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Using Differential Privacy to Protect the United States Census

By Matthew R. Francis

In 2006, Netflix hosted a competition to improve its algorithm for providing movie recommendations to customers based on their past choices. The DVD rental and video streaming service shared anonymized rental records from real subscribers, assuming that their efforts to remove identifying information sufficiently protected user identities. This assumption was wrong; external researchers quickly proved that they could pinpoint personal details by correlating other public data with the Netflix database, potentially exposing private information.

This fatal flaw in the Netflix Prize challenge highlights multiple issues concerning privacy in the information age, including the simultaneous need to perform statistical analyses while protecting the identities of people in the dataset. Merely hiding personal data is not enough, so many statisticians are turning to differential privacy. This method allows researchers to extract useful aggregate information from data while preserving the privacy of individuals within the sample.

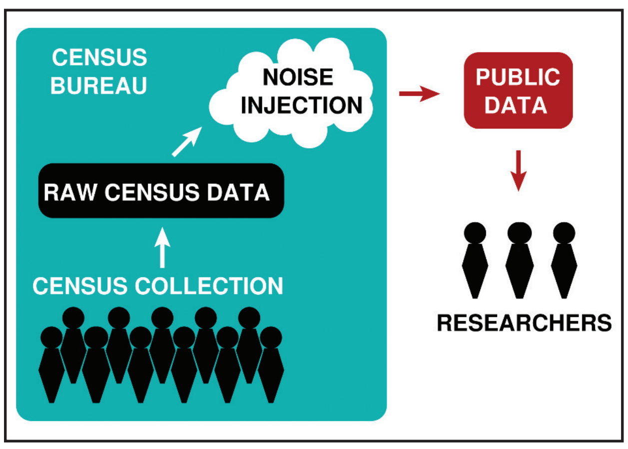

Figure 1. *Differential privacy protects information provided to the U.S. Census (represented by the blue box). The census injects noise into the data before releasing it to the public; this prevents the exposure of identifying information to anyone analyzing the results. Figure courtesy of Matthew Francis.*

"Even though researchers are just trying to learn facts about the world, their analyses might incidentally reveal sensitive information about particular people in their datasets," Aaron Roth, a statistician at the University of Pennsylvania, said. "Differential privacy is a mathematical constraint you impose on an algorithm for performing data analysis that provides a formal guarantee of privacy."

Differential privacy protects individuals in a dataset from identification by injecting noise into the data. This yields a probability distribution, from which it is nearly impossible—in a practical sense—to determine a particular record's presence within the set. In other words, anyone performing statistical analysis will not be able to link a specific person to their details contained in the database.

Such protections are more than simply good ideas. The U.S. Census Bureau is legally obligated to guard the privacy of the people it surveys for civil rights reasons, including voting protections. To that end, the 2020 U.S. Census will implement differential privacy, marking a major shift in how the U.S. government handles information propagation.

John Abowd, chief scientist and associate director for research and methodology at the U.S. Census Bureau, spearheaded this change. During his presentation at the American Association for the Advancement of Science 2019 Annual Meeting, which took place earlier this year in Washington, D.C., Abowd highlighted the current inadequacies of privacy measures implemented during the 1990 Census.

"We don't have the option of saying that it's broken and we can't fix it, so we'll keep using it," Abowd said. "So we searched for a technology that could provably address this vulnerability. And there's exactly one: differential privacy."

From Flipping Coins to Noise Injection

To understand the philosophy behind the differential privacy paradigm, Roth proposes two nearly-identical worlds. "The same data analysis is run in both worlds," he said. "But while your data is included in the analysis in one world, it is not in the other. There should be no statistical test that can reliably distinguish these two worlds better than random guessing. So that's a strong guarantee of privacy: I can learn things about the population generally but I cannot learn anything about you because I cannot even tell whether I had access to your data."

A simple version of differential privacy is a technique called "randomized response." Consider a yes or no question that reveals embarrassing or illegal activity, e.g., "Have you used drugs in the past month?" Before answering, flip a coin; if it turns up tails, answer the question truthfully. If it lands on heads, flip a second coin and record a "yes" upon a second heads and a "no" upon tails, regardless of the true

See **Differential Privacy** *on page 3*

Mathematical Models for Him (and for Her Too)

By Anita Layton

Picture someone having a heart attack. Do you imagine a man doubled over, clutching his chest? In Hollywood films, this "classic heart attack" is almost always portrayed by a male. Despite the fact that heart disease is the leading killer of women worldwide, a misconception that the ailment is a man's illness persists. That incorrect assumption has unfortunate implications in medical care for women.

Gender biases and false impressions are by no means limited to heart attack symptoms. Such prejudices exist throughout the healthcare system, from medical research to disease diagnosis and treatment strategies. Historically, clinical research studies have primarily focused on men and utilized male animal models. The Physicians' Health Study—a landmark Harvard Medical School analysis founded in 1982 to examine aspirin's effect on heart disease—initially enrolled over 22,000 participants. None of these participants were female. In the 1970s, the U.S. Food and Drug Administration (FDA) banned women of childbearing age from participating in phase I clinical trials. The ban remained in effect for 20 years and was only lifted in 1993.

Why would researchers choose to exclude half of the population? Female menstrual cycles and fluctuating hormones, which scientists fear may limit the reliability and reproducibility of their findings, might be to blame. Cost is another likely deterrent, as replicating experiments in both sexes requires double the resources. For these reasons, researchers often conduct experiments in men and assume that the findings apply to women as well.

Does this mentality pose a problem? Consider a scenario in which the FDA approved a drug that was tested exclusively in cats. Given that cats share about 90 percent of our genes, would you be comfortable taking the drug without human data? Of course not; biological differences between people and animals could lead to unexpected, undesirable drug reactions.

Women face a similar dilemma when it comes to medical research. While the physiological differences between males and females are undoubtedly much smaller than those between humans and cats, important—albeit subtle—variations nonetheless exist. For example, men are generally larger than women. As a result, a recommended dosage calculated for an average-sized man may cause an overdose in small women. Major differences also exist in the kidneys, which can affect how the body excretes some drugs.

Because of these gender disparities, many diseases affect men and women in dissimilar ways and elicit different responses to treatment. One notable example is high blood pressure, also known as hypertension.

See **Him (and for Her Too)** *on page 2*

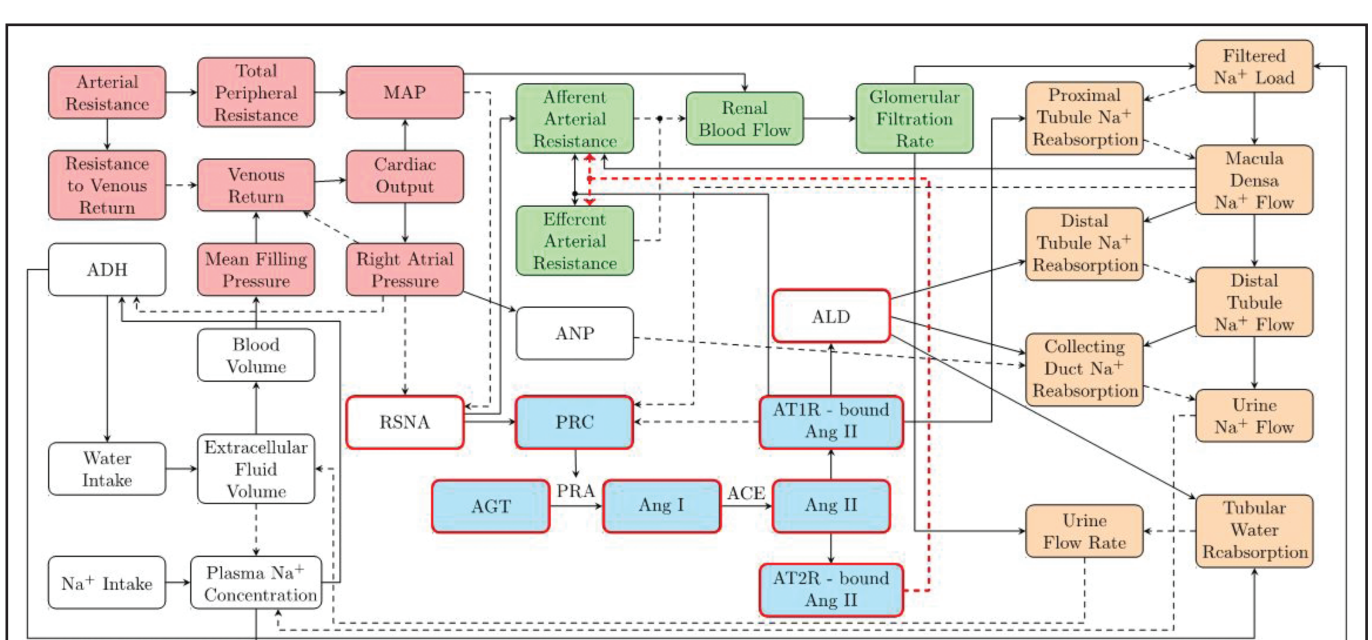

Figure 1. *Schematic model of blood pressure regulation. Pink nodes denote variables that describe cardiovascular function, green nodes denote renal hemodynamics, orange nodes denote renal sodium handling and urine production, and blue nodes denote the renin-angiotensin system. Each node typically consists of a system of ordinary differential and algebraic equations. Red outlines indicate sex-specific model components. Notations are defined as follows: anti-diuretic hormone (ADH); mean arterial pressure (MAP); atrial natriuretic peptide (ANP); renal sympathetic nerve activity (RSNA); plasma renin concentration (PRC); plasma renin activity (PRA); angiotensinogen (AGT); angiotensin I (Ang I); angiotensin II (Ang II); angiotensin II type 1 receptor bound angiotensin II (AT1R-bound Ang II); angiotensin II type 2 receptor bound angiotensin II (AT2R-bound Ang II); and aldosterone (ALD). Figure courtesy of [3].*

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Him (and for Her Too)

Continued from page 1

According to a report[1] from the American Heart Association, hypertension affects one in three U.S. adults and is the primary cause of premature death in the developed world. Scientists have long known that men generally experience higher blood pressure and are at greater risk for heart and kidney disease. Despite this fact, doctors often prescribe the same medication for both males and females suffering from high blood pressure. This one-size-fits-all approach is problematic; even though women with hypertension are more likely to seek treatment and follow their medication regimens, only 45 percent of treated women get their blood pressure under control (compared to 51 percent of treated men).

His and Her Blood Pressure Regulation Models

Blood pressure regulation involves multiple systems; as such, it can benefit from studies related to systems biology. In 1972, Arthur Guyton, Thomas Coleman, and Harris Granger pioneered computational modeling of the circulatory system for blood pressure regulation in their seminal work, often referred to as the Guyton model [2]. The model consists of a large set of coupled ordinary differential and algebraic equations that describe how different regulators operate synergistically in the circulatory system. Model components include cardiovascular function, circulatory dynamics, renal hemodynamics, kidney function, respiratory function, neurohormonal feedback, autonomic nervous system activity, and electrolyte balance. For instance, the kidneys effectively regulate blood pressure by decreasing reabsorption and thus increasing sodium and water excretion in response to elevated blood pressure. This causes extracellular fluid volume to go down, which in turn lowers blood pressure.

