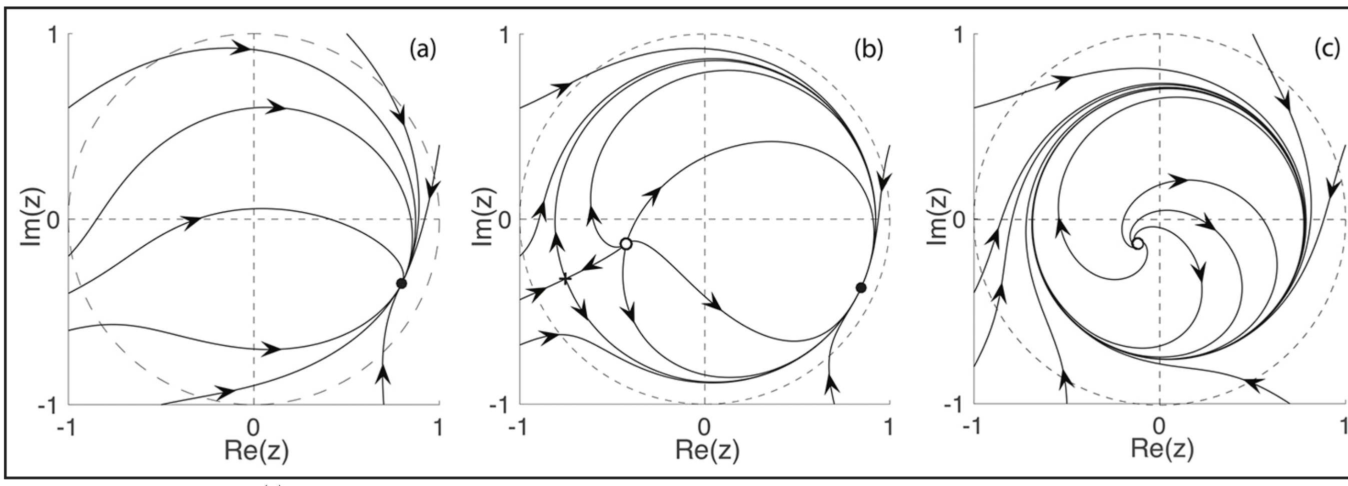

Figure 1. *Trajectories of $z(t)$ from (2) for three different parameter sets. Image Credit: [4].*

The model (2) has three parameter regimes with qualitatively different dynamics, as depicted in Figure 1. We expect a healthy person's circadian rhythm to entrain with the external 24-hour period, which corresponds to a stable fixed point in the *z*-phase-space (the black dot in Figure 1a and 1b). An individual whose circadian rhythm is not synched with the external 24-hour period corresponds to a *z*-phase-portrait, as shown in Figure 1c, where the individual's circadian phase relative to the phase of the external drive continually drifts around a closed curve, a periodic orbit in *z*. There is a difference between dynamics corresponding to Figure 1a and 1b; through a saddle-node bifurcation, Figure 1b has—besides the stable fixed point—two other fixed points, one unstable (shown as an open circle) and one a saddle (shown as a cross). The unstable manifold of the saddle forms a loop, along which *z* can approach the stable fixed point (black dot) from two opposite directions.

To analyze recovery of a healthy individual from jet lag, we assume that the traveler is entrained to his/her pre-travel time zone (*z* at the stable fixed point) before the trip. For simplicity, we also assume that the traveler's cross-time-zone travel is very fast, and model it as a discontinuous change of *p* in (1). Thus, immediately after travel, the state variable *z* is suddenly displaced by a rotation $(|z|\,\mathrm{fixed})$ by the angle $[p(initial) - p(final)]$. Depending on where the trip ends, *z* moves back to the stable fixed point either by advancing or delaying its phase. We are particularly interested in the east-west asymmetry in the direction of recovery and the time it takes for the recovery to occur. We use a 'typical' set of parameters representative of a typical healthy individual. This set of parameters yields the dynamics shown in Figure 1a, where only one fixed point is stable. The mean oscillation period of SCN cells when external drive is absent is taken to be 24.5 hours, consistent with experimental observations. This computation surprisingly indicates that the small amount (~30 minutes) by which the natural SCN period exceeds 24 hours in a typical human is sufficient to explain the rather noticeable east-west asymmetry of jet lag.

Edward Ott will present the Jürgen Moser Lecture, "Emergent Behavior in Large Systems of Many Coupled Oscillators," at the SIAM Conference on Applications of Dynamical Systems (DS17), to be held in Snowbird, UT, this May. He will also organize and give a talk at a minisymposium on "Using reservoir computers to learn dynamical systems," while Michelle Girvan will speak at a minisymposium titled "Symmetry, Asymmetry, and Network Synchronization."

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The authors are all affiliated with the Institute for Research in Electronics and Applied Physics at the University of Maryland, College Park. Additionally, Thomas Antonsen and Edward Ott hold joint appointments in the Department of Physics and the Department of Electrical and Computer Engineering. Michelle Girvan holds appointments in the Department of Physics and the Institute for Physical Science and Technology. Zhixin Lu is a research graduate student in the Chemical Physics Program.

Request for Proposals for the 2018 NSF-CBMS Regional Research Conferences in the Mathematical Sciences

Proposal Due Date: April 28, 2017

To stimulate interest and activity in mathematical research, the National Science Foundation intends to support up to 10 NSF-CBMS Regional Research Conferences in 2018. In the 48-year history of this NSF-CBMS Regional Research Conference series, a total of 358 such conferences have been held.

Submission of Proposals

Full detailed information about submission of proposals is given in NSF Program Solicitation 13-550, which—along with additional information about the NSF-CBMS Regional Research Conference series—can be found at www.nsf.gov/funding/pgm_summ.jsp?pims_id=504930.

Obituaries

By Xing Cai, Jan S. Hesthaven, Martin Peters, Marie E. Rognes, and Aslak Tveito

Professor Hans Petter Langtangen, a brilliant and beloved scientist and educator, passed away last October after an 18-month fight with cancer. Hans Petter's research revolved around numerical methods and scientific software tools for continuum mechanical problems. His approach was truly interdisciplinary, combining mathematics, statistics, and computer science to address problems in physics, geoscience, physiology, and medicine. Hans Petter was an unusually inspirational and visionary man, always motivating colleagues and students with his enthusiasm, encouragement, inspiration, and insight. He will be deeply missed by all those who knew him.

Hans Petter was born on January 3, 1962, in Vinderen, a neighborhood in Oslo, Norway. He received his M.Sc. and Ph.D. degrees in mechanics from the Department of Mathematics at the University of Oslo in 1985 and 1989, respectively. Hans Petter's research interests at the time encompassed computational methods for fluid flow, multiphase flows in porous media, and stochastic mechanics, topics that would follow him throughout his career.

Hans Petter started his career at Stiftelsen for industriell og teknisk forskning (SINTEF) in 1990, before becoming an assistant professor at the University of Oslo in 1991. Around that time, he began his groundbreaking work in applying C++ to implement partial differential equation solvers, which led to the renowned Diffpack library. Hans Petter was promoted to full professor of mechanics in 1998; he became a professor of scientific computing in the Department of Informatics in 1999.

In 2001, Hans Petter helped found Simula Research Laboratory, and remained a cornerstone of the research and educational environment there for several years. In 2007, he established the Center for Biomedical Computing (CBC), a Norwegian Centre of Excellence dedicated to the development and application of novel simulation technologies to better understand complex physiological processes affecting human health. At the CBC, Hans Petter continued his pivotal roles as a visionary driver and advocate for Python-based numerical software, in particular through the FEniCS Project, an open-source platform for automated scientific computing. In addition, he spearheaded the use of mathematical modelling and numerical simulation in new application areas associated with cardiovascular and neurological disorders, such as stroke and dementia.

Hans Petter was a singularly-talented, passionate, and much-beloved educator for over three decades. Since the early 2000s, he played a central role in the Computing in Science Education initiative at the University of Oslo. This project, which had wide international impact, revolutionized the integration of programming and simulation in mathematics and basic science education at the university. In 2016, Hans Petter received the Olav Thon Foundation Prize for Excellence in Teaching for his pioneering role and innovative methods in the teaching of programming and several other fields.

Hans Petter was a brilliant and prolific writer. He wrote a number of well-recognized books to accompany his courses, including *A Primer on Scientific Programming with Python*, which introduces programming via the Python language, and *Computational Partial Differential Equations: Numerical Methods and Diffpack Programming*, which teaches finite element methods to generations of students. His courses, ranging from introductory to graduate level, became the most popular courses in the Department of Informatics at the University of Oslo. Hans Petter supervised nearly 100 M.Sc./Ph.D. students, and was exceptionally dedicated to mentoring and advancing young researchers and scientists. He was an outstanding lecturer.

In addition to those that served as teaching material for his courses, Hans Petter authored several other books (he completed four during his last year), published upwards of 60 papers in international journals and over 60 peer-reviewed book chapters and conference papers, and gave more than 130 scientific presentations. He served as editor-in-chief of the *SIAM Journal on Scientific Computing* from 2011 to 2015, and was on the editorial boards of six other international journals. Hans Petter was also a member of the Norwegian Academy of Science and Letters and the European Academy of Science.

Hans Petter Langtangen, 1962-2016. Photo credit: Simula/Sverre Jarild.

A conference on computational science and engineering in memory of Hans Petter is planned for October 23-25, 2017, in Oslo. In the meantime, condolences and tributes from friends and colleagues are welcome at his memorial page.[1] As evidenced by these remembrances, Hans Petter and his warmth, enthusiasm, sense of humour, and drive left a mark on everyone he interacted with. Colleagues looked forward to even routine meetings with him; a serendipitous discussion with Hans Petter tended to brighten anyone's day, and an email from him was a source of encouragement and inspiration. We will miss him deeply.

[1] hpl-memorial.simula.no

Xing Cai is a chief research scientist at Simula Research Laboratory in Norway and a professor of scientific computing at the University of Oslo. He is a former student of Hans Petter Langtangen at both the masters and doctoral levels. Jan S. Hesthaven is a professor of mathematics and Dean of Basic Sciences at the École Polytechnique Fédérale de Lausanne (EPFL) and currently serves as editor-in-cheif of the SIAM Journal on Scientific Computing, succeeding Hans Petter. Martin Peters is executive editor of mathematics and computational science and engineering at Springer in Heidelberg, Germany. Marie E. Rognes is a chief research scientist at Simula Research Laboratory and a former student of Hans Petter at both masters and doctoral levels. Aslak Tveito is managing director of Simula Research Laboratory and a professor of scientific computing at the University of Oslo.

