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In the article on page 8, Evelyn Sander and James A. Yorke tackle the topic of chaos and its various manifestations and mathematical definitions.

Figure 2 (above) from the article shows eight basins of attraction for the forced–damped pendulum. The geometric complexity of the basins results from the chaos on their boundaries.

Tensor Decompositions in Smart Patient Monitoring

By Sabine Van Huffel

Substantial progress in both data recording technologies and information processing in recent years has enabled the acquisition and analysis of large amounts of biomedical data. Extraction of the underlying information patterns relevant to patient health, called "sources," is a core problem in biomedical information processing. Blind source separation (BSS) is the task of finding such source signals and the mixing mechanism, given only the raw signals, hence "blind." Rapid advances in healthcare diagnostics and medical technologies are opening new challenges for information processing.

Biomedical signal processing has moved away from vector processing (classical single-channel time- and frequency-domain analysis, despite the continued use in medical practice) to matrix processing. At that level, the BSS problem is solved by decomposition of the data matrix into a sum of interpretable rank-1 terms. This problem is underdetermined, however, unless strong assumptions on the sources are imposed, such as statistical independence (with use of independent component analysis (ICA) methods) or nonnegative components (with use of nonnegative matrix factorization (NMF) methods). Matrix-based BSS methods have become increasingly popular for artifact removal [4] and even for separating stimulus-related activity in multimodal data, such as electro-encephalography (EEG) and functional magnetic resonance imaging (fMRI) [9]. These methods are too restrictive, however. It may be possible to develop more powerful information systems by facing the following challenges:

■ **From matrix to tensor.** For maintaining structural information, higher-order tensors are very attractive. They generalize vectors and matrices to multiway tables of numbers [1]. The BSS problem is solved via tensor decomposition. Most well known is the canonical polyadic decomposition (CPD; also known as canonical decomposition or parallel factor decomposition) [6], which decomposes a tensor in rank-1 terms (Figure 1 on page 5, top). CPD is unique under mild conditions, which makes it a broadly applicable key tool for BSS. The assumptions needed for source extraction are very natural.

See **Tensor Decompositions** *on page 5*

Control, Intuition, Existence, and Regularity

By Francis Clarke

Two concentric rings are dipped in a soapy solution so as to form a surface of revolution (see Figure 1). What is the profile x(·) of the resulting soap film? This famous *minimal surface* problem amounts to identifying the function $x(t)$ that minimizes the integral functional

$$x(\cdot) \mapsto \int_a^b x(t)\sqrt{1+x'(t)^2}\, dt$$

under the constraint that x have prescribed values at a and b. Leonhard Euler solved this instance of the basic problem in the *calculus of variations* in 1744, finding that the (physically observed) curve is a *catenary*. This is a smooth function, and the soap film visibly exists; thus, no issues of regularity or existence would seem to arise at all.

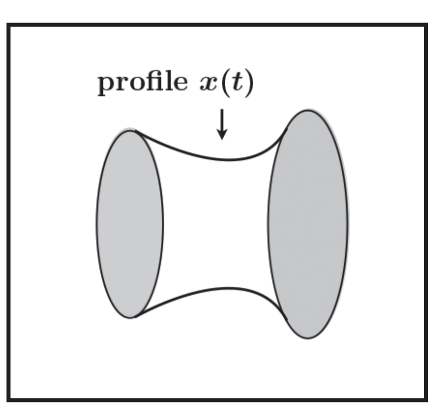

Figure 1. *A soap film spanned by two concentric rings.*

But what happens if we gradually increase the distance between the two rings? At some point, our physical intuition tells us, the bubble pops. Why so? Has the solution to the problem simply ceased to exist?

The reality is more complex. Almost a century later, Goldschmidt explained (in 1831) that, in fact, the minimal surface has folded *onto* the rings and has become the union of two disks—that is, the surface of revolution generated by the "broken curve" shown in Figure 2, one that has two "corners" (points of nondifferentiability). To our knowledge, this heralds the first appearance in analysis of a nonsmooth function (and irreversible dynamics).

There is no element of "control" in the problem above; that is, there is no structure one seeks to maximize, will not be made explicit here; see [1,3]. Instead, let's take a look at what turns out to be the answer. It has the form of a *feedback synthesis*, in which the choice of control values depends on the current state.

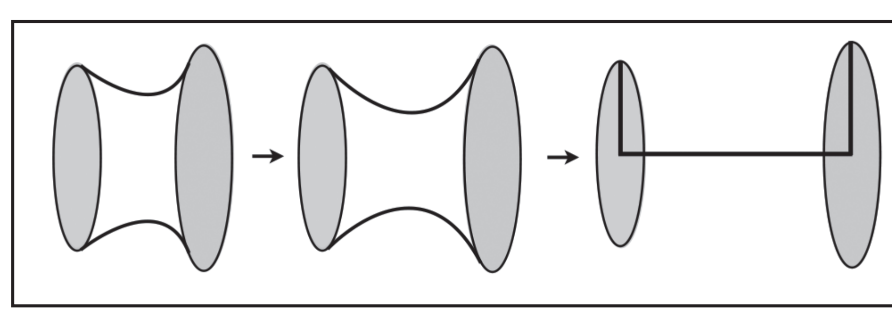

Figure 2. *Does the soap film disappear?*

whereby the state $x(\cdot)$ corresponds to the choice of a control function $u(\cdot)$ via certain dynamics, such as that of a *standard control system*

$$x'(t) = f\big(x(t), u(t)\big) \text{ a.e,}$$
$$u(t) \in U \text{ a.e.} \qquad (1)$$

(For the shape of soap bubbles, nature does the controlling, so to speak.) But the insight of Goldschmidt prefigures two of the central topics in control theory, a descendant of the calculus of variations: existence and regularity.

Let's examine these issues for an optimal control problem that arises in modeling renewable resources. There are two state variables: x (which we may think of as a measure of a fish population) and y (available boats, which are subject to depreciation). We choose directly two controls: $u(t)$ (boats sent out at time t to catch fish, a value between 0 and $y(t)$), and $I(t)$ (investment at time t in new boats). It is assumed that investment can have immediate effect; thus I may be an impulse control. The dynamics linking the choice of the control functions (u, I) to the resulting states (x, y) (that is, the function f in (1)), as well as the (net infinite-horizon discounted) return that

The optimal synthesis is indicated in Figure 3 (see page 3), for the case in which the initial values of x (fish) and y (boats) lie at the point designated by the letter **a** (many fish, few boats). It makes sense to invest in boats (at a certain cost, of course) from such a point: We move to the point **b** via an impulse purchase. (The dotted curve, whose provenance is explained later, determines **b**.) Subsequently, between **b** and **c**, we use all the boats we have ($u = y$); the number of boats is decreasing through wear and tear (depreciation). Once **c** is reached, there is a change of tactic: We cease to use all available boats ($u < y$) in order to maintain the fish population at a certain level x_S.

As experience shows, some economists will grumble at this stage that it must have been wasteful to buy so many boats initially, since some are not being used now. However, the level x_S, it turns out, corresponds to a *short-term* equilibrium that is a known (and economically accepted) feature of the solution when investment and depreciation are absent; so there is an economic argument for it. At the point **d**, however, where the boat level y_S is attained, economic intuition is even more seriously challenged: We change our minds about

See **Control** *on page 3*

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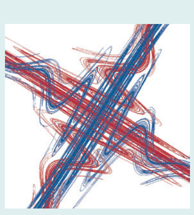
3 **Bringing *SIAM News* to Life for Nearly Three Decades**
Gail Corbett bid goodbye to SIAM and *SIAM News* in July, leaving behind a stellar publication largely shaped by her ability to grasp the social and intellectual networks that drive mathematics, as Paul Davis eloquently puts it.

4 **How *Interstellar* Was Born**
Jim Case reviews *The Science of Interstellar*, a book that captures the underlying science behind Christopher Nolan's blockbuster film. As far-fetched as wormholes into distant planets and universal blights caused by giant dust storms can be, the book's author, Kip Thorne, assures us that the movie adheres to firmly established laws of the universe.

6 **A SIAM Special Function at NIST**
Willard Miller, Jr. writes about the vital role of special functions and orthogonal polynomials in the construction of mathematical models of real-world systems that can be solved analytically and explicitly, and how this was evident at the 13th International Symposium on Orthogonal Polynomials, Special Functions and Applications.

7 **Moving SIAM to the Forefront of Technology in the Information Age**
Upon his retirement after more than three decades, James L. Goldman, SIAM's inaugural director of information systems, recaps the history of the use of technology at SIAM and shares other thoughts.