Even mathematical biologists often fail to account for sex differences. Indeed, the Guyton model and its many variants—published over the last four and a half decades—are all gender neutral. To investigate sexual dimorphism and its implications in antihypertensive therapy, our group recently published the first and only set of sex-specific computational models for blood pressure regulation [3]. These models represent sex differences in the renin-angiotensin system (RAS) [4], a signaling pathway that interacts with the kidneys and plays a crucial role in blood pressure regulation as well as the less excitable and more easily repressed female renal sympathetic nervous activity. The schematic in Figure 1 (on page 1) outlines sex-specific components in red.

We use the RAS to illustrate the structure of each model component. Figure 2 portrays the RAS reaction cascade, which starts with angiotensinogen (AGT). Renin, angiotensin-converting enzyme (ACE), and neutral endopeptidase activity facilitate AGT's conversion into different forms of angiotensin. Renin catalyzes the first reaction in the RAS and is often considered to be the cascade's driving force. The products of the RAS—angiotensin (1-7) (denoted Ang (1-7)) and Ang II—bind to receptors and impact the brain, heart, kidneys, vasculature, and immune system.

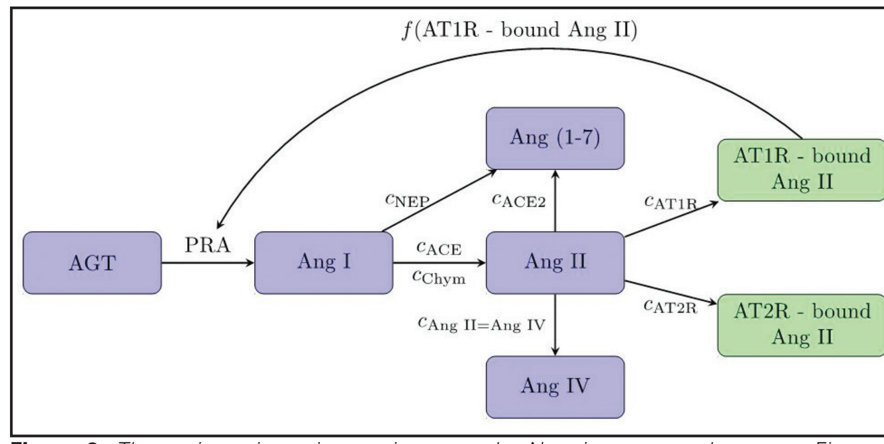

Figure 2. *The renin-angiotensin reaction cascade. Notations are analogous to Figure 1. Figure courtesy of [4].*

Receptor-bound Ang II regulates blood pressure via its effects on the kidneys. ACE and chymase convert Ang I to Ang II at rates c_{ACE} and c_{Chym} respectively. ACE2 then converts Ang II into Ang (1-7) at rate c_{ACE2}. Ang II also binds to the receptors AT1R and AT2R at corresponding rates of c_{AT1R} and c_{AT2R}. Assuming that Ang II has a half-life of $h_{\text{Ang II}}$, the rate of change of Ang II concentration is given by

$$\frac{d[\text{Ang II}]}{dt} = (c_{\text{ACE}} + c_{\text{Chym}})[\text{Ang I}] - (c_{\text{ACE2}} + c_{\text{Ang II=Ang IV}} + c_{\text{AT1R}} + c_{\text{AT2R}})[\text{Ang II}] - \frac{\ln(2)}{h_{\text{AngII}}}[\text{Ang II}].$$

We specify analogous rate equations for AGT, Ang I, Ang (1-7), Ang IV, AT1R-bound Ang II, and AT2R-bound Ang II.

To formulate sex-specific RAS models, we consider the system at steady state and seek to determine the reaction rate parameters (i.e., the cs), which will presumably differ between the sexes. We first identified male and female hormone levels (i.e., Ang I, Ang II, etc.) from experimental literature. Half-lives (the hs) for the hormones are known, and solving a linear system separately for males and females yields the rate parameters (or cs) [4].

We can then incorporate these sex-specific RAS models into the blood pressure regulation models, which are useful for understanding contrasting male and female responses to various hypertensive stimuli [3]. Simulation results suggest that the severity of hypertension induced for a given pathophysiological perturbation may vary significantly between men and women. They also indicate that stronger renal sympathetic nervous activity-mediated regulation of afferent arteriole tone in women is primarily responsible for their resistance to hypertension. Renal sympathetic nervous activity is elevated in response to high blood pressure. This results in a higher degree of afferent arteriole dilation in females, leading to a larger increase in renal blood flow and consequential urine output.

Sex as a Biological and Mathematical Variable

Sex and gender differences exist in many other ailments besides high blood pressure. Heart disease is a classic example, as men and women have disparate prevalences, symptoms, comorbidities, and treatment responses. For instance, women are more likely to report pain associated with heart attack somewhere other than the chest. Multiple sclerosis is another example: females are more susceptible to the condition, but it progresses more severely in males. Pain can also affect men and women differently. Failure to properly account for these gender discrepancies often leads to misdiagnosis and inappropriate treatment in women. Computational modeling thus plays an important role in identifying the most suitable treatment for each sex or gender.

Policymakers have attempted to close the gender gap in medical research. Recently-implemented rules from the National Institutes of Health (NIH) mandate the incorporation of sex as a biological variable in NIH-sponsored research, and have increased the number of females in experimental and clinical studies. While modeling has seen similar progress, the overall number of sex-specific computational models remains low. A search for "sex-specific computational model kidney" on PubMed yields only two publications, both of which belong to our group [1, 5].

A comprehensive understanding of sex and gender's impact on health and disease is key to the ultimate development of effective sex-based therapies, and mathematical modeling can be a major contributor. Model analysis that highlights sex and gender differences will facilitate the larger effort of precision medicine.

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[1] https://www.heart.org/idc/groups/heart-public/@wcm/@sop/@smd/documents/downloadable/ucm_319587.pdf

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Dynamics, Information, and Organization: The Origins of Computational Mechanics

By James P. Crutchfield

Computational mechanics defines pattern and structure with the goal of detecting and quantifying the organization of complex systems. The field developed from methods introduced in the 1970s and early 80s to (i) identify strange attractors as the mechanisms that drive weak fluid turbulence via the reconstruction of attractor geometry from measurement time series, and (ii) estimate effective theories directly from complex time series. Such estimation foundered in selecting a representational basis without first-principle guidance. Computational mechanics addressed this weakness by providing a mathematical and operational definition of structure. The result is a principled means of discovering patterns in natural systems. Its applications include information measures for complex systems, a structural hierarchy of intrinsic computation, quantum compression of classical processes, intelligence in Maxwellian demons, and evolution of computation and language.

The rise of dynamical systems theory and the maturation of the statistical physics of critical phenomena in the 1960s and 70s led to a new optimism that complicated and unpredictable occurrences in the natural world were actually governed by simple, nonlinearly-interacting systems. Moreover, new mathematical concepts and increasingly powerful computers provided an entrée to understanding the emergence of such phenomena over time and space. The overarching lesson was that intricate structures in a system's state space amplify microscopic uncertainties, guiding and eventually attenuating them to form complex, spatiotemporal patterns. In short order, this new perspective on complex systems raised questions surrounding the quantification of their unpredictability and organization.

By themselves, qualitative dynamics and statistical mechanics were mute to this challenge. The first hints of addressing this lay in Andrey Kolmogorov's introduction of computation theory and Claude Shannon's information theory into continuum-state dynamical systems. These innovations demonstrated that information has an essential role in physical theories of complex phenomena — a role that is complementary to (and as equally important as) energy. They yielded a new algorithmic foundation to randomness generated by physical systems: incompressible behavior is random. A bona fide measure of complex systems' unpredictability was thus established.

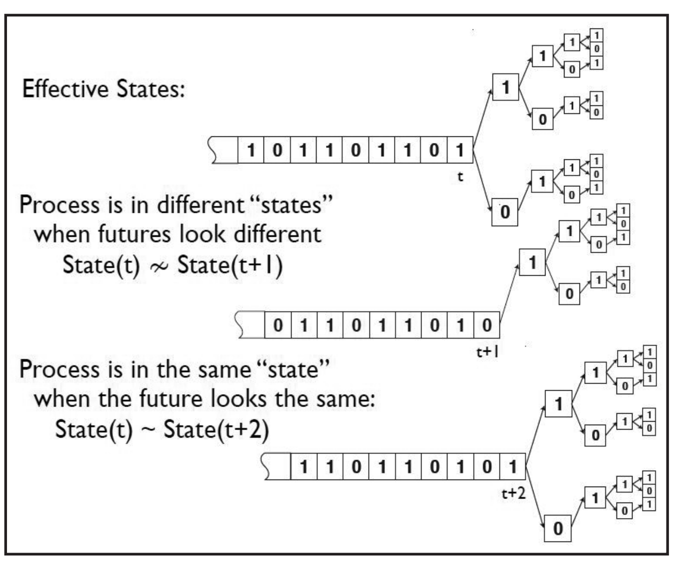

Figure 1. *Causal equivalence future conditional distributions. Figure courtesy of James Crutchfield.*

Yet information generation comprises only one aspect of complex systems. How do such systems store and process that information? How is that information expressed and remembered in structure? The first uses of information and algorithmic concepts side-stepped questions concerning the structure and organization of complex systems' internal mechanisms. Delineating their informational architecture is a subtle task.

Even when we know their governing equations of motion, truly complex systems generate patterns over long temporal and spatial scales. For example, the Navier-Stokes partial differential equations describe the local-in-time-and-space balance of forces in fluid flows. A static pressure difference leads to material flow. However, the Navier-Stokes equations themselves do not directly describe fluid structures such as vortices, vortex pairs, vortex streets, or vortex shedding, let alone turbulence. These patterns are *emergent*, and are generated at spatiotemporal scales far beyond those directly specified by the local, instantaneous equations of motion.

Two questions pertaining to emergent patterns immediately arise, which is where the subtlety comes into play. We see that something new has originated, but how do we objectively describe its structure and organization? More prosaically, how do we discover patterns in the first place?

By refining the reconstruction methods developed to identify chaotic dynamics in fluid turbulence, computational mechanics provided an answer that was both simple and complete: a complex system's architecture lies in its causal states. A *causal state* is a set of histories, each of which leads to the same set of futures (see Figure 1). It is a simple dictum — do not distinguish histories that point to identical predictions of the future.

The causal states and their corresponding transition dynamics yield a canonical representation: the *ϵ-machine*. A system's ϵ-machine is its unique optimal predictor of minimal size. The historical information stored in the causal states of a process quantifies its level of structure. A process's ϵ-machine is its effective theory — its equations of motion at the level of emergent patterns. Focusing only on optimal process prediction leads to a notion of structure in terms of stored information and symmetry; this is a notable aspect of the ϵ-machine's construction. Predictability and organization are inextricably intertwined. Researchers cannot discuss or properly measure one without reference to the other.

A system's ϵ-machine minimal representation solves the challenge of quantifying emergent organization. The answer lies in a complex system's *intrinsic computation*, which addresses three simple questions: (i) How much of the past does a process store? (ii) In what architecture is that information stored? (iii) How is the stored information used to produce future behavior?

The answers are straightforward. The stored information is in the causal states, the ϵ-machine's states and transitions explicitly lay out the process architecture, and the process's Shannon-Kolmogorov entropy rate monitors information production.

