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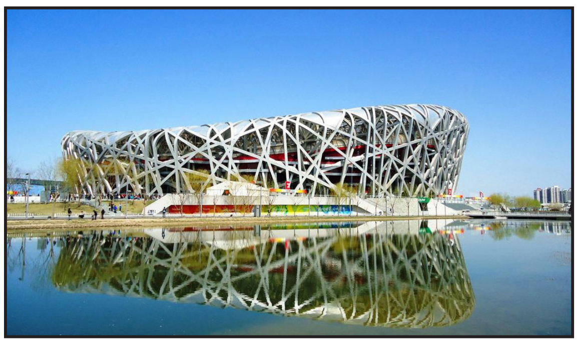
Program for ICIAM 2015 Takes Shape

Winter-worn residents of some parts of the world might want to think ahead six months to the eighth International Congress on Industrial and Applied Mathematics, which will be held in Beijing, August 10–14. Key elements of the program, including the 27 invited speakers (selected by the Scientific Program Committee and approved by the ICIAM Board), the names of the five ICIAM prize recipients, and details about Éva Tardos's Olga Taussky-Todd Lecture, have been posted at www.iciam2015.cn/.

An important date is rapidly approaching: Proposals for poster sessions are due no later than April 30, which is also the deadline for early registration. The full ICIAM 2015 program will be posted by the end of May.

A reminder to the SIAM community: Several prize presentations and lectures that are normally part of the Annual Meeting have been worked into the ICIAM schedule. Jennifer Tour Chayes of Microsoft Research New England will give the John von Neumann Lecture in Beijing (her invited talk at the 2014 SIAM Annual Meeting in Chicago can be viewed at SIAM Presents, https://www.pathlms.com/siam/courses/480/sections/718). Carlos Castillo-Chavez, Eitan Tadmor, Gerhard Wanner, and other as yet unnamed prize recipients will also be honored. The AWM–SIAM Sonia Kovalevsky Lecture will be given at ICIAM.



ICIAM sessions will be held at the China National Convention Center, in the Beijing Olympic Green.

Confirmed ICIAM satellite meetings in China include the International Conference on Numerical Mathematics and Scientific Computing in Nanjing, August 16–19; others have been scheduled for Korea (SIAM Conference on Applied Algebraic Geometry, Daejeon, August 3–7; see article on page 6) and Japan (Mathematics for Nonlinear Phenomena: Analysis and Computation, Sapporo, August 16–18).

2014 National Medal of Science

High Honors for Versatile Engineer/Mathematician

As a recipient of the 2014 National Medal of Science, Thomas Kailath of Stanford University was cited not only for "transformative" scientific contributions to the fields of information and systems science, but also for mentoring activities and entrepreneurial ventures that proved influential in industry.

A longtime (since 1975) member of SIAM, Kailath has written frequently over the years for a variety of SIAM publications. His research monograph *Indefinite-Quadratic Estimation and Control: A Unified Approach to* H^2 *and* H^∞ *Theories*, with Babak Hassibi and Ali H. Sayed, for example, appeared in 1999 as Volume 16 in the SIAM Studies in Applied and Numerical Mathematics series.

SIAM executive director James Crowley fondly recalls a dinner with Kailath at an Indian restaurant in Palo Alto in the early 1990s. Beyond the very pleasant meal and conversation, Crowley remembers Kailath's description of frequent changes in his research focus—a new emphasis every decade—in the course of his career. "This is certainly borne out by the diversity and richness of Kailath's research contributions," Crowley says.

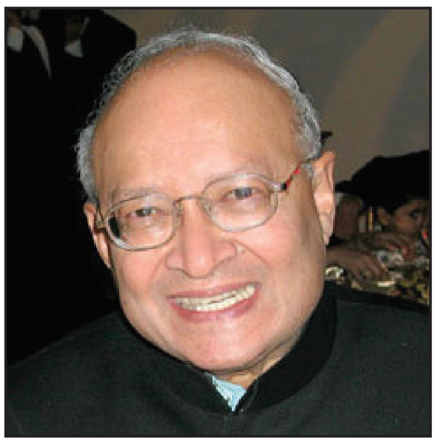


Photo courtesy of Thomas Kailath.

