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Special Issue on Quantum Computing

*This **special issue** highlights research that connects applied mathematics and computational science with quantum computing, and overviews timely developments and trends in the field.*

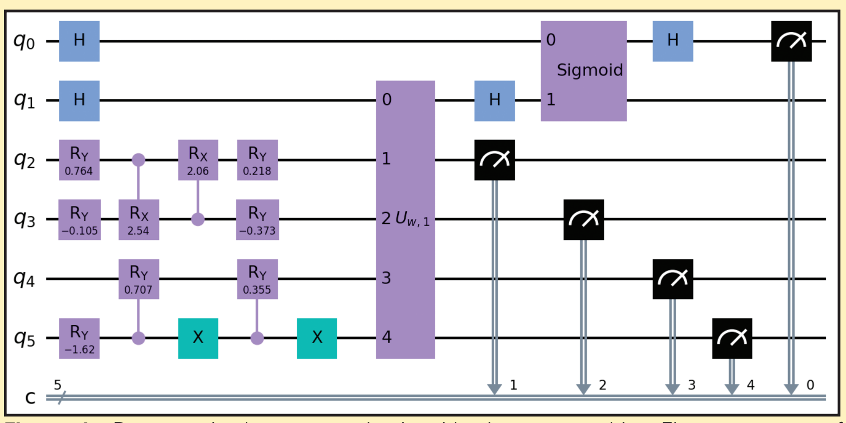

Figure 1. *Parameterized quantum circuit with six quantum bits. Figure courtesy of Antoine Jacquier, Oleksiy Kondratyev, Gordon Lee, and Mugad Oumgari.*

On page 2, David Hyde and Alex Pothen introduce Part I of the SIAM News Special Issue on Quantum Computing by surveying the many exciting concepts and technical developments that appear throughout the issue.

In an article on page 6, Antoine Jacquier, Oleksiy Kondratyev, Gordon Lee, and Mugad Oumgari explore quantum computing's potential impacts on the discipline of financial mathematics and delve into the details of computation — including the construction of quantum circuits (see Figure 1).

What Can Quantum Computers Do for Applied Mathematicians?

By Giacomo Nannicini

As applied mathematicians, we are familiar with the standard model of computation that is embodied by Turing machines. The Church-Turing thesis postulates that any physically realizable computation can be performed by a Turing machine, while the extended version suggests that any such computation can be performed *efficiently* by a probabilistic Turing machine. Although the original Church-Turing thesis is widely accepted, quantum computers challenge the veracity of the extended version; these computers represent a reasonable, physically realizable model of computation, but we do not yet know whether a probabilistic Turing machine can efficiently simulate them. The general belief is that it cannot, but as with many other fundamental questions in computational complexity theory, this belief may very well be disproven.

Computational Model

On the surface, quantum computers are programmed much like classical (i.e., non-quantum) machines; they have a state that evolves through the application of operations, and they ultimately output some information based on the final state. However, the state, operation, and output components behave differently from their classical counterparts. Here, we concisely describe these components; more details are available in the literature [8].

Let $\otimes$ denote the tensor product, which is the same as the Kronecker product in this context:

$$A \otimes B = \begin{pmatrix} a_{11} & \dots \\ a_{21} & \dots \\ \vdots & \ddots \end{pmatrix} \otimes B = \begin{pmatrix} a_{11}B & \dots \\ a_{21}B & \dots \\ \vdots & \ddots \end{pmatrix}.$$

The basic unit of information for a quantum computer is the quantum bit (qubit).

See **Quantum Computers** *on page 4*

Bridging the Worlds of Quantum Computing and Machine Learning

By Somayeh Bakhtiari Ramezani and Amin Amirlatifi

The emergence of machine learning—particularly deep learning—in nearly every scientific and industrial sector has ushered in the *era of artificial intelligence* (AI). On a parallel trajectory, quantum computing was once considered largely theoretical but has now become a reality. The fusion of these two powerful disciplines has created an unprecedented avenue for innovation, ultimately giving rise to *quantum machine learning* (QML). This novel concept promises to revolutionize computational science, data analytics, and predictive modeling in a wide variety of areas, from optimization to pattern recognition (see Figure 1).

Quantum computing offers the necessary computational horsepower to speed up complex machine learning algorithms, and machine learning provides a toolkit for the optimization of quantum circuits or the decoding of quantum states. Here, we postulate as to how QML—especially quantum deep learning and quantum large language models (QLLMs)—can redefine the future of machine learning.

How Quantum Computing Can Benefit Deep Learning

Quantum Speedup in Learning Algorithms: Quantum computing can have an immediate and substantial impact on algorithmic speedup, which is particularly relevant for machine learning and deep learning applications. A number of QML algorithms are modeled after Grover's algorithm, which offers quadratic and exponential speedup in unstructured search problems; support vector machines and several clustering methods exemplify this improvement [8]. Grover-like speedup could potentially reduce the training time for large neural networks.

Quantum Neural Networks (QNNs): Traditional neural networks face computational limitations, especially as they grow in size and complexity. In contrast, QNNs leverage quantum advantages—such as superposition and entanglement—to carry out computations more efficiently [9]. Hybrid quantum-classical networks have shown promising results in proficiently tackling machine learning tasks despite the initial limitations of QNNs.

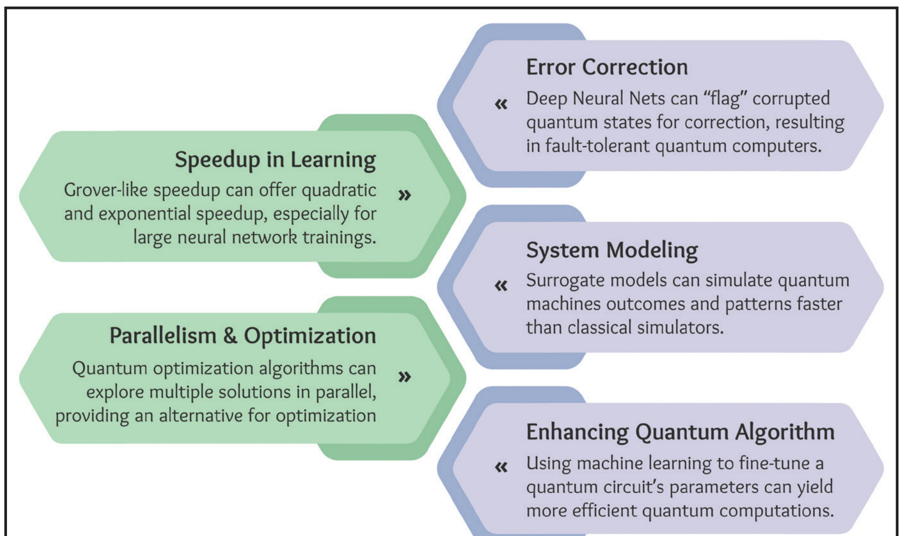

Figure 1. *The mutual benefits of quantum computing (in green) and machine learning (in blue) have resulted in a new concept called quantum machine learning, which will influence the future directions of fields like computational science, data analytics, and predictive modeling. Figure courtesy of the authors.*

Quantum Natural Language Processing (QNLP): Deep learning and large language models like GPT-4 are becoming integral parts of our world, with applications that range from natural language processing to decision-making algorithms. While these models are undeniably transforming various fields, there is a growing but often overlooked concern about their environmental impact. Training extensive AI/machine learning models requires significant computational resources and generates a substantial carbon footprint. In fact, a 2019 study estimated that training a single large neural network could emit the same amount of carbon that five cars produce over their entire lifetimes [10]. This alarming reality, which illuminates the significant environmental costs that are often overshadowed by technological advancements, calls for an immediate reassessment of the sustainability of current machine learning practices — especially in light of the global urgency to combat climate change.

Quantum computing might offer a more energy-efficient way to train and deploy large language models. Preliminary research in QNLP indicates the potential ability of quantum states to capture semantic relationships between words, which could lay the foundation for more advanced natural language processing systems [6]. QLLMs will likely impact this research area, especially when simulating human-like conversation with high accuracy.

See **Machine Learning** *on page 3*

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An Introduction to Quantum Computing and Applied Mathematics

By David Hyde and Alex Pothen

Over the last decade, quantum computing has steadily become a global research priority. In 2018, the U.S. federal government created the $1.2-billion National Quantum Initiative Act[1] to spur quantum research and development. And in 2023, the U.S. National Institute of Standards and Technology[2] identified quantum information technologies as a critical and emerging technology for prioritization[3] (alongside domains like artificial intelligence and machine learning, clean energy generation, and semiconductors). The current emphasis on quantum computing (see Figure 1) has inspired multiple new funding opportunities across science, technology, engineering, and mathematics.

With this backdrop in mind, it is important to recognize the deep ties that exist between applied mathematics and quantum computing. The primary languages of quantum mechanics and quantum computing are optimization and theoretical and numerical linear algebra — all of which are foundational competencies of SIAM's membership. And given the constraints of near-term quantum computers, scientific computing topics like domain decomposition have significant relevance in the quantum realm. In the other direction, quantum computing also has the potential to meaningfully impact the work of the SIAM community, with prospective applications from portfolio optimization in financial mathematics to the prediction of chemical phenomena via variational quantum eigensolvers.

In light of such exciting possibilities, we assembled a panel of experts in quantum computing and applied and industrial mathematics to illuminate the synergies between these research areas and ultimately foster new collaborations. This international cohort of researchers includes academics, national laboratory scientists, and practitioners from industry. These individuals have prepared a collection of articles for *SIAM News* that explore particular components of the intersection between applied mathematics and quantum computing.