At first blush it may not be apparent, but computational mechanics parallels basic physics in this way. Physics tracks various types of energy and monitors their transformation into each other. Similarly, computational mechanics explores the kinds of information inherent in a system and the ways in which such information transforms into other variations. Although the ϵ-machine describes a mechanism that generates a system's statistical properties, computational mechanics captures more than mere generation. And this is why the field was so named. It was an extension of statistical mechanics that went beyond analysis of a system's statistical properties to capture its computation-theoretic characteristics: how a system stores and processes information, and how it intrinsically computes.

A synopsis of the main concepts underlying computational mechanics necessarily neglects its intellectual history. From where did this mix of ideas originate? What

See **Computational Mechanics** *on page 4*

Differential Privacy

Continued from page 1

answer. Numerically, if your real answer is "yes," the algorithm assigns a "yes" response to you roughly three-quarters of the time. The probable fraction f of "yes" answers in the randomized dataset is

$$f \simeq \frac{1}{4}(1+2p),$$

where p is the actual fraction of "yes" answers, and the estimate's accuracy increases with the number of survey participants. Analysis of the resulting data ideally estimates the rate of the activity in question without exposing individuals to retaliation for truthful responses [1].

Differential privacy extends this concept to more complex datasets. Researchers determine the level of privacy they require, which is often sufficiently quantified by a single parameter ε. Formally speaking, consider two datasets $\{D, D'\}$ that differ by a small amount (one record in the basic case). An algorithm M is ε-differentially private if the probability P of extracting particular data x from both databases obeys the following inequality:

$$P[M(D)=x] \leq \exp(\varepsilon) P[M(D')=x].$$

This simple randomized response example is "ln 3-differentially private" because obtaining a truthful "yes" answer is three times more probable than getting a false "yes" answer: $\varepsilon = \ln 3$.

More sophisticated privacy algorithms inject noise into data generated with the Laplace distribution

$$\mathrm{Lap}(x \mid b) = \frac{1}{2b}\exp(-\frac{|x|}{b}),$$

where $b \propto 1/\varepsilon$ determines the scale of the distribution from which random values are drawn. The privacy guarantee is meaningless for large ε values because D and D' are too different, but small nonzero ε values can provide very strong privacy guarantees.

With regard to the census, it is essential that individuals not be linked to their geospatial location for a number of legal and ethical reasons. The differential privacy algorithm tested on past census datasets injects noise on multiple levels of the geographic hierarchy used in census tabulation (see Figure 1, on page 1). "Oftentimes, people who use data are initially taken aback when we say we're going to introduce noise into their data," Roth said. "But it is important to remember that data is already noisy due to sampling error. The goal of differential privacy is to introduce just a bit more error, enough to hide with statistical uncertainty the influence that single data points have on any statistic we are releasing."

He points out that a type of noise was already added to census data from 1990, 2000, and 2010. However, external analysts did not know the nature of that noise, which required them to either pretend that they had the raw data or make assumptions about the algorithm. "If you want to construct statistically valid confidence intervals around statistics computed from the data, you need to understand the process by which any noise was introduced," Roth noted. "The guarantees of differential privacy are such that the mechanism that adds these perturbations can be made public. We are not getting any privacy guarantees from trying to keep things secret, and that's a benefit for science."

Future-proofing Privacy

Demographic data is people. Protecting the privacy of these people is at odds with sharing their data for research purposes, so privacy protections often involve "security through obscurity." This is accomplished by hiding certain database fields from users—as with the Netflix Prize contest—or randomizing fields using a non-public algorithm, as the U.S. Census Bureau did with data from the previous three censuses.

However, when Census Bureau researchers accounted for modern algorithms and computing power, they discovered the inadequacy of these measures. Like with Netflix, security through obscurity collapsed when other public data sources were combined with the last census. This discovery created a sense of urgency during preparation for the 2020 Census.

"The system that was used to protect the 2010 Census [is] clearly no longer best practice," Abowd said. "It does not provide protection against vulnerabilities that have emerged in the ensuing decade. The Census Bureau didn't invent those vulnerabilities; they are an inevitable feature of the 21st-century information age."

The protection afforded by differential privacy is sufficiently strong for Abowd to call it "future proof." "The mathematical principle that underlies the risk measure used in differential privacy is a worst-case analysis," he continued. "Once you apply the protections, you can release the data and never have to worry about a stronger attack undoing what you did because you have basically allowed for the strongest possible attack. If the worst case happens, the protection is even stronger than ε."

Differential privacy is not a magic guarantee of protection, and no algorithm can prevent compromise if a database contains enough identifying information to render noise injection worthless. Additionally, differential privacy cannot counter political manipulation. Census Bureau experts raised concerns over U.S. Secretary of Commerce Wilbur Ross's recent proposal to add a citizenship question to the 2020 Census; they feared that people from vulnerable populations would opt out of responding, which would damage the data's quality independent of privacy concerns [2].

Nevertheless, differential privacy may offer the best possible option for balancing openness, analytic usefulness, and privacy protections for individuals in the U.S. Census and other realistic datasets.

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Exploring New Avenues for Engagement

Data Mining in/for Africa at SDM19

By Reginald E. Bryant and Sekou L. Remy

The 2019 SIAM International Conference on Data Mining (SDM19), which took place earlier this year in Calgary, Canada, introduced a first-of-its-kind simulcast workshop on data mining in Africa. The workshop was simultaneously livestreamed between Nairobi, Kenya—home of IBM Research-Africa—and Calgary, and supported multiple other locations since workshop attendees were co-located in Japan, the U.K., and the U.S. The event featured several research sessions, a keynote presentation, and two panels.

The organizing committee consisted of Reginald E. Bryant, Sekou L. Remy, Tonya Nyakeya, and Evalyn Kemunto of IBM. Attendees at the Nairobi site represented various organizations from academia and industry, and traveled from as far away as Uganda. Prior to the simulcast portion, they networked and experienced two on-site demonstrations, courtesy of Remy and Joreen Arigye of Fenix International.

IBM researcher Aisha Walcott officially opened the event with a brief discussion about her career as a scientist and her recent work in applying artificial intelligence to Africa's challenges. Featured speaker Michael Gitau of the Universitat Autònoma de Barcelona then presented his research with autoencoder data representation — a machine learning/deep learning technique. Using data from 220 patients with end-stage renal disease, Gitau reconstructed the condition's temporal evolution to predict patient mortality.

A Panel of Domain Experts

During the "Panel of Domain Experts" segment of the workshop, discussion ensued in the areas of finance and healthcare — fields in which researchers are currently applying data science in East Africa. The two panelists, Moses Alobo of the African Academy of Sciences and John Olukuru of Strathmore University and @iLabAfrica, acquired their expertise by overseeing and commissioning data science projects.

In the open-question portion of the panel, Alobo highlighted a particular instance in which patient outcomes incentivize the collection and sharing of genetic health data. He referenced a dataset presently being amassed by the Human Heredity & Health in Africa consortium.[1] Large western pharmaceutical companies can currently only collect genetic data with small variability from people of African descent (the data is closely tied to West Africa due to historic circumstances). Such limited data becomes problematic when doctors engineer and globally administer personalized drugs. Since this information does not account for the genetic variability on the African continent, individualized, gene-based drugs may have unintended consequences.

As an example, Alobo spoke about captopril, an antihypertension medication administered to a particular population in Kenya. While Western studies indicated that black patients require higher doses of captopril to manage hypertension (compared to their white counterparts), those higher doses proved to be too high for Kenyans and unintentionally initiated hypotension. While the aforementioned situation is just one case, other adverse drug reactions could occur if research studies continue to draw from non-representative data samples.

Alobo's comments also exposed incentives for individuals and companies in the healthcare sector to share data for mutual benefit. However, he urged caution when establishing sharing mechanisms; people own their genetic data and should receive proper remuneration when pharmaceutical companies leverage said data to create medications that may ultimately be sold back to the patients.

Spurred by an audience question, the panel then broached the topic of politics in regards to technology concerning societal good. An attendee expressed frustration at resistance to the development of a water-credit technology for the disadvantaged community in Kibera, Kenya amidst a shifting political climate. Olukuru acknowledged the challenges of interacting with government officials, and encouraged individuals to be patient and work on convincing decision-makers of the technology's value. He further emphasized the importance of establishing trust as a technology producer; one must demonstrate and prove rather than simply build and tell.

Specific to Kenya, innovation that uses or incorporates assets from large companies like Safaricom (a major mobile network operator) can bias officials against startup technology. Without proper differentiation, the tools offered by startups may be indistinguishable from Safaricom technologies.

A Panel of Data Science Practitioners

"A Panel of Data Science Practitioners" convened following hors d'oeuvres and one-on-one conversations among Kenyan participants. Panelists included Leonida Mutuku of Intelipro, Samuel Kamande of Ajua (formerly mSurvey), and Chris Orwa of I&M Bank Kenya.

In response to the opening question that asked whether financial institutions in emerging markets are investing their energy into "front offices" (financial/loan products) or "back offices" (customer due diligence), Orwa offered insight into the operations of local and correspondent banks in East Africa. Most banks in the region are currently focused on improving back-office operations with machine learning and data science, rather than concentrating on front-office operations like customer experiences and new product offerings.

Kamande took the audience on Ajua's naturally-evolving data science journey. Initially established as a way to collect data from disenfranchised, often-overlooked communities, the company has since grown into a customer management platform that effectively serves the needs of consumers throughout the socio-economic strata in Kenya and other emerging economies around the globe.

Mutuku stressed the importance of data science education. Future competitive data scientists must have complementary business skills and be part of a team that is capable of building a machine learning/data science pipeline to address real industry issues, not just fleeting desires.

Aisha Walcott of IBM Research-Africa delivers the opening address at the simulcast workshop on data mining in Africa, which took place in Nairobi, Kenya and was livestreamed during the 2019 SIAM International Conference on Data Mining in Calgary, Canada, earlier this year. Photo courtesy of Tonya Nyakeya.

Final Observations

Late into the evening hours as the workshop drew to a close, people continued to arrive. It is worth noting that the event was held on the day after a state holiday, and traffic from town was at an all-time high.

Many participants would likely agree that this pioneering workshop was a learning experience for the organizers, attendees, and SDM19 organizing committee. During several points throughout the program, vibrant conversations resonated with themes that resurfaced from SDM19's agenda.

Moving forward, an additional goal for the workshop is to stimulate communities of practice and learning, and allow distributed participants to contribute more seamlessly to the meeting objectives. We hope to explore approaches that will accomplish these tasks more effectively and support increasingly personal interactions. However, the ball is rolling; awareness is the step towards this aim. As members of the organizing committee, we were happy to facilitate one of three featured workshops at SDM19, but even more excited to report on developments in Africa on the world stage.

Reginald E. Bryant and Sekou L. Remy are research scientists at IBM Research-Africa. They were members of the organizing committee for the Simulcast Workshop on Dating Mining in/for Africa at the 2019 SIAM International Conference on Data Mining.

[1] https://h3africa.org/

Computational Mechanics

Continued from page 3

is their historical context? What problems drove their invention? Revisiting the conditions that inspired computational mechanics reveals how this history resonates with the ensuing science.