Sayed, Kailath's frequent co-author and former student, now a member of the faculty at UCLA, points out that throughout Kailath's work, the interest for the SIAM community lies in his ability to "exploit in insightful and often magical ways the mathematical structure underlying problems in many areas, whether signal processing, control or information theory, or semiconductor manufacturing." That high-level assessment

See **Kailath** *on page 3*

The Race to Compute High-order Gauss–Legendre Quadrature

By Alex Townsend

A typical quadrature rule is the approximation of a definite integral by a finite sum of the form

$$\int_{-1}^{1} f(x)dx \approx \sum_{j=1}^{n} w_j f(x_j), \qquad (1)$$

where $x_1, \ldots, x_n$ and $w_1, \ldots, w_n$ denote the quadrature *nodes* and *weights*, respectively.

In 1814 Gauss [3] described a particularly ingenious choice for the nodes and weights that is optimal in the sense that for each n it exactly integrates polynomials of degree $2n - 1$ or less. It can be shown that no other quadrature rule with n nodes can do as well or better. Today, we call this Gauss–Legendre quadrature because of pioneering work of Jacobi showing that the nodes are the zeros of the degree-n Legendre polynomial $P_n(x)$ and $w_k = 2(1 - x_k^2)^{-1}[P_n'(x_k)]^{-2}$.

There is a catch. For large n there is no explicit closed-form expression for the Gauss–Legendre nodes or weights. And Gauss knew this. To demonstrate it practically, he calculated (by hand!) the nodes and weights to 16 digits for $n = 7$. Ever since, and especially since the advent of the modern computer, there has been an unofficial race to compute the nodes and weights for larger and larger n to more and more digits. It's a race that the famous Golub–Welsch algorithm never led. Ignace Bogaert of Ghent University emerged recently with a new, winning algorithm. Here is a race report. (See Figure 1.)

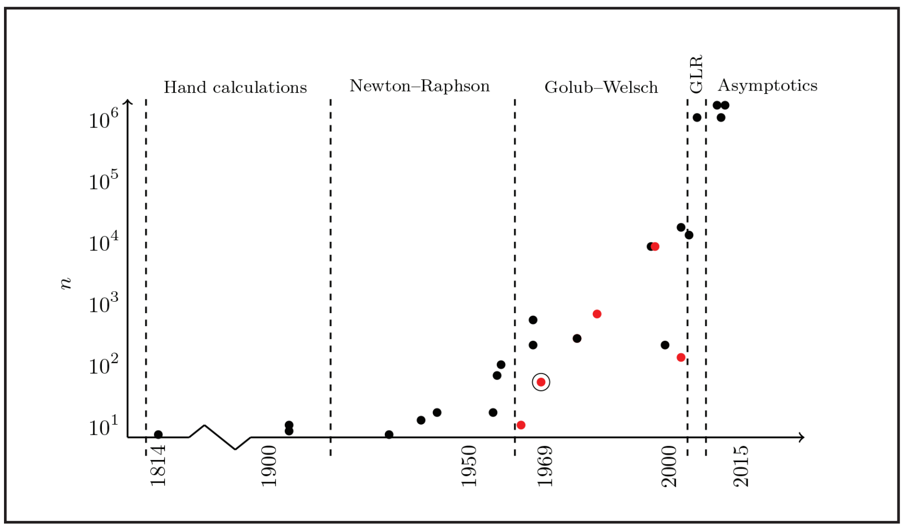


Figure 1. *The 100-year race for high-order Gauss–Legendre quadrature. A dot represents published work, located by the publication year and the largest Gauss–Legendre rule reported therein. A red dot indicates a paper based on variants of the Golub–Welsch algorithm. The dot for Golub and Welsch (1969) is circled. For a list of the papers used, see http://math.mit.edu/~ajt/GaussQuadrature/.*

Hand calculations led the way for over a century. Tallquist (1905), Moors (1905), Nyström (1930), and Bayly (1938) used fountain pens and dogged determination to calculate the quadrature nodes for $n \leq 12$. Eventually, presumably with a small army of human calculators, Lowan, Davids, and Levenson (1942) tabulated the nodes and weights for $1 \leq n \leq 16$ for the Mathematical Tables Project.