The seven articles in this series are divided across two subsequent issues of *SIAM News*. In this first installment, Giacomo Nannicini (University of Southern California) introduces the fundamentals of quantum computing and overviews several problems that may be well suited for quantum computers. Somayeh Bakhtiari Ramezani and Amin Amirlatifi (both of Mississippi State University) investigate the interplay between quantum computing and machine learning. Lin Lin (University of California, Berkeley) discusses the importance of end-to-end complexity for quantum algorithms. And lastly, Antoine Jacquier (Imperial College London), Oleksiy Kondratyev (Abu Dhabi Investment Authority), Gordon Lee (Bank of New York Mellon Corporation), and Mugad Oumgari (Lloyds Banking Group) address the connections between the quantum world and financial mathematics. See Figure 2 for a list of common acronyms that appear throughout the four articles in this issue.

We hope that these bite-sized surveys provide an accessible starting point for SIAM members to pursue new ideas and collaborations in the realm of quantum computing, especially as numerous funding agencies continue to emphasize the importance of quantum technologies. We encourage interested readers to contact the authors and explore the references that are mentioned in these works. Finally, we look forward to sharing the next set of articles about the intersection of quantum computing and applied mathematics in the forthcoming May issue of *SIAM News*.

Pending funding, SIAM will hold the SIAM Quantum Intersections Convening – Integrating Mathematical Scientists Into Quantum Research in October 2024. The goal of this convening is to foster and increase the involvement and visibility of mathematicians and statisticians in quantum science research and education. Stay tuned for additional details!

David Hyde is an assistant professor of computer science at Vanderbilt University. His research interests include computational physics, cloud computing, computer graphics, and quantum computing. Alex Pothen is a professor of computer science at Purdue University. His research interests include combinatorial scientific computing, graph algorithms, and parallel computing. Pothen received SIAM's George Pólya Prize in Applied Combinatorics in 2021 and is a Fellow of SIAM, the American Mathematical Society, and the Association for Computing Machinery.

NISQ	Noisy intermediate-scale quantum
QAOA	Quantum Approximate Optimization Algorithm
QLLM	Quantum large language model
QML	Quantum machine learning
QNLP	Quantum natural language processing
QNN	Quantum neural network
QPU	Quantum processing unit
QSDP	Quantum semidefinite programming
QSP	Quantum signal processing
QSVT	Quantum singular value transformation
Qubit	Quantum binary digit (quantum bit)
VQE	Variational quantum eigensolver

Figure 2. *List of common quantum computing acronyms that appear throughout the articles in this issue. The acronyms are also defined within the text itself.*

Figure 1. *This quantum computer at Lawrence Berkeley National Laboratory is exploring quantum's potential to enable ground-breaking computational power. Figure courtesy of the University of California, Lawrence Berkeley National Laboratory.*

[1] https://www.quantum.gov
[2] https://www.nist.gov
[3] https://www.nist.gov/news-events/news/2023/09/nist-seeks-input-implementation-national-standards-strategy-critical-and

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Quantum Advantages and End-to-end Complexity

By Lin Lin

Rapid advancements in quantum computing offer unparalleled opportunities for the scientific computing community. However, it is quite difficult to fully harness the potential of quantum computers and outperform classical computers in scientific computing. It may be tempting to think that n quantum bits (qubits) can encode 2^n complex amplitudes—which would suggest exponential quantum speedups—but the reality is more subtle. Every quantum algorithm must interact with classical processing systems, which means that we need to thoughtfully consider input-output models and the specific requirements of quantum algorithms when evaluating quantum complexities. Due to the inherent constraints of quantum devices, we can only achieve significant quantum advantages for problems that have a limited amount of input and output data.

Let us divide the quantum cost into three main categories: input, output, and running costs. A quantum algorithm typically begins with a standard state such as $|0^n\rangle$; a unitary matrix then transforms this state to prepare the input state. The *input cost* is the quantum gate complexity that is required to implement this unitary matrix, and the *output cost* pertains to the quantum measurement process — which is generally performed on one or more qubits at the end of the algorithm. The number of necessary repetitions to carry out the quantum algorithm determines the output cost. Finally, the *running cost* refers to the expense that is incurred by executing the quantum algorithm a single time (excluding the cost of preparing the input state). In order to conduct a comprehensive end-to-end analysis of quantum advantage, we must consider all three of these costs. We also have to compare the quantum algorithm's performance with that of the best available classical algorithms. A recent survey systematically investigated this end-to-end complexity for a wide range of quantum applications [5].

Shor's algorithm serves as a great example of end-to-end quantum advantage because it effectively tackles the prime factorization problem, which challenges classical computers. This algorithm excels at end-to-end complexity in multiple ways: (i) It has minimal input and output costs since it only involves integers; (ii) it maintains a running cost that is polynomial in relation to the integer's bit length; and (iii) it significantly surpasses the best classical algorithm for the task, which has a super-polynomial cost in the number of bits.

Hamiltonian simulation—which finds numerous applications in quantum physics and chemistry—is another method that could achieve a quantum advantage. During this process, an initial state $|\psi_0\rangle$ evolves over time t to $|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$. For a system with n qubits, the size of the Hamiltonian matrix H is 2^n but the amount of information in H is typically only polynomial in n. We begin with simple initial states that are prepared at a polynomial cost in n; the best algorithm for simulating quantum dynamics up to time t with precision ϵ only queries the unitary encoding of H (known as a block encoding) $\mathcal{O}(\|H\|_2 t + \log(1/\epsilon))$ times [7, 10]. The output emphasizes accurate approximations of observables that are associated with $|\psi_t\rangle$—i.e., $\langle\psi_t|O|\psi_t\rangle$—rather than reconstructions of all of the information in $|\psi_t\rangle$. The cost of measuring these observables

See **End-to-end Complexity** *on page 5*

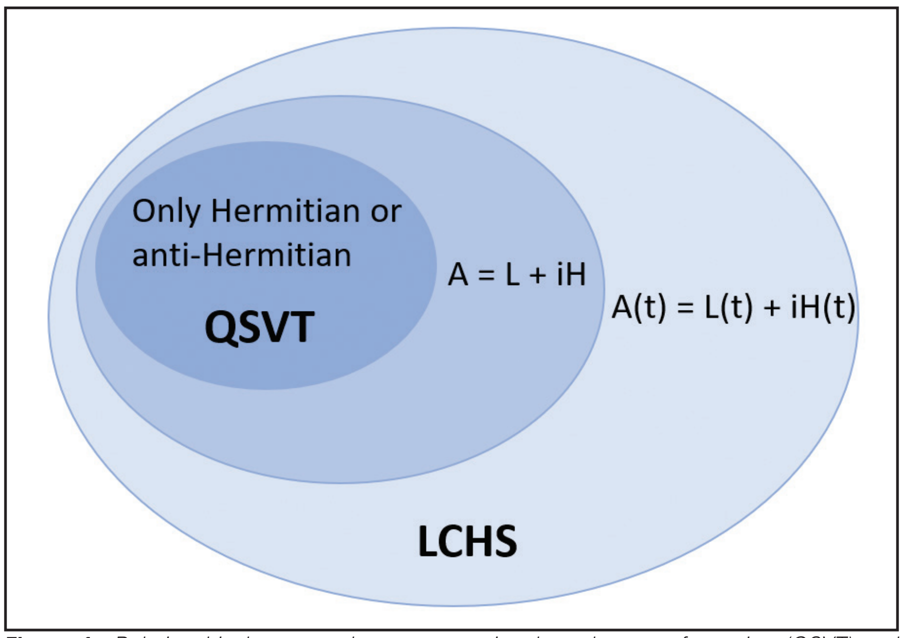

Figure 1. *Relationship between the quantum singular value transformation (QSVT) and the linear combination of Hamiltonian simulation (LCHS) for the simulation of e^{-At}. Figure courtesy of Dong An of the Joint Center for Quantum Information and Computer Science.*

Machine Learning

Continued from page 1

Quantum Parallelism and Optimization: A defining feature of quantum computing is its ability to perform parallel computations through superposition — an invaluable property for optimization problems, which are the underlying theme of machine learning algorithms. The Quantum Approximate Optimization Algorithm (QAOA) is potentially able to optimize complex functions that are classically hard to solve [4]. QAOA employs quantum parallelism to simultaneously explore multiple solutions, thus providing a much-needed alternative for the optimization of deep learning models. Another promising quantum algorithm is the variational quantum eigensolver (VQE) [7]. Many machine learning algorithms hinge on the solution of eigenvalue problems, so the VQE could significantly expedite these calculations on quantum hardware.

How Deep Learning Can Benefit Quantum Computing

Machine Learning for Quantum Error Correction: Quantum error correction is vital in the creation of reliable quantum computers. In classical computers, error correction is relatively straightforward; errors usually arise when binary digits (bits) flip from 0 to 1 or vice versa, and several techniques—such as parity checks—can identify and correct them. But in quantum computing, phase errors and other quantum decoherence mechanisms may also disrupt the delicate quantum states. Although traditional quantum error correction techniques—like surface codes and cat codes—are effective, they require extensive resources.

Machine learning methods have shown promise in error detection and correction. We can train deep neural networks to identify quantum errors by learning the intricate patterns through which these errors typically manifest. By doing so, the networks can effectively "flag" corrupted quantum states for correction — even when traditional error correction techniques are computationally expensive or less efficient. This approach could bring fault-tolerant quantum computers closer to reality.

Quantum System Modeling: Deep learning can also assist with the modeling and simulation of complex quantum systems. We can use quantum data to train surrogate models that simulate the original system's behavior at a faster pace than other classical simulators; analysis of such models may lend insight into behaviors that are difficult to study directly. These surrogate models can pinpoint patterns and properties within quantum systems that otherwise may not be readily identifiable, potentially leading to advancements in molecular science, quantum chemistry, materials science, and other related fields [3].