Part II of this article, to be published in the November issue of SIAM News, will detail the author's personal interest in computational mechanics and its extensions and applications to nonlinear physics.

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James P. Crutchfield teaches nonlinear physics at the University of California, Davis, directs its Complexity Sciences Center, and promotes science interventions in nonscientific settings. He is mostly concerned with patterns: what they are, how they are created, and how intelligent agents discover them. His website is http://csc.ucdavis.edu/~chaos/.

Scientific Machine Learning: How Julia Employs Differentiable Programming to Do it Best

By Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B. Shah

SOFTWARE AND PROGRAMMING

Julia, an advanced dynamic programming language for numerical computing, has solved the "two-language problem" by allowing developers to write low-level, high-performance code and high-level "scripting" code in a single language. It is significantly faster than Python and R, and considerably more productive than C, C++, and Fortran. In addition to these successes, Julia continues to bring novel technologies to computational scientists. One of its most exciting new capabilities is *differentiable programming* (∂P). Recent work to leverage Julia's compiler allows for the computation of efficient and accurate derivatives of arbitrary Julia programs. This is no mere party trick; program differentiation enables scientists to combine modern machine learning with the computational models developed from centuries of domain knowledge. Much as Julia provides the best of productivity and performance, ∂P offers scientific programmers tremendous benefits—in terms of both performance and accuracy—over black box approaches to machine learning.

```
using Pkg
Pkg.add("Zygote")
Pkg.add("ForwardDiff")
using Zygote, ForwardDiff

function s(x)
    t = 0.0
    sign = -1.0
    for i in 1:19
        if isodd(i)
            newterm = x^i/factorial(i)
            abs(newterm)<1e-8 && return t
            println("i=",i)
            sign = -sign
            t += sign * newterm
        end
    end
    return t
end
```

Figure 1. *Taylor series expansion for* $\sin(x)$.

At first glance, a casual practitioner might think that scientific computing and machine learning are different fields. Modern machine learning has made its mark through breakthroughs in neural networks. The applicability of such networks to solving a large class of difficult problems has led to the design of new hardware and software that process extremely high quantities of labeled training data while simultaneously deploying trained models in devices. In contrast, scientific computing—a discipline as old as computing itself—tends to use a broader set of modeling techniques that arise from underlying physical phenomena. Compared to the typical machine learning researcher, computational scientists generally work with smaller volumes of data but more computational range and complexity. However, deeper similarities emerge if we move beyond this superficial analysis. Both scientific computing and machine learning would be better served by the ability to differentiate, rather than the building of domain-specific frameworks. This is the purpose of ∂P.[1]

∂P concerns computing derivatives in the sense of calculus. Scientists have calculated derivatives since Isaac Newton's time, if not before, and machine learning has now made them ubiquitous in computer science. Derivatives power self-driving cars, language translation, and many engineering applications. Most researchers have likely written derivatives—often painfully by hand—in the past, but derivatives for scientific challenges are increasingly beyond reach.

Julia's approach has one overarching goal: enable the use domain-specific scientific models as an integral part of researchers' machine learning stack. This can be in place of—or in addition to—big training sets. The idea is profoundly compelling, as these models embody centuries of human intelligence. Julia makes it possible for scientists to apply state-of-the-art machine learning without discarding hard-won physical knowledge. In a recent blog post, Chris Rackauckas discusses the essential tools of modern scientific machine learning and finds Julia to be the language best suited for scientific machine learning [2].

Julia can perform automatic differentiation (AD) on eigensolvers, differential equations, and physical simulations. Loops and branches do not present obstacles.

[1] As 2018 Turing Award winner Yann LeCun said, "Deep learning est mort! Vive differentiable programming."

Researchers calculate derivatives in customized ways for a variety of problem areas, yet ∂P can greatly simplify the experience. We list a few and invite readers to share more:

1. Surrogate modeling: Running scientific simulations is often expensive because they evaluate systems using first principles. Allowing machine learning models to approximate the input-output relation can accelerate these simulations. After training neural networks or other surrogate models on expensive simulations once, researchers can use them repeatedly in place of the simulations themselves. This lets users explore the parameter space, propagate uncertainties, and fit the data in ways that were previously impossible.

2. Adjoint sensitivity analysis: Calculating the adjoint of an ordinary differential equation (ODE) system requires solving the reverse ODE `λ'=λ'*df/du+df/dp`. The term `λ'*df/du` is the primitive of backpropagation. Therefore, applying machine learning AD tooling to the ODE function f accelerates the scientific computing adjoint calculations.

3. Inverse problems: For many parameterized scientific simulations, researchers speculate about the parameters that would make their model best fit. This pervasive inverse problem is difficult because it requires the gradient of a large, existing simulation. One can train a model on a simulator, then use the simulator to quickly solve inverse problems. However, doing so requires generating massive amounts of data for training, which is computationally expensive. Scientists can learn much more quickly and efficiently by differentiating via simulators.

See **Julia** *on page 6*

Julia

Continued from page 5

4. Probabilistic programming: Inference on statistical models is a crucial tool. Probabilistic programming enables more complex models and scaling to huge data sets by combining statistical methods with the generality of programming constructs. While AD is the backbone of many probabilistic programming tools, domain-specific languages lack access to an existing ecosystem of tools and packages. In a general-purpose language, ∂P has the benefit of higher composability, access to better abstractions, and richer models.

Julia's flexible compiler—which can turn generic, high-level mathematical expressions into efficient native machine code—is ideal for ∂P. The `Zygote.jl` and `Cassette.jl` packages provide a multipurpose system for implementing reverse-mode AD, whereas `ForwardDiff.jl` provides forward mode. Julia runs efficiently on central processing units, graphics processing units (GPUs), parallel computers, and Google TPUs (tensor processing units), and is ready for future processors. Because of its composability, the combination of two packages that respectively offer the ability to conduct ∂P and run on GPUs automatically yields the capacity to perform ∂P on GPUs without additional effort.

∂P: Differentiate Programs, not Formulas: sin(x) Example

We begin with a very simple example to differentiate $\sin(x)$, written as a program through its Taylor series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

The number of terms is not fixed, but instead depends on x through a numerical convergence criterion. This runs on Julia v1.1 or higher (see Figure 1, on page 5).

While we could have written the Taylor series for sine more compactly in Julia, we used a loop, a conditional, a print, and function calls to illustrate more complex programs. AD simply works, and that is powerful. Let's compute the gradient at x=1.0 and verify that it matches cos(1.0) (see Figure 2).

∂P: Differentiating a Trebuchet

One may wonder if this concept operates successfully on a more complex example. What if we replace tabulated data with a physical model? An example that has captivated attention—perhaps due to its appearance in battle scenes from *Game of Thrones*—is the trebuchet, a medieval battle catapult (see Figure 3) [1]. We perform ∂P on the trebuchet, which—despite its deceptively-simple appearance—is a non-trivial system for the purposes of simulation. A differential equation models the distance of a projectile given angle, weight, and wind speed. The human operator can control the projectile's weight and angle for a set wind speed.

This illustration combines a neural network with the trebuchet's dynamics (see Figure 4). The network learns the dynamics in a training loop. For a given wind speed and target distance, it generates trebuchet settings (the mass of the counterweight and angle) that we feed into the simulator to calculate the distance. We then compare this distance to our target and backpropagate through the entire chain to adjust the network's weights. This is where ∂P arises. The training is quick because we have expressed exactly what we want from the model in a fully-differentiable way; the model is trained within a few minutes on a laptop with randomly-generated data. Compared to solving the inverse problem of aiming the trebuchet by conducting parameter estimation via gradient descent (which takes 100 milliseconds), the neural network (five microseconds) is 20,000 times faster. The `Trebuchet.jl` repository contains all the necessary code and examples.

The `DiffEqFlux.jl` project, which further explores many of the ideas in neural ODEs, is also noteworthy. For example, it allows use of an ODE as a layer in a neural network and provides a general framework for combining Julia's ODE capabilities with neural networks.

Leading institutions like Stanford University; the University of California, Berkeley; and the Massachusetts Institute of Technology continue to utilize Julia for both research and teaching in introductory and advanced courses. We hope our new ∂P capabilities will help researchers combine ideas in machine learning with those in science and engineering, and ultimately lead to novel breakthroughs.

Jeffrey Bezanson, Stefan Karpinski, and Viral B. Shah—creators of the Julia Language—received the James H. Wilkinson Prize for Numerical Software at the 2019 SIAM Conference on Computational Science and Engineering, which took place earlier this year in Spokane, Wash. The prize recognized Julia as "an innovative environment for the creation of high-performance tools that enable the analysis and solution of computational science problems."

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Jeff Bezanson earned his Ph.D. from the Massachusetts Institute of Technology (MIT) and currently designs programming languages for work and play. He is chief technology officer (CTO) in charge of language design at Julia Computing. Alan Edelman is a professor of mathematics, member of MIT's Computer Science & Artificial Intelligence Laboratory, and principal investigator of the Julia Lab. Stefan Karpinski is currently CTO in charge of open-source strategy at Julia Computing. He has worked as a data scientist and software engineer at Etsy, Akamai Technologies, and Citrix Systems. Viral B. Shah holds a Ph.D. from the University of California, Santa Barbara. He developed Circuitscape for conservation, is co-author of Rebooting India, and is the chief executive officer of Julia Computing.

Figure 3. *A trebuchet. Public domain image.*

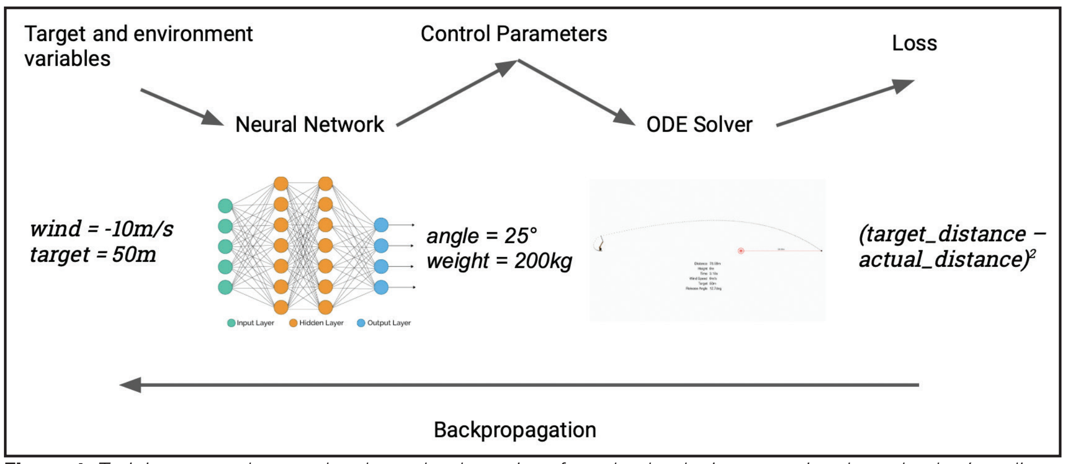

Figure 4. *Training a neural network to learn the dynamics of a trebuchet by incorporating the trebuchet's ordinary differential equations into the loss function. Figure courtesy of [1].*

```
julia> ForwardDiff.derivative(s, 1.0) # Forward Mode AD
i=1
i=3
i=5
i=7
i=9
i=11
0.540302303791887

julia> Zygote.gradient(s, 1.0) # Reverse Mode AD
i=1
i=3
i=5
i=7
i=9
i=11
(0.5403023037918872,)

julia> cos(1.0)
0.5403023058681398
```

Figure 2. *Forward and reverse mode automatic differentiation (AD) of the code in Figure 1. The AD packages seamlessly handle loops, conditions, function calls, and much more.*

The Difficulties of Addressing Interdisciplinary Challenges at the Foundations of Data Science (Part II of II)

By *Michael W. Mahoney*

Part I[1] of this article, which published in the September issue of SIAM News, described the National Science Foundation's (NSF) Transdisciplinary Research in Principles of Data Science (TRIPODS) program, as well as structural, justification, and cultural challenges that arise from this effort.