A decade later computers were beginning to dominate tedious hand calculations, and large tabulations of nodes and weights were profitably published. The most popular algorithm for computing Gauss nodes was the Newton–Raphson method for finding the roots of $P_n(x)$ with a three-term recurrence used to evaluate P_n and P_n'. Huge strides were made. Gawlik (1958) briefly led with $n = 64$ before Davis and Rabinowitz (1958) got $n = 96$, and finally Stroud and Secrest (1966) achieved $n = 512$. This was the golden age for Gauss–Legendre quadrature.

By the 1960s orthogonal algorithms for eigenproblems were hot off the press and Gene Golub was becoming famous. The Golub–Welsch algorithm [5]—featuring both QR and Golub—was momentous. It quickly overshadowed the earlier (1963) result of Rutishauser. Contrary to popular belief, however, the Golub–Welsch algorithm is not, and never was, the state-of-the-art algorithm for computing Gauss–Legendre quadrature rules in terms of accuracy and speed. Yet, by elegantly bringing together eigenproblems and Gauss quadrature, it radically changed how the world computed integrals. Before 1969, a few would compute quadratures by carefully extracting the tabulated values from Stroud and Secrest (1966) and calculating (1). After 1969, all were computing Gauss nodes and weights for themselves. Tabulations were already falling out of favor across the computational sciences; the Golub–Welsch algorithm made it so for Gauss nodes and weights as well. This makes 1969 a year to remember for more than just the moon landing.

In the years that followed, only a handful of experts noticed improvements to the details of the Newton–Raphson approach produced by Lether (1978), Yakimiw

See **Race Report** *on page 3*

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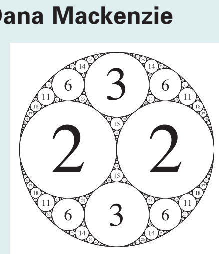
4 **Untangling the Threads of a Heroic and Complicated Life**
James Case re-reads the 30-year-old biography of Alan Turing by Andrew Hodges, reissued to coincide with the appearance of an Oscar-nominated movie. The movie is "loosely based on" the book, which chronicles Turing's career in highlights occurring roughly every five years—among them the decoding of messages from the German navy's Enigma device (1940) and his 1945 design for the Automatic Computing Engine.

4 **Last Bastion of Purity in a Corrupt World?**
A book on "the mathematical life" elicits from reviewer Ernest Davis comparisons with the lives of neuroscientists, librarians, violin makers, journalists, photographers, rock gardeners. . . .

6 **Geometry, Invariants, and the Search for Elusive Complexity Lower Bounds**
SIAG/Algebraic Geometry chair Jan Draisma takes on the challenge of presenting current research in the field to the SIAM community at large. Drawing on sessions in geometric complexity theory at a Simons Institute program, he guides readers through "the best-known lower bound to date on the determinantal complexity of the permanent" and on to "another notorious question"—the complexity of matrix multiplication.

8 **MAA Chauvenet Prize Awarded to Dana Mackenzie**



7 **Professional Opportunities**

7 **Announcements**

Science and Industry Advance with Mathematics*

TALK OF THE SOCIETY
By James Crowley

These are exciting times, as we witness the growth of applied mathematics and its increasing relevance to so many sectors of the economy and to our daily lives. And yet a note of regret crept in when we learned in recent weeks that we will soon lose a staunch ally in supporting applied and computational math: the IMA at the University of Minnesota.

I am struck by a recent flood of stories on how mathematics and computing are making new inroads into increasing productivity in the economy. One of the latest to cross my screen notes that "the single greatest instrument of change in today's business world, the one that is creating major uncertainties for an ever-growing universe of companies, is the advancement of mathematical algorithms and their related sophisticated software."† The article states that "To some degree, every company will have to become a math house" in order to exploit the efficiencies of algorithms for data analysis.

While it is true that the growth of data science is phenomenal, and that statistics is now reported to be the fastest growing STEM major,‡ we should not forget the important role played by other parts of our discipline. I am reminded of one class of models we (at least in this part of the world) hear about frequently at this time of year: weather prediction models.