Quantum Algorithms: Hybrid quantum-classical algorithms that utilize both quantum computers and classical machine learning models are forging new paths for the solution of complex problems in optimization, data analysis, and the like. One major machine learning application in quantum computing is the optimization of traditional quantum algorithms. For example, reinforcement learning can fine-tune a quantum circuit's parameters and yield more efficient and effective quantum computations [3].

Open Problems in the Era of Noisy Intermediate-scale Quantum Computing

A key challenge that presently impacts quantum computing in general (and QML in particular) is the limitation of existing quantum hardware. Current gated quantum computers are predominantly classified as noisy intermediate-scale quantum (NISQ) devices. These devices often have a limited number of quantum bits (qubits)—ranging from tens of qubits to a few hundred—though machines with several thousand qubits are under development. Computers in this transitional period are not yet fully fault tolerant and are constrained by physical limitations like decoherence and gate errors, which affect their ability to maintain high-quality entanglement and achieve a high circuit depth. Despite these issues, NISQ devices can perform certain computational tasks more efficiently than their classical counterparts. Furthermore, the limited number of qubits and relatively large error rates complicate the implementation of complex QNNs on these machines. Even before the issue of algorithmic design, NISQ computers must handle intrinsic imperfections — such as the aforementioned decoherence and gate errors [1].

While QML in the NISQ era faces unique challenges—especially concerning hardware limitations—it also presents exciting research opportunities for interdisciplinary collaborations between computer scientists, applied mathematicians, and physicists. A variety of techniques are paving the way for increased QNN compatibility with NISQ-era devices, including variational circuits, error mitigation, and hybrid models. As these methods mature, the prospect of QML implementation in near-term quantum computing becomes even more promising.

One of the most popular approaches in this regard is the use of *variational circuits*: shallow quantum circuits that are adaptable to NISQ-era constraints. This tactic classically optimizes the circuit parameters, while the quantum component of the computation executes specific subroutines [7]. *Error mitigation* techniques, such as zero-noise extrapolation, also help to reduce the effect of noise in the system. By running the same quantum operation multiple times with varying noise levels, users can estimate and correct for the impact of errors. A third approach for NISQ-era quantum computers integrates quantum computing into classical neural networks as *hybrid quantum-classical models* [2]. Doing so allows the quantum portions of the model to focus on specific tasks that suit them well—such as complex optimizations—while offloading other tasks to the classical system. Finally, we note that effective methods for *quantum data encoding*—i.e., encoding classical data into quantum states—remain an open problem. Current approaches either suffer from inefficiencies or lack the ability to capture the richness of classical data [5].

Concluding Thoughts

As we venture deeper into the realms of AI and quantum mechanics, the convergence of these two technologies offers unparalleled potential. The synergistic relationship between quantum computing and machine learning necessitates a concrete interdisciplinary framework wherein quantum physicists, computer scientists, and applied mathematicians can work together to develop robust, scalable, and applicable quantum algorithms for machine learning.

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Quantum Computers

Continued from page 1

The *state* of a q-qubit quantum computer is a unit vector in $(\mathbb{C}^2)^{\otimes q} = \mathbb{C}^{2^q}$. Because this vector space has 2^q standard basis elements, we conventionally label them as q-digit binary strings that are denoted by $|j\rangle$ for $j \in \{0,1\}^q$; a "ket"—i.e., a symbol included in brackets $|\cdot\rangle$—is simply a column vector. In quantum mechanics, practitioners often graphically represent a 1-qubit state $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \alpha_0\begin{pmatrix}1\\0\end{pmatrix} + \alpha_1\begin{pmatrix}0\\1\end{pmatrix}$, $\alpha_0, \alpha_1 \in \mathbb{C}$ on a sphere (see Figure 1). Such a representation is only up to a global phase factor $e^{i\phi}$, but this factor is unimportant due to the laws of measurement.

The *operations* that evolve the state correspond to quantum circuits, which are unitary matrices $U \in \mathbb{C}^{2^q \times 2^q}$ (see Figure 2). We typically express circuits in terms of basic gates — i.e., certain 2×2 or 4×4 complex unitary matrices that constitute the "assembly language." The composition of basic gates follows the standard rules for tensor products and matrix multiplication. Consider a 2-qubit system; applying the gate U onto the first qubit and the gate V onto the second qubit, followed by the gate W onto the first qubit, is equivalent to applying the matrix $(W \otimes I)(U \otimes V) = WU \otimes V$ to the entire quantum state. A *measurement* of the state $|\psi\rangle = \sum_{j=0}^{2^q-1} \alpha_j |j\rangle$ is a special, non-unitary operation whose outcome is a random variable X with sample space $\{0,1\}^q$ and $\Pr(X=j) = |\alpha_j|^2$. We only gain information about a quantum state from measurements, and the state collapses to $|j\rangle$ if we observe j as the outcome of a measurement.

A quantum algorithm contains quantum circuits and subsequent measurements. In order for a quantum algorithm to be efficient, it must use a polynomial number of resources — i.e., a polynomial number of qubits and basic quantum gates (the assembly language). Based on this explanation, several differences between classical and quantum computers are readily observable. First, describing the state of a quantum computer requires that we specify an exponential-sized complex vector (i.e., 2^q for a q-qubit system), whereas describing the state of a classical computer simply requires a linear-sized binary vector. But given the effect of measurements, we can only extract a linear amount of information (in terms of the number of qubits) from the exponential-sized complex vector — so from a q-qubit state, we obtain q bits of information after a measurement. Second, all operations (except measurements) that a quantum computer applies are linear and reversible; a unitary matrix U satisfies $UU^\dagger = U^\dagger U = I$, where $\dagger$ denotes the conjugate transpose. Though these properties may seem restrictive, a universal quantum computer is Turing-complete in that it can compute any Turing-computable function while only requiring some polynomial number of additional resources.

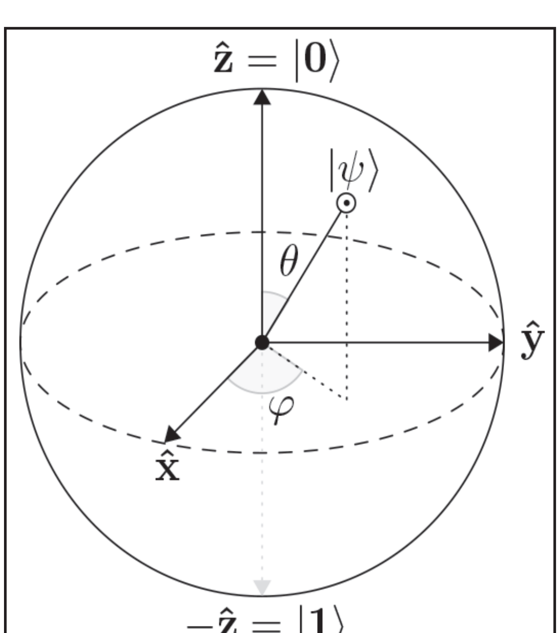

Figure 1. *In quantum computing, the Bloch sphere is a possible graphical representation of the state of a quantum bit. Figure courtesy of the author.*

Practical Uses of Quantum Computers

From a practical viewpoint, existing quantum computers are still far from faithfully reproducing the ideal model of quantum computation. Nonetheless, the research community has been persistently seeking strong use cases for quantum computers — many of which will resonate with the SIAM community. Here, we present some of the relevant problems. The following list is not exhaustive, nor can it be, as this area of research is highly active; the takeaway is that quantum computers excel at certain tasks and perform poorly at others. Because classical algorithms and quantum computers offer different tradeoffs for many interesting computational problems, researchers often study quantum approaches in search of potential advantages.

Richard Feynman originally proposed the concept of quantum computers to simulate the time evolution of a quantum mechanical system [5]. Mathematically, this idea is akin to implementing a circuit that acts as e^{-iHt} on the state vector, where the matrix H and scalar t are input data. Quantum computers can solve this problem in time polynomial in the number of qubits [2], whereas no efficient classical algorithm has been discovered so far. The problem is "prototypical" for the class of problems that are efficiently solvable by a quantum computer. It finds direct applications in quantum physics and chemistry (i.e., the simulation of quantum dynamics) and is a core component of many quantum algorithms.

Quantum computers can also aptly estimate certain eigenvalues. Consider a q-qubit unitary U (recall that U is a $2^q \times 2^q$ matrix) and an eigenvector $|\psi\rangle$ of U. The phase estimation algorithm determines an ε-approximation of the eigenvalue of $|\psi\rangle$ with $\mathcal{O}(1/\varepsilon)$ applications of U and a number of gates that is polynomial in q, whereas a classical algorithm would generally need to perform a matrix-vector operation with the (exponentially-sized) matrix U.

There are several quantum algorithms for the solution of linear systems, beginning with seminal work in 2009 [6]. Multiple possible input models are also in use; for example, the sparse oracle access model describes matrix entries via maps that indicate the position of nonzero elements and their values. However, this model is not necessarily the most efficient approach for every scenario. Let $A \in \mathbb{R}^{m\times m}$, $b \in \mathbb{R}^m$, $\varepsilon > 0$, and $z = A^{-1}b$. The natural "quantum encoding" of the solution z is the state $|\psi\rangle = \frac{1}{\|z\|}\sum_{j=0}^{m-1} z_j |j\rangle$. A quantum linear systems algorithm produces a state $|\phi\rangle$ so that $\||\phi\rangle - |\psi\rangle\| \leq \varepsilon$. The runtime of such an algorithm is polylogarithmic in m but depends (at least linearly) on the linear system's condition number κ [9]. The fastest known runtime for the sparse access input model is $\tilde{O}(d\kappa)$ (ignoring all polylogarithmic factors), where d is the maximum number of nonzero elements in each row of A.