A Bike and a Catenary

MATHEMATICAL CURIOSITIES

By Mark Levi

There is a surprising connection between the catenary (the shape of the hanging chain, given by the hyperbolic cosine) on the one hand, and the pursuit curve, also known as the tractrix (as illustrated in Figure 1) on the other. The tractrix is defined by the property that every tangent segment *RF* to a given line *MN* has fixed length; it is the track of the bike's rear wheel *R* when its front wheel *F* follows a straight line. Our bike is just a moving segment *RF* of *fixed length*, which we take to be 1, with the velocity of *R* constrained to the line *RF*.

Figure 2 summarizes the connection: all normals to the tractrix are tangent to the catenary (so that the tractrix is the *involute* of the catenary). Equivalently, if we let a string ATR_0 hug the catenary and—keeping the end *A* fixed—unwind the end *R* while holding the string taut, *R* will sweep a tractrix.

Yet another way to put it: as the bike in Figure 1 moves as shown, the line of its rear axle remains tangent to the catenary. And the tangency point *T* is the center of curvature of the bike's rear track.

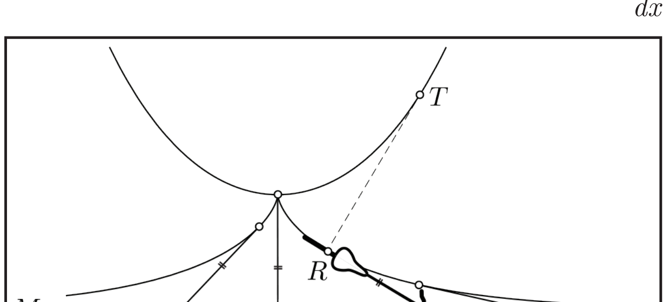

Figure 1. *The catenary and the tractrix.*

To prove this connection, consider an arbitrary position of the "bike" *RF* in Figure 2, and let *T* be the point of intersection of the normal through *R* and the normal to *MN* at *F*. The point *T* is automatically the center of curvature of the tractrix at *F*; leaving out the proof of this fact (which I will address in next month's column), we show that *T* traces out a catenary, i.e., that $y = FT = \cosh x$, where *x* is the coordinate of *F* on the line. From Figure 3,

$$y = \frac{1}{\sin\theta}. \qquad (1)$$

According to Figure 4,

$$\theta' = \frac{d\theta}{dx} = -\sin\theta. \qquad (2)$$

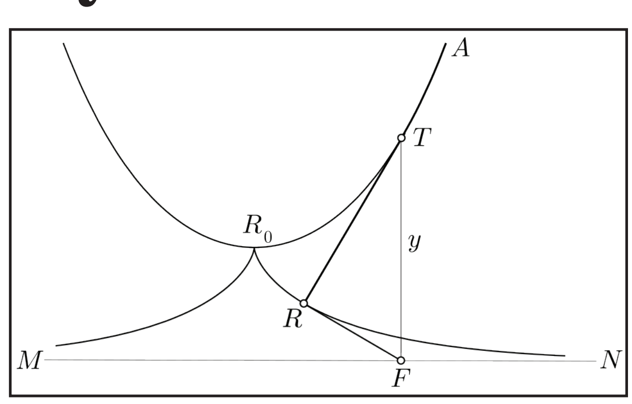

Figure 2. *As the string unwraps from the catenary, its end R describes the tractrix.*

Indeed, the angular velocity of the "bike" is given by the difference of the sideways velocities of *F* and *R* divided by the length $|RF| = 1$; θ decreases in the figure, which explains the minus sign. Differentiating (1) and using (2), we get

$$y' = \frac{dy}{dx} = \cot\theta.$$

One more differentiation and one more use of (2) gives

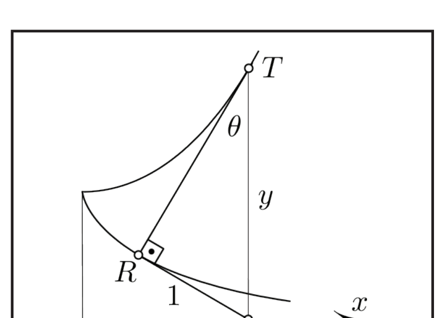

Figure 3. *Explanation of (1).*

$$y'' = -\frac{1}{\sin^2\theta}(-\sin\theta) = \frac{1}{\sin\theta} = y,$$

so that *y* is a combination of $\cosh x$ and $\sinh x$. And since $y(0)=1$ and $y'(0) = 0$, we determine that $y = \cosh x$, the equation of a catenary, as claimed.

In conclusion, here is an intriguing consequence of the catenary-tractrix connection. Consider the two surfaces of revolution generated by spinning each curve around the line *MN* in Figure 1. The surface of revolution of the catenary has zero mean curvature (it is the shape of a soap film spanning two circular hoops), while the surface of revolution of the tractrix has constant Gaussian curvature −1 (a pseudosphere). Is this just a coincidence, or a sign of something deeper?

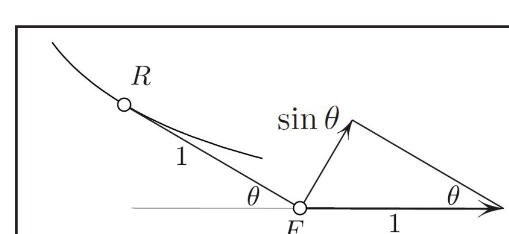

Figure 4. *Explanation of (2).*

The figures in this article were provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

Modeling Vegetation Patterns in Vulnerable Ecosystems

By Lakshmi Chandrasekaran

On a cold, rainy January morning in a café in downtown Chicago, I met Mary Silber, a leading scientist who applies mathematics to understand repetitive patterns of vegetation, which alternate rhythmically with bands of bare soil. The vegetation that Silber and her team study grows in a part of the world that is quite unlike Chicago – the dry and arid Horn of Africa.

Parts of this region, such as areas in Somalia and Ethiopia, receive very little rain throughout the year. With the world's population currently at 7 billion and projected to rise to 9.6 billion by 2050,[1] food sustainability—ensuring that we produce enough food for everybody to eat—becomes especially important. It is thus imperative to globally increase the percentage of arable land available for food creation beyond the current 28%.[2] Such an increase involves targeting new areas, such as deserts, which exist in numerous parts of the world.

But surely studying the flora of a region falls under an ecologist's domain – why would a mathematician possibly be interested in this problem?

Silber, whose career trajectory started with a Ph.D. in physics from the University of California, Berkeley, has never been one to settle for something conventional. She is currently a professor in the Department of Statistics at the University of Chicago and director of a new graduate program called "Committee on Computational and Applied Mathematics."

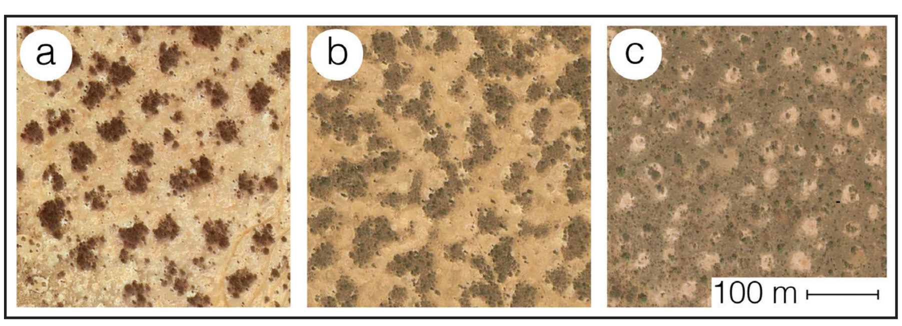

Figure 1. *Aerial images of flat terrain vegetation patterns in Sudan.* **1a.** *Spot (11.6280, 27.9177).* **1b.** *Labyrinth (11.1024, 27.8228).* **1c.** *Gap patterns (10.7549, 28.5955). Images © Google, DigitalGlobe. Image credit: Karna Gowda.*

Over the past few decades, Silber built her expertise by using dynamical systems to study pattern formation in fluid mechanics. Over time, however, she grew restless and craved newer ventures—real-world problems where she could apply her skills—ultimately shifting her focus to problems relevant to climate change, such as vegetation patterns.

A bird's-eye view is necessary to study the vegetative dynamics of any region. "You can only make out the vegetation pattern from the air because of its scale," Silber said. The instability of the Horn of Africa makes it a challenging and interesting region to study. But what drew Silber's group to the region in the first place was the beauty of the landscape when viewed from above, whether via modern satellite images or early aerial photographs.

However, the vegetation project was not devoid of challenges. "Equations unknown, parameters unknown, time scales over a century or less, and spatial scales of about hundreds of meters or kilometers," Silber said, all of which are unlike classical fluid mechanics problems. And, of course, the unpredictability that comes with studying our planet. "Carefully controlled experiments? No! This is Earth – we don't repeat things!" Silber exclaimed with a laugh, pointing to the most difficult parameter to control in this problem.

In short, the vegetation problem does not present itself well to testing in controlled, pristine research settings and is prone to much heterogeneity, with a lack of physical monitoring on the ground. Nevertheless, Silber toyed with this challenge, seeking a mathematical workaround for the experimental drawbacks. And in 2012, Karna Gowda, a student from Silber's 'Mathematical Modeling in the Earth Sciences' class at Northwestern University, where she taught until quite recently, expressed interest in working on the project. "Karna has been the main driver behind this work," Silber said proudly.

Gowda began with the question, "What vegetative pattern sequences can possibly occur when we set up the simplest problem we can think of?" Silber and her colleagues addressed this issue in [2]. They used a system of equations describing the amplitude of Fourier modes on a hexagonal lattice, which permits vegetative patterns that resemble spots, stripes, and gaps (see Figure 1).

"The aim here was to find a bifurcation theoretic framework to allow us to investigate the transition sequence emerging in a variety of different conceptual models that had been proposed," Silber said. Analysis revealed that the gaps→labyrinths→spots pattern was only one of a few different scenarios that could possibly occur in the simple generic setup, and that these patterns occurred when certain conditions are met (a topic explored in their subsequent paper).

Is there a historical precedent to modeling vegetation patterns? Silber mentioned physicist Ehud Meron at Ben-Gurion University of the Negev in Israel as someone who has championed the development of mathematical frameworks for investigating vegetation patterns. She also referred me to her colleague Arjen Doelman, a professor at Leiden University's Mathematical Institute in the Netherlands and an expert at using mathematical models to predict vegetation patterns.