Here in Philadelphia, a large overnight snow storm was predicted in early February; the city responded with advanced cancellation of public transportation and many flights, only to wake up to a light dusting rather than the predicted foot-plus of snow. The essentials of the storm were predicted remarkably well—the front would race across Pennsylvania, form a low-pressure cell off the coast, pull in moisture from the south, and move north toward New England, all of which happened. But the details—in particular, the formation of the low-pressure cell about 100 miles to the east of the predicted location—were off by enough to change the weather completely.

Forecasts will improve. They rely on models—the PDEs that describe the physics of the atmosphere—as well as on data. The models are solved by numerical methods run on high-performance computers. Improvements will come, yes, from better data, but they alone won't result in better predictions. Improvements in the accuracy of numerical codes are also required, and they in turn rely on better models, greater resolution of the codes, and faster computing. This is but one example. Even as we embrace the surge of interest in fields like data science, we know that modeling, analysis, and computing will continue to play important roles.

It is exciting to live at the nexus of advances in data science, modeling, algorithms, and high-performance computing. At the end of January, I had the opportunity to share some of that excitement with the House Science Committee's Subcommittee on Energy. I had been invited to testify on the value of research supported by the Department of Energy's Office of Science, in particular by its Advanced Scientific Computing Research (ASCR) program.

Testifying for a congressional committee is an interesting experience. As one of four witnesses, I was humbled to share the table with Norman Augustine, former CEO of Lockheed Martin; Roscoe Giles of Boston University; and Dave Turek, vice president for technical computing at IBM. Each of us had five minutes to speak, facing colored lights that warned us as our time was ending ("Red light, green light . . . all around the town"). A gallery of folks sitting behind us took notes and tweeted as we spoke. Following our testimony, each of the seven members of Congress in attendance had five minutes either to speak or to use for questions and answers.

SIAM submitted written testimony, which was vetted through our Committee on Science Policy. In truth, the written testimony is far more detailed than anything that could be communicated in a short oral presentation. The oral testimony and the accompanying written document are part of a process. We participate in the hope that the information we provide will support ASCR's important research programs, especially those in applied mathematics and computer science.

And this takes me back to the Institute for Mathematics and its Applications. We received the IMA's Year in Review report for 2014 this week. In the introduction, current IMA director Fadil Santosa noted that NSF funding for the IMA in its present form, as one of eight mathematics institutes, will end in two years. This means that the IMA, long a valued institute and resource for many in the SIAM community, will change dramatically or possibly even cease to exist. While we recognize that NSF's Division of Mathematical Sciences needs to review and revise its portfolio, we will greatly miss an institute that has served the applied and industrial math community since 1982. As Santosa wrote in his introduction,

Bob O'Malley as SIAM President (1991–92)

> "The IMA has been a major force in applied mathematics. . . . The IMA has been an enabler of interdisciplinary collaborations involving mathematicians and has forever changed the culture of mathematics research. It has also led the way in this country in developing a field now recognized as industrial mathematics."

■ ■ ■

I would like to end by taking a few inches to thank Bob O'Malley for his dedicated service as editor of the *SIAM Review* Book Review section. Younger readers may not know that Bob is a former president of SIAM (1991–1992); when his term ended, he continued to play an active role in SIAM activities. In 2000 he took over as *SIAM Review* book review editor from Bruce Kellogg. Until the beginning of this year, when he was succeeded by David Watkins, Bob ran the section.

Tim Kelley, the editor-in-chief of *SIAM Review*, isn't a person who panics easily, but he came close to that state when told that Bob would be stepping down. The reason, of course, is that Bob did a marvelous job as editor of the book reviews, and Tim realized that it would be difficult to find anyone who came even close to combining Bob's passion for books and his understanding of the SIAM community.

As his colleague Mark Kot noted,

> "When I ask Bob if he has received any exciting new books, he usually pulls a new book from one of his stacks, mentions an interesting connection, and tells an interesting story about the author. And then, more often than not, he says something really nice about the book. He continues to find new gold in countless old topics."

On behalf of the entire SIAM community: Thanks, Bob!

*An alternative interpretation of the acronym SIAM that dates back about twenty years but seems, periodic setbacks notwithstanding, at least as appropriate as ever.

†"The Algorithmic CEO," by Ram Charan, *Fortune*, January 22, 2015; http://for.tn/1ynLsMP.