The overarching purpose of these algorithms for linear systems is to implement the matrix function $f(x) = 1/x$, which implicitly computes an eigendecomposition of A and takes the reciprocal of each eigenvalue in the corresponding eigenspace. Two important factors merit consideration in this endeavor. First, we must account for the cost of the oracles that describe the entries of A and prepare a state encoding b. These oracles can be inexpensive if A and b admit efficient algorithmic descriptions, but they may take a time that is proportional to the total number of nonzero elements in less favorable scenarios — resulting in a corresponding increase in runtime. Second, the solution z cannot be read directly because it is encoded as a quantum state. If we wish to extract a classical description of the solution, we must perform a potentially expensive operation called *quantum state tomography* [10].

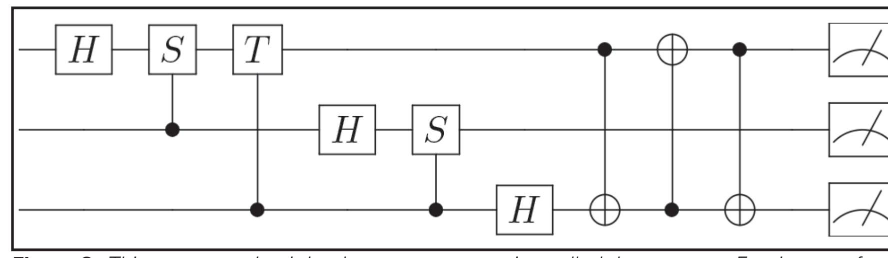

Figure 2. *This quantum circuit implements an operation called the quantum Fourier transform. All operations correspond to unitary matrices except for the last operation on each wire, which indicates a measurement. Figure courtesy of the author.*

It is also possible to efficiently implement other matrix functions besides the inverse on a quantum computer. This prospect is best understood in the framework of *block encodings* [7]. A block encoding of a matrix A is a quantum circuit that, in some subspace, acts on the quantum state as A (possibly rescaled). While a quantum circuit is always a unitary operation, A need not be unitary or even square in this case. We can utilize a variety of tactics to implement a block encoding of some given matrix A in a data-driven way. From this block encoding, we can then implement approximations of polynomial functions of A. The construction of the Gibbs state $e^A / \mathrm{Tr}(e^A)$ for Hermitian A is a particularly notable scenario. Gibbs states are important in many branches of applied mathematics, including machine learning and optimization. In some cases, a block encoding of an $n \times n$ Gibbs state can be constructed in times as fast as $\mathcal{O}(\sqrt{n})$! The speedup is quite large compared to classical approaches, although the stated runtime is only achievable under very specific, favorable conditions.

Finally, quadratic quantum speedups via amplitude amplification yield faster algorithms for many problems [3]. One such example is *unstructured search* (also known as Grover's algorithm), which searches over a set with no structural property so that the only possible search approach is to examine all of the elements in the set. Another example is *mean estimation*, which computes the mean of a univariate or multivariate random variable [4]. These approaches are strongly related to quantum walks, which also admit quadratic speedups when compared to classical random walks [1]. The speedups rely on specific input models and may incur additional conditions, so it is important to pay attention to the details.

The aforementioned problems represent only a tiny fraction of active research areas, but hopefully this overview will generate some excitement about quantum computing. Quantum algorithms can be understood purely through linear algebra and often offer different tradeoffs than classical algorithms, which means that they are potentially useful under the right conditions. However, we must overcome many challenges—in subjects such as hardware design and engineering, algorithms and software, and practical considerations like numerical stability—to bring this source of potential to fruition. Classical computers will likely remain the best choice for most computational problems, but if quantum computers can accelerate even just a few key issues in practice, that alone could be worth the time and exploratory investment.

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MIT SIAM Student Chapter Hackathon Utilizes Open-access Energy Data

By Bianca Champenois and Sanjana Paul

During the first weekend in February, undergraduate and graduate students from the greater Boston area came together at the Massachusetts Institute of Technology (MIT) to take part in the Global Energy Monitor (GEM) Hackathon.[1] The event was co-hosted by Bianca Champenois, president of the MIT SIAM Student Chapter,[2] and Sanjana Paul, executive director of Earth Hacks.[3] Hackathons generally serve as programming contests during which participants work in small groups to teach each other new skills and develop interesting projects that pertain to a certain theme, all while competing against other teams for prizes. The projects can take many forms, ranging from the creation of code repositories and website mockups to new datasets and hardware prototypes. The GEM Hackathon encouraged participants to utilize the open-access energy data in the GEM databases[4] to tackle questions about worldwide energy data availability and estimate solar resource potential.

Students chose between two open-ended challenge statements and applied their mathematics, modeling, programming, mapping, visualization, and storytelling skills to develop feasible solutions. The first challenge asked attendees to design a tool that would allow individuals from anywhere in the world to learn about the power plants in their vicinity, and the second challenge tasked them with analyzing and combining multiple datasets to compare the potential of solar power against existing real-world implementations. The assignments were intentionally broad so that students could use their creativity to generate new perspectives. Represented schools at the GEM Hackathon included MIT, Bentley University, Boston University, Brandeis University, Bunker Hill Community College, Northeastern University, and Simmons University. Because the backgrounds and majors of participating students varied widely, team members were able to exchange perspectives and apply a variety of skill sets in an interdisciplinary setting.

The two-day event, which took place during the Independent Activities Period at MIT, included workshops and talks to support students' projects and experiences. To wrap up the first day of festivities, Hackathon organizers created a custom version of GeoGuessr[5]—a popular geography game wherein players guess the locations of various Google Street View images—that focused on power plants around the world. During this activity, participants learned about different types of power plants and observed their physical appearances in real life. Exploring new landscapes via Google Street View also offered a fresh perspective on the size and impact of global energy projects — and gave hackers a break from project development to socialize and have some fun.

Another session familiarized students with Social Explorer:[6] a tool that provides access to U.S. demographic data. Alejandro Paz, a Librarian for Energy and Environment at MIT, explained the history and mechanics of the tool and gave a demonstration. He also overviewed all of the resources and datasets that are available to students via MIT's libraries.

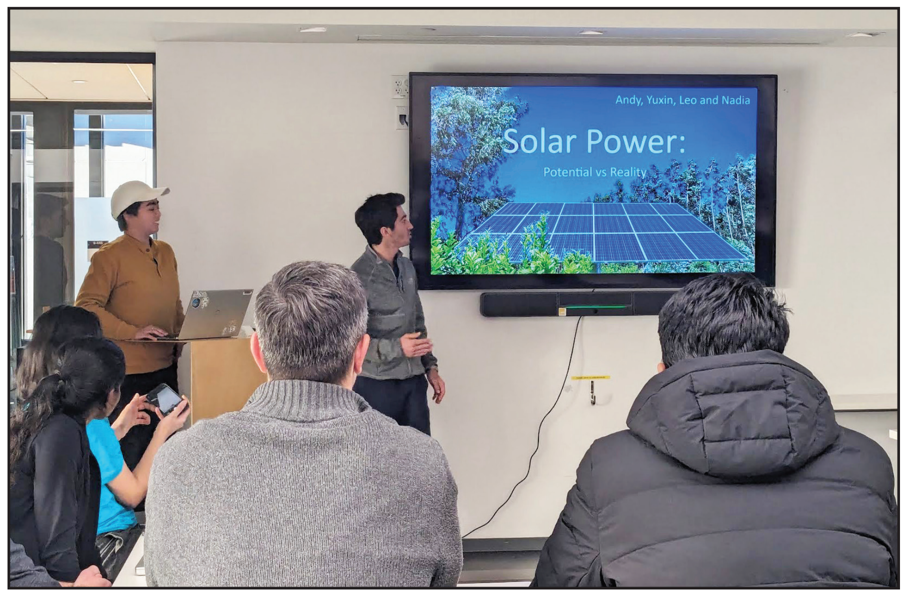
The GEM Hackathon was further bolstered by mentor support, including that of Ted Nace (founder and executive

See **Hackathon** *on page 7*

1 https://gem-hackathon.devpost.com
2 https://web.mit.edu/siam/www
3 https://earthhacks.io
4 https://globalenergymonitor.org

5 https://www.geoguessr.com

6 https://www.socialexplorer.com



During the Global Energy Monitor Hackathon, which took place at the Massachusetts Institute of Technology in early February, the "Plane Watchers" team presents their project about solar power. The students utilized multiple datasets to compare the potential of solar power with existing real-world implementations. Photo courtesy of Bianca Champenois.

End-to-end Complexity

Continued from page 3

is again polynomial in n and $1/\epsilon$. Given these parameters, classical algorithms cannot reliably and accurately compute such dynamical properties over an extended duration t at a polynomial cost in n. This shortcoming sets a strong foundation for the potential of quantum speedups during the simulation of quantum dynamics.

Does quantum computing demonstrate a clear, end-to-end advantage in other domains besides prime factorization and quantum dynamics simulation? While many applications still lack a solid foundational basis, rapid progress is certainly occurring across various areas.