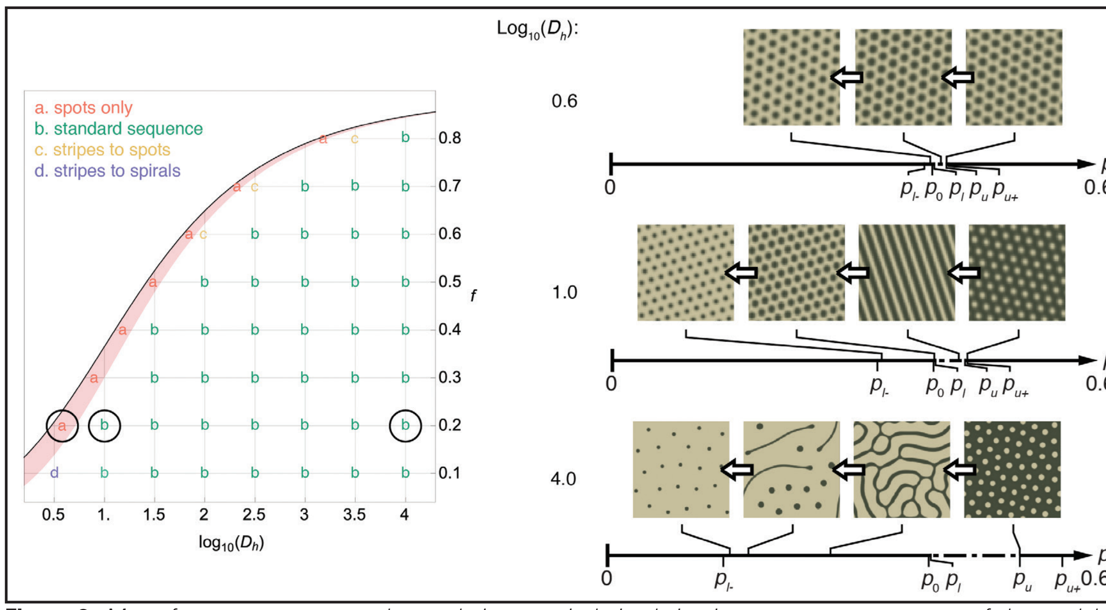

Figure 3. *Map of pattern sequences observed via numerical simulation in a two-parameter space of the model by Rietkerk et al. [4]. The parameter f controls differential infiltration between bare and vegetated areas, and D_h is the surface water diffusion parameter. A "spots only" sequence occurs in a thin red region of the parameter space predicted by analytical theory, while analogs of the "standard" gaps→labyrinths→spots occur in most simulations elsewhere. Example simulations are shown for three distinct parameter sets circled in the left panel, corresponding to f = 0.2 and D_h = 0.6, 1.0, and 4.0. Image credit: Karna Gowda.*

Doelman described the Klausmeier model [3], a system of advection-diffusion equations used to study banded vegetation. "The Klausmeier model is the oldest model, and it is a simple one. I prefer to think of it as a conceptual or even 'toy' model," he said.

The equations governing the Klausmeier model describe how the dynamics of water (W) and plants (N) change by interacting with each other:

$$\frac{\partial W}{\partial T} = A - LW - RWN^2 + V\frac{\partial W}{\partial X}. \qquad (1)$$

(1) describes dynamics of water supply change over time as a function of rainfall, loss of water due to evaporation, infiltration by plants, and transport of water downhill.

$$\frac{\partial N}{\partial T} = RJWN^2 - MN + D\Big(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\Big)N. \qquad (2)$$

(2) describes the dynamics of plant growth over time as a function of water absorption by plants, distribution of plants over a given area, and loss of vegetation due to animal grazing or lack of water.

Successfully solving the Klausmeier model recapitulates some vegetation patterns on certain terrains, such as sloped surfaces. However, this simple model has a limitation: it cannot be generalized to predict patterns from all kind of terrains, without modifications.

"The local topography of the environment in the Klausmeier model comes in through the spatial derivative (transport) terms, and specifically, the advection term in (1)," Gowda said. "The model in its original form assumes that we are looking at a sloped surface on which the water travels at a constant speed V." But in places with flat terrain, such as fairy circles of Namibia,[3] advective runoff may be insignificant.

So Silber and her team turned to the Reitkerk model, more realistic for their study. "It distinguishes the differences between ground water and surface water," Doelman said. Several equations govern the Reitkerk model:

$$\frac{\partial h}{\partial T} = p - I(n)h + D_h\nabla^2 h. \qquad (3)$$

(3) describes how the dynamics of surface water (h) change with respect to rainfall, diffusion, and loss of water via infiltration. The infiltration term $I(n)$ captures the loss of surface water through soil absorption in the presence of vegetation. Infiltration positively influences vegetative growth, which in turn influences infiltration, thus creating a feedback loop.

$$\frac{\partial w}{\partial T} = -vw + I(n)h - \gamma G(w)n + D_w\nabla^2 w. \qquad (4)$$

(4) describes the dynamics of water (w). It involves surface water infiltration and loss of water due to evaporation, diffusion, and transpiration by plants.

$$\frac{\partial n}{\partial T} = -\mu n + G(w)n + \nabla^2 n. \qquad (5)$$

(5) quantifies the growth of vegetation (n) as a function of soil water availability, dispersal of plants in a given area, and plant mortality.

Starting with a uniform vegetative cover, Gowda and colleagues examined when the vegetative patterns transition to patches with decreasing rainfall. Mathematically, these transitions occur between the lower and upper Turing points (see Figure 2).

These results, published last year in [1], use numerical simulations that match qualitatively with the analytical predictions in [2]. Key highlights of this work include understanding the transitions in patterns as a function of change, both in the rate of infiltration and its interaction with the amount of available vegetation (see Figure 3).

But the Reitkerk model has limitations too. "The models we used are idealized, since different types of vegetation are thrown into a single biomass variable," Gowda said. "For instance, if a drought hits a certain type of vegetation more than others, how is that going to affect the pattern? That is hard to predict."

Rather than predict how vegetation patterns develop in arid regions, this model

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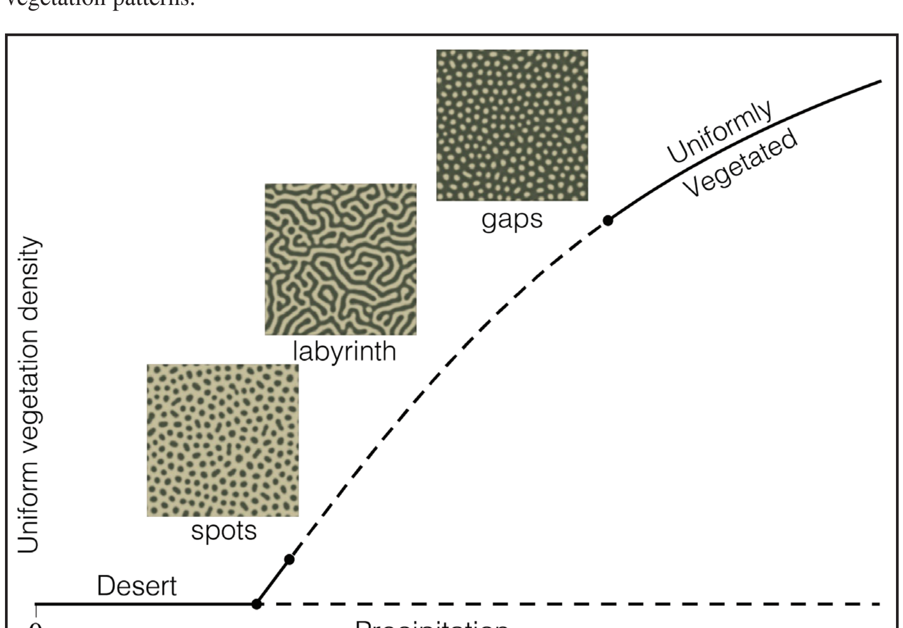

Figure 2. *Uniform vegetation equilibria of the model by von Hardenberg et al. [5] plotted as a function of precipitation, with patterned solutions shown in insets. Image credit: Karna Gowda.*

[1] http://www.pewresearch.org/fact-tank/2014/02/03/10-projections-for-the-global-population-in-2050/

[2] https://ourworldindata.org/land-use-in-agriculture/

[3] https://www.nytimes.com/2017/01/19/science/fishing-for-clues-to-solve-namibias-fairy-circle-mystery.html

Analyzing Multiple Time Scales in Two-Dimensional Fluids Using Dynamical Systems

By Margaret Beck and C. Eugene Wayne

The time scales over which fluids evolve have a critical effect on the physical systems in which they occur. These time scales arise from an interplay of different effects, some of which—like Lagrangian coherent structures—tend to stabilize the flow, while others, such as shear (inviscid) damping or viscous damping, tend to break down structures, at least on small-length scales. A variety of recent investigations, many of which involve a dynamical systems point of view, have begun to shed light on the origin of these time scales in the case of two-dimensional fluid flows. Since the basic questions of existence and uniqueness for two-dimensional fluid flows are well understood, one can ask more detailed queries about their evolution. Moreover, these flows have a tendency to form large, vortical structures on both laboratory and geophysical scales, as seen in Figure 1, a satellite photo of the Gulf Stream. Dynamical systems theory is well suited to answer these types of questions since invariant families of solutions often appear to organize the dynamics, effectively creating the multiple time scales and observed asymptotic behavior [3, 5]. In simple settings, invariant manifolds [4, 9] can even characterize this organization.