‡http://fortune.com/2015/02/10/college-major-statistics-fastest-growing/.

siam news

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We mourn the death of

SIAM founding director

and

longtime managing director

I. Edward Block.

February 18, 2015

Kailath

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reflects a dynamic career that has, in fact, been characterized by periodic shifts in focus among different fields.

Especially gratified by the National Medal citation's reference to "distinctive and sustained mentoring of young scholars," Kailath commented to *SIAM News* on the importance of students in his career. In the early years, he said, "almost all my papers were single-authored. But then I realized that if I wanted to address new topical challenges, the best way to do that in a university was to work with groups of brilliant students—and that is what enabled me to change the major focus of my research roughly every decade."

Kailath's graduate work at MIT, from 1957 to 1961, on the characterization and identification of random linear time variant channels and on communication via such channels had gained him an international reputation before he joined Stanford as an associate professor in 1963. As director of the Information Systems Laboratory from 1971 to 1981, he was instrumental in building it into a world-class center for communications, control, and signal processing research. He is currently the Hitachi America Professor of Engineering Emeritus in the electrical engineering department at Stanford.

A decade-by-decade perspective on Kailath's research activities at Stanford begins with the development of an algorithm for exploiting the availability of noiseless feedback and new techniques in the theory of signal detection. *Linear Systems*, his influential 1980 textbook, resulted from his work during the previous decade using state-space techniques to model and understand the behavior of dynamical systems. That was followed by a decade's work on multiple-antenna signal processing and the design of VLSI arrays/architectures for signal processing, along with development of the concept of displacement structure.

An overview of his work on the latter can be found in his and Sayed's extensive 1995 *SIAM Review* article, in which they relate the development of fast computational algorithms for matrices that have what Kailath named "displacement structure." An important example is a fast triangularization procedure for such matrices (generalizing a 1917 algorithm of Schur). As they point out in the abstract of their article, "this factorization algorithm has a surprisingly wide range of significant applications going far beyond numerical linear algebra." Examples include inverse scattering, analytic and unconstrained rational interpolation theory, digital filter design, algebraic coding theory, and adaptive filtering. A collection of review articles on displacement structure can be found in the volume *Fast Reliable Algorithms for Matrices with Structure*, edited by Kailath and Sayed and published by SIAM in 1999.

Displacement structure was a focus for Kailath and his colleagues for a number of years. The group's algorithms for solving complex design problems, in which the matrices are very large, Sayed says, are an order of magnitude faster than other algorithms and continue to be widely studied today. And in the process of discovering the algorithms, he says, Kailath's group discovered fascinating connections with several other areas of mathematics, including interpolation theory and orthogonal polynomials.

In the 1990s, Kailath and colleagues turned their attention to smart antenna technology for wireless communication, as well as to resolution-enhancement techniques for optical lithography in semiconductor manufacturing. The latter work used techniques from signal processing and communications to break a barrier of 100 nm, perceived at that time as the minimum line width achievable by optical lithography. In 1996 Kailath and a group of graduate students formed the company Numerical Technologies, which went public in 2000 and was acquired by Synopsis in 2003.

In the course of his career, Kailath and his students have also made a variety of contributions to probability and statistics. Kailath is an emeritus fellow of the Institute of Mathematical Statistics. He is also a member of the inaugural class of SIAM fellows, cited for his contributions to linear algebra, systems and control, and their applications in engineering.

Another important textbook, *Linear Estimation*, by Kailath, Sayed, and Hassibi, appeared in 2000. Widely used today as a reference in the broad area of state-space estimation theory, the book offers a unified and motivated window on a range of results in this area that emanated from Kailath's group.

Students today, Sayed points out, "have the benefit and convenience of rushing to search for articles and references online"; this casts the rewards of his own experience with Kailath as an adviser in a new light. "At a time when students used to visit libraries more often, Kailath's ability to pinpoint references with precision in archival publications was simply amazing. His immersion in the science of his work was also contagious and lives with me to this day."

A Lagrange Multiplier Problem Without Multipliers

MATHEMATICAL CURIOSITIES

By Mark Levi

Here is a calculus-free solution of the standard vector calculus problem

Minimize

$$f(x, y, z) = ax^2 + by^2 + cz^2 \qquad (1)$$

subject to $g(x, y, z) = x + y + z = 1$, *where a, b, and c are positive constants.*

Solution. Figure 1 shows three springs with Hooke's constants a, b, and c, connected end to end. If each spring is assumed to have relaxed length of zero, (1) gives twice the potential energy of the system.