Consider a seemingly simple variation of the simulation of quantum dynamics, where iH is replaced with a general matrix A that acts on n qubits. Such problems appear when simulating certain open quantum systems. The goal is to simulate $|\psi_t\rangle = e^{-At}|\psi_0\rangle$ and subsequently measure an observable $\langle\psi_t|O|\psi_t\rangle$. We can always decompose a general matrix A (through a Cartesian decomposition) as $A=L+iH$, where $L=(A+A^\dagger)/2$, $H=(A-A^\dagger)/(2i)$, and $A^\dagger$ represents the Hermitian conjugate of A. Both L and H are Hermitian matrices. If L is positive semidefinite, the matrix 2-norm satisfies $\|e^{-At}\|_2 \leq 1$ and the norm of the final state satisfies $\||\psi_t\rangle\|_2 \leq 1$. The input can still be a simple state that is prepared at a polynomial cost in n. When $\|L\|_2$ is small, the dynamics only differ slightly from the Hamiltonian simulation problem, thus suggesting that the general task of estimating the observable $\langle\psi_t|O|\psi_t\rangle$ could be hard for classical computers. But if the norm $\||\psi_t\rangle\|_2$ decreases rapidly with respect to t, estimating $\langle\psi_t|O|\psi_t\rangle$ with a multiplicative accuracy of ϵ requires an increased number of repetitions — thereby raising the output cost. To establish a quantum advantage over this non-Hermitian simulation problem, we must find a duration t that is sufficiently demanding for classical computers yet feasible for quantum computers. The lack of current knowledge about the difficulty of practically relevant non-Hermitian Hamiltonians calls for further research in this area.

We have discussed input cost, output cost, and the potential difficulties that classical solvers face. The remaining aspect of end-to-end analysis is the running cost — specifically, the simulation e^{-At} on a quantum computer. This task is actually quite challenging. One significant advancement in quantum algorithms from the past decade is the development of the quantum singular value transformation (QSVT) [7]. Consider the singular value decomposition of $A=U\Sigma W^\dagger$. Since U and W are unitary matrices, implementing a singular value transformation like $Uf(\Sigma)W^\dagger$ mainly requires that we address the non-unitarity of $f(\Sigma)$. This notion is a key innovation in both QSVT and quantum signal processing [10]. When $A=iH$, the singular value decomposition directly relates to the eigenvalue decomposition $A=VDV^\dagger$ in which V is also unitary, thus allowing QSVT to perform the Hamiltonian simulation e^{-iHt}. But for a more general A, the eigenvalue decomposition becomes $A=VDV^{-1}$ and V is simply an invertible matrix. Because the simulation task $e^{-At}=Ve^{-Dt}V^{-1}$ is intrinsically an eigenvalue decomposition problem, techniques such as QSVT are not applicable.

Given this restriction, how do we prepare the state $|\psi_t\rangle = e^{-At}|\psi_0\rangle$ on a quantum computer? The leading approach is somewhat complex and perhaps counterintuitive. It begins by treating the problem like an ordinary differential equation (ODE): $\frac{\mathrm{d}|\psi(s)\rangle}{\mathrm{d}s} = -A|\psi(s)\rangle,\ \psi(0)=|\psi_0\rangle$ on $0 \leq s \leq t$. We then discretize this ODE over time and convert it into a large linear system of equations. We solve the resulting linear system with a quantum linear system solver, such as the renowned Harrow-Hassidim-Lloyd algorithm [8] or a more recent near-optimal solver [3, 4, 9]. The ODE itself is solvable via a traditional time-marching strategy, similar to the type that is employed in standard numerical ODE solvers. Although direct implementation leads to an excessively high output cost due to diminishing success probability, we developed a time-marching strategy that can partially mitigate this issue [6].

One timely advancement was a significant simplification of the simulation of non-unitary quantum dynamics [2]. If L is positive semidefinite, then

$$e^{-At} = \int_{\mathbb{R}} \frac{1}{\pi(1+k^2)} e^{-i(kL+H)t}\mathrm{d}k. \quad (1)$$

This formula generalizes the scalar identity $e^{-|x|} = \int_{\mathbb{R}} \frac{1}{\pi(1+k^2)} e^{-ikx}\mathrm{d}k$ to the matrix setting. Since the matrices H, L do not commute in general, the proof of (1) is *not* based on the spectral mapping theorem, which evaluates a matrix function $f(A)=Vf(D)V^{-1}$ via the eigendecomposition $A=VDV^{-1}$ [2]. This identity expresses e^{-At} as a linear combination of Hamiltonian simulation (LCHS) problems of the form $e^{-i(kL+H)t}$. We can even generalize LCHS to time-dependent $A(t)$, where the time-ordered propagator $\mathcal{T}e^{-\int_0^t A(s)\mathrm{d}s}$ replaces e^{-At} (see Figure 1, on page 3). The LCHS approach both streamlines the simulation process and achieves optimal query complexity with respect to the initial state preparation, which reduces the input cost. A recent study generalized the LCHS formalism to a family of identities that can express linear non-unitary evolution operators as a linear combination of unitary evolution operators [1]. This work is the first approach to solve linear differential equations with both optimal state preparation cost and near-optimal scaling in matrix queries on all parameters.

To fully harness the potential of quantum computers and achieve a quantum advantage in the coming years, we must develop innovative methods to map various problems into suitable quantum frameworks. We thus welcome both theoretical and empirical discussions of the end-to-end complexities of these solutions. Problems that already exhibit some degree of "quantumness" may have a head start, as classical hardness is easier to argue. At the same time, significant advancements might arise as we apply quantum computers to classical problems — much like the revolution in prime number factoring and cryptography due to Shor's algorithm in the 1990s.

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Quantum Computing for Financial Mathematics

By Antoine Jacquier, Oleksiy Kondratyev, Gordon Lee, and Mugad Oumgari

The discipline of financial mathematics has experienced many periods of rapid development that are often followed by relative calm. As a synthetic discipline at the cross section of applied mathematics, financial theory, computer science, prudential regulation, and other fields, financial mathematics benefits from discoveries and breakthroughs in all of these areas. The birth of financial mathematics is often attributed to Louis Bachelier's doctoral thesis, "The Theory of Speculation," which he defended in 1900 under the supervision of Henri Poincaré. Bachelier was the first person to utilize a stochastic process (later called *Brownian motion*) to model financial assets. Since then, financial mathematics has felt the influence of stochastic calculus (e.g., Itô's lemma and the Girsanov theorem), control theory (e.g., the Kalman filter), and statistics (e.g., the Kolmogorov-Smirnov test) while also benefitting from progress in microprocessors, financial deregulation, ultrafast communication, and object-oriented programming.

Quantum computing, which promises enormous computing power at a very low cost, marks the start of a new chapter for financial mathematics. All financial problems—pricing, risk management, credit scoring, discovery of trading signals, data encryption, portfolio optimization, and so forth—are computational in nature, and financial mathematics seeks to provide the most efficient and convenient tools for these types of computations.

A computation is a function that transforms information, or the transformation of one memory state into another. In classical digital computing, the fundamental memory unit is a *binary digit* (bit) of information. *Logic gates* are functions that operate on bits of information — namely Boolean functions, which can be combined into *circuits* that perform additions, multiplications, and more complex operations. But is Boolean logic the sole or most general way to realize digital computation? The answer is clearly "no." Classical computing is just a special case of a more general computational framework that we now call *quantum computing*. A classical bit is a two-state system that can exist in one of two discrete deterministic states, traditionally denoted as 0 and 1. All classical bits are independent, in that flipping the state of a given bit does not affect the states of other bits. To generalize these two features of classical computing, we can permit the bit to exist in a superposition of the two states and allow the states of different bits to entangle (a certain form of correlation). It is therefore clear how quantum computing got its name; superposition and entanglement are the key characteristics of quantum system states, and it is tempting to perform these computations with the *controlled evolution of a quantum system*, i.e., by running a *quantum computer*.

Superposition and entanglement are also responsible for the extraordinary power of quantum computing. They allow for more general computation, a broader definition of the memory state as compared to classical digital computing, and a wider range of possible transformations of such memory states. The fundamental memory unit in quantum computing is the *quantum bit* (qubit). Mathematically, a qubit's state is a unit vector in the two-dimensional complex vector space. Norm-preserving unitary operators (unitary matrices) that act on qubit states serve as *quantum logic gates*. Once a computation is complete and the *quantum circuit* (a sequence of quantum logic gates) has transformed the initial system state, we can measure the qubit states by projecting them onto the basis states (see Figure 1, on page 1). Qubits in their basis states correspond to classical bits, as all superpositions have collapsed. The remainder of the computational protocol can occur classically after the readout of the bitstring from a quantum computer.

Why have researchers not utilized this superior mode of computation until very recently? Although quantum mechanics was formulated nearly a century ago, the realization of quantum mechanical rules in the computational protocol of classical digital computers requires an enormous amount of memory. Exponential gains in computing power are offset by exponential memory requirements.

In order to efficiently perform quantum computations, we must use the ability of actual quantum mechanical systems to encode information in their states. For instance, we can describe the state of a quantum system that consists of n entangled qubits by specifying 2^n probability amplitudes: a massive amount of information that would be impossible to store in classical memory. Decades passed before quantum processing units (QPUs)—devices that control quantum mechanical systems as they perform computations—became technologically feasible.

Current state-of-the-art QPUs contain several hundred qubits, and the qubit fidelity is still insufficient for fault-tolerant computation. However, the size and qubit fidelity of these systems are already sufficient enough to be useful. Two qubit types in particular stand out as the most developed and most promising: qubits made of superconducting circuits with a coherence time of $\sim 10^3 \mu s$ [10] (see Figure 2), and qubits made of trapped ions with a coherence time of $>10^8 \mu s$ [2] (see Figure 3). These qubit characteristics indicate that we are approaching the threshold beyond which various error correction algorithms become feasible, meaning that we may finally enter the era of fault-tolerant quantum computing.