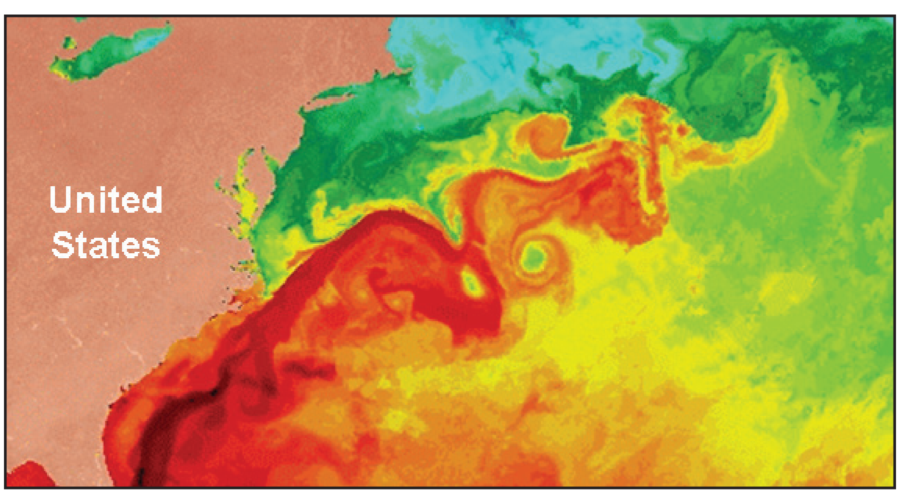

Figure 1. *Satellite photo of the Gulf Stream. Image courtesy of NASA.*

A particularly important example of this is the two-dimensional incompressible Navier-Stokes equation with small viscosity, $0 \ll \nu < 1$:

$$\partial_t \omega + \mathbf{u} \cdot \nabla \omega = \nu \Delta \omega,$$

$$\omega = \omega(x,t), \quad x \in \Omega \subseteq \mathbf{R}^2 .$$

Here, ω is the vorticity of the fluid and $\mathbf{u}$ is the fluid velocity, recoverable from the vorticity via the Biot-Savart law. In other words, $\omega = (\nabla \times \mathbf{u}) \cdot (0,0,1)$. When $\nu = 0$, the equation reduces to the Euler equation, which has infinitely many stationary solutions. Though these no longer remain stationary states for positive (but small) vorticity, it is reasonable to believe that they still play an important role in the longtime evolution of the Navier-Stokes equation. However, most stationary states of the Euler equation are surprisingly never observed in the Navier-Stokes evolution. Instead, a small number of the Euler states become quasi-stationary states of Navier-Stokes, and only a subset of these seem to have long-term influence. As a first "guess" at the time scales over which the viscosity makes itself felt, one can note that the two-dimensional Navier-Stokes equations on $\mathbb{R}^2$ have a family of exact solutions known as the Oseen vortices, given by

$$\omega^O(x,t) = \frac{A}{1+t} e^{-\frac{|x|^2}{4\nu(1+t)}} .$$

From this formula, it seems as if the viscosity should be perceptible on a time scale

$$t_{visc} \sim \frac{1}{\nu} .$$

However, numerical experiments indicate that vortices and other large-scale characteristic structures emerge in the flow on a much shorter time scale. For instance, in the numerical simulation of Figure 2 [10], the viscous time scale would be $t_{visc} \sim 1500$, but large-scale vortical structures emerge on a much shorter time scale. Understanding the origin of these scales is currently a question of great interest.

There is presently no mathematical theory predicting which of the Euler solutions will play the most important role in the viscous evolution. However, a finite subset of these quasi-stationary states correspond to an explicit family that decays on the viscous time scale $\mathcal{O}(e^{-\nu t})$ and can be described by the lowest four Fourier modes, $\{e^{\pm ix}, e^{\pm iy}\}$. Bar states (also known as Komogorov flow, a type of shear flow) are solutions that vary only in the *x* or only in the *y* direction, while dipoles vary in both directions. Researchers have observed, both experimentally and numerically, that most initial conditions lead to solutions which originally experience rapid evolution to either a bar state or a dipole, followed by slow decay to the background rest state (zero solution). A classical approach to analyzing such behavior begins by linearizing the Navier-Stokes equation about a bar state or dipole and attempting to determine the rate of convergence to the state, which should correspond to the observed initial period of rapid evolution. This type of linearization near a bar state [5] suggests that the rapid evolution occurs on the time scale $\mathcal{O}(e^{-\sqrt{\nu} t})$, at least at the linear level. Interestingly, the linearization leads to a highly non-self-adjoint operator, making it unclear whether the multiple time scale phenomenon is spectral or pseudospectral. A dynamical systems perspective is useful in this analysis because it permits a separation between the decay rate to the invariant family, at $\mathcal{O}(e^{-\sqrt{\nu} t})$, and the decay rate within the family, at $\mathcal{O}(e^{-\nu t})$.

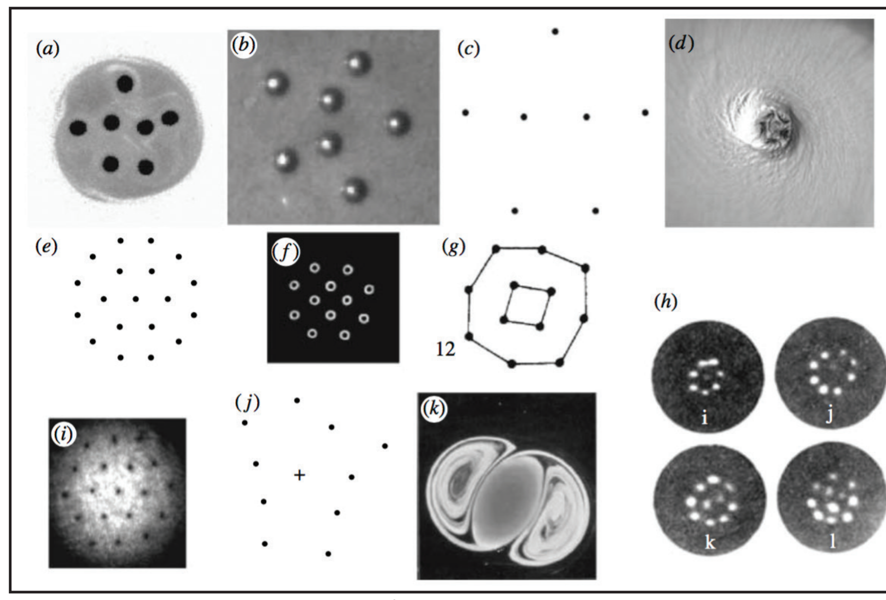

Figure 3. *Experimental illustrations of point vortices. Image credit: [1], by permission of the Royal Society.*

A related work also analyzes the rapid convergence to bars and dipoles using a dynamical systems perspective [3]. Researchers take the two-dimensional Navier-Stokes equation, written in Fourier space, and formally project that system onto the lowest eight modes: the lower four contain the bars and dipoles and the next four model the effects of all higher modes. They then use classical dynamical systems techniques, including invariant manifolds and estimates involving Duhamel's formula, to study the resulting eight-dimensional ordinary differential equation (ODE). This method focuses on understanding the effects of perturbing the domain from a square torus, represented by a parameter $\delta = 1$, to a rectangular torus, represented by $\delta \neq 1$. The parameter δ controls whether a particular invariant manifold is a center ($\delta = 1$), stable ($\delta < 1$), or unstable ($\delta > 1$) manifold, which then determines if the dominant quasi-stationary state was a dipole, *y*-bar state, or *x*-bar state, respectively. In this ODE model, the initial period of rapid decay notably occurred on the time scale $\mathcal{O}(e^{-t/\nu})$ instead of time scale $\mathcal{O}(e^{-\sqrt{\nu} t})$, which researchers observed in the previously-mentioned work.

The dynamical systems perspective sheds light not only on the question of multiple time scales in fluids, but on other aspects of their motion as well. For example, one can analyze the stability and interaction of vortices in the planar Navier-Stokes equation with limit $\nu \to 0$ using a point vortex model [11]. A key aspect of that work is its ability to capture the higher-order effects of vortex interaction, showing that for motions in which the centers of vorticity were initially well-separated, the essentially inviscid motion of the vortex cores accurately described the overall nature of the flow until the distance between vortices became comparable to the size of the vortex core. Interestingly, such configurations of near point vortices appear in a host of experimental circumstances (see Figure 3) [1].

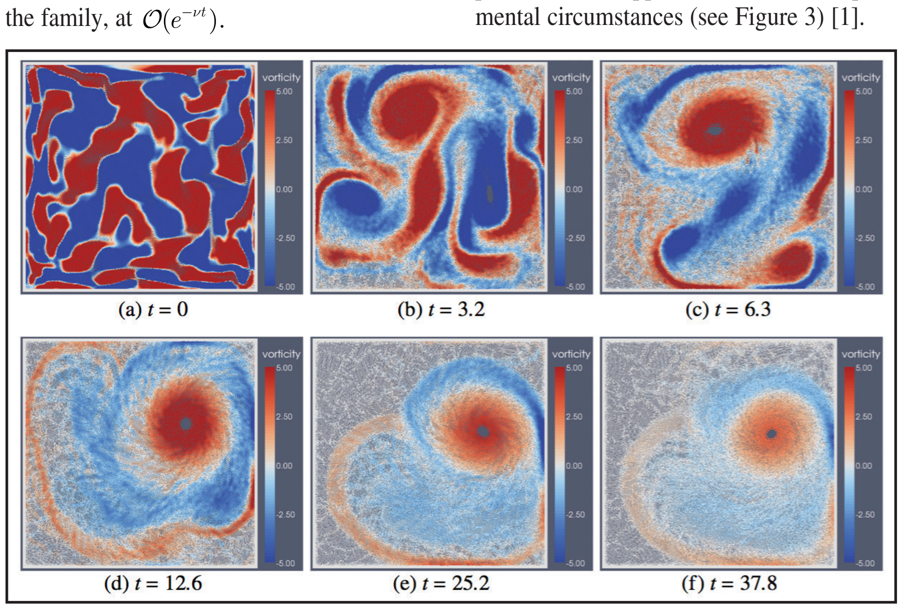

Figure 2. *A numerical simulation of a two-dimensional flow at six different times. Image credit: [10], by permission of John Wiley & Sons, Inc.*

In work more closely related to the above discussion about bar state metastability [6], researchers studied solutions of the two-dimensional Navier-Stokes equations in a neighborhood of Couette flow, a particular type of shear flow in a channel. Using careful partial differential equation (PDE) estimates in Gevrey spaces, they were able to treat the full nonlinear problem. This work is particularly interesting because it identifies precisely different time scales associated with an initial period of so-called inviscid damping—in which the Euler equations essentially govern flow—followed by a rapid evolution due to enhanced diffusion and then a final, slow period of convergence to the Couette flow, during which viscosity dominates. The intermediate period of enhanced diffusion relates to the hypocoercivity in [5, 8], and is further connected to the phenomenon of Taylor dispersion, which also occurs in the channel setting but for different boundary conditions. Originally studied in the 1950s, Taylor dispersion is another example of a situation in which shearing in the ambient flow field enhances dispersive or dissipative effects.