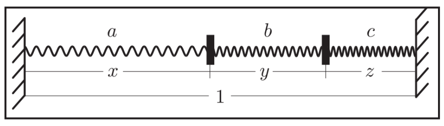

Figure 1. *The form (1) is minimized when the tensions are equal.*

If this energy is minimal, the system is in equilibrium and the tensions of the springs are thus equal:

$$ax = by = cz. \qquad (2)$$

We have solved the problem by showing that the minimizing lengths should be in inverse proportion to the corresponding coefficients ("the stiffer, the shorter"), implying

$$x = \frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}},$$

with similar expressions for y and z.

We can compare this with the "standard" solution: Lagrange's necessary condition for the minimum

$$\nabla f = \lambda \nabla g \qquad (3)$$

yields

$$\langle 2ax, 2by, 2cz \rangle = \lambda \langle 1, 1, 1 \rangle,$$

which is the same as our result (2) (without the physical interpretation).

Incidentally, Lagrange's method (3)—for general functions f and g—admits a simple static interpretation. Consider a particle constrained frictionlessly to the surface g = constant and subject to the force field $\mathbf{F} = -\nabla f$ in space; we thus interpret f as the potential energy, which we are trying to minimize. But minimality at some M means that M is an equilibrium, where $\mathbf{F}$ must be cancelled by the reaction force $\mathbf{R}$:

$$\mathbf{F} = -\mathbf{R}. \qquad (4)$$

Without friction, the reaction is normal to the surface: $\mathbf{R} = \lambda\nabla g$, and (4) is the same as (3). The Lagrange condition (3) is thus a special case of Newton's first law.

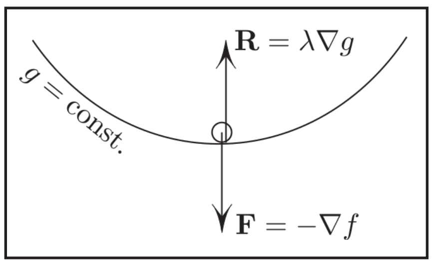

Figure 2. *Lagrange's relation (3) as the equilibrium condition (4).*

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(1996), Petras (1999), and Swarztrauber (2003). While the Golub–Welsch algorithm was computing a few hundred nodes and weights, the Newton–Raphson approach was computing thousands. Many, still unaware of the developments after 1969, have concluded that Gauss–Legendre quadrature is not computationally feasible for large n. Attention has shifted to adaptive and piecewise quadrature schemes.

In 2007 Glaser, Lui, and Rokhlin described a ground-breaking algorithm that can compute a million quadrature nodes in a handful of seconds [4]. Accolades should have followed, but the algorithm failed to awaken much interest. A few years later Bogaert, Michiels, and Fostier [2] and Hale and Townsend [6] showed that the Newton–Raphson method for finding the roots of $P_n(x)$ with careful evaluation of P_n and P_n' by asymptotic formulas could be just as fast and more accurate than anything the world had seen before.* The golden age had returned. Figure 2 shows the quadrature error (see ε_{quad} in [6] for the exact definition) and the timings for five historically important algorithms. It was after careful numerical comparisons like these that the race was fully appreciated.

The epilogue was written by Bogaert a few months ago [1]. He derived explicit asymptotic formulas for the Gauss–Legendre nodes and weights that are accurate to 16 digits for any $n \geq 20$. Using his formulas, I just computed one billion and two Gauss–Legendre nodes and weights on my laptop. This is a world record! So large is this rule that nodes that are near neighbors of ±1 are identical to 15 decimal places. It now takes less than a millisecond to compute a thousand nodes and less than a tenth of a second to compute a million.

Ignace Bogaert is the winner of the 100-year race. Bravo!

It was a fun race with a deserving winner. We are now searching for applications that require thousands of nodes and weights. Our algorithms are poised for use. If you have an application in mind, please email ajt@mit.edu.

One million Gauss–Legendre nodes and weights—no problem. But how will we use them?

*See [6] for computing Gauss–Jacobi, Gauss–Lobotto, and Gauss–Radau quadratures.