While perhaps not immediately obvious, many seemingly innocuous decisions can provide a strong selection bias toward/against certain areas in terms of interdisciplinary efforts. The following actions all possess the potential for such selection bias: encouraging interdisciplinary interactions without understanding conflicting recognition and reward requirements; expecting publication immediately after one has vocalized an idea versus after he/she has clarified all of the details; deciding on a complex interdisciplinary effort's final form quickly versus deliberately; requiring attendance throughout most or all of a long program; and listing authors alphabetically, based on contribution, or according to some other rule. Ignoring these issues—albeit often inadvertently—invariably undermines interdisciplinary efforts. Encouraging the research community to address these concerns in ways that draw strength from the diversity of researchers—and do not undermine their cultural sensibilities—continues to be a primary challenge.

Broader NSF Context

Discussions on this topic at the 2016 workshop on Theoretical Foundations of Data Science (TFoDS): Algorithmic, Mathematical, and Statistical[2] occurred within a broader context of conversations at the NSF regarding the organization's long-term research agenda. That same year, the NSF proposed its 10 Big Ideas,[3] a set of "long-term research and process ideas that identify areas for future investment at the frontiers of science and engineering." One of these ideas—"Harnessing the Data Revolution" (HDR)[4]—focuses on both fundamental research in data science and engineering as well as the development of a 21st-century, data-capable workforce to help researchers exploit the Big Data revolution. This was part of the NSF's effort toward "Growing Convergence Research," another big idea that seeks to integrate multiple disciplines to advance scientific discovery and innovation. The timing was right, and the NSF's first major investment toward HDR was the TRIPODS program.[5]

With TRIPODS—and following the suggestions of the TFoDS workshop report—the NSF made a call for institutes on the foundations of data science. However, it was not overly-prescriptive as to what that means. Instead, it split the program into two phases—Phase I and Phase II (described in Part I of this article)—so that the community could determine what it desired from an institute that spanned three rather culturally different areas. This two-phase structure also permits a ramp-up period before full-scale institute activities—like research, education, workforce development, visitor hosting, and direction setting—begin. Phase I principal investigators (PIs) are addressing additional challenges, such as how to design institutes that do not grate against the standards of one or more of the communities, and instead yield a true synergy of all of the three disciplines' best capabilities.

Each of the 12 Phase I institutes approaches these challenges and its individual mission in somewhat different ways, ultimately acting as a type of "experimental trial" for struggles and successes. One of TRIPODS' more unusual aspects is the occurrence of a monthly PI call and annual PI meeting. These allow for frank discussion of what is and is not working at different Phase I institutes and within the general community. Indeed, one of TRIPODS' most valuable aspects is its facilitation of a camaraderie between leading researchers with diverse backgrounds interested in similar challenges.

Interdisciplinary and "Antedisciplinary" Balance

TRIPODS' emphasis on interdisciplinary foundations and its address of cultural challenges associated with cross-cutting research are discussion points at the PI calls and meetings.

The three core areas of TRIPODS—statistics, mathematics, and theoretical computer science—focus on questions that are relevant to the theoretical foundations of data science, but they do so in very different and sometimes incomparable ways. Each Phase I institute is adopting its own approach. The Foundations of Data Analysis Institute at the University of California, Berkeley will initially concentrate on four deep theoretical challenges: the possibility of a general complexity theory of inference in the context of optimization; the power of stability as a computational-inferential principle; the value of randomness as a statistical and algorithmic resource in data-driven computational mathematics; and the principled combination of science-based and data-driven models. These foundational challenges straddle existing cultures of disciplinary research. Each is situated squarely at the interface of theoretical computer science, theoretical statistics, and applied mathematics; and each is directly relevant to a wide range of very practical data science problems.

From the perspective of addressing cultural challenges associated with cross-cutting research, TRIPODS and the accompanying "TRIPODS model" offer the very real possibility of a "test case" on engineering funding for what Sean Eddy of Harvard University calls "antedisciplinary science" (where "ante" means "before," not "anti" as in "against," although the two are clearly related). He defines this as "the science that precedes the organization of new disciplines, the Wild West frontier stage that comes before the law arrives" [1]. The way in which data science will evolve—e.g., whether the field will look more like present-day computational science or computer science—remains to be seen. Nevertheless, Eddy's discussion, which relates to the National Institutes of

See **Data Science** *on page 9*

[1] https://sinews.siam.org/Details-Page/the-difficulties-of-addressing-interdisciplinary-challenges-at-the-foundations-of-data-science-part-i-of-ii

[2] http://www.cs.rpi.edu/TFoDS/

[3] https://www.nsf.gov/news/special_reports/big_ideas/

[4] https://www.nsf.gov/cise/harnessingdata/

[5] https://www.nsf.gov/news/news_summ.jsp?cntn_id=242888

One Year Amounts to a Career of Countless Opportunities

By Jennifer Pearl

I am many things: a theater fan, a big-picture thinker, and a mathematician. More specifically, I am a mathematician who takes great joy in working at the intersection of policymaking and science, technology, engineering, and mathematics (STEM). As director of the Science & Technology Policy Fellowships (STPF) of the American Association for the Advancement of Science (AAAS)—a program that recruits and places doctoral-level STEM professionals in government agencies—I am fortunate to utilize my background as part of an important opportunity to serve the country.

Mathematicians are in high demand in Washington, D.C. They understand the difference between causation and correlation, and bring a skeptic's mindset to the table; these characteristics help inform sound policy decisions.

Like math, public policy aims to solve problems — specifically those pertaining to a group of people. One can best solve such challenges by considering all potential impacts and providing evidence for the effectiveness of any given solution. Once a policy is adopted, practitioners must consider issues such as the allocation and distribution of resources necessary to achieve a goal. Government policymaking runs the gamut of societal affairs, from education and defense to fiscal practices and human health. The latest advancements in mathematical methods contribute to better the government through modeling, problem-solving, economic decision-making, and risk assessment. The result is a government in constant (and growing) need of analytical expertise.

Mathematical physicist and STPF fellow Gordon Aiello spoke aptly about the matter. "A common thread among applied mathematicians is a desire to work on stimulating, interdisciplinary questions that arise from real-world demands," he said. "The scope and magnitude of the challenges being addressed at the federal level provide fellows with a treasure trove of topics to pursue and learn from — with the added benefit of knowing that their work will be used to inform policy for the sake of our nation's posterity." Aiello began his placement at the U.S. Department of Defense (DoD) Office of Monetary Affairs in September.

The STPF program offers participants hands-on professional development in public policy and provides the government with increased access to STEM professionals. Fellows develop the critical ability to communicate recommendations and solutions in all settings, ranging from the policy world to private and multilateral sectors. They also make connections that expand their professional networks by an order of magnitude!

Fellows are outstanding mathematicians, statisticians, scientists, and engineers at any career stage — from newly-minted Ph.D.s to seasoned professionals. The year-long program runs from September through August and enrolls over 250 individuals who represent a broad range of backgrounds and disciplines.

More than two dozen scientific partner societies sponsor fellowship placements in Congress. SIAM works with other mathematical and computer science organizations to recruit data science fellows, and the American Mathematical Society (AMS) finances a congressional fellow. The AAAS sponsors numerous placements in Congress and roughly 20 executive and judicial branch agencies each year, which are open to professionals from all STEM disciplines.

Fellows are sprinkled across Washington in all three branches of government, where they work on nearly every issue under the sun. There is no one "typical" fellowship, though all involve an immersive experience that teaches participants to utilize analytical skills for the betterment of the country and introduces them to a lifelong network of scientists with similar motivations.

As a fellow in the Department of Defense's Office of International Cooperation, Jessica Libertini coordinated the signing of the Chapeau Agreement that allows for the U.S. and the Netherlands to more rapidly enter into bilateral defense cooperation agreements. The treaty was signed by the ambassadors and witnessed by Dutch Prime Minister Mark Rutte. Here Libertini fills in the treaty's date and location information at the Dutch Embassy. Photo courtesy of Jessica Libertini.

Applied mathematician and faculty member Jessica Libertini recently completed a fellowship with a plan to return to academia and help applied mathematics and engineering students consider a broader set of career paths. As a fellow in the DoD's Office of International Cooperation, she strengthened DoD links with foreign partners in places like Indonesia, Singapore, and Lithuania, and coordinated a bilateral treaty between the U.S. and the Netherlands. "I am now equipped to teach math using problems that are not only meaningful, but also couched in realistic ways," she said. "These problems demonstrate the value of mathematics far better than pushing symbols around a page. My students may be involved in policy one day, and I want to increase the chances of mathematicians making impactful decisions."

Sponsored by the AMS, applied mathematician Margaret Callahan served one year in the office of Senator Amy Klobuchar, where she quickly got up to speed on her portfolio issues: education, workforce development, and public health. Callahan is currently a fellow at the U.S. Department of State, where she evaluates data on global conflict and protests in Venezuela and the Central African Republic to map international security challenges and inform policy development. She credits the fellowship for providing her with an opportunity to advocate for science and bring scientific rigor to the policymaking process.

During his fellowship at the National Science Foundation (NSF) Directorate for Computer & Information Science & Engineering, algebraist Tyler Kloefkorn witnessed firsthand both the NSF's methods to support basic research in science, as well as mathematics' value in science and science policy. He also learned how to integrate best practices in education and research from the larger science arena into the mathematics discipline. Partly due to his experience at the NSF, Kloefkorn now routinely uses linear algebra and statistics to better understand problems and communicate solutions. Today he is a program officer at the National Academies of Sciences, Engineering, and Medicine.

After completing the program, STPF fellows become members of a closely-knit corps of 3,000+ alumni comprised of policy-savvy STEM leaders in academia, government, industry, and nonprofit arenas. The STPF alumni body includes numerous noteworthy names. Karoline Pershell—current executive director of the Association for Women in Mathematics—served in the U.S. Department of State and now directs strategy and evaluation at a technology company. Carla Cotwright worked in the U.S. Senate and is presently a data scientist at the DoD. In 2015, former DoD fellow DJ Patil was appointed as the nation's first-ever chief data scientist, a position he held for two years. Catherine Paolucci served as a fellow in Congress and at the NSF, and recently accepted a faculty position at the University of Florida.

It is worth mentioning that I, too, was a STPF fellow! The fellowship opened my eyes to the amazing breadth of work and expertise required by the federal government, and became a pivotal point in my career.