Researchers have recently attacked this problem from two different perspectives using dynamical systems ideas. In [7], hypocoercivity methods are used to analyze the decay enhancement in a variety of shearing flows. In [2], more classical dynamical systems methods like invariant manifolds play a key role, but not in an entirely straightforward way. Although the PDE does not seem to possess an invariant manifold—in fact, evidence suggests that it does not—its solutions are shown to be well-approximated by solutions to an ODE that does possess a center manifold, on which the enhanced diffusion can be computed explicitly. This matches Taylor's original formal calculations from the 1950s.

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Mathematics in Space

Increasing Our Understanding of the Celestial

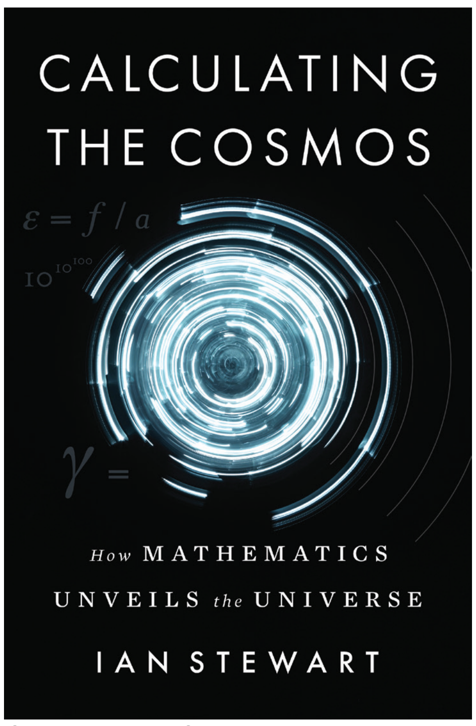
Calculating the Cosmos: How Mathematics Unveils the Universe. *By Ian Stewart. Basic Books, New York, NY, October 2016. 360 pages, $27.99.*

BOOK REVIEW
By James Case

From the beginning of time, writes Ian Stewart in his latest popular mathenscience[1] book, men have looked at the night sky and questioned, "What's going on up there?" In fact, virtually all proposed answers to this question have been abandoned in favor of better ones, which are then abandoned in term. Indeed, with the advance of the physical and mathematical sciences, the ability to discriminate between good and bad guesses has grown markedly, rendering not-yet-discredited guesses increasingly hard to dismiss. In *Calculating the Cosmos: How Mathematics Unveils the Universe*, Stewart offers an extensive catalogue of noteworthy conjectures concerning the nature and extent of the cosmos, beginning with some of the earliest and culminating in current "best guesses."

SIAM members are likely familiar with many of the earlier notions, such as Ptolemaic cosmology, which placed Earth at the center of the universe—surrounded by an invisible "celestial sphere," to which fixed stars were attached—while the sun, the moon, and the planets travelled without collision on separate concentric spheres. The Copernican Revolution challenged medieval orthodoxy with evidence that the sun lies at the center of the universe, while Earth and other planets rotate around it in orbits consistent with Isaac Newton's inverse-square law. Relativity and Edwin Hubble's discovery of an expanding universe spawned yet another round of cosmological guesswork. More recently still, the succession of manned and unmanned missions that followed *Sputnik I*'s trip to space in October 1957 has given birth to a phalanx of more modern guesses, no few of which were quickly debunked. Stewart follows space exploration closely, and offers an up-to-date summary of what scientists have learned.

Chapter 1 introduces gravity, conic sections, *N*-body problems, general relativity, and the historic realization that nature obeys mathematical laws. After pointing out that Newton solved the two-body problem, and that the three-body problem appears insoluble, Stewart reveals that current investigators continue to discover new and unexpected consequences of Newton's laws. He mentions particularly a family of planar orbits—the simplest being shaped like a figure eight—around which three equal point masses can pursue one another indefinitely, along with a corkscrew-shaped orbit that spirals around the line segment joining the centers of a binary star. The spirals are loose near the middle of the segment but crowd together by the stars at the ends, somewhat resembling a slinky toy stretched only in the middle. There is some evidence that an exoplanet named Kepler-b may be trapped in such an orbit.



Calculating the Cosmos: How Mathematics Unveils the Universe. By Ian Stewart. Courtesy of Basic Books.

Chapter 2 concerns the origins of the solar system. The current best proposition attributes its formation to the collapse of a giant gas cloud, in which non-uniformities in the initial distribution of matter, together with gravitational attraction, caused gaseous clumps to form and then congeal into solid bodies. These bodies frequently collided and grew in size as smaller bodies were drawn to larger ones. The vast number of craters on the moon, Mercury, and Mars attests to the frequency of such collisions in the early universe.

The growth of computing power in the 1980s, along with the development of accurate computational techniques, allowed scientists to model the collapse of giant gas clouds as *N*-body problems. A realistic application of this method requires a few hundred billion bodies, rendering the calculations infeasible. Hence, smaller numbers are used. Crude integration techniques cannot be trusted here, since they neglect such physical realities as the conservation of energy and angular momentum. If such oversight were to decrease overall energy, for instance, rather than conserve it, the effect would resemble friction – closed planetary orbits would be replaced by decaying ones that spiral into the sun.

The development of *symplectic integrators*, numerical methods specifically designed for the integration of ordinary differential equations in Hamiltonian form: $\dot{p} = -H_q(p,q)$, $\dot{q} = -H_p(p,q)$, has largely surmounted such difficulties. Since these methods preserve the symplectic 2-form $dp \wedge dq$, they also preserve linear and angular momentum *exactly*, along with total energy. As a result, symplectic integrators permit the accurate simulation of systems of mutually-gravitating bodies in free space over extremely long periods of time.

Stewart writes extensively about the rings of Saturn, the study of which has a long and surprisingly complex history. Originally observed by Galileo in 1610, they seemed at first to look like separate moons, and later like ears on a face. But Christiaan Huygens, armed with a better telescope, was able to report in 1655 that Saturn "is surrounded by a thin flat ring, nowhere touching, inclined to the ecliptic." By 1666, Robert Hooke could observe shadows, both of the globe upon the ring and the ring upon the globe, showing what's in front of what. Then in 1787, Pierre-Simon Laplace pointed out that a single wide flat ring would break apart since, by Johannes Kepler's third law, the outer portions must rotate more slowly than the inner ones. He thus concluded that the wide flat ring must be composed of several concentric ringlets, each rotating at a different speed. Next in 1859, James Clerk Maxwell showed that even a narrow flat ringlet is unstable, since the slightest disturbance causes such a surface to buckle, ripple, and bend, immediately snapping like a dry piece of spaghetti with the application of distortive forces. Could the rings be composed of fluid? No, because as Sophie Kovalevsky showed in 1874, fluid rings would also be unstable. It was not until around 1895 that telescopes improved to the point where observers could declare Saturn's rings to be composed of a truly vast number of small (presumably solid) orbiting bodies.

According to Stewart, no military plan survives contact with the enemy, and no astronomical theory survives contact with better observations. Man's knowledge of Saturn changed forever in 1980, when *Voyager I* started sending back pictures of the rings. The images soon revealed, for instance, that one of the rings is not circular, and that dark fuzzy "spokes" seem to emanate from the center of the planet and rotate within the "wheel" formed by the rings. Nothing previously noted concerning the rings had lacked circular symmetry. *Voyager II*, which had launched before *Voyager I* but was moving more slowly, confirmed both observations some nine months later. The *Voyager* missions also revealed that some of the rings appear to be braided, some exhibit strange kinks, and some are incomplete, consisting of discrete, roughly-circular arcs separated by gaps. Before the *Voyager* encounters, Earth-bound astronomers had observed that Saturn possessed nine moons; *Voyager* increased the number to 30. Today it's 62, 53 of which now have official names. The Cassini probe,[2] currently orbiting Saturn, provides a stream of data on the planet, its rings, and its moons.

Later chapters discuss—in something like layman's terms—the location of asteroids in the solar system; the rings of Saturn; the overall curvature of space; the Big Bang theory; the whereabouts of dark matter and dark energy; and a great deal more. For anyone who hasn't kept up with space exploration, *Calculating the Cosmos* is an exceedingly pleasant and highly informative read.

James Case writes from Baltimore, Maryland.

[1] Term coined by the elder President Bush to describe a subject in which few American students excel.

[2] https://saturn.jpl.nasa.gov/

Vegetation

Continued from page 6

could instead predict the increasing desertification in drought-prone areas. "The final 'catastrophe' of desertification is preceded by 'mini-catastrophies' in which the pattern undergoes significant changes, say, half of the stripe patterns disappear at a very fast time scale," Doelman said. "These are model predictions, and we're presently working with ecologists to validate this."

To help constrain the model and determine what satellite images could actually measure and quantify, Gowda has been studying the available literature on these vegetation patterns from the past 60 years. He is currently looking at British aerial survey data of dry lands in Somalia, collected during World War II. "Our goal is to try and construct some record of dynamics," he said.

Another aspect of the problem involves analyzing the role of terrain-topography in influencing patterned vegetation. "We could see that the shallow topography was playing a role, and we want to bring together mathematical modelers in this area with eco-hydrologists for an exchange of ideas," Silber said. "This meeting of the minds is what we hope will happen during the minisymposium[4] that Sarah Iams and Punit Gandhi have organized for the SIAM Conference on Applications of Dynamical Systems (DS17), to be held in Snowbird, UT, in May 2017."

With both Silber and Doelman's groups as key players in the mathematical study of vegetation in arid ecosystems, it should be very exciting to hear their sessions, as well as plenary lectures by Silber and Doelman, at DS17. Check out more details on the conference and register![5]

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Lakshmi Chandrasekaran received her Ph.D. in mathematical sciences from the New Jersey Institute of Technology. She is currently pursuing her masters in science journalism at Northwestern University, and is a freelance science writer whose work has appeared in several outlets. She can be reached on Twitter at @science_eye.

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Time Scales

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Margaret Beck is an associate professor of mathematics at Boston University. C. Eugene Wayne is a professor of mathematics at Boston University.