8 **Chaos and its Manifestations**



2 **Obituaries**

7 **Professional Opportunities**

7 **Announcements**

Obituaries

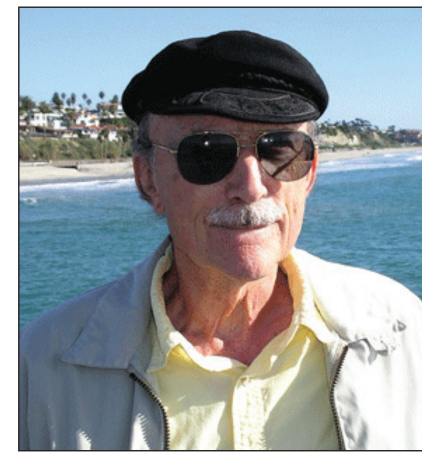


Photo by Tom Haigh.

Charles Lawson, 1931–2015

Charles Lawson, who worked at the Jet Propulsion Laboratory (JPL) at Caltech from 1960 until his retirement in 1996, died on July 2, 2015, in Laguna Woods, California.

Chuck was born in 1931 in Weiser, Idaho. He went to the University of California at Berkeley for his undergraduate degree, a BS in optometry. He then went into the U.S. Army, after which—thankfully—he enrolled at UCLA, receiving an MSc in mathematics under Peter Henrici and, in 1961, a PhD under Theodore Motzkin. While at UCLA he had experience with the SWAC (the Standards Western Automatic Computer), built (completed in 1950) by the West Coast branch of the National Bureau of Standards.

Chuck began to support the use of computers at JPL shortly before receiving his PhD. In 1968, when I joined his group, it was clear to people all over the lab that Chuck was the person to see about issues connected with the use of computers, especially with respect to numerical mathematics. It was interesting that people would often come in with one problem and leave with a solution to a different problem—the problem they should have been trying to solve in the first place. If you are involved in consulting, this is a good thing to keep in mind. Of course it also helps if you have a good background, a clear understanding of the issues involved, and an ability to explain things in understandable terms. Chuck had all of these qualities to a high degree, together with a very agreeable personality.

When I joined Chuck's group, our primary charter was to do research in numerical mathematics, but all of us were interested in creating software that could be used by the projects. Although the funding from NASA headquarters was primarily for research in computational mathematics, our work in developing mathematical software was tolerated, and the usefulness of that work later became key to continued support for the group. Chuck did some early work on computer approximation, which together with contributions of others, appeared in the book *Computer Approximations*. He was responsible for the early use of orthogonal transformations for matrix computations at JPL, work that led to another book, *Solving Least Squares Problems*, co-authored with Richard Hanson. That book is now part of the SIAM Classics in Applied Mathematics series.

Early in the 1970s, I had occasion to look at the ugly assembly code generated by our UNIVAC Fortran compiler. That motivated me to write assembler routines for what we would now call ddot and daxpy, which showed that significant speedups could be achieved in this way. I had thought that this would be more useful for the software libraries we were developing at JPL. Chuck had a much better idea—he expanded the functionality and got a number of other people involved. The result was the first BLAS paper, which seems to have spawned a minor industry, spearheaded largely by Jack Dongarra. That later work was followed by more significant gains than possible with just the simple vector operations in the original paper.

It was Chuck who decided that we should develop a library of mathematical software for use at JPL. Work he did in 1976 on C1 interpolation to scattered data in the plane was included in the MATH77 library in 1991. Later, he took the lead in making that work available in C as well. These libraries, MATH77 and mathc90, are still in use at JPL.*

In later years, with a change in management came the requirement that a charge number be attached to all work. An engineer or scientist, instead of just dropping by to get some help, had to ask a supervisor for a charge number to cover our group's time. After the new procedure was adopted, our group was much less effective, and funding gradually dried up. In what I regard as an act of extraordinary generosity, Chuck started to take on other jobs at JPL, in effect giving me the position he had created.

Chuck was a competitive swimmer in college, and his later interests were varied and intense. He enjoyed folk dancing to the point of teaching classes in it, and in his later years the ukulele was his passion. He is survived by his wife Dottie, children Michael, Brian, Melanie, and Marcella, and numerous grandchildren.

I feel blessed to have had Chuck as a boss and as a friend.—*Fred Krogh, fkrogh@mathalacarte.com (with helpful input from Brian Lawson, Richard Hanson, Van Snyder, and Barbara Horner-Miller).*

Additional information can be found in Tom Haigh's extensive interview with Lawson; http://history.siam.org/oralhistories/lawson.htm.

*These libraries were recently made available at http://netlib.org/math with an open-source license.

A Cycloid is a Tautochrone
A Short Proof

MATHEMATICAL CURIOSITIES
By Mark Levi

In 1659 Christiaan Huygens answered a question that came to him as he watched a swinging chandelier in a church: What curve has the property that a bead sliding along it under uniform gravity and with no friction will oscillate with a period independent of the amplitude?

The answer turned out to be the cycloid generated by a circle, as illustrated in Figure 1. Several solutions to this problem have been found (http://en.wikipedia.org/wiki/Tautochrone_curve); Abel's is particularly remarkable [1].

Presented here is a very short geometrical proof of the tautochronous property of the cycloid. It is based on the fact that $\mathbf{v} \perp \mathbf{r}$, as explained in Figure 1.*

To prove that the cycloid is a tautochrone, it suffices to show that the arclength distance s from the bottom of the cycloid behaves as a harmonic oscillator:

$$a = -ks,$$

where $a = \ddot{s}$, for some constant k. (This idea, which I had learned from Henk Broer, is attributed to Lagrange.) Because $a = 0$ when $s = 0$, we just need to verify that

$$da = -k\, ds. \qquad (1)$$

But $da = d(g\cos\theta) = -g\sin\theta\, d\theta$. And from Figure 2 we have $ds = D\sin\theta\, d\theta$. Comparing these expressions for da and ds proves (1) with $k = g/D$. QED

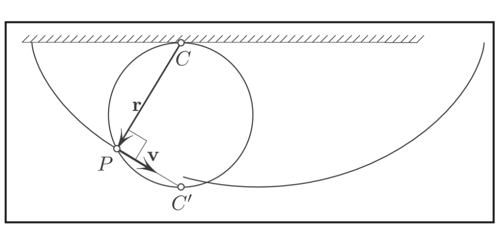


Figure 1. *The contact point C is an instantaneous center of rotation of the rigid wheel, and thus* $\mathbf{v} \perp \mathbf{r}$.

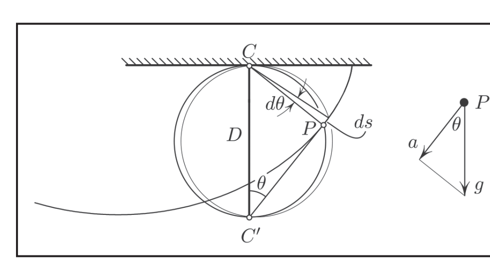


Figure 2. *Proof that a = –ks, using the fact that PC′ ⊥ PC.*

*Incidentally, building on this fact, the line of velocity of every point on a rolling wheel (in the ground reference frame) passes through the topmost point of the wheel. A pebble stuck to the tire always aims straight at, or straight away from, the topmost point of the wheel!

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[1] M. Levi, *Classical Mechanics with Calculus of Variations and Optimal Control*, AMS, Providence, Rhode Island, 2014.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University. *The work from which these columns are drawn is funded by NSF grant DMS-1412542.*

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Control

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maintaining the fish level at x_S and return to using all available boats ($u = y$), even though this drives x below the level x_S that we had been respecting; hmm. . .

Later, the value of x eventually returns

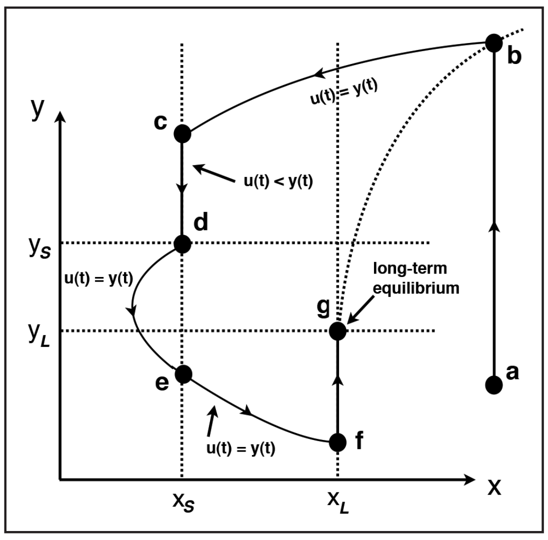

Figure 3. *An optimal feedback synthesis.*

to x_S (at point **e**), boats having sufficiently depreciated to allow this. Subsequently we tolerate a low fleet level, using all boats, allowing x to increase beyond x_S to a certain value x_L, at point **f**. Then we make an immediate purchase of boats to arrive at the point **g** defined by $y = y_L$, following which we employ a constant level of continuous investment in order to remain, happily ever after, at **g**. The value x_L is revealed to be the *long-term* optimal stock level which one attains by a circuitous route.