The great thing about mathematics is that there is no limit to the number of ways one can use it. Some element of math is baked into nearly everything, like federal agencies' allocation of funds to improve health, or modeling the optimal supply of passports for individuals whose identification was lost in national disasters. Additionally, the broader ability to examine an ill-defined or complicated system and create some structure to help understand it is valuable in all areas of government.

The question is, what do you want to do with *your* math? Do you want to understand how your skills can be beneficial in a federal context? Are you interested in helping shape good policy? Learn more about STPF fellowships[1] or register for a live chat with fellows.[2] The deadline to apply for fellowships is November 1.

Jennifer Pearl is director of the Science & Technology Policy Fellowships (STPF) program of the American Association for the Advancement of Science (AAAS). Prior to coming to the AAAS, Pearl spent 12 years at the National Science Foundation, where she served in the Division of Mathematical Sciences and the Office of International Science & Engineering. She was an AAAS STPF fellow from 2002 to 2003.

[1] http://www.stpf-aaas.org/
[2] https://www.aaas.org/news/2019-chat-series

A Mirage on the Road and the Poincaré Metric

Here I present a few miscellaneous observations prompted by the boredom of a long drive on a sunny summer day.

Estimating the Temperature of a Hot Highway Surface from the Distance to a Mirage

With unabashed departure from reality, I assume a perfectly flat highway and an air temperature that is solely dependent on height. The hot air near the asphalt is less dense and supports a higher speed of light. According to Fermat's principle, light picks the path of least time; it is therefore beneficial for the rays to bend towards the highway, where they travel faster and save time (see Figure 1). With the temperature—and thus the speed of light—depending only on height, Snell's law (derivable from Fermat's principle) applies. According to this law, the sine of angle θ between the tangent to the ray and the vertical varies along any ray in direct proportion to the speed of light:

$$\sin\theta / c = \text{const.} \qquad (1)$$

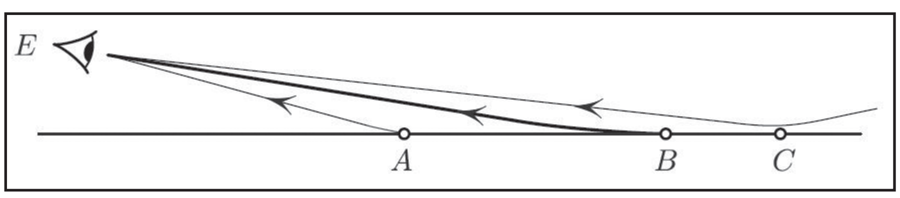

Figure 1. B is the farthest point of the highway visible to the eye E.

We can roughly estimate the index of the hot air's refraction at the hot road surface. Let B in Figure 1 be the farthest point on the road surface from the eye E that is still visible to E. The ray BE is hence tangent to the road at B, so that $\theta_B = \pi/2$. By Snell's law, $\sin\theta_E / c_E = \sin\theta_B / c_B = 1/c_B$, so that the indices of refraction (the reciprocals of the speed of light) satisfy

$$n_B = n_E \sin\theta_E. \qquad (2)$$

We can now eyeball $\sin\theta_E$ as the ratio of E's height to distance EB, and thus estimate n_B/n_E. This ratio is a monotone function of the road's temperature, which one can therefore estimate in principle.

Interestingly, the way in which c depends on height does not affect angle θ_E; only the values of c at the ray's endpoints matter. Of course, all of this is highly idealized due to the assumptions established at the outset.

Rays Near the Road and the Poincaré Metric

Assuming the speed of light c to be a linearly decreasing function of height (presumably not too unreasonable an assumption for a small strip near the hot road surface), what is the shape of the rays? By Fermat's principle, a ray traveling between two points picks the path γ that minimizes[1] travel time $\int_\gamma dt = \int_\gamma ds/c$. Using dilation and translation, we can achieve $c(y) = y$ up to a constant factor; y measures the (rescaled) downward distance from some horizontal line above the ground (see Figure 2). This is the line where c would have vanished if the speed of light had continued its decrease with increasing height beyond the narrow strip near the road surface. We choose the line $c=0$ as the zero level and count the distance y downwards from it.

[1] More precisely, the path is a minimum of the time functional only if there are no conjugate points along the ray connecting the two points. For $c(y) = y$, this no-conjugate-points condition automatically holds for any pair of endpoints (the Poincaré metric has negative curvature) [1].

The travel time along γ is $\int_\gamma ds/y$, up to the scaling factor. This is precisely the length of γ in the Poincaré metric ds/y! The Poincaré metric is thus the simplest imaginable non-Euclidean metric, corresponding to the simplest nonconstant speed of light: linear function $c(x,y) = \alpha x + \beta y$, reducible to $c(y) = y$ by rotation and dilation.

MATHEMATICAL CURIOSITIES
By Mark Levi

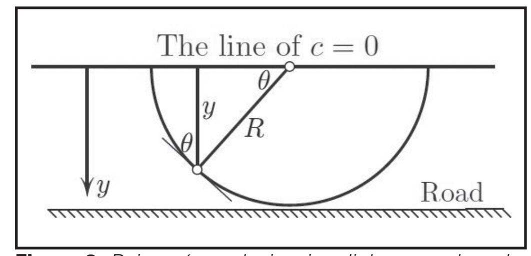

Figure 2. *Poincaré geodesics, i.e., light rays when the speed of light $c(y) = y$, are semicircles: proof with few words.*

Why are the Poincaré Geodesics Circular?

Consider an arbitrary ray with the speed of light $c = c(y) = y$; the ray satisfies Snell's law (1). Let us presumptuously denote the constant in the right-hand side of (1) by R^{-1}, so that our ray satisfies

$$\frac{\sin\theta}{y} = \frac{1}{R}, \quad \text{i.e.,} \quad R\sin\theta = y.$$

But a semicircle of radius R centered at a point on the line $y=0$—as in Figure 2—satisfies the exact same relationship as the ray. Now this relationship is a first-order ordinary differential equation in disguise for the function $y = y(x)$, where x is the horizontal coordinate. According to the uniqueness theorem, our ray coincides with the semicircle that shares a point and a tangent to the ray at that point. We have thus shown that Poincaré geodesics/light rays are semicircular arcs centered at the zero velocity line.

As a concluding remark, the upside-down counterpart of internal reflection of the highway mirage is the internal reflection observed by divers (see Figure 3).

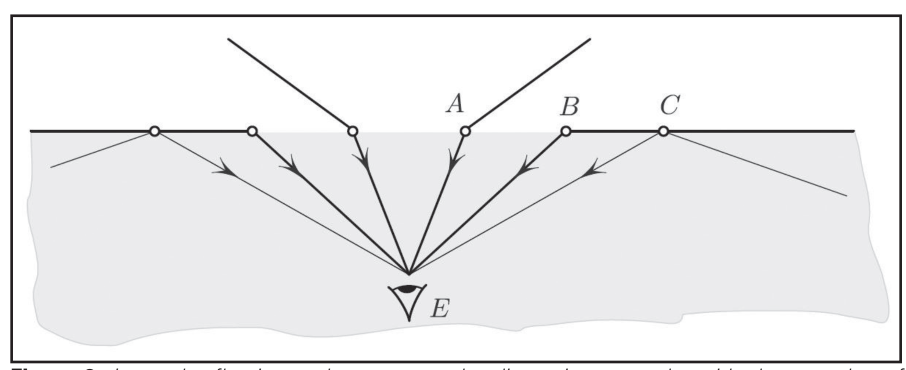

Figure 3. *Internal reflection under water — the discontinuous and upside-down analog of the highway mirage.*

The figures in this article were provided by the author.

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Celebrating James H. Wilkinson

On the occasion of the centenary of the birth of James H. Wilkinson (1919-1986), the University of Manchester held a conference to discuss current developments and future challenges in the field of numerical linear algebra. Incidentally, four of SIAM's Past Presidents were in attendance. The two-day event, titled "Advances in Numerical Linear Algebra: Celebrating the Centenary of the Birth of James H. Wilkinson," featured several invited talks. Sven Hammarling (University of Manchester) shared his personal thoughts on Wilkinson in his opening address. Margaret Wright (New York University) revealed interesting anecdotes from Wilkinson's lecture notes pertaining to courses he taught at Stanford University. Nicholas Higham (University of Manchester) described a series of talks that Wilkinson delivered with Cleve Moler (MathWorks) in 1973 during an Eigensystem Workshop at Argonne National Laboratory, videos of which are now available.[1]

Slides from the various talks are available on the conference webpage,[2] and select videos are accessible via YouTube.[3] Read more about the event online.[4]

[1] https://nickhigham.wordpress.com/2019/06/05/the-argonne-tapes/
[2] https://nla-group.org/advances-in-numerical-linear-algebra-2019/
[3] https://www.youtube.com/playlist?list=PLACiWdJ40LRNjhKDxpNzwW16yt0Fd8cPi
[4] https://nla-group.org/2019/06/19/highlights-of-advances-in-numerical-linear-algebra-conference/

Four SIAM Past Presidents gathered at "Advances in Numerical Linear Algebra: Celebrating the Centenary of the Birth of James H. Wilkinson" a conference that took place at the University of Manchester earlier this year. From left to right: Cleve Moler (MathWorks), Margaret Wright (New York University), Lloyd N. Trefethen (University of Oxford), and Nicholas Higham (University of Manchester). Photo courtesy of Gian Maria Negri Porzio.

SIAM News and *SIAM Review* Go Green

To help reduce SIAM's carbon footprint and lower costs, SIAM members will be able to modify the delivery options for their *SIAM News* and *SIAM Review* (*SIREV*) subscriptions, starting with this fall's membership renewal cycle. Individuals can also change their delivery options on their online profiles.[1]

Free student members will need to opt in to continue receiving print issues of *SIAM News*. All issues of *SIAM News*—along with supplemental blog articles—are freely available online,[2] and table-of-contents notifications are emailed to all SIAM members and individuals subscribed to *SIAM News* alerts. Subscribe to *SIAM News* alerts here.[3]

SIAM members who currently receive a paper copy of *SIREV* will have the option to opt out of receiving print copies, but will retain access to the online version, which is freely available to all members.

[1] https://my.siam.org
[2] https://sinews.siam.org/
[3] https://sinews.siam.org/Subscribe

Data Science

Continued from page 7

Health's funding of computational biology is relevant to the discussion of cross-cutting research in the foundations of data and beyond. "Focusing on interdisciplinary teams instead of interdisciplinary people reinforces standard disciplinary boundaries rather than breaking them down," he writes. "An interdisciplinary team is a committee in which members identify themselves as experts in areas besides the actual scientific problem at hand, and abdicate responsibility for the majority of the work because it's not in their field."

Many TRIPODS PIs are wrestling with these challenges, both in their own research agendas and their efforts to create Phase II TRIPODS institutes that are broadly useful to the community. Are the specifics of the current Phase I–Phase II structure the best way to coalesce community understanding of what should comprise a longer-term institute (either for foundations of data or more general cross-cutting challenges)? How can such institutes encourage interdisciplinary people as well as interdisciplinary teams? How can they highlight the broad usefulness of interdisciplinary foundational work without diluting its foundational content? How can we ensure that current design decisions do not deter substantial participation by one of the disciplines of interest? Of course, more obvious issues—ensuring that the whole is more than the sum of its parts and exploring novel ways to extend NSF funding—also exist.