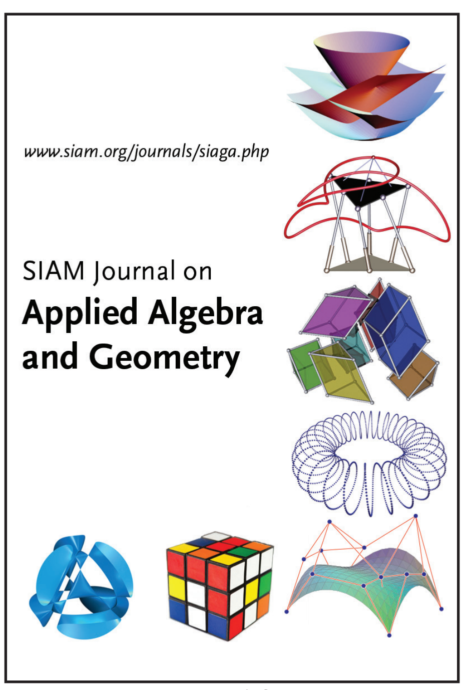
SIAGA: A New Window for Algebra and Geometry

By Bernd Sturmfels

The *SIAM Journal on Applied Algebra and Geometry* (*SIAGA*) is the latest member in the outstanding family of journals published by SIAM. *SIAM News* readers are already familiar with the "storyboard" behind the new journal, thanks to Anna Seigal's inspiring three-part article from last year, "A SIAGA of Seven Pictures."[1]

SIAGA offers a new home for exciting themes in the core of mathematics and their emerging applications. The journal's creation was the result of a thorough planning process that dates back several years. Its impetus came from the SIAM Activity Group on Algebraic Geometry (SIAG/AG), which held its inaugural conference in Raleigh, NC, in October 2011. Following the second meeting in Fort Collins, CO, in August 2013, SIAG/AG members formed a committee to discuss the possibility of a new journal. The committee wrote a proposal under the leadership of Frank Sottile and Thorsten Theobald, and with strong support from the publications committee and SIAM's board and council, the journal received its final approval in December 2015.

SIAGA's mission is to publish "research articles of exceptional quality on the development of algebraic, geometric, and topological methods with strong connection to applications." The journal covers mathematical subjects such as algebraic geometry, algebraic topology, algebraic and topological combinatorics, differential geometry, convex and discrete geometry, commutative and noncommutative algebra, multilinear and tensor algebra, number theory, representation theory, and symbolic and numerical computation. Areas of application include biology, data science, coding theory, complexity theory, computer graphics, computer vision, control theory, cryptography, machine learning, game theory and economics, geometric design, optimization, quantum computing, robotics, statistics, and social choice.



The inaugural volume of SIAM's newest journal, the SIAM Journal on Applied Algebra and Geometry (SIAGA), published in February 2017.

Douglas Arnold, former president of SIAM and director of the Institute for Mathematics and its Applications, played a decisive role by supporting these developments. "Just a decade ago, algebra and geometry would have seemed strange directions for SIAM, and the title of the journal something of an oxymoron," he writes. "But now, after the formation of the algebraic geometry activity group in 2009, the establishment of a biennial conference series, and the resulting influx of people, ideas, and interaction, applied algebra and geometry have become core areas for SIAM. This signals a change in scientific culture for which SIAM has been an important catalyst. The new *SIAM Journal on Applied Algebra and Geometry* is both a recognition and a natural outcome of this change. It's great for math and science, and it's great for SIAM."

Jan Draisma, former chair of SIAG/AG and an associate editor of *SIAGA*, also attributes the journal's birth to the two expanding disciplines. "The SIAM activity group in algebraic geometry unites the rapidly growing communities of algebraists and geometers fascinated by applications and scientists in need of new algebro-geometric techniques," he says. "*SIAGA* is quickly becoming the journal of choice for the very best of their combined research."

The new journal began taking submissions in March 2016. A team of three corresponding editors and 26 associate editors is handling the growing number of submissions in a timely and professional manner. Many referees are contributing excellent reports, ensuring high standards for acceptance. Sottile, inaugural chair of SIAG/AG and a corresponding editor for *SIAGA*, emphasizes the journal's influence. "This journal, because of its focus and editorial board, can get quality reports for papers that are interdisciplinary and require refereeing from two or more perspectives," he says.

By the end of 2016, authors had submitted 90 manuscripts for publication to *SIAGA*, and the journal was ready for its inaugural all-electronic volume. The first nine articles were published in February 2017. This initial batch touches upon a wide range of subjects, such as complexity theory, convex optimization, frame theory, graphical models, machine learning, numerical analysis, projective geometry, signal processing, and tensor methods. A focus on models governed by nonlinear algebraic constraints remains a common thread. "We are overjoyed to see so many high-quality submissions, especially from our junior colleagues," says Alicia Dickenstein, vice president of the International Mathematical Union and a *SIAGA* corresponding editor.

To increase *SIAGA* readership well beyond the current community, the editorial board reaches out to everyone interested in studying nonlinear problems that arise in the aforementioned application areas. We hope that the new approaches and research presented in the journal will be of interest to many scientists, engineers, and industrial mathematicians.

Now we will briefly introduce the authors and articles featured in the first volume of *SIAGA*. The article "On the geometry of border rank decompositions for matrix multiplication and other tensors with symmetry," by Joseph M. Landsberg and Mateusz Michalek, offers a new approach to tensors with symmetry, with focus on complexity lower bounds for matrix multiplication. Michael Kech and Felix Krahmer advance our understanding of inverse problems by deriving "Optimal injectivity conditions for bilinear inverse problems with applications to identifiability of deconvolution problems." Jameson Cahill, Dustin Mixon, and Nate Strawn resolve a longstanding problem in applied harmonic analysis with "Connectivity and irreducibility of algebraic varieties of finite unit norm tight frames." Diego Cifuentes and Pablo Parrilo achieve dramatic speed-ups when solving algebraic equations by using "Chordal networks of polynomial ideals." Peter Bürgisser connects numerics and algebraic geometry in an article titled "Condition of intersecting a

See **SIAGA** *on page 10*

[1] https://sinews.siam.org/Details-Page/applied-algebra-and-geometry-a-siaga-of-seven-pictures-2

SIAM Committee on Science Policy Discusses Impact of Incoming U.S. Presidential Administration

By Karthika Swamy Cohen, Miriam Quintal, and Eliana Perlmutter

The SIAM Committee on Science Policy meets biannually to monitor developments in federal and/or state governments, as well as new policy directions of interest to SIAM and its members. The committee also helps increase visibility of applied mathematics and SIAM in the federal government and scientific community.

At its fall 2016 meeting last November, the committee met with key decision-makers at federal agencies to better understand the environment for research funding related to applied mathematics and computational science, especially in regards to the new U.S. presidential administration and its potential impact on federal research priorities.

Representatives from the Department of Energy (DOE), the National Institutes of Health (NIH), the Defense Advanced Research Projects Agency (DARPA), and the National Science Foundation (NSF)'s Division of Mathematical Sciences (DMS) and Office of Integrative Activities presented updates on new research initiatives, personnel changes, and the fiscal year (FY) 2017 budget.

The historic election of Donald Trump as the 45th U.S. president will have significant implications for scientific funding and research. The Republican Party's retention of majorities in both the U.S. House of Representatives and the Senate will also have a major impact, as the U.S. will see one-party governance for the first time since former President Barack Obama's first two years in office.

The extent of the White House's potential involvement with setting the legislative agenda isn't clear, except in a few key areas emphasized by Trump, such as tax reform, infrastructure, and healthcare. Neither Trump's campaign nor his transition team have offered specifics with regard to proposed policies in education, research, science, or technology. Hence, it is hard to assess the amount of emphasis (or lack thereof) that a Trump administration will place on universities and the research community.

As a result, as the Trump transition team and new administration refine their policy agenda and prioritize actions for early legislative activities in the coming months, it will be critical for universities, scientific societies, and organizations to carefully assess Trump's positions as more details emerge, and define strategies to best concentrate energy with respect to key priorities. Opportunities for engagement will likely arise once science officials and lower-level agency leadership are chosen.

Based on information from policy advisors and the Trump team's responses to questionnaires such as Science Debate,[1] it seems that a Trump administration will likely value investment in basic academic research. Past Republican administrations have also placed a higher priority on basic research, rather than applied research, environmental sciences, and social and behavioral sciences.

On November 18, 2016, Trump's transition team and congressional Republican leaders came to an agreement to extend the current Continuing Resolution (CR), which would prolong present funding levels for most governmental agencies to fund government operations through the end of March 2017. The extension includes certain funding increases or anomalies, such as $872 million to boost medical research and drug approval efforts, $10 billion in additional war funding for military and diplomatic efforts, $4.1 billion in disaster relief, $170 million to help repair the lead-contaminated water system of Flint, Mich., and $45 million for continued healthcare benefits for retired coal miners.

With regard to committees, Congress's composition has remained largely the same, except for a few key leadership positions and the loss of some science champions in the Senate. It is thus imperative to identify and nucleate new champions. While it is hard to know who the science liaisons will be in the new administration, we can assume that they will report to those without a technical background. Hence, it is important to craft a message in a way that will be understandable to laypersons.

Additionally, with some knowledge of the new administration's priorities—such as defense, cybersecurity, and infrastructure—mathematicians can begin to focus on advocacy in those areas.

Dr. Steven Binkley, Deputy Director for Science Programs in the Office of Science at the DOE, offered an overview of research initiatives, programs, and personnel changes at the department. He described the Advanced Scientific Computing Research (ASCR) program's exascale computing focus, with large scientific data as a central theme. Binkley also mentioned that the open position for director of the Advanced Computing Technologies Division could have great impact.

Dr. Steven Lee, a physical scientist at the DOE, broke down the ASCR's three main research themes: algorithms, models, and data. He noted that the shift of some applied math activities to the Exascale Computing Project had left an opening for new ideas and directions. Lee discussed ideas to reinvigorate the applied math program, including organizing workshops, soliciting input from the community on ASCR research themes, and hosting a SIAM event to increase community involvement.

Dr. Michael Vogelius, director of the DMS, talked about recently-introduced DMS programs, including Transdisciplinary Research in Principles of Data Science (TRIPODS) and Algorithms for Modern Power Systems (AMPS). He also provided details on two joint programs: Algorithms for Threat Detection (ATD), with the National Geospatial-Intelligence Agency, and the joint NSF/NIH initiative on Quantitative Approaches to Biomedical Big Data (QuBBD). Other initiatives include public-private partnerships in centers for quantitative biology and DMS-funded internship opportunities for mathematical sciences graduate students at national labs. Vogelius also gave an update on the Mathematical Sciences Research Institutes, for which an open competition is planned, with proposals likely due in 2019. The DMS will put forward a solicitation in 2017 to call for new centers of mathematical biology.