To assert the optimality of this scheme without proof, given its various counter-intuitive aspects, simply won't do. (A strictly numerical solution wouldn't yield much insight.) How to produce such a proof? If we know a priori that our optimal control problem does have a solution (existence theory), if we dispose of rigorously true necessary conditions that apply to the problem at hand, and if we can analyze them to deduce the above strategy, then that would constitute a satisfactory proof by the *deductive method*. It is a fact that many optimization problems can be solved this way, which explains the very practical importance of existence theorems, as well as necessary conditions that are fully proved under precise hypotheses.

The ingredients of the deductive approach are lacking here. But informal use of the necessary conditions known as the Pontryagin maximum principle leads to the various dotted lines in Figure 3, which can then be used to construct the solution on a speculative basis. It needs to be confirmed, however. There is a famous *inductive* method for doing this, that of *verification functions* (see [5]). Given a proposed solution (found, perhaps, by guesswork or dubious means), it is based upon finding a function that satisfies the Hamilton–Jacobi partial differential equation (or inequality) and that is related to the putative solution in a certain way. Then the very existence of this function verifies that the proposed solution is correct. There is a hitch, however, and it's a question of regularity. Generally, in control (as in this example), the verification function will need to be *nonsmooth*. Then the solution concept for the PDE necessarily involves generalized derivatives; such topics form part of the subject often referred to as *nonsmooth analysis* (see [2,5]).

We observe that in the optimal strategy found above, the dependence of the optimal control values on the current state (the feedback law) is discontinuous. There's nothing unusual about that; it has been a feature of optimal control since the beginning of the theory (that is, since the 1950s). Engineers, though, tend to look askance at discontinuous feedbacks, for various reasons. Their intuition tells them that they result from demanding the very best solution to a problem and that by settling for more reasonable (suboptimal) feedbacks, or by approximating, it will always be possible to use continuous feedbacks. Their experience in linear systems theory, that highly successful bedrock of engineering control design, bolsters this thinking. It is a surprising revelation of recent years, however, that this intuition is not fully correct, once one strays from the classic linear setting, as sometimes one must. Let's be a bit more specific, by looking at what is arguably the most basic issue in control systems: the design of stabilizing feedback.

Suppose that the control system (1) is nicely controllable to the origin (in a certain sense). Is it then stabilizable by feedback? Translation: Is there a function $u(x)$ taking values in the control set U so that the differential equation $x'(t) = f(x(t), u(x))$ is stable (its trajectories go to 0)? Note that no optimality of any kind is involved here; we simply require a "reasonable" feedback law that will have the effect of driving the system automatically to zero. Nonetheless, it turns out that, in general, we must have recourse to *discontinuous* feedback functions $u(x)$ in order to achieve stabilization. This is so even for such simple, bilinear, mechanically relevant systems as the classical nonholonomic integrator. It is true that there are potential pitfalls in using discontinuous feedbacks. And their implementation must certainly be carefully studied. (The feedback law may be discontinuous, but no actual physical motion of the system itself will be, of course.) But discontinuous feedbacks can offer some real advantages [4,6].

So far, we have seen problems where irregularity arises *indirectly*: The solution has corners, the verification function is nonsmooth, and the optimal synthesis or the stabilizing feedback is discontinuous. Nonsmoothness also can arise directly, as an intrinsic part of the problem from the start. Consider, for example, the simplest possible RC electrical network (see Figure 4), but replace the resistor with a diode—that is, a resistor for which the proportionality constant in Ohm's law depends on the direction of the current. The function f in the differential equation (1) describing this circuit is nondifferentiable (it is an exercise to show this). Another (more hidden) example of intrinsic nonsmoothness arises in the well-known engineering problem of minimizing the maximum eigenvalue of a matrix (relative to some of its entries). Again, this function, and others arising in optimal design, will generally be nonsmooth [2].

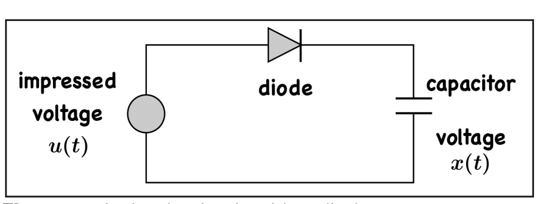

Figure 4. *A simple circuit with a diode.*

In summary, we have seen why it is crucial to be able to solve certain control problems analytically, that intuition cannot always be counted on, and how existence is a central issue. And we have seen various ways, both indirect and intrinsic, in which nonsmoothness arises and is unavoidable. Is this to be deplored? Not at all, for it is a fact of nature; to quote the Old Testament: *Consider God's handiwork: who can straighten what He hath made crooked?* [Ecclesiastes 7:13].

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Francis Clarke is recipient of the 2015 W.T. and Idalia Reid Prize and a professor in the Institut Camille Jordan at the Université de Lyon. This article is adapted from the Reid Prize lecture he delivered on July 9 in Paris at the SIAM Conference on Control and its Applications.

Bringing *SIAM News* to Life for Nearly Three Decades

SIAM News existed before SIAM found Gail Corbett, but she found for SIAM the newsjournal that SINEWS is today, a compelling communicator of mathematics and its applications within and far beyond the boundaries of our society. SINEWS keeps us together as it tells our story all around the globe to business, industry, government, and academia.

Gail Corbett's first issue of SIAM News *from March 1987 when she began as associate editor of the membership's newsjournal.*

I became involved with SINEWS, along with a few other volunteers (Ed Rogers, Jim Frauenthal), before Gail's time but after it had moved out of Ed Block's kitchen and into SIAM's offices in the Architect's Building. All of us subscribed wholeheartedly to Ed's vision of telling non-mathematicians of the accomplishments and importance of applied mathematics. None of us could move from that vision to a self-sustaining, much less thriving, publication.

Gail Corbett accomplished that, bringing SINEWS to life, a slow, arduous, and ultimately brilliant accomplishment. She understands writers, mathematicians, mathematics, and how each entity interacts with the others.

She managed a changing cast of writers, some of them mathematicians, some not. Some of them wrote about what they did or knew. Some of them learned, then wrote. Some delivered what they had promised when it was due. Others drifted off like satellites into deep space, eventually falling silent, mission incomplete. But she filled issue after issue with more and better material even as the pace of publication increased.

A more critical skill was her grasp of the social and intellectual networks that drive mathematics, especially the interdisciplinary networks of applied mathematics. No one could work with her for long without being struck by her intuitive understanding of the shapes and connections of important ideas and of the roles of specific individuals in mastering those relationships.

Reading a recent piece* in the *NY Times Magazine* by Gareth Cook about the UCLA Fields Medalist Terry Tao brought to mind those insights of Gail's. Cook wrote,

> "Tao told me that his view of mathematics has utterly changed since childhood. . . . It turned out that the work of real mathematicians bears little resemblance to the manipulations and memorization of the math student. . . . The ancient art of mathematics, Tao has discovered, does not reward speed so much as patience, cunning, and . . . the sort of gift for collaboration and improvisation that characterizes the best jazz musicians."

Perhaps we shouldn't be surprised that a textile artist and fluent Francophile should understand so well how and what we do, likely better than many of us understand ourselves, certainly with a mastery that has led *SIAM News* to become so effective and so highly regarded for telling our stories.

Gail, thank you for taking time away from your loom to build SINEWS for us. Enjoy a long and happy retirement. We'll miss you.—*Paul Davis, Professor Emeritus, Worcester Polytechnic Institute*

After more than twenty-eight years, Gail Corbett retired as SIAM News *editor on July 31, 2015. Karthika Swamy Cohen takes over from Gail as the new managing editor and will oversee publication of the newsjournal along with Sara Murphy, associate editor.*

*G. Cook, *The singular mind of Terry Tao*, NY Times Magazine, July 24, 2015.