While Google demonstrated so-called *quantum supremacy* on a specially designed problem in 2019, recent work has exhibited clear signs of *quantum advantage*: productive applications of quantum computers to real-world problems that classical computers have trouble handling. That being said, emulators play a pivotal role in the current quantum ecosystem due to the scarcity and cost of full-fledged quantum computers. These emulators simulate quantum operations on classical hardware and enable researchers and developers to design, test, and refine quantum algorithms without direct access to a quantum machine, thus propelling state-of-the-art research and bypassing current limited availability. Given their parallel processing capabilities, graphics processing units have emerged as the go-to hardware for the emulation of quantum systems. Their architecture is well suited to handle the matrix operations that are fundamental to quantum mechanics. In recent years, large technology companies have begun to create public frameworks for quantum computing. For example, IBM's `Qiskit`[1] allows any Python user to implement and test quantum algorithms; Google Quantum AI provides the `Cirq`[2] framework that lets developers create, edit, and invoke quantum circuits on real and simulated quantum devices; the Microsoft Azure Quantum Development Kit[3] includes the Q# language, which developers can use to write quantum algorithms that run on classical simulators; and Xanadu's `PennyLane`[4] is specifically designed to implement quantum machine learning (QML) tools.

Several aspects of quantum computing are especially relevant to financial mathematics problems.

Optimization

Digital quantum computing allows practitioners to solve NP-hard combinatorial optimization problems with variational methods, such as the variational quantum eigensolver and the Quantum Approximate Optimization Algorithm [3]. Both algorithms can address a wide range of finance-related optimization problems [6]. Moreover, variational algorithms are noise resistant and therefore suitable for the current generation of noisy intermediate-scale quantum computers [9]. Classically hard optimization problems naturally lend themselves to implementation on *analog quantum computers* that realize the principles of adiabatic quantum computing. The flagship financial use case is discrete portfolio optimization, which demonstrates the first experimental evidence of a quantum speedup [11].

Quantum Machine Learning

The combination of quantum computing and artificial intelligence will likely generate some of the most exciting opportunities, including a wide range of possible applications in finance. We have already seen promising results with parameterized quantum circuits that were trained as either generative models (such as the quantum circuit Born machine [7]) or discriminative models (such as quantum neural networks). Possible use cases include market generators, data anonymizers, credit scoring, and the creation of trading signals. The quantum generative adversarial network (GAN) is another generative QML model with significant potential [1]. Much like classical GANs, quantum GANs comprise a generator and a discriminator with the ability to distinguish quantum states. Since each quantum state encodes a probability distribution, researchers can use the quantum GAN discriminator to verify whether the datasets in question came from the same probability distribution. This technique has direct applications to time series analysis, the detection of structural breaks, and alpha decay monitoring.

Partial Differential Equation Solvers

In 2009, Aram Harrow, Avinatan Hassidim, and Seth Lloyd devised a quantum algorithm that can surpass classical computation times when solving linear systems [5]. Linear systems are ubiquitous

See **Financial Mathematics** *on page 8*

1 https://www.ibm.com/quantum/qiskit
2 https://quantumai.google/cirq/start/intro
3 https://learn.microsoft.com/en-us/azure/quantum/overview-what-is-qsharp-and-qdk
4 https://www.xanadu.ai/products/pennylane

one-qubit gate		two-qubit gate	
Gate time:	$\sim 10^{-2}$µs	Gate time:	$\sim 10^{-2}$-10^{-1}µs
Fidelity:	99.99%	Fidelity:	99.9%

Figure 2. *Coherence time and fidelity of superconducting circuits.*

one-qubit gate		two-qubit gate	
Gate time:	∼ 1-10µs	Gate time:	∼ 10µs
Fidelity:	99.9999%	Fidelity:	99.9%

Figure 3. *Coherence time and fidelity of trapped ion circuits.*

Electrical Resistance and Conformal Maps

MATHEMATICAL CURIOSITIES

By Mark Levi

I would like to elaborate on the observation in my April 2023 article, titled "Conformal Deformation of Conductors."[1] Imagine a current-conducting sheet: negligibly thin, homogeneous, and isotropic. Let us cut a square out of the sheet and measure the resistance, as in Figure 1. The following fact is both fundamental and almost trivial:

$$\boxed{\text{Squares of all sizes have the same resistance.}} \qquad (1)$$

Indeed, dilation of the square changes the distance that the current must travel, and by the same factor as the width; these two effects cancel each other out — but *only* in $\mathbb{R}^2$. In $\mathbb{R}^3$, for example, dilating a cube by a factor λ divides the resistance (between the opposite faces) by λ, and in $\mathbb{R}$ the resistance multiplies by λ.

From now on, let the resistance of the square $=1$ ohm. Geometrically, resistance is a *measure of elongation*: a rectangle whose resistance $=1$ must then be a square (see Figure 2).

A classical theorem in complex analysis states that two annuli (see Figure 3) are conformally equivalent—i.e., they can be mapped onto one another by a conformal $1-1$ map—if and only if they have the same ratio of radii. A more general theorem states that two doubly connected "annuli" (like those in Figure 4) are conformally equivalent if they have the same modulus: a certain number that is associated with the region. I would like to point out that that the modulus is simply the electrical resistance.

[1] https://sinews.siam.org/Details-Page/conformal-deformation-of-conductors

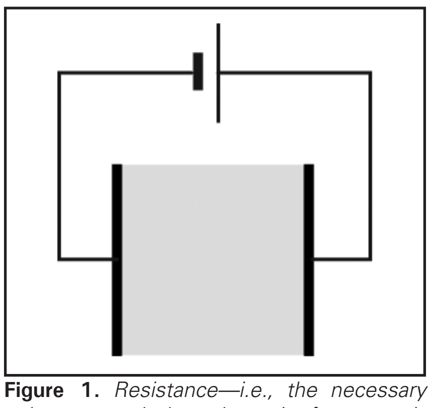

Figure 1. *Resistance—i.e., the necessary voltage to push through a unit of current—is measured between opposite sides (coated with a perfect conductor).*

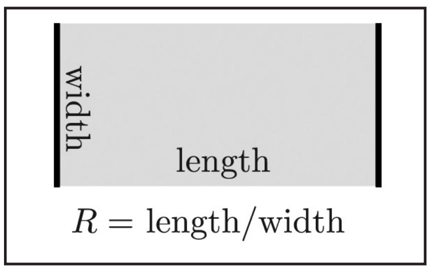

Figure 2. *If $R=1$, the rectangle is a square.*

To rephrase these theorems: *Two annular regions A and A' (as in Figure 4) are conformally equivalent ($A\sim A'$) if and only if they have the same electrical resistance between their inner and outer boundaries*:

$$R(A)=R(A'). \qquad (2)$$

Idea of the Proof

In order to construct a conformal map $A\leftrightarrow A'$, let us push the current by applying voltages $V=0$ to the inner boundary and $V=1$ to the outer boundary.[2] For a large integer n, consider the equipotential lines h_i, $i=0,\ldots n$ that are spaced by the potential difference $1/n$ (see Figure 5); h_0 is the inner boundary and h_n is the outer boundary. Fix an arbitrary line v_0 of steepest descent of the electrostatic potential—the line of current—and let v_1 be the line of steepest descent that is chosen so that the current through the channel v_0v_1 is $1/n$. Continue adding current lines v_j, as in Figure 5, and stop at $j=m$ when the current through the channel v_mv_0 becomes $<1/n$. This last channel plays no role in the limit of $n\to\infty$.

[2] By doing so, we consider the solution of the Dirichlet problem in the annulus with prescribed boundary values 0 and 1.

We divided the annulus into $n\times m$ infinitesimal curvilinear rectangles Q_{ij}, which we enumerate by the rectangle's layer i, $1\le i\le n$ and the channel j, $1\le j\le m$ (see Figure 5).

I claim that each curvilinear rectangle Q_{ij} *is a square in the limit of* $n\to\infty$. Indeed, the resistance is

$$R(Q_{ij})=\frac{\text{voltage drop}}{\text{current}}=\frac{1/n}{1/n}=1,$$

and a rectangle for which resistance $=1$ is a square (as indicated in Figure 2).

What is the resistance of A? Each channel has resistance n (being a stack of n squares), and with m channels in parallel,

$$R(A)=\frac{n}{m},$$

ignoring a small error due to the resistance of the last channel v_mv_0. The resistance therefore has an almost combinatorial meaning.

To construct the map $A\leftrightarrow A'$, we divide A' into $n\times m'$ squares Q'_{ij}. If $R(A)=R(A')$, then $m=m'$; this allows a $1-1$ assignment of Q'_{ij} to Q_{ij}. The result is a discrete conformal map since it takes squares to squares.

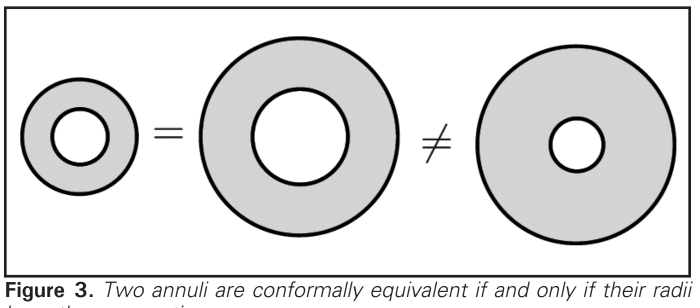

Figure 3. *Two annuli are conformally equivalent if and only if their radii have the same ratios.*

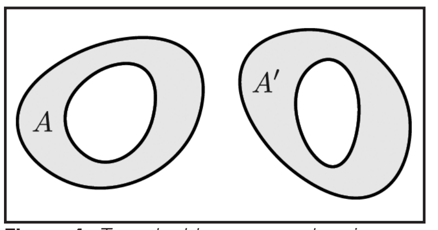

Figure 4. *Two doubly connected regions are conformally equivalent precisely when they have the same resistance between their inner and outer boundaries.*

Showing the Converse

$A\sim A'$ implies $R(A)=R(A')$. We divide A into "squares" Q_{ij} as before, with $1\le i\le n$ and $1\le j\le m$. The conformal equivalence induces a division of A' into "squares" (by conformality) with the same $m'=m$ (since the map is $1-1$). Therefore, $n/m=n/m'$ and $R(A)=R(A')$. In short, (1) demonstrates that *the resistance is a conformal invariant*, as was already mentioned in my April 2023 article.