Dr. Suzi Iacono, head of the NSF's Office of Integrative Activities, gave an overview of the 10 "big ideas" for future NSF investments, defining a set of research agendas and processes that will require collaborations with industry, agencies, scientific societies, research institutions, and universities. The six research ideas include harnessing data for 21st-century science and engineering; shaping the human-technology frontier; understanding the rules of life (i.e., predicting phenotypes from genotypes); recognizing the next quantum revolution (physics); navigating the new Arctic (including a fixed and mobile observing network); and exploring windows on the universe: multimessenger astrophysics. The three process ideas include more convergent research; support for midscale infrastructure (instruments or facilities that cost between $10 million and $100 million); and NSF 2050, a common fund to seed large, ambitious projects. The tenth idea is NSF's Inclusion across the Nation of Communities of Learners of Underrepresented Discoverers in Engineering and Science (INCLUDES) program, which aims to transform science, technology, engineering, and mathematics (STEM) education and career pathways to make them more widely inclusive and more reflective of the diversity of U.S. society.

Dr. Susan Gregurick, division director of the National Institute of General Medical Sciences (NIGMS) Division of Biomedical Technology, Bioinformatics, and Computational Biology (BBCB) at the NIH, spoke about NIH programs related to computation and mathematics, including the Biomedical Information Science and Technology Initiative (BISTI) and the Maximizing Investigators' Research Award (MIRA). The NIH also has many interagency partnerships involving computing, such as the Brain Research through Advancing Innovative Neurotechnologies (BRAIN) Initiative, the National Strategic Computing Initiative (NSCI), the Interagency Modeling and Analysis Group (IMAG), and the-DMS collaboration with the NSF. The NIH Big Data to Knowledge (BD2K) program has created an NIH Commons pilot to test a virtual platform for sharing data, using computing services, and accessing large public data sets. The goal is to make the Commons available to any researcher with an NIH grant by the fall of 2017. Gregurick also noted that NIH priorities for FY 2017 include "Applying Big Data and Technology to Improve Health."

Drs. Fariba Fahroo, Carey Schwartz, and Hava Siegelmann, program managers at DARPA, summarized the role and structure of their agency. They highlighted DARPA's long history of applied math and emphasized that conducted research is targeted to improve national security. Fahroo remarked that Stefanie Tompkins, director of the Defense Sciences Office (DSO), has been tremendously supportive of math as a foundation of programs within the DSO, and that this office has a specified focus area in "Mathematics/Modeling/Design." The panel also remarked that the Information Innovation Office (I2O) uses a significant amount of mathematics. Siegelmann highlighted a new program that she is developing intended to support efforts to create a continuously learning and evolutionary computer.

While much is still unknown regarding Congressional research and funding priorities at this time, the goal of SIAM and other scientific societies remains the same: raise visibility of applied mathematics in the federal government, define policy agendas, and conduct outreach and advocacy to influence congressional legislation and federal programs.

Karthika Swamy Cohen is the managing editor of SIAM News. Miriam Quintal is SIAM's Washington liaison at Lewis-Burke Associates LLC. Eliana Perlmutter is a Legislative Research Assistant at Lewis-Burke Associates LLC.

[1] http://sciencedebate.org/20answers

SIAGA

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projective variety with a varying linear subspace." In "On Fano schemes of toric varieties," Nathan Ilten and Alexandre Zontine show how to solve binomial equations in terms of linear forms.

"The geometry of rank-one tensor completion" reveals the work of Thomas Kahle, Kaie Kubjas, Mario Kummer, and Zvi Rosen. In a mathematical contribution to deep learning, titled "Dimension of marginals of Kronecker product models," Guido Montufar and Jason Morton prove that restricted Boltzmann machines are identifiable. Greg Blekherman, Rainer Sinn, and Mauricio Velasco advance convex geometry and polynomial optimization by addressing the intriguing question, "Do sums of squares dream of free resolutions?"

As the inaugural issue approached publication, the *SIAGA* team was also busy with articles to appear in the near future. For instance, next in line is an article on mathematical neuroscience that grew out of a 2014 Mathematics Research Communities program. Carina Curto, Elizabeth Gross, Jack Jeffries, Katherine Morrison, Mohamed Omar, Zvi Rosen, Anne Shiu, and Nora Youngs answer the question, "What makes a neural code convex?"

We are still hoping to branch out further and attract truly outstanding submissions from a wider range of communities. For instance, we'd like to receive more articles from fields such as applied topology, cryptography, geometric modeling, differential geometry, and mathematical biology. As the journal approaches its steady state, the future looks bright. *SIAGA* will serve as a window for first-rate research that transcends the historic division of mathematics into "pure" and "applied." In the immediate future, it welcomes outward-looking authors and all readers with a taste for algebra, geometry, and topology.

Bernd Sturmfels is a professor of mathematics, statistics, and computer science at the University of California, Berkeley. Starting in summer 2017, he will be director of the Max Planck Institute for Mathematics in the Sciences in Leipzig, Germany. He serves as the editor-in-chief of SIAGA.

Help Wanted at *SIAM Review*

By Desmond J. Higham, Tim Kelley, and David S. Watkins

SIAM Review (*SIREV*) is one of the few outlets for in-depth, expert overviews of recent books in applied and industrial mathematics. What makes a good book review? "A good book review doesn't simply list the chapter titles and walk through the contents," SIAM President Nick Higham said. "It gives a general feel for what the book is about, what the book's distinctive features are, and how it fits into the landscape of existing books on the subject. It assesses whether the book is suitable for its target audience, comments on the accuracy of the contents (potential readers need to be warned if there are mathematical errors or many typos), and discusses the book's usability (Good index? Useful bibliography? Suitable ordering of topics and cross-referencing?). Finally, it tells a non-expert reader something they didn't already know about the field in question, such as what the most active research topics and questions are (or no longer are)." Vice President at Large Ilse Ipsen also added a few thoughts. "Good book reviews are informative, insightful, and can help in deciding whether to buy a book, be it a research monograph or textbook," she said.

For good reason, authors of journal articles are bound by the conventions of technical writing. The writer of a book review, however, has the opportunity to be interesting as well as authoritative: to take a more personal stance on what's important, provide anecdotes and historical asides, and speculate about future developments. Margaret Wright, a past president of SIAM and former editor-in-chief of *SIREV*, echoed these views. "In these days of information overload, more than ever people seeking to know more about a subject need help finding the right book," she said. "A bland summary of the table of contents is easy enough to find for ourselves, but what we want most is an honest and fair review by an expert (even when opinionated). And this is precisely what *SIREV* book reviews give us."

David Watkins will step down as Book Review Section Editor of *SIREV* when his terms ends in December 2017. The journal needs a book lover to take over the job starting on January 1, 2018. Perhaps you are that person.

The role is suitable for a senior applied mathematician with broad interests who enjoys books, likes to write, has some editorial experience, and is prepared to serve a three-year term. *SIREV* publishes quarterly, and the Book Reviews section has around 10 reviews (20 pages) per issue. The editorial board for the section currently consists of six members. The new section editor may wish to increase this number to handle a growing workload and reflect the diversity (demographic, disciplinary, and geographic) of applied and industrial mathematics.

When asked to serve SIAM as a volunteer, most people want a job description. Watkins has one: *I look at new applied math titles as they publish and make decisions about whether or not to review. I've also been doing some catch-up, reviewing books that are two-three years old but still merit a review. For each book I decide to review, I also choose an editor (or myself) to handle it.*

About once every three months, I send each of my editors a list of perhaps three or four books for which I would like them to seek reviewers. They have veto power; if they think a book isn't worth reviewing, we won't review it.

For those titles that I decide to handle myself, I look for appropriate reviewers and send out requests to review. I get many rejections, but I keep trying until I find someone who agrees to do the job. Many people who decline are at least willing to suggest other possible reviewers. The internet is also a great aid in the search for appropriate reviewers.

In the 21 months since I took over this job, I personally have handled about 70 books, not all of which were reviewed. I like to review books, and I try to provide at least one review per issue. The next editor is under no obligation to do this.

As the reviews come in, I collect them, make minor edits, and send in a batch for the next issue once every three months. When the galleys come back, I read them (and so do the review authors), make any last corrections, and write the introduction for that SIREV issue.

The nature of this job is changing, and will continue to steadily evolve. I do things a lot differently from my predecessor, Bob O'Malley. My successor will surely make still more changes. Bob had every book come across his desk. He decided which to review and solicited reviewers. Once Bob found a reviewer, he mailed the book out (with some local secretarial help). I ask publishers not to send me physical books, but some do anyway. If they want to send a book, they can send it to the SIAM office. Brittni Holland, SIAM's editorial associate, lets me know what we have received. My editors and I rely on publishers' websites for the information we need to make decisions about each book. Once we have a reviewer, Brittni mails him/her the book. Sometimes, and increasingly frequently, Brittni contacts the publisher and has the publisher send the book directly to the reviewer. Sometimes we deal with e-books, and I expect that to increase. We will probably end up with an entirely electronic operation eventually.

By reading book reviews, SIAM members explore the state of the art in research, keep abreast of emerging topics, and discover new teaching resources. By writing book reviews, authors can offer their slant on a subject, present insights and personal opinions, and draw connections between fields. At the top of this pyramid is the Book Review Section Editor, who steers the scope, content, and style of the section, and in doing so obtains a unique perspective on the field.

Persons interested in this role may informally contact Des Higham (d.j.higham@strath.ac.uk) for further information. Mike Miksis, SIAM's Vice President for Publications, hopes to have the next Book Review Section Editor approved by mid-summer 2017.

Des Higham is a numerical analyst at the University of Strathclyde in Glasgow. He has research interests in stochastic computation, network science, and city analytics, and is a SIAM Fellow and a Fellow of the Royal Society of Edinburgh. He is also the editor-in-chief of SIAM Review. Tim Kelley chairs the SIAM Board of Trustees and has served as editor-in-cheif of SIAM Review and the SIAM Journal on Optimization. He is Drexel Professor of Mathematics at North Carolina State University. David Watkins is currently Book Review Section Editor for SIAM Review and a professor of mathematics at Washington State University.

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Call for Nominations for the 2017 Ostrowski Prize

The aim of the Ostrowski Foundation is to promote the mathematical sciences. Every second year it provides a prize for recent outstanding achievements in pure mathematics and in the foundations of numerical mathematics. The value of the prize for 2017 is 100.000 Swiss francs.