How *Interstellar* Was Born

The Science of Interstellar. *By Kip Thorne, W.W. Norton, New York, 2014, 336 pages, $24.95.*

BOOK REVIEW
By James Case

The idea for the recent blockbuster movie *Interstellar* was hatched over dinner in October 2005 by astrophysicist Kip Thorne and his former girlfriend Lynda Obst, by then an A-list Hollywood producer. Wishing to make a science fiction movie about both the end of (human) life on Earth and space travel through wormholes—a concept pioneered by Thorne in the 1960s—Obst wondered if he would care to collaborate on such a project. Would he! Before long he resigned the Feynman Professorship (Emeritus) at Caltech to devote more time to the project.

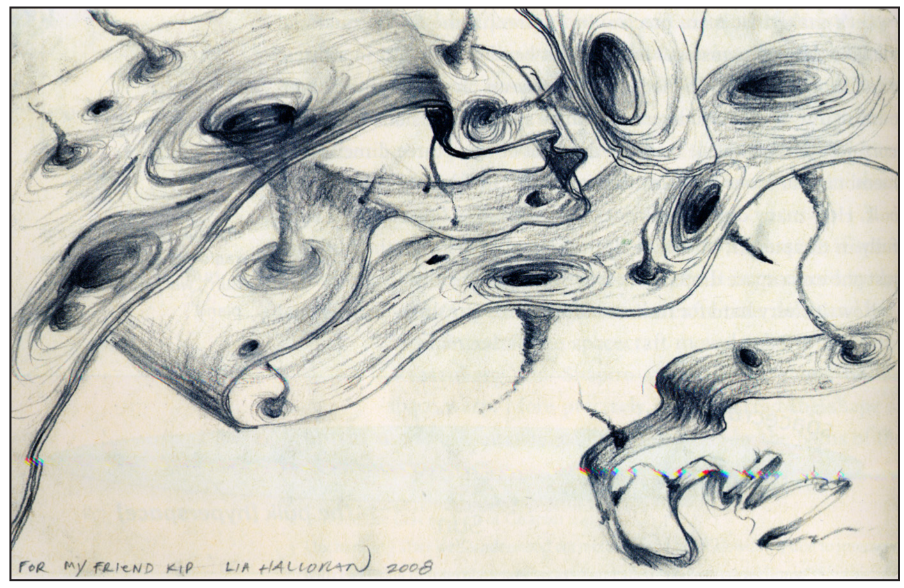
Within months of their dinner conversation, Obst had persuaded Stephen Spielberg to sign on as director and had scheduled a brainstorming session with Spielberg, Spielberg's father, Thorne, and 13 of his Caltech colleagues to discuss scientific content. Thorne proposed two guidelines:

1. Nothing in the film would violate firmly established laws of physics or firmly established knowledge of the universe.
2. Speculations (however wild) about ill-understood physical laws and the nature of the universe would spring from real science—ideas that at least a few respectable scientists regard as possible.

The resulting movie was directed by Christopher Nolan, rather than Spielberg, who cited more pressing commitments and left the project. It opened to mixed reviews in November 2014, with an all-star cast that included Matthew McConaughey, Anne Hathaway, Michael Caine, and Jessica Chastain. Ancillary products include the book under review, a novelization of the film by Greg Keyes, an official script notable for the cryptic "sound bites" in which onscreen characters attempt to explain complicated science to the theatre audience, and a downloadable version of the film itself.

Thorne's book explains the science underlying the movie. Chapters I, II, and IV–VI describe current knowledge of the cosmos, with an emphasis on black holes, wormholes, singularities, gravitational anomalies, and the fifth dimension, while Chapter III describes a sequence of events on Earth that at some future point could force mankind to seek a home elsewhere in space.

Briefly, global warming could generate giant dust storms (larger than the ones that menaced even East Coast cities during the Dust Bowl of the 1930s), paving the way for a "universal blight" that would strike one weakened crop after another until, as is revealed in the movie's opening scene, corn (maize) alone remains to feed humanity. Soon after that, as one of the characters in the film predicts, that crop too, would seem certain to fail.



Black holes and wormholes extending out of the brane into and through the bulk. Figure 4.5 in The Science of Interstellar.

In reality, the world has never known such a blight. Although some, like the one responsible for the Irish Potato famine of the 1840s, have devastated specific plant species, none has displayed an ability to progress from one species to another. Far-fetched as the idea may be, agronomists seem unable to deny that so aggressive a blight is at least possible.

The rest of the story seems even more far-fetched. For a manned mission to reach a potentially habitable planet, the writers postulate a wormhole connecting a location near Saturn to one in a distant galaxy, put in place by an unknown race of benefactors known only as "they." These more advanced beings appear to reside in the higher dimensional space wherein our four-dimensional space–time is embedded and to communicate among themselves by means of gravity waves. In the language of cosmology, our own space–time is known as the *brane* (short for membrane), while the higher dimensional space in which it is embedded is called the *bulk*. Superstring theorists suggest that the weak, strong, and electro-magnetic forces of nature operate only in the brane, while gravity operates throughout the bulk. They also suggest that the bulk has many more than five dimensions but, for his expository purposes, Thorne elects to ignore that aspect of their speculations.

For ease of visualization, Thorne makes frequent use of diagrams depicting a two-dimensional brane embedded in a three-dimensional bulk. One of his more fanciful renditions, prepared by artist friend Lia Halloran, appears above. In it, black holes appear as (roughly conical) singularities of curvature in the brane, while wormholes resemble singularities joined together in pairs, vertex to vertex, connecting one sheet of the brane with another. These wormhole-forming pairs, it is now known, are unstable objects that (unless held open by outside forces) soon degenerate into separate black holes.

Whereas ordinary celestial objects such as stars and planets are but shallow depressions in the brane, sufficiently massive stars may (as they exhaust their nuclear fuel) implode into singularities of curvature in space–time. Indeed it was shown (by J. Robert Oppenheimer in 1939) that, if the implosion is exactly spherical, the imploding object must (i) create a spherical black hole around itself, (ii) create a curvature singularity at the hole's center, and (iii) subsequently get swallowed up into the singularity, leaving behind no matter at all. The resulting black hole is made entirely of warped space–time.

In his quest for a general theory of relativity, Einstein became convinced that *Everything likes to live where it will age most slowly, and gravity pulls it there.* Accordingly, his field equations imply that time has to slow down in the presence of massive objects (i.e., space–time warps), and

See **Interstellar** *on page 5*

Tensor Decompositions

continued from page 1

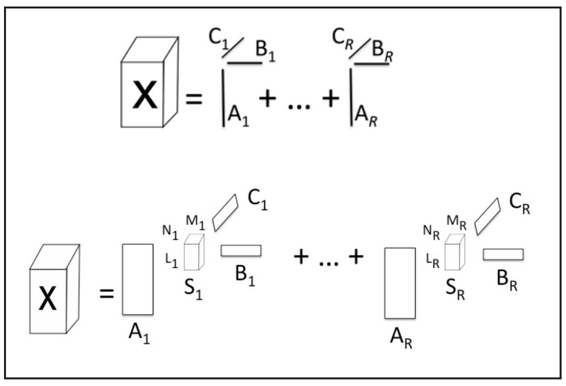


Figure 1. *Decomposition of a third-order tensor. Top, CPD, atomic terms; bottom, BTD, molecular terms. Adapted from [3].*

■ **From rank-1 to low-rank.** It is important to observe that a rank-1 structure is, in fact, very restrictive. Except for strength, no other source variations can be modeled. The decomposition may still be unique when it involves terms that are more general and/or realistic than rank-1 terms. We denote such generalized decompositions as block term decompositions (BTD) [3]. (See Figure 1, bottom.) BTDs make it possible to model more variation (shape, delay, . . .) and provide more detail. In addition, BTDs allow broad source modelling [2].

De Lathauwer's team implemented most tensor decompositions, including CPD and BTD, into a powerful open-source, easy-to-use, optimization-based Matlab toolbox; see www.tensorlab.net. Moreover, factorizations can be coupled or fused with each other. Factors can be shared, and any structure can be imposed. Tensorlab is the core of our solution of tensor-based BSS problems. A few case studies illustrate the process.

Extracting the Epileptic Component from EEG

Because seizures exhibit oscillatory behavior that is almost stable in localization and frequency during an entire episode (2–10 sec), they fit a trilinear structure. In Figure 2, we construct a third-order tensor via wavelet transform of all EEG channels and show that CPD reliably identifies one epileptic source with $R = 2$ and no need for artifact removal [4].