The figures in this article were provided by the author.

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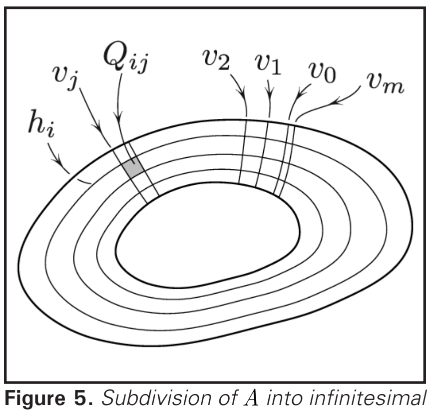


Figure 5. *Subdivision of A into infinitesimal squares Q_{ij}. Concentric "horizontal" lines h_i are equipotentials. Steepest descent "vertical" current lines v_j are added in a counterclockwise direction until the last line v_m. The "square" Q_{ij} is bounded by h_{i-1}, h_i and v_{j-1}, v_j.*

Hackathon

Continued from page 5

director of GEM) and Wesley Hamilton (senior software developer at PTC[7] and former member of the University of North Carolina, Chapel Hill SIAM Student Chapter). Hamilton helped to host a Datathon4Justice[8] at the University of Utah in 2021 and thus brought valuable experience to the MIT event. Students consulted the mentors for assistance and support throughout the course of the Hackathon.

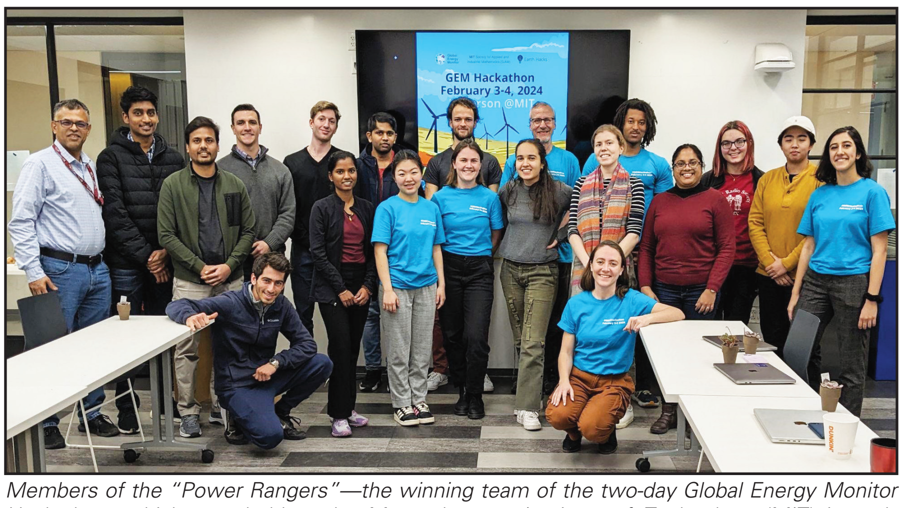
The winning team, named the "Power Rangers," created a website that summarizes the landscape of energy projects in a region of interest. Their work built upon an existing codebase that was initially intended to map the locations of Chipotle restaurants in a given neighborhood, though the students added many new features that incorporated further data analysis and insights. GEM intends to stay in touch with all participants to oversee the implementation of their projects beyond the prototyping phase.

At the event's conclusion, students returned to their studies with newfound confidence in their ability to program with the Structured Query Language, employ the `pandas` Python library,[9] utilize the Google Maps application programming interface, and perform regressions. Overall, the GEM Hackathon served as a great reminder of the importance of computational mathematics and data science in climate and environmental-based projects.

[7] https://www.ptc.com/en

[8] https://sinews.siam.org/Details-Page/datathons4justice-address-social-justice-issues-with-data-science

[9] https://pandas.pydata.org



Members of the "Power Rangers"—the winning team of the two-day Global Energy Monitor Hackathon, which was held at the Massachusetts Institute of Technology (MIT) in early February—gather with Hackathon organizers and participants for a group photo. The programming contest, which asked participating students to examine worldwide energy data availability and solar resource potential, was co-hosted by the MIT SIAM Student Chapter and Earth Hacks. Photo courtesy of MIT's Department of Urban Studies and Planning.

Bianca Champenois is a Ph.D. candidate in the joint mechanical engineering and computational science/engineering program at the Massachusetts Institute of Technology (MIT). Her research involves data-driven fluid mechanics for ocean and atmospheric modeling. Champenois is also president of the MIT SIAM Student Chapter.

Sanjana Paul is the co-founder and executive director of Earth Hacks: an environmental hackathon organization. She is currently a graduate student in environmental policy and planning at MIT. Paul holds bachelor's degrees in electrical engineering and physics and has worked on projects that range from atmospheric science software engineering to building decarbonization policies at the municipal level.

Take Advantage of SIAM's Visiting Lecturer Program

Hearing directly from working professionals about research, career opportunities, and general professional development can help students gain a better understanding of the workforce. SIAM facilitates such interactions through its Visiting Lecturer Program (VLP), which provides the SIAM community with a roster of experienced applied mathematicians and computational scientists in academia, industry, and government. Mathematical sciences students and faculty—including SIAM student chapters—can invite VLP speakers to their institutions to present about topics that are of interest to developing professional mathematicians. Talks can be given in person or virtually.

The SIAM Education Committee[1] sponsors the VLP and recognizes the need for all members of our increasingly technological society to familiarize themselves with the achievements and potential of mathematics and computational science. We are grateful to the accomplished individuals who have graciously volunteered to serve as visiting lecturers.

Points to consider in advance when deciding to host a visiting lecturer include the choice of dates, speakers, topics, and any additional or related activities (such as follow-up discussions). Organizers can reach out directly to speakers and must address these points when communicating with them. Read more about the program and view the current list of participants online.[2]

[1] https://www.siam.org/about-siam/committees/education-committee

[2] https://www.siam.org/students-education/programs-initiatives/siam-visiting-lecturer-program

High School Mathematical Contest in Modeling Explores Dandelions and Electric Buses

By Kathleen Kavanagh and Benjamin Galluzzo

The Consortium for Mathematics and Its Applications[1] (COMAP)—which has been providing educators with mathematical modeling resources for more than four decades—held its annual High School Mathematical Contest in Modeling[2] (HiMCM) from November 1-14, 2023. The final judging session took place in January and acknowledged nine outstanding winning teams from around the world for their impressive solution papers.[3] All current middle and high school students are eligible to register for this two-week competition, though teams where all members are 14.5 years old or younger may instead choose to partake in the Middle Mathematical Contest in Modeling (MidMCM). Participating HiMCM teams—comprised of up to four students from the same school—have a 14-day window to select and download one of two open-ended, real-world problems; collaborate and employ mathematical modeling techniques to develop a solution; and electronically submit their final papers to COMAP. MidMCM follows the same protocol but only consists of a single problem.

A total of 967 teams from 417 schools and 18 countries/regions competed in the 2023 HiMCM. SIAM members Kathleen Kavanagh and Benjamin Galluzzo (both of Clarkson University) each wrote one of the two prompts,[4] which were respectively titled "Dandelions: Friend? Foe? Both? Neither?"[5] and "Charging Ahead with E-buses."[6]

Kavanagh authored *Problem A*, which focused on invasive species and asked students to predict the spread of dandelions over the course of one, two, three, six, and 12 months given the initial presence of a single "puffball" next to an open field (see Figure 1). In addition to spatial considerations, most of the teams accounted for seed release, seed travel time, germination time, and dandelion growth phases. The problem also challenged students to analyze the dandelion's potential success under different climate conditions, such as varying levels of wind, temperature, and humidity. Finally, participants created ranking systems that assigned an impact factor to an invasive species, then tested their models on dandelions and two other plants of their choice. The breadth and depth of the solution papers were outstanding; they utilized techniques that ranged from simple linear models to finite element simulations and susceptible-infectious-recovered systems of equations.

In *Problem B*, Galluzzo prompted teams to address the global shift towards electric buses (e-buses) as a sustainable urban transportation solution in light of growing concerns about air pollution and climate change. The problem asked students to devise models that assessed the ecological and financial impacts of a shift to e-buses while accounting for factors like initial costs, operational expenses, and governmental incentives. After selecting a metropolitan area of their choice, teams used their models to generate a 10-year roadmap for the hypothetical transition to a fully electric bus fleet, paying strict attention to complications like charging infrastructure and range limitations. They then crafted concise policy recommendation letters that articulated their strategies and recommendations to transportation officials, emphasizing the necessity of a holistic approach to sustainable transit.

Readers might also be familiar with COMAP's sister competitions for undergraduate students: the Mathematical Contest in Modeling (MCM) and Interdisciplinary Contest in Modeling (ICM).[7] MCM/ICM take place annually in February and provide students with an opportunity to work on a team and improve their modeling, problem-solving, and writing skills. In 2024, more than 30,000 teams from across the world participated in these international contests.

SIAM members can support and promote mathematical modeling competitions in a variety of ways. COMAP's slate of competitions and the MathWorks Math Modeling Challenge[8] (M3 Challenge)—a program of SIAM with MathWorks as its title sponsor—are constantly seeking challenge questions for future competitions. If you have an idea for a real-world problem that is well suited for mathematical modeling and you would like to submit it for consideration, HiMCM coordinators and the M3 Challenge Problem Development Committee will work with problem authors to tailor their questions for the appropriate audience and locate any relevant data. Both organizations also routinely look for judges to review student submissions and select the winners.