The prize has been awarded every two years since 1989. The most recent winners are Ben Green and Terence Tao in 2005; Oded Schramm in 2007; Sorin Popa in 2009; Ib Madsen, David Preiss, and Kannan Soundararajan in 2011; Yitang Zhang in 2013; and Peter Scholze in 2015.

See **https://www.ostrowski.ch/index_e.php** for the complete list and further details.

The jury invites nominations for candidates for the 2017 Ostrowski Prize. Nominations should include a CV of the candidate, a letter of nomination, and two-three letters of reference.

The chair of the jury for 2017 is Gil Kalai of the Hebrew University of Jerusalem, Israel. Nominations should be sent to **kalai@math.huji.ac.il** by May 15, 2017.

A New Twisting Somersault

By Holger R. Dullin

Platform and springboard diving are among the most beautiful Olympic sports. A fascinating outcome from our study of aerial motion and its mathematical description is the suggestion of a new twisting somersault [5] with five full twists, called "513XD." The Fédération internationale de natation (FINA) diving code states that 513XD has three half-somersaults and $X = 10$ half-twists. This dive has not yet been performed, but we believe that it is humanly possible and divers will perform it at world-class competitions in the future. "Bodies in Space," the ARC-funded research project that supported this work, is being carried out jointly with the New South Wales Institute of Sports in Sydney, Australia.

What mathematics led to the discovery of this tricky new dive?

Non-rigid Body Dynamics

Differential equations describing the motion of the human body have been used in biomechanics for a long time. Biomechanists typically model the body as a collection of rigid pieces coupled by joints. Inspired by geometric mechanics, our approach exploits the symmetry of the equations and, most importantly, clearly separates the shape of the athlete from the overall orientation. It all starts with Euler's equation

$$\dot{\mathbf{L}} = \mathbf{L} \times \mathbf{\Omega}, \qquad (1)$$

where $\mathbf{L} = \in \mathbb{R}^3$ is the angular momentum vector and $\mathbf{\Omega} \in \mathbb{R}^3$ is the angular velocity vector, both in a body frame. An orthogonal matrix $R \in SO(3)$ describes the body's orientation. The matrix R transforms vectors from the body frame to the space-fixed frame, so that $R\mathbf{L}$ gives the constant angular momentum in space. Differentiating the constant vector $R\mathbf{L}$ with respect to time yields $\dot{R}\mathbf{L} + R\dot{\mathbf{L}} = 0$, and solving for $\dot{\mathbf{L}}$ gives Euler's equation (1). Differentiating $R^t R = \text{id}$ demonstrates that $R^t \dot{R}$ is antisymmetric. The usual hat-map from $\mathbb{R}^3$ to $so(3)$ identifies the angular momentum as $\hat{\mathbf{\Omega}} = R^t \dot{R}$. The essential point is that (1) is true even for a shape-changing body, as long as angular momentum conservation holds.

Computing the angular momentum for a shape-changing body [3] gives

$$\mathbf{L} = I\mathbf{\Omega} + \mathbf{A}, \qquad (2)$$

where I is the body's tensor of inertia and $\mathbf{A} \in \mathbb{R}^3$ is a momentum shift vector. The difference between rigid and non-rigid body dynamics now appears. When the shape is constant, the vector $\mathbf{A}$ is zero and Euler's rigid body equations are thus recovered. When the shape is changing, the symmetric moment of inertia tensor $I(t)$ and the momentum shift vector $\mathbf{A}(t)$ encompass all the complexity of a particular coupled rigid body model for the human body. Given a particular shape change, one can compute both $I(t)$ and $\mathbf{A}(t)$; explicit formulas for this are available in [3]. Maurice Raymond Yeadon first derived similar equations for the description of the twisting somersault, as described in his collected papers [6].

Let us first describe a somersault. Fix a coordinate system in the body's trunk where the x-axis points out of the chest, the y-axis points to the left, and the z-axis points towards the head. By definition, a somersault is a motion where R is a rotation about a fixed $\mathbf{L} = (0, l, 0)^t$, which is an eigenvector of I. When the diver pulls into a tuck position, the corresponding eigenvalue of I decreases and the angular momentum $\mathbf{\Omega}$ increases, as determined by (2). While the shape changes, the momentum shift $\mathbf{A}$ is non-zero but parallel to $\mathbf{L}$, so that all three vectors remain parallel throughout the dive and—in the body frame—the solution remains at the equilibrium point $\dot{\mathbf{L}} = 0$.

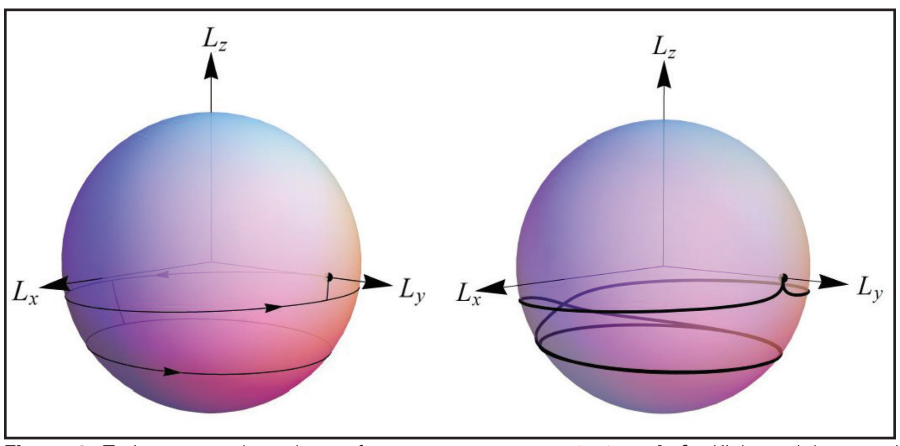

Figure 1. *Trajectory on the sphere of constant momentum* $|\mathbf{L}| = l$. **Left.** *Kick model, second kickoff by a half twist.* **Right.** *Realistic model. Image credit: [5].*

The Kick Approximation

A twisting somersault dive begins with a somersault, followed by a shape change for which $\mathbf{A}$ is not parallel to $\mathbf{L}$. This motion moves $\mathbf{L}$ away from the equilibrium point and the body starts twisting, with vector $\mathbf{L}$ revolving about the z-axis. To understand the dynamics, let's consider a fast shape change, which makes $\mathbf{A}$ arbitrarily large in the kick-limit. In this limit, (1) and (2) yield

$$\dot{\mathbf{L}} \approx -\mathbf{L} \times I^{-1}\mathbf{A}, \qquad (3)$$

a linear, time-dependent differential equation. The shape change is simple to integrate when it occurs such that the direction of $I^{-1}\mathbf{A}$ remains constant, and the solution is a rotation about that direction. Moving a stretched arm in the yz-plane produces a rotation R_x about the x-axis. In a typical twisting somersault, the first arm motion starts the twisting and—following a full number of twists—reversing the arm motion stops it.

We observed that when the second arm motion is performed after a half twist instead of a full twist, it has the opposite effect: instead of stopping the twist, it speeds it up. The reason for this is geometrically simple. Let α be the amount of rotation generated by the arm motion. While $R_x(\alpha)R_z(2\pi)R_x(-\alpha)$ is the identity, $R_x(\alpha)R_z(\pi)R_x(-\alpha) = R_x(2\alpha)R_z(\pi) = R_z(\pi)R_x(-2\alpha)$ is not. So the initial $\mathbf{L} = (0, l, 0)^t$ moves closer to the pole in the second case. The twisting motion speeds up as $\mathbf{L}$ gets closer to the pole, and thus the second arm kick after a half-twist increases the twisting speed (see Figure 1).

The Full Model

This argument is no doubt approximate, and the true motion is more complicated than the analysis indicates. However, it does convey the central idea. More details and a full numerical simulation, in which the arm motions are performed with realistic speed, confirm the mechanism's validity [5].[1] When the second arm motion involves both arms—one down, and one up, like the wings of a windmill (see Figure 2)—it achieves an effect roughly twice as big. We amplify that effect in [1], where a rotating disc replaces the arms. A reverse arm motion occurring a full number of twists later stops the high-speed twist, and a fourth arm motion stops the twisting altogether, as in Figure 1.

[1] View an animation of the 513XD dive at goo.gl/Xvi7pD.

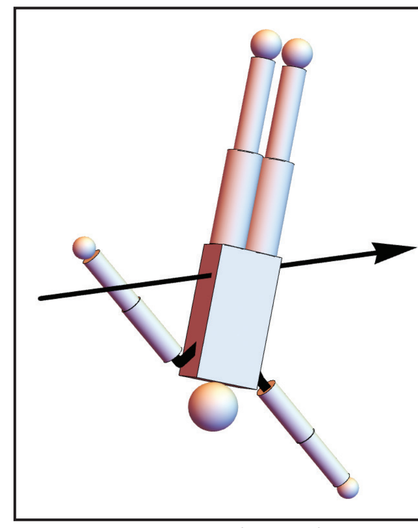

Figure 2. *513XD dive after the first 1/2 twist and 1/2 somersault is complete while arms are in windmill motion. The black vector is the constant angular momentum vector. Image credit: [5].*

But how much does the body rotate in space? A diver must perform a half-integer number of somersaults for the dive to be successful overall. Symmetry reduction to the body frame eliminates the rotation about the fixed angular momentum vector in space. But geometric mechanics teaches us that one can recover the missing somersault angle as a combination of a geometric phase and a dynamic phase from data of the reduced equations alone. Richard Montgomery [4] does this for rigid bodies, and Alejandro Cabrera and La Plata [2] do the same for non-rigid bodies. We extend these formulas to our setting in [3], and show that within a certain limit, the ratio of the number of somersaults to the number of twists is a rotation number of the integrable Euler top. All of this leads to a good theoretical understanding of the twisting somersault; now we hope to find a volunteer athlete to try the new 513XD dive—with five full twists—in practice!

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Holger R. Dullin is a professor of applied mathematics at the University of Sydney in Australia. His research interests are Hamiltonian dynamical systems, including integrable systems, fluid dynamics, (non-) rigid body dynamics, and semi-classical quantization.

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The International Council for Industrial and Applied Mathematics (ICIAM) Prize Committee calls for nominations for the five ICIAM prizes (the Collatz Prize, the Lagrange Prize, the Maxwell Prize, the Pioneer Prize, and the Su Buchin Prize) to be awarded in 2019. Each ICIAM Prize is truly international and has its own special character. We therefore welcome nominations from every part of the world. A nomination should consider the specifications for a particular prize[1] and contain the following information:

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[1] See http://www.iciam.org/iciam-prizes