When seizures are nonstationary (e.g., varying in space or frequency during the episode), CPD is too restrictive. In that case, use of BTD improves the extracted seizure component, as shown in [5]. We applied wavelet transform or Hankel expansion to organise the EEG data in a tensor. With the former approach, we were able to model nonstationary seizures evolving in either frequency or spatial distribution; the latter was useful for extracting the epileptic pattern obscured by severe artifacts. Nevertheless, successful use of this technique in practice depends on blind selection of appropriate model parameters.

Monitoring Neonatal Brain Recovery from EEG

We have worked to automate monitoring of the vulnerable brain in the Neonatal Intensive Care Unit following hypoxic insult or brain injury, after which seizures often emerge. We developed CPD-based algorithms similar to those described above and successfully extracted the onset of seizures [4]. In addition, we sought to monitor brain recovery through automated quantification of the abnormality in one-hour EEG segments (called "EEG grading"). Our proposed holistic approach is shown in Figure 3. When we simultaneously decomposed all such tensors in a training set of 33 neonates via higher-order discriminant analysis (HODA), more relevant features were selected, thereby improving EEG grading accuracy (89%) and increasing robustness to the presence of artifacts [7].

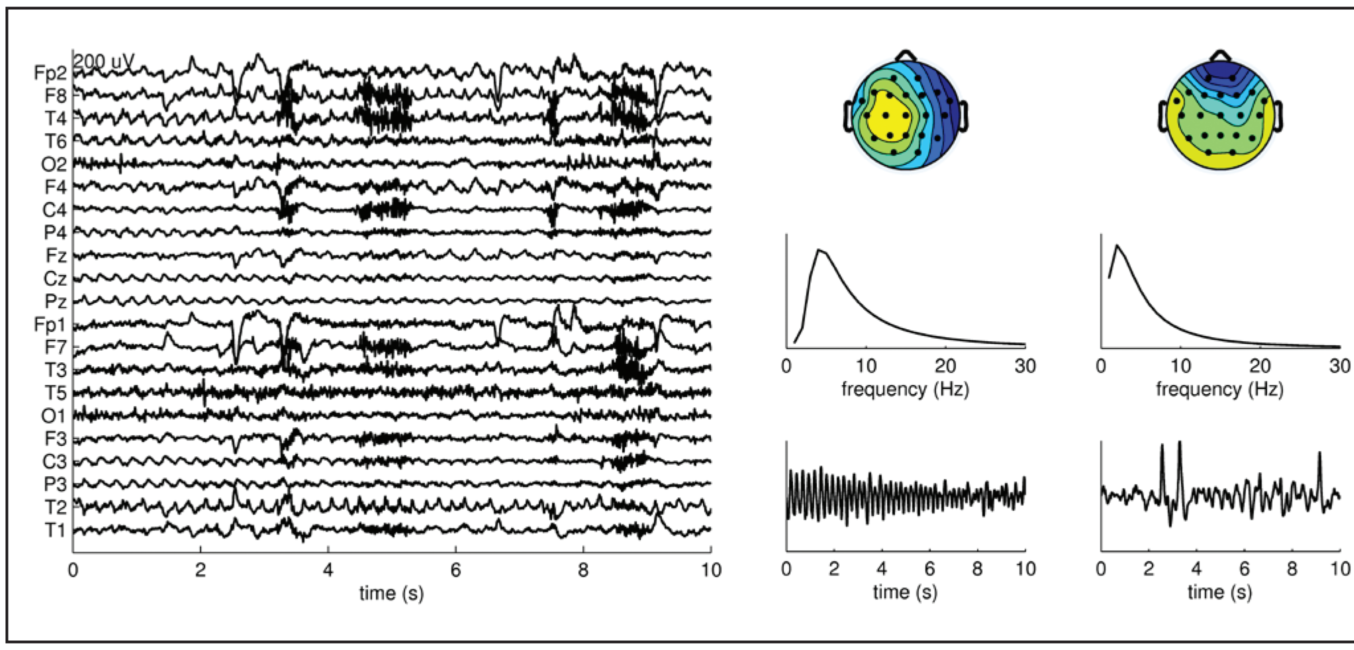


Figure 2. *At left, CPD of wavelet transformed EEG showing 10 sec onset of a right temporal lobe complex partial seizure. Muscle (at 5 and 9 sec) and eye blink artefacts (sharp wave) are visible. At right, the spatial (top), frequency (middle), and temporal modes (bottom) are shown. The epileptic atom is on the left, the eye blink artefact on the right. Adapted from [5].*

Combining EEG and fMRI to Study Cognitive Function

Symmetric, data-driven fusion of EEG and fMRI has the potential to reveal and characterize the consecutive steps of cognitive brain processing with high spatiotemporal resolution. The methodological challenge is to define meaningful yet efficient multiway representations, decompositions, and constraints for the coupling of the two modalities.

Using a well-known visual detection task, we successfully applied Joint-ICA: structuring of EEG and fMRI data together in one matrix, followed by joint decomposition with ICA [8]. This algorithm was extended to more complicated task paradigms (e.g., contour integration) and to a multichannel set-up. Extensions to a tensor-based framework are currently under investigation.

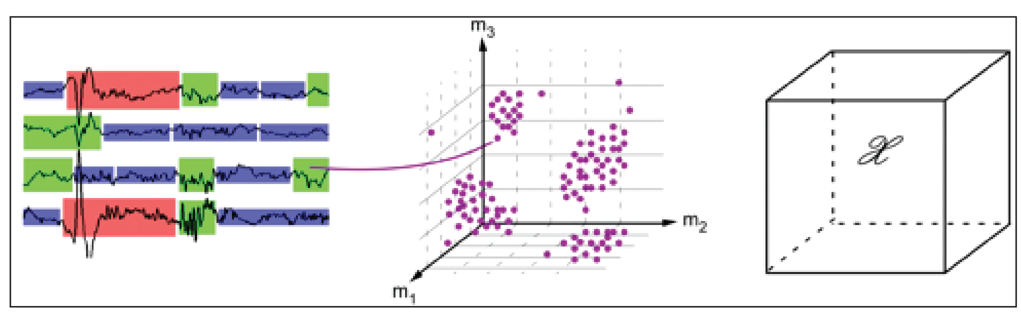


Figure 3. *The algorithm performs adaptive segmentation, according to the EEG amplitude (left: red, green, and blue indicate respectively high, moderate, and low amplitude EEG) and maps the segments' features (amplitude, duration, spatial distribution) into a multidimensional histogram (middle) which is stored in a tensor (right). Courtesy of Vladimir Matic.*

In summary, we have shown that tensor decompositions can be highly useful in biomedical data processing. CPD is now the main approach; the advantages of BTD are emerging. Tensors still have great unexplored potential in smart patient monitoring.

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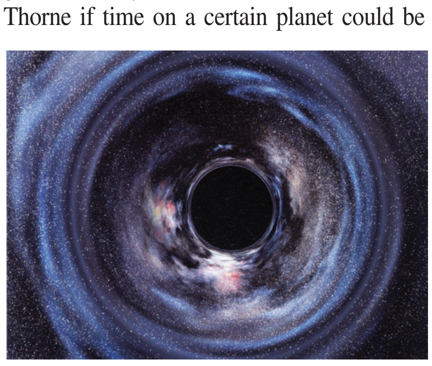
Interstellar

continued from page 4

Thorne describes some of the experimental evidence confirming this aspect of the general theory. When director Nolan asked Thorne if time on a certain planet could be slowed to the extent that a single hour on its surface could correspond to seven years on Earth, Thorne initially doubted that it could. However, after playing around with the field equations, Thorne discovered such slowing to be possible in the environs of a black hole spinning at 99.8% of the limiting rate of spin imposed by the fact that nothing can exceed the speed of light. In time, Thorne was able to develop a specialized set of field equations that enabled the project's technically sophisticated special effects team to produce a picture of a massive and rapidly spinning black hole silhouetted against a star-studded patch of night sky (see image at left).



The stars in Gargantua's galaxy as seen around the black hole's shadow, depicted in Figure 3.3 in the book. The black hole bends the light rays coming from each star, "gravitationally lensing" the galaxy.

This, according to Thorne, is new science! It shows with great precision how "gravity lensing" would distort any picture one might take of the firmament beyond such an object. Indeed, Thorne is currently writing a technical paper describing the new science discovered during the making of *Interstellar*.