While many challenging questions remain, participants of the current TRIPODS program are diligently tackling these questions to establish the foundations of data science.

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Michael W. Mahoney is affiliated with the International Computer Science Institute and the Department of Statistics at the University of California, Berkeley.

ICIAM 2019 Panel Explores Academic and Industrial Careers in Mathematical Sciences

By Sven Leyffer, Volker Mehrmann, Jill Pipher, Amy E. Radunskaya, Wil Schilders, and Carlos Vázquez

The 9th International Congress on Industrial and Applied Mathematics (ICIAM 2019) in Valencia, Spain, featured a special panel on mathematical science careers in academia and industry. Panelists Sven Leyffer (Argonne National Laboratory), Volker Mehrmann (European Mathematical Society (EMS) and Technische Universität Berlin), Jill Pipher (American Mathematical Society and Brown University), Ami E. Radunskaya (Association for Women in Mathematics and Pomona College), and Wil Schilders (European Consortium of Mathematics in Industry (ECMI) and Eindhoven University of Technology) represented a wide range of backgrounds and institutions. Carlos Vázquez (University of A Coruña) organized the panel.

Panelists engaged in hearty discussion about the present and future role of mathematicians in academia and industry, as well as possible improvements in the training of applied mathematicians to ensure successful careers in both sectors.

U.S.-based Careers in the Mathematical Sciences

Pipher opened the panel by reporting on AMS data pertaining to numbers and field distribution of Ph.D.s, career trajectories, demographic dispersion, and tendencies in the mathematical and statistical sciences. Notable trends included the slight but possibly significant decline in the percentage of Ph.D.s awarded to women over the last decade; the continued dearth of advanced degrees in mathematics among underrepresented minorities in the U.S.; the growth in non-research faculty positions at research universities; and increased student demand for mathematics, computational/statistical, and computer science education relevant to data science careers.

For some years, the U.S. job market for newly-minted Ph.D.s has offered many more postdoctoral positions in academia than follow-up, tenure-track jobs. Pipher briefly discussed ways in which mathematics departments can prepare students for the various rewarding non-academic careers accessible to them. SIAM's career resources page[1] is a terrific repository of information for students looking to learn about specific fields, job openings, and companies that routinely hire mathematicians.

European Careers in the Mathematical Sciences

Mehrmann spoke about the substantial inhomogeneity that currently exists in Europe for both the academic and industrial job markets. In some countries, companies have trouble filling positions because many options are available for industry careers. In other countries, no job openings are available. A similar situation is occurring in academia; some European countries offer ample opportunities at the assistant professor level but fewer prospects for higher positions, while others have almost no reasonably-paid openings. The EMS and the European Service Network of Mathematics for Industry and Innovation (EU-MATHS-IN)[2] provide job portals for prospective employees. Many national societies also actively involve more young scientists in the organizations themselves, and work to rectify the gender imbalance. However, the asymmetrical situation in Europe causes much south-north movement among early-career professionals.

Careers in All Sectors

Radunskaya reminded attendees that mathematicians are needed in all facets of society. For example, teaching mathematics to young students is critical and should be considered a valued career trajectory. Mathematicians also play a role in policymaking and decision-making for nearly every area and endeavour, including medicine and energy. Professional societies and individual mathematicians must expose the broader community—from neighbors to politicians and business owners—to the utility and joyful playfulness of mathematics. A better awareness of mathematics' role in society might also encourage heightened participation from groups that are currently underrepresented in the field.

CAREERS IN MATHEMATICAL SCIENCES

Careers at National Laboratories

Leyffer reported on careers within the U.S. and European national laboratories networks. 14 Department of Energy labs exist in the U.S., and a similar number of institutes—such as the Max-Planck[3] and Fraunhofer[4] institutes and research units in Germany, the National Institute for Research in Computer Science and Automation (INRIA)[5] in France, and the Rutherford Appleton Laboratory[6] in the U.K.—operate in Europe. The U.S. laboratories offer a range of opportunities for young mathematicians, like student internships and prestigiously-named postdoctoral fellowships that allow participants to pursue independent research agendas within the labs for up to two years. These positions are open to applicants from most countries, and many labs often hire new employees from the pool of former students and postdocs. Staff members tend to work on 100 percent soft-money projects and collaborate with domain and computer scientists. These projects are often large, multi-institutional, and multidisciplinary, thus requiring participants who can both assimilate new information quickly and communicate well with other domain scientists. Current applications include the power grid in addition to data analysis for physics experiments and beamlines.

Unlike academic job environments, the labs specifically emphasize the development of open-source, production-quality software that runs on high-performance computing systems. Leyffer expects future projects to pivot towards artificial intelligence and machine learning for physics-based models, as well as heterogeneous and emerging computer architectures like quantum, neuromorphic, and low-precision devices.

Laboratories typically require that their employees have Ph.D.s as evidence of inde-

See **ICIAM 2019 Panel** *on page 11*

[1] https://www.siam.org/careers/resources
[2] https://www.eu-maths-in.eu/EUMATHSIN
[3] https://www.mpg.de/en
[4] https://www.fraunhofer.de/en/institutes.html
[5] https://www.inria.fr/en/
[6] https://stfc.ukri.org/about-us/where-we-work/rutherford-appleton-laboratory/

ICIAM 2019 Panel

Continued from page 10

pendent thinking and the ability to serve as principal investigators on research grants. Communication and negotiation skills are important when working in a multidisciplinary environment, as is overall confidence when searching for and inquiring about job opportunities. Companies often hire mathematicians for their analytical abilities, not for specific knowledge in a narrow area. Over time, applied mathematicians in national labs become jacks-of-all-trades and branch out into new fields. Leyffer observed that lab salaries are typically competitive with industry; this is especially true for applied mathematicians, whose salaries are on par with computer scientists.

Industrial Careers in the Mathematical Sciences in Europe

Schilders discussed ways in which EU-MATHS-IN supports mathematics careers within industry. The aforementioned job portal on its website is dedicated to maths appointments in the industrial sector. Students can browse job opportunities and industrial companies can post open positions. Appointments related to the European Union's European Industrial Doctorates program are also posted here; these positions require that Ph.D. students spend at least 50 percent of their time in industry. Schilders, who spent 30 years in industrial environments (first at Philips in Eindhoven and then at NXP Semiconductors), also explained the dual ladder system that makes it possible for researchers to pursue careers as managers and experts.

The ECMI, which is closely related to EU-MATHS-IN, organizes so-called European Study Groups with Industry (ESGI) in many European countries. These weeklong workshops begin with presentations of five or six industrial challenges. The participating mathematicians then split into groups, work on the challenges all week, and present their final results to the industrial participants. ESGI is one of the best ways to get involved with industry, and can even lead to job offers and industrial careers.

Schilders also provided information on the yearly "Speed Dating with Industry" event at the Dutch Mathematical Congress, which provides a space for students to ask questions and converse with industry representatives about job opportunities. The Agency for Interaction in Mathematics with Business and Society—the French network within EU-MATHS-IN—organizes career fairs each year that draw 50 to 60 companies and 1,200 to 1,400 students from all across France.

Open Q&A Session

Following their initial presentations, the panelists engaged with an active audience on a range of questions about mathematical science careers in academia and industry, and a comparison between the two.

Addressing a question about the number of mathematics positions at universities and research centers, Leyffer cited the growth in artificial intelligence and machine learning at Google, Microsoft, and U.S. schools like the Georgia Institute of Technology and Lehigh University. He also added that applied mathematicians can find relevant opportunities in industrial engineering, statistics, operations research, and computer science departments.

Mehrmann spoke about the European Union's industrial mathematics professorships, which frequently attract candidates with industrial experience. He talked specifically about the transition between sectors, noting that postdoctoral researchers from labs often move back to academia, and even hold joint appointments with an emphasis on industry.

Schilders admitted that moving from industry to academia is generally difficult and usually happens late in one's career. In many cases, academic appointments in the Netherlands involve only one or two days per week. If one hopes to return to academia from industry, he/she must maintain an updated academic profile by attending conferences and publishing papers whenever possible. Leyffer added that transitioning to academia is typically easier in industrial engineering departments or national labs, whose cultures are most similar to industry.

During his closing remarks, Mehrmann shared a telling statistic; several recent studies have estimated that applied mathematics' contribution to a country's gross domestic product is between 15 and 25 percent, which is a strong base for a healthy profession. He encouraged the audience to urge ICIAM member societies to foster applied math by putting more effort into support, advertisement, and education of young scientists.

Sven Leyffer is a senior computational mathematician at Argonne National Laboratory who works on nonlinear optimization. He currently serves as secretary of the International Council for Industrial and Applied Mathematics. Volker Mehrmann is a professor of numerical mathematics at the Technische Universität Berlin, president of the European Mathematical Society, and co-founder of the European Service Network of Mathematics for Industry and Innovation (EU-MATHS-IN). Jill Pipher is Vice President for Research and Elisha Benjamin Andrews Professor of Mathematics at Brown University. She began her term as president of the American Mathematical Society in February 2019. Ami E. Radunskaya is a professor of mathematics at Pomona College. She is the most recent past-president of the Association for Women in Mathematics and is a co-director of the EDGE Program (Enhancing Diversity in Graduate Education). Wil Schilders is a professor of scientific computing for industry at the Eindhoven University of Technology, director of the Dutch Platform for Mathematics, and co-founder and current president of EU-MATHS-IN. Carlos Vázquez is a full professor of applied mathematics at the University of A Coruña and a member of the organizing committee for the 9th International Congress on Industrial and Applied Mathematics (ICIAM 2019).

Participants share their insights during the "Careers in Mathematical Sciences to Academia and Industry" panel at the 9th International Congress on Industrial and Applied Mathematics (ICIAM 2019), which took place earlier this year in Valencia, Spain. From left to right: Wil Schilders (European Consortium of Mathematics in Industry and Eindhoven University of Technology), Sven Leyffer (Argonne National Laboratory), Ami Radunskaya (Association for Women in Mathematics and Pomona College), Volker Mehrmann (European Mathematical Society and Technische Universität Berlin), Jill Pipher (American Mathematical Society and Brown University), and Carlos Vazquez (University of A Coruña). Photo courtesy of ICIAM 2019 organizing committee.

Multiscale Simulation of Flow and Transport in Porous Media

By Olav Møyner

Flow in porous media is omnipresent. It exists in blood running through our capillaries, air moving through our lungs, humidity in the clothes we wear and sanitary products we use, fertile soil that sustains our crops, subsurface aquifers and filter systems that provide clean water, and deep geological formations that yield hydrocarbons and thermal energy. Modelling to describe and understand such systems is therefore significant. Here I discuss fluid flow in porous rock, which is relevant to the geosciences, climate studies, and various other areas.

Conservation of mass and Darcy's law describe the flow of a single, incompressible fluid as follows:

$$\nabla \cdot \vec{v} = q, \qquad \vec{v} = -\mathbf{K}\nabla p, \tag{1}$$

where $\vec{v}$ is discharge per unit area (often known as Darcy flux or velocity), p signifies fluid pressure, and q represents volumetric source terms.