Additionally, U.S. teams that score well in either HiMCM or M3 Challenge may be invited to potentially represent the U.S. in the International Mathematical Modeling Challenge[9] (IM^{2}C): a competition that allows each participating country/region to nominate up to two representative teams that then tackle a difficult math modeling problem over five consecutive days. For more information about HiMCM and M3 Challenge, please reach out to **himcm@comap.org** and **m3challenge@siam.org**.

M3 Challenge is an annual mathematical modeling competition for U.S. high school juniors and seniors and sixth form students in England and Wales. Participating teams of three to five students have 14 consecutive hours during Challenge Weekend to tackle a complex, real-world problem and produce a report that explains and justifies their solutions. Registration is completely free.

The 2024 M3 Challenge Final Event will take place on April 29th in New York City. The finalist teams and Technical Computing awardees—having submitted their papers in early March—will present their work to a live panel of judges and compete for more than $100,000 in scholarship funds. Stay tuned for an article about the winning team's solution in the June issue of SIAM News!

Kathleen Kavanagh is a professor of mathematics at Clarkson University and the former Vice President for Education at SIAM. Benjamin Galluzzo is an associate professor of mathematics at Clarkson University. He is the director of the Consortium for Mathematics and Its Applications' High School Mathematical Contest in Modeling.

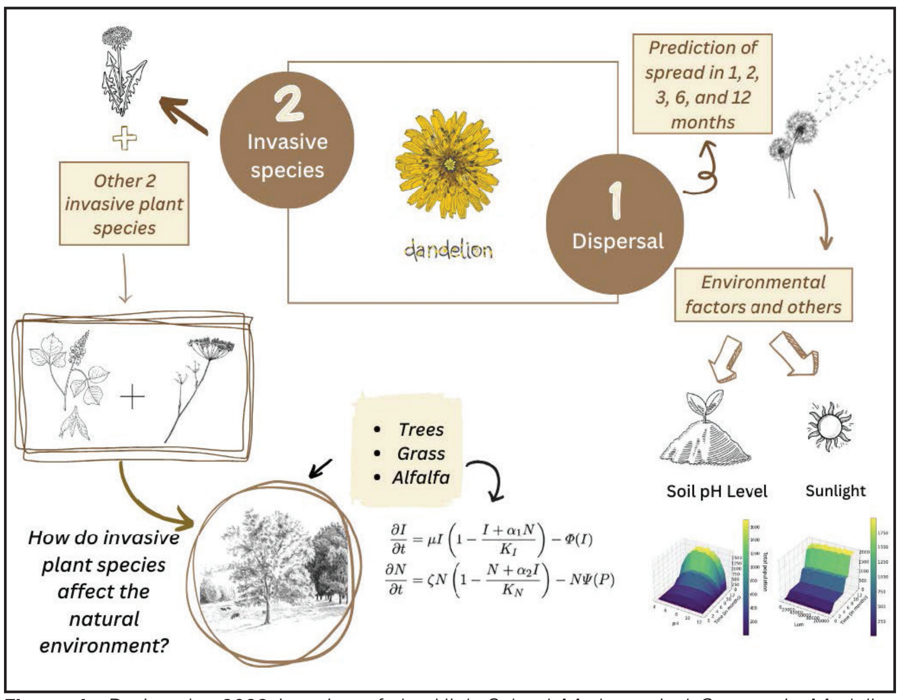


Figure 1. *During the 2023 iteration of the High School Mathematical Contest in Modeling (HiMCM), which took place in November, one problem prompted students to predict the spread of dandelions while accounting for numerous influencing factors. Figure courtesy of HiMCM Team 13845 from BASIS International School Guangzhou.*

[1] https://www.comap.org
[2] https://www.comap.org/contests/himcm-midmcm
[3] https://www.contest.comap.org/highschool/contests/himcm/2023results.html
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[7] https://www.comap.org/contests/mcm-icm
[8] https://m3challenge.siam.org
[9] https://www.immchallenge.org

Financial Mathematics

Continued from page 6

across applications, and many aspects of mathematical finance rely on the ability to solve these systems. The solution of partial differential equations (PDEs) is a particularly important application. In fact, a quantum algorithm for linear PDEs can efficiently price European and Asian options in the Black-Scholes framework [4].

Quantum Monte Carlo

Another quantum algorithm can accelerate Monte Carlo methods in a very general setting [8]. This algorithm estimates the expected output value of an arbitrary randomized or quantum subroutine with bounded variance, ultimately achieving a near-quadratic speedup over the best possible classical algorithm.

Quantum Semidefinite Programming

Quantum semidefinite programming (QSDP) is based on the observation that a normalized positive semidefinite matrix is naturally representable as a quantum state. On a quantum computer, operations on quantum states are sometimes computationally cheaper than classical matrix operations; this idea prompted the development of quantum algorithms for semidefinite programming. In finance, QSDP is potentially useful for maximum risk analysis and robust portfolio construction [6].

§

After decades of theoretical results, quantum computing is progressively becoming a reality. While full-scale quantum computers are not yet ready to replace their classical counterparts, they are nonetheless already useful in both speeding up specific procedures in classical algorithms (bringing forth the *hybrid classical-quantum era*) and providing new ways of thinking about old problems (so-called *quantum-inspired algorithms*). Rather than succumbing to quantum skepticism, we should instead embrace quantum computing as a valuable new tool that will help us more accurately address the numerous problems in quantitative finance.

The views and opinions expressed in this article are those of the authors and do not necessarily reflect the views and policies of their respective institutions.

Acknowledgments: Antoine Jacquier is supported by Engineering and Physical Sciences Research Council grants EP/W032643/1 and EP/T032146/1.

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Antoine (Jack) Jacquier is a professor of mathematics at Imperial College London. His research focuses on quantum computing as well as stochastic analysis and volatility modeling in finance. Jacquier also serves as a scientific consultant and advisor for various finance and technology companies. Oleksiy Kondratyev is the Quantitative Research and Development Lead at Abu Dhabi Investment Authority (ADIA). Prior to joining ADIA, he held quantitative research and data analytics positions at Standard Chartered, Barclays Capital, and Dresdner Bank. Kondratyev received the Risk Magazine Quant of the Year Award in 2019. Gordon Lee is head of the Markets Quants team at the Bank of New York Mellon Corporation. Mugad Oumgari is a managing director at Lloyds Banking Group. He received a postgraduate research degree in mathematics and a master's degree in economics from the London School of Economics and Political Science.

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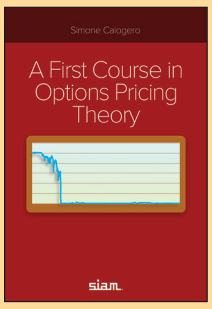

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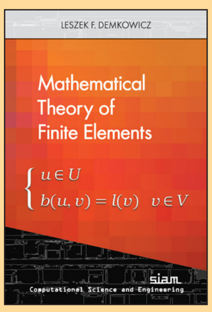

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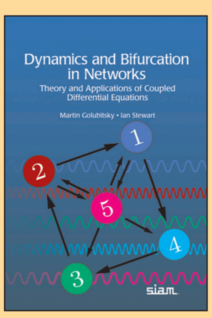

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Martin Golubitsky and Ian Stewart

In recent years there has been an explosion of interest in network-based modeling in many branches of science. This book attempts a synthesis of some of the common features of many such models, providing a general framework analogous to the modern theory of nonlinear dynamical systems. How networks lead to behavior not typical in a general dynamical system and how the architecture of the network influences this behavior are the book's main themes. It is the first book to describe the formalism for network dynamics developed over the past 20 years. The authors introduce a definition of a network and the associated class of "admissible" ordinary differential equations, in terms of a directed graph whose nodes represent component dynamical systems and whose arrows represent couplings between these systems; develop connections between network architecture and the typical dynamics and bifurcations of these equations; and discuss applications of this formalism to various areas of science.

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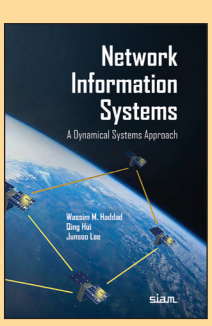

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New

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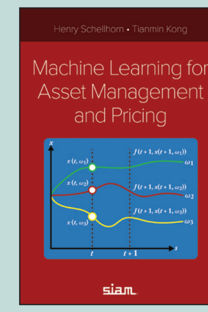

This textbook covers the latest advances in machine-learning methods for asset management and asset pricing. Recent research in deep learning applied to finance shows that some of the techniques used by asset managers (usually kept confidential) result in better investments than the more standard techniques. Cutting-edge material is integrated with mainstream finance theory and statistical methods to provide a coherent narrative. Coverage includes an original machine learning method for strategic asset allocation; the no-arbitrage theory applied to a wide portfolio of assets as well as other asset management methods; and neural networks and other advanced techniques.

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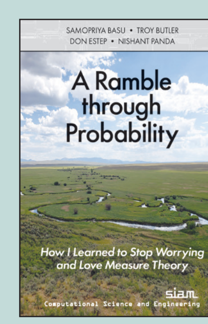
Samopriya Basu, Troy Butler, Don Estep, and Nishant Panda



Measure theory and measure-theoretic probability are fascinating subjects. Proofs describing profound ways to reason lead to results that are frequently startling, beautiful, and useful. Measure theory and probability also play roles in the development of pure and applied mathematics, statistics, engineering, physics, and finance. This book traces an eclectic path through the fundamentals of the topic to make the material accessible to a broad range of students. It brings together the key elements and applications in a unified presentation aimed at developing intuition; contains an extensive collection of examples that illustrate, explain, and apply the theories; and is supplemented with videos containing commentary and explanations of select proofs on an ancillary website.

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