Thorne cofounded the Laser Interferometer Gravitational Wave Observatory (LIGO) project in 1983 along with Rainer Weiss at MIT and Ronald Drever at Caltech. The project he envisioned and spent two decades bringing to fruition is now an international collaboration of 900 scientists in 17 nations. With gravitational wave detectors in Hanford, Washington, and Livingston, Louisiana, in the U.S., a third location planned in India, and similar detectors under construction in Italy and Japan, it will form a worldwide network to explore the universe using gravitational waves. In the book, Thorne explains in particular how the movie's NASA personnel might have used LIGO measurements to detect the presence of a wormhole near Saturn.

As interesting as these recent discoveries are, they fall well short of explaining how the hero of the story, a NASA-trained space pilot/engineer named Cooper—played onscreen by Matthew McConaughey—manages to survive his fall into the black hole Gargantua and return through the wormhole in a tesseract (a four-dimensional hypercube) provided by "them" in time to witness his daughter's death on a space station orbiting Saturn. It does even less to explain how Cooper manages to send data recorded inside the black hole backward through time to that very daughter (a celebrated physicist in adulthood) in time for her to figure out how one might exploit "gravitational anomalies" to propel county-sized space stations into solar orbit. Thorne supplies the missing explanations in the final chapters of this well-written book, a must read for all who refuse to abandon the dream of travel through interstellar space.

Jim Case writes from Baltimore, Maryland.

A SIAM Special Function at NIST

By Willard Miller, Jr.

The 13th International Symposium on Orthogonal Polynomials, Special Functions and Applications was held at the National Institute of Standards and Technology, Gaithersburg, Maryland, June 1–5, 2015. OPSFA-13 was jointly sponsored by the SIAM Activity Group on Orthogonal Polynomials and Special Functions and the NIST Applied and Computational Mathematics Division, with support from the National Science Foundation and NIST. NIST, formerly called the National Bureau of Standards, was a particularly appropriate host for the symposium (the first held in the U.S.): NBS/NIST has been a leader in the development of handbooks and other reference materials that provide basic formulas and notation for special functions, as well as numerical tables and software.

There is no technical definition of special functions. Basically, they are useful functions, occurring so frequently in the sciences that it becomes imperative to collect them in handbooks, with descriptions of their properties and codifying notation. Most of these functions are the "special functions of mathematical physics" (arising as solutions of second-order linear differential and difference equations in chemistry, physics, and engineering, perhaps via separation-of-variables methods or as families of orthogonal polynomials); examples include hypergeometric and *q*-hypergeometric functions. More recently, Painlevé functions (satisfying second-order nonlinear differential equations), exceptional polynomials (the Askey scheme and beyond), and other functions have risen to handbook status.

NBS/NIST has played an invaluable role in codifying and disseminating information about special functions. Work on the NBS tables started in 1938, culminating in the 1964 publication of the NBS handbook (edited by Milton Abramowitz and Irene A. Stegun). The earlier NBS tables were largely numerical, oriented toward practical computations. In recognition of the emerging predominance of mathematical software, the 1964 handbook focused more on formulas, with tables of numerical values filling fewer than half its pages.

Walter Gautschi and the late Frank Olver played major roles in developing the material for the handbook, of which more than a million copies are probably in print. It remains the most highly cited mathematics publication of NIST.

In 2010 the Digital Library of Mathematical Functions succeeded the 1964 handbook; DLMF is available as a handbook and also online (http://dlmf.nist.gov/). Almost no tables of numbers appear in DLMF, which contains more than twice as many formulas as the old handbook. The online version is continually updated; it imparts information about the behavior of these functions through interactive graphics. Frank Olver provided leadership on subject matter for the project. (He was the founding editor of *SIAM Journal on Mathematical Analysis* (1970), which before a change in editorial policy was among the most prestigious journals covering special functions.) The other DLMF editors are Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Part of this year's symposium was devoted to assessments of the history and continuing updates of the project, and discussions of plans for the future.

The OPSFA International Symposia date back to 1984. At the first meeting, held in Bar-Le-Duc, France, the invited speakers were the luminaries J. Dieudonné, "Fractions contiuées et polynômes orthogonaux dans l'œuvre de E.N. Laguerre"; W. Hahn, "Über Orthogonalpolynome die linearen Differenzengleichungen genügen"; G. Andrews and R. Askey, "Classical orthogonal polynomials"; and W. Gautschi, "Some new applications of orthogonal polynomials."

Karl Liechty received the Szegö Prize "for his original work in the asymptotic analysis of orthogonal polynomials arising in models from statistical mechanics, in particular the six-vertex model and a model of non-intersecting random paths." Photo by Walter Van Assche.

The community of researchers who use special functions is divided into two overlapping categories: those whose main interest is the special functions themselves and their properties, and those who are motivated by other branches of the sciences but encounter special functions and orthogonal polynomials in these pursuits. This meeting was no exception. Four of the ten plenary talks focused on the core body of the theory of special functions and orthogonal polynomials: 1) *Charles Dunkl*: vector-valued Jack and Macdonald polynomials; 2) *Mourad Ismail*: *q*-polynomials; 3) *Nico Temme*: asymptotic and computational aspects of special functions; 4) *Teresa Pérez*: multivariate orthogonal polynomials and moment functionals. Other research areas furnished the motivation for the remaining six: 5) *Percy Deift*: random matrices, Riemann–Hilbert problems, Airy functions; 6) *Wadim Zudilin*: number theory, Riemann zeta function; 7) *Craig A. Tracy*: random matrices; 8) *Sarah Post*: symmetries in mathematical physics; 9) *Olga Holtz*: polynomials with real roots, optimization theory; and 10) *Lauren Williams*: models of particles on lattices. (See the full program at http://meetings.siam.org/program.cfm?CONFCODE=FA15.)

Every two years SIAG/OPSF, joint with SIAM and the OPSFA symposium series leadership, awards the Gábor Szegö Prize to an early-career researcher for outstanding research contributions in the area of orthogonal polynomials and special functions. This year the prize was awarded to Karl Liechty of DePaul University (see photo), who delivered the Gábor Szegö Prize lecture: "Tacnode Kernels and Lax Systems for the Painlevé II Equation."

The SIAM Activity Group on Orthogonal Polynomials and Special Functions was founded in 1990, under the leadership of Charles Dunkl. The group's newsletter (http://math.nist.gov/opsf/) has appeared since 1993, and the Gábor Szegö Prize was launched in 2010, thanks to then SIAG/OPSF chair Francisco Marcellán.

In this era of machine computation and computer simulation, special functions and orthogonal polynomials remain vibrant areas of research for reasons far beyond the intrinsic beauty of the theory. The construction of realistic mathematical models of real-world systems that can be solved analytically and explicitly with adjustable parameters through the use of special functions plays a vital role in the understanding of the structure of these systems. This role was clearly demonstrated at OPSFA-13.

Willard Miller, Jr., is a professor emeritus of the School of Mathematics at the University of Minnesota.

Moving SIAM to the Forefront of Technology in the Information Age

As SIAM's inaugural director of information systems, James L. Goldman helped launch our first steps in this area. Upon his retirement after 30 years, Jim leaves us with his thoughts and musings on working for SIAM and on how far the organization has come both in terms of technology and as a society. See Jim's complete article at sinews.siam.org.

I was extremely lucky to have come to SIAM 30 years ago and to have been able to maintain my association with such a great organization through my career. To have been hands-on in a small organization, working at the frontier of what information systems would become, and at the forefront of putting scientific publishing online in the dawn of the World Wide Web was an amazing experience. Our first website went online in 1994, only three years into the life of the web. SIAM's journals went online in 1997, making us clearly one of the earliest adopters of that approach to publishing. In 2003, our Dynamical Systems activity group transitioned from a traditional website to a collaborative and community-driven shared-contribution model. It is called "DSWeb," and while shared authorship is fairly common today, in 2003 it was an early example of this type of collaboration. It should be gratifying to all in the SIAM community just how often our society has led the way in employing technology to support its mission.

Photo by Diane Fryer.

The SIAM office is a wonderful place to work, and those who might have any doubts about my claim need look no further than the percentage of our staff who have been with SIAM for 10, 20, or more years. Clearly, we are able to hold on to some very talented and motivated people. Having the chance to work with our officers, committee volunteers, SIAGs, members, editors, and authors as well as with our peers at other scientific societies, with subscription agencies and index database publishers, and with conference sponsors and interdisciplinary leadership committees is very rewarding. Few realize just how much the SIAM staff is intertwined with all aspects of the international scientific community on a daily basis.

Retiring from a long career at SIAM doesn't mean I'm retiring from a life in technology. I continue to serve on the IEEE Computer Society's Digital Library Committee and as an active volunteer in the world of amateur radio, where my focus is on low-speed, long-distance wireless data networks and remote control applications. I also lead a college-level educational foundation, and I serve on the boards of two national historical societies. But the big bonus of retirement is that I will get to spend more time in, around—and sometimes under—airplanes. An active pilot since 2003, I have more recently gotten into the mechanical/maintenance side of aviation, and these two interests are enough to make me wonder how I ever found the time to work for SIAM!

As a longtime SIAM member, I continue to support our society and the evolution of its pivotal information systems infrastructure.—*James L. Goldman, PhD, SIAM Information Systems, 1985–2015*

Professional Opportunities

Send copy for classified advertisements to: Advertising Coordinator, SIAM News, *3600 Market Street, 6th Floor, Philadelphia, PA 19104–2688; (215) 382–9800; fax: (215) 386–7999; marketing@siam.org. The rate is $3.00 per word (minimum $375.00). Display advertising rates are available on request.*

Advertising copy must be received at least four weeks before publication (e.g., the deadline for the November 2015 issue is September 30, 2015).

Advertisements with application deadlines falling within the month of publication will not be accepted (e.g., an advertisement published in the November issue must show an application deadline of December 1 or later).

Students (and others) in search of information about careers in the mathematical sciences can click on "Careers and Jobs" at the SIAM website (www.siam.org) or proceed directly to

www.siam.org/careers

Williams College

Department of Mathematics and Statistics

The Williams College Department of Mathematics and Statistics invites applications for two tenure-track positions in statistics, beginning fall 2016, at the rank of assistant professor. (In an exceptional case, a more advanced appointment may be considered.)

The department seeks highly qualified candidates who have demonstrated excellence in teaching and research and who are committed to working with an increasingly diverse student body. The candidates will become the fourth and fifth tenure-track statisticians in the department, joining a vibrant and active statistics group with a newly established statistics major. The teaching load is two courses per 12-week semester and a winter term course every other January. In addition to excellence in teaching, the chosen candidate will be expected to have an active and successful research program. The candidate should have a PhD by the time of appointment. The department welcomes applications from members of groups traditionally underrepresented in the field.

Applicants may apply at interfolio.com by uploading a curriculum vitae and having three letters of recommendation on teaching and research uploaded to http://apply.interfolio.com/30206. Teaching and research statements are also welcome. Evaluations of applications will begin on or after November 15 and will continue until the position is filled. All offers of employment are contingent upon completion of a background check. Further information is available upon request. For more information on the Department of Mathematics and Statistics, applicants should visit http://math.williams.edu/.

Williams College is a coeducational liberal arts institution located in the Berkshire Hills of western Massachusetts. The college has built its reputation on outstanding teaching and scholarship and on the academic excellence of its approximately 2,000 students. Please visit the Williams College website (http://www.williams.edu). Beyond meeting fully its legal obligations for non-discrimination, Williams College is committed to building a diverse and inclusive community where members from all backgrounds can live, learn, and thrive.

North Carolina State University

Department of Mathematics

The Department of Mathematics at North Carolina State University invites applications for one or more tenure-track positions beginning Fall 2016, depending on the availability of funding. The department seeks exceptionally well-qualified individuals with research interests compatible with those in the department in the general area of applied mathematics. Applicants must have a PhD in the mathematical sciences or related areas, a successful postdoctoral experience, an outstanding research program, a commitment to effective teaching at the undergraduate and graduate levels, and demonstrated potential for excellence in both research and teaching.

The Department of Mathematics has strong research programs in applied and pure mathematics. Many members of the department participate in interdisciplinary programs and research groups. Information about the department is available at http://www.math.ncsu.edu. Among the areas of interest are dynamical systems, control, numerical analysis including numerical linear algebra, and stochastic systems and applications.

Applicants should submit their application materials at http://www.mathjobs.org/jobs/ncsu. Applications should include a curriculum vitae, at least three letters of recommendation, and a description of current and planned research. After submitting these materials, applicants will receive instructions to complete a faculty profile at http://jobs.ncsu.edu/. For questions concerning the position, contact: math-jobs@math.ncsu.edu. Priority will be given to applications received by November 15, 2015.

North Carolina State University is an Equal Opportunity and Affirmative Action Employer and welcomes all persons without regard to sexual orientation. The College of Sciences welcomes the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners.

Texas Tech University

Department of Mathematics and Statistics

The Department of Mathematics and Statistics at Texas Tech University invites applications for the position of department chair and professor beginning fall 2016. The salary will be competitive and commensurate with qualifications and experience.

Candidates must have demonstrated outstanding vision, leadership, and scholarship; possess strong commitments to interdisciplinary research and educational activities; and be a collegial motivator and advocate for faculty. A PhD in mathematics or statistics is required. The successful candidate is expected to work with the faculty to develop the department under a strategic vision; foster excellence in research and teaching; and provide appropriate service to the college, university, and profession.

The department currently has 45 full-time faculty members with active research groups in pure and applied mathematics and in statistics. Six degrees are offered: BA, BS, MA, MS, and PhD in mathematics and MS in statistics, as well as several interdisciplinary and combined undergraduate and graduate degrees. For further information regarding the Department of Mathematics and Statistics, applicants should refer to the department's website: http://www.math.ttu.edu.

Applicants should apply at http://www.texastech.edu/careers/ using Requisition 4369BR. Applicants should submit a detailed letter of application along with a current resumé, including externally funded research, teaching, administrative experience, publications, and four letters of professional reference. Questions about the position and/or the application process can be directed to mathchairsearch@ttu.edu.

Review of applications will begin immediately. Applications will be accepted until the position is filled, with those received prior to October 1, 2015, assured full consideration.

Texas Tech University, located in west Texas in the city of Lubbock, was founded in 1923. The student population is 35,000 and is anticipated to grow to 40,000. Texas Tech is a state-designated national research university. Its strategic plan charts a course for becoming a great public research university by 2020. The university has the characteristics of a Carnegie-classified doctoral granting university with very high research productivity.

As an Equal Employment Opportunity/Affirmative Action employer, Texas Tech University is dedicated to the goal of building a culturally diverse faculty committed to teaching and working in a multicultural environment. We actively encourage applications from all those who can contribute—through their research, teaching, and/or service—to the diversity and excellence of the academic community at Texas Tech University. The university welcomes applications from minorities, women, veterans, persons with disabilities, and dual-career couples.

Amherst College

Department of Mathematics

Amherst College has two tenure-track assistant professor positions in mathematics, one with a preference for applied mathematics. The department seeks applicants who can teach and encourage students of diverse backgrounds, including first-generation students, international students, and students with varying mathematical preparation. Responsibilities include teaching two courses per semester and supervising undergraduate theses.

Requirements include a PhD in mathematics or a related field, a strong commitment to research, and a passion for teaching. Applicants should submit a cover letter, curriculum vitae, list of publications, research statement, teaching statement, and at minimum three letters of recommendation, with at least one specifically addressing teaching, to MathJobs.Org. Applications will be accepted until the positions are filled; those received by December 1, 2015, will be guaranteed consideration.

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Old Dominion University

Graduate Programs in Computational and Applied Mathematics

The Department of Mathematics and Statistics at Old Dominion University invites outstanding students to apply for graduate assistantships in its vigorous applied mathematics and statistics MS and PhD programs for the 2016–2017 academic year. Old Dominion faculty are active in various areas of applied and computational mathematics, scientific computing, analysis, and numerical analysis; they play a key role in the university's multidisciplinary modeling and simulation graduate program, and the Center for Computational Science. Old Dominion faculty enjoy an excellent funding record, including grants from NSF, NIH, NASA, AFOSR, the U.S. Army, and other state and federal funding agencies. For further information, applicants should see http://ww2.sci.odu.edu/math/academics/grad.shtml.

22nd Century Mathematics and Mechanics: Seven Decades and Counting. . .

This workshop will celebrate seven decades of accomplishments in mathematics and mechanics and their impact in the materials sciences, and will explore current and future directions of research. The event is specially aimed at young researchers in the field.

It will take place at the Valley River Inn, Hotel and Conference Center, in Eugene, Oregon, starting at 1pm on October 23, 2015 and ending at noon on October 25, 2015.

Travel support from the National Science Foundation is available for young researchers (graduate students, postdocs, and assistant professors) to attend the meeting. Please apply directly to the webpage of the IMA. Participants are encouraged to bring a poster and participate in the poster blitz session.

For information about the meeting, please consult the web page: http://www.ima.umn.edu/2015-2016/SW10.23-25.15/.

Chaos and its Manifestations

By Evelyn Sander and James A. Yorke

Typical dynamical systems can either have simple trajectories, such as steady states or periodic or quasiperiodic orbits, or they can be chaotic. While simple behavior is well understood, chaos is defined in so many different ways that is it confusing and difficult to get a reasonable answer to the simple question: What is the definition of chaos? We assert that this is the wrong question to ask.

Chaos does not have a satisfactory single mathematical definition, not because it is not a single mathematical concept, but rather because it has many mathematical manifestations in many different situations. In this regard, we are reminded of the 2500-year-old parable of the blind monks and the elephant. Each monk touches a different part of the elephant; a monk feeling a tusk thinks the elephant is a spear, while others encounter different manifestations of the elephant—a rope-like tail, perhaps, or column-like legs. When we observe a chaotic system, we are blind to the enormity of the concept and observe only a limited number of aspects of the system's chaotic nature.

In this article we discuss a variety of the manifestations and mathematical definitions of chaos. Our underlying message is that although it is possible to find pairs of definitions, one of which indicates that a system is chaotic and the other that it is not, this phenomenon is the exception, rather than the rule. We conjecture that, typically, the different forms of chaos are equivalent.

Arguably, the earliest observation of mathematical chaos occurred in the context of Newtonian mechanics. Henri Poincaré wrote a proof of the stability of the solar system, a result for which he won a substantial prize from King Oscar II of Sweden in 1890. After the prize was awarded, Lars Edvard Phragmén pointed out a substantial omission: Poincaré had assumed that only simple behavior could occur in a situation in which, in fact, it is possible to have chaotic behavior and what is now known as a transverse homoclinic orbit. In particular, there is a Cantor set of points with the property that the trajectory through almost any one of them occasionally comes arbitrarily close to that steady state both forward and backward in time, but with large excursions along the way.

Poincaré's proof of the stability of the solar system completely collapsed, and the question remains open today (although the high likelihood of the survival of the planets in their orbits has been verified numerically to the expected time of the sun's demise). Poincaré then undertook a major rewrite of the paper, in the course of which he discovered the underlying complexity of dynamical systems. In his words, "If one seeks to visualize the pattern formed . . . a chain-link network of infinitely fine mesh . . . one will be struck by the complexity. . ., which I am not even attempting to draw" [3]. Thankfully, the existence of computers makes it possible to depict this situation (which is shown in Figure 1).

Following this path of ideas, Stephen Smale showed in 1967 that even in a simple two-dimensional map known as a horseshoe map, one sees the same complexity as in transverse homoclinic orbits; in fact, horseshoe maps are embedded in any system containing a transverse homoclinic orbit. This aspect of chaos is useful when visualization is difficult but analysis is tractable, as for delay differential equations and partial differential equations. Chaos of this type can often be transient—that is, it cannot be observed through direct simulations of a trajectory or in physical experiments. Furthermore, the characterization is not quantitative.

In the course of numerical experiments for models of two very different natural systems, Yoshisuke Ueda (in 1961) and Edward Lorenz (in 1963) observed a robust but highly irregular topology in the trajectories of the systems. This "fractal topology" is a hallmark of attracting chaotic systems known as strange attractors; their measurement has since been made more precise in the form of attractor-dimension calculations. A general definition of strange attractors is still lacking, however, except for the subtype of rank-one attractors. Transverse homoclinic orbits often play a key role in shaping the geometry of strange attractors, and these concepts have been rigorously shown to be tightly related in many subclasses of systems.

Rather than providing full knowledge of a system, experiments often give rise to time-series data. In 1975, Jerry Gollub and Harry Swinney demonstrated that the time-series data from Taylor–Couette fluid flow experiments had the chaotic hallmark of a broad power spectrum. In fact, the closely related Lyapunov exponent is the current most commonly used metric for characterizing chaos; using added information about the behavior of nearby orbits, the Lyapunov exponent indicates the degree of stretching along solutions. For experimental high-dimensional chaos, it is not possible in general to observe this stretching. The concept of stretching also appears in the definition of scrambled sets of Li and Yorke. Aside from stretching, another way to measure chaos is the possession of positive entropy.

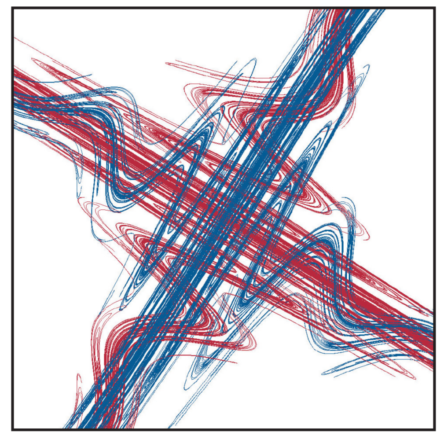

Figure 1. *A homoclinic tangle with the chaotic complexity described by Poincaré.*

If the number of "distinct" orbits of period n is proportional to a^n, then the entropy is proportional to $\log(a)$. As with chaos, there are a variety of types of entropy. Each of the above-mentioned concepts can be shown to be distinct, although they appear to overlap in all but rather extreme cases, as discussed in [2].

The importance of non-attracting chaos is often overlooked, as it cannot be seen either by direct simulation or experimental observation. This behavior, though not stable, can organize the underlying behavior of the system and can be robust under changes to the system parameters. Such is the case for chaotic saddles. Figure 2 (see page 1), for example, shows the basins of eight attractors for the forced–damped pendulum. The eight attractors are simply periodic and fixed points. The chaotic saddle on the boundary of the basins causes the geometric complexity. The progression of the basins as the forcing changes can be seen in our YouTube video [4].

Each manifestation or definition of chaos mentioned here comes with its own settings in which it can be verified, and with its own strengths and shortcomings, in terms of both numerical and theoretical uses. It is true that the non-equivalence of these mathematically precise formulations can be shown, but we choose to focus on the similarities, reiterating that the concept of chaos is too big for one single mathematical definition to suffice. We end by quoting a statement made during the controversy about Pluto as it was stripped of planethood, but which applies just as well to the chaos definition controversy: "Nature abhors a definition. Try to lock something into too small a box, and I guarantee nature will find an exception" [1].

References

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[3] H. Poincaré, *New Methods of Celestial Mechanics*, *Vol. 13*, American Institute of Physics, College Park, Maryland, 1993.

[4] E. Sander and J. Yorke, *Pendulum gyrations,* 2014; http://youtu.be/JyhaHyCvgw4 and http://math.gmu.edu/~sander/EvelynSite/pendulum-gyrations.html.

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This article is a summary of our recent paper [5] and is based on the talk Evelyn Sander gave at the 2015 SIAM Conference on Applications of Dynamical Systems. Readers can find a more substantial reference list in the paper.

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Photo courtesy of Boston University.

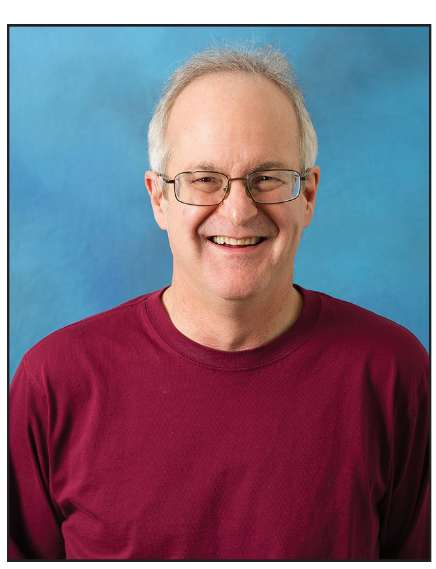

Photo courtesy of the University of Pittsburgh.

The $100,000 Mathematical Neuroscience Prize from Israel Brain Technologies was awarded to Nancy Kopell (left) of Boston University for her work in mathematical analysis of the nervous system functions and to Bard Ermentrout from the University of Pittsburgh for his classic work in mathematical biology. Each received a $100,000 prize at the ceremony held on March 11 at the BrainTech Conference in Tel Aviv. Both recipients are well known to SIAM: Kopell gave the John von Neumann lecture on rhythms of the nervous system at ICIAM 2007. Ermentrout's talk on modeling and analysis of large-scale activity in the brain (see http://bit.ly/1J3Mdi0) was well received at the 2015 SIAM Conference on Applications of Dynamical Systems.