Alternatively, we can write (1) as a second-order, elliptic Poisson equation $-\nabla \cdot (\mathbf{K}\nabla p) = q$. The positive-definite tensor $\mathbf{K}$—called permeability—represents the rock's ability to transmit fluids, which is key to determining flow patterns in aquifers and hydrocarbon reservoirs. Sedimentary rocks develop over millions of years via a layering process that compacts different sediments into a highly complex structure. As a result, permeability exhibits a strong multiscale nature—with a wide range of spatial correlations that have no clear scale separation—and up to eight or nine orders-of-magnitude variations. This makes it surprisingly difficult to discretize and solve (1) with sufficient accuracy, since important changes often occur over distances that may be significantly shorter than feasible grid sizes in flow simulators.

The mathematical geosciences community has therefore devised a number of upscaling or homogenization techniques that aim to capture essential features of the permeability on a coarse scale through a mixture of analytical and numerical methods. Over the past two decades, many researchers have specifically focused on so-called *multiscale methods* that attempt to solve reduced-order pressure equations on a coarse scale — using numerically-computed basis functions that consistently incorporate subscale permeability variations. If (1) is discretized with a cell-centered, finite-volume scheme as linear system $A\mathbf{p} = \mathbf{q}$ for fine-scale pressure $\mathbf{p}$, a multiscale method instead seeks to solve a coarser problem. This occurs via the approximation $\mathbf{p} \approx B\mathbf{p}_c$, where $\mathbf{p}_c$ is a pressure that one can map onto the fine scale by basis matrix B. Applying linear restriction operator R finds the coarse system

$$A_c\mathbf{p}_c = \mathbf{q}_c, \quad A_c = RAP, \quad \mathbf{q}_c = R\mathbf{q}. \tag{2}$$

Naturally, the approximation's quality and efficiency strongly depend on the choices one makes to compute B's basis functions.

This procedure is a specialized algebraic two-level solver and does not offer much computational gain over a standard algebraic multilevel solver if we only wish to compute the pressure once. The main interest in multiscale methods arises from multiphase flow simulations, illustrated as follows by the simple prototype problem of incompressible, immiscible, two-phase flow:

$$\nabla \cdot \vec{v} = q_p, \quad \vec{v} = -\lambda(S)\mathbf{K}\nabla p,$$
$$\phi\frac{\partial S}{\partial t} + \nabla \cdot (\vec{v} f(S)) = q_s. \tag{3}$$

Here, the total mobility λ models permeability reduction in the presence of multiple fluid phases, and S is the volume fraction of the wetting phase. To solve (3) and advance p and S in time, one must first solve the Poisson problem $-\nabla \cdot (\lambda\mathbf{K}\nabla p) = q$ multiple times with variations in λ, source terms, and boundary conditions. Additionally, (3) frequently appears as a forward problem in the highly heterogeneous, data-dense problems that pertain to uncertainty quantification, forecasting, and optimization.

Many research papers have proposed and analyzed various forms of multiscale methods for solving (3). However, a significant gap exists between the idealized, regular, structured grids in the literature and the highly-complex grid models that geologists use to describe real rock formations. The most widespread format in industry is a semi-structured design composed of hexahedra that may degenerate, shift up and down, or collapse to zero volume to account for erosion, faults, and other deformations in the deposited rock volumes.

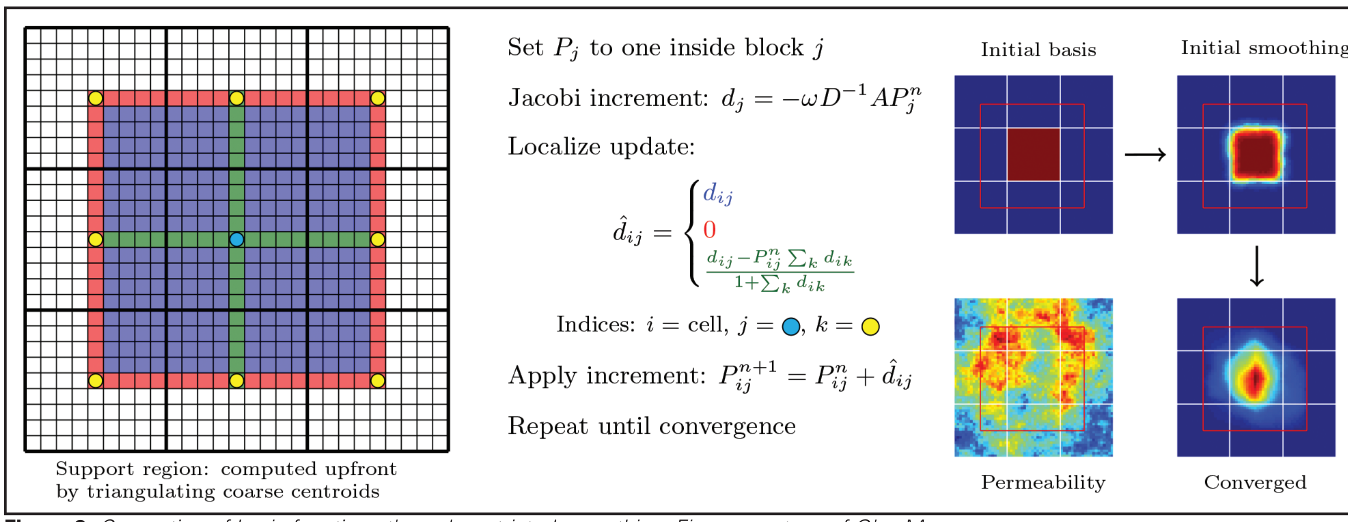

Figure 2. *Generation of basis functions through restricted smoothing. Figure courtesy of Olav Møyner.*

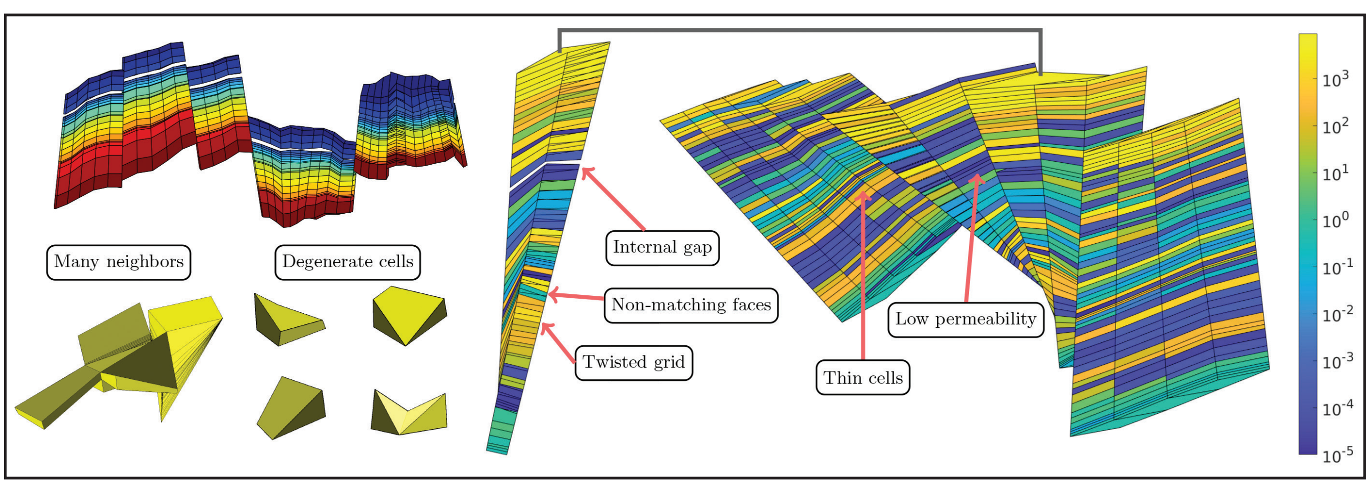

Figure 1. *Illustration of some of the challenges present when grids model geological porous media. Figure courtesy of Olav Møyner.*

Scientists also frequently build grids from more unstructured elements; such grids may contain local refinements that adapt to well trajectories and important geological characteristics. One can normally assume that a typical grid will consist of general polyhedral cells, non-matching interfaces, large aspect ratios, multiple neighboring cells, and several orders of magnitude of variation in cell-to-cell permeability. Figure 1 depicts a few representative examples taken from engineering models. Posing local problems to produce acceptable basis functions on these grids is a substantial challenge that is essential to implementation of multiscale methods for various applications, including engineering computations in hydrocarbon recovery, carbon dioxide sequestration, geothermal energy processes, and natural gas storage. Furthermore, typical industrial applications use multiphase, multicomponent flow models that are significantly more intricate than (3) and harbor further complexity and submodels that account for wells and various forms of surface and control facilities.

As a part of an ongoing industrial collaboration, we worked to adapt existing multiscale methods to more general grids. After several partial successes, we discovered a method [3] that was sufficiently robust on general grids. The approach—illustrated in Figure 2—is based on techniques from algebraic multigrid [4] and has a surprisingly simple implementation, as requested by our industrial partners. A quick turnaround—execution within a few weeks of delivering the method manuscript—was made possible by a very flexible software environment and continuous validation of models with full industrial complexity. We easily sketched out, prototyped, and tested new models in the same environment so that seemingly promising novel ideas could "fail fast" on real data and be discarded when needed.

This is typical when applying mathematics to issues in industry. While mathematicians tend to excel at solving difficult problems, targeting the *right* difficult problem is a challenge. Whereas simplification of a problem is an essential part of its solution, one must possess significant domain knowledge to delineate essential aspects from the more mundane details. Reinventing all parts necessary to ride the buggy is often time-consuming and impractical for smaller groups of researchers. Thus, the distance from research to industrial application may seem unnecessarily long. The computational geosciences group at SINTEF, where I work as a researcher, is committed to producing open-source software. For example, we freely release our research platform—the Matlab Reservoir Simulation Toolbox (MRST)—as a "batteries included" package for reservoir simulation [1-2]. So far, it has contributed to 140 master's and Ph.D. theses and 270 scientific papers by external authors. We thus believe that MRST is a significant asset for the research community and a great starting point for scientists interested in testing their problems on porous media applications.

This article is based on the author's SIAM Activity Group on Geosciences Early Career Prize lecture, entitled "Multiscale Simulation of Porous Media Flow: Obstacles, Opportunities, and Open-source." He presented this talk at the 2019 SIAM Conference on Mathematical & Computational Issues in the Geosciences, which took place earlier this year in Houston, Texas.

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Olav Møyner is a researcher at SINTEF Digital and a VISTA postdoctoral fellow at the Norwegian University of Science and Technology. He is currently a visiting scholar in the Department of Energy Resources Engineering at Stanford University.

Jack Dongarra Elected as Foreign Fellow of the Royal Society

SIAM Fellow and longtime member Jack Dongarra of the University of Tennessee (UT) was named a Foreign Fellow of the Royal Society earlier this year. He is University Distinguished Professor of Computer Science in the Electrical Engineering and Computer Science Department at UT, and director of both UT's Innovative Computing Laboratory and the Center for Information Technology Research.

Jack Dongarra of the University of Tennessee.

Dongarra is also a Distinguished Research Staff member in the Computer Science and Mathematics Division at Oak Ridge National Laboratory, a Turing Fellow at Manchester University, and an adjunct professor in the Computer Science Department at Rice University. He specializes in numerical algorithms in linear algebra, advanced computer architectures, parallel computing, programming methodology, and tools for parallel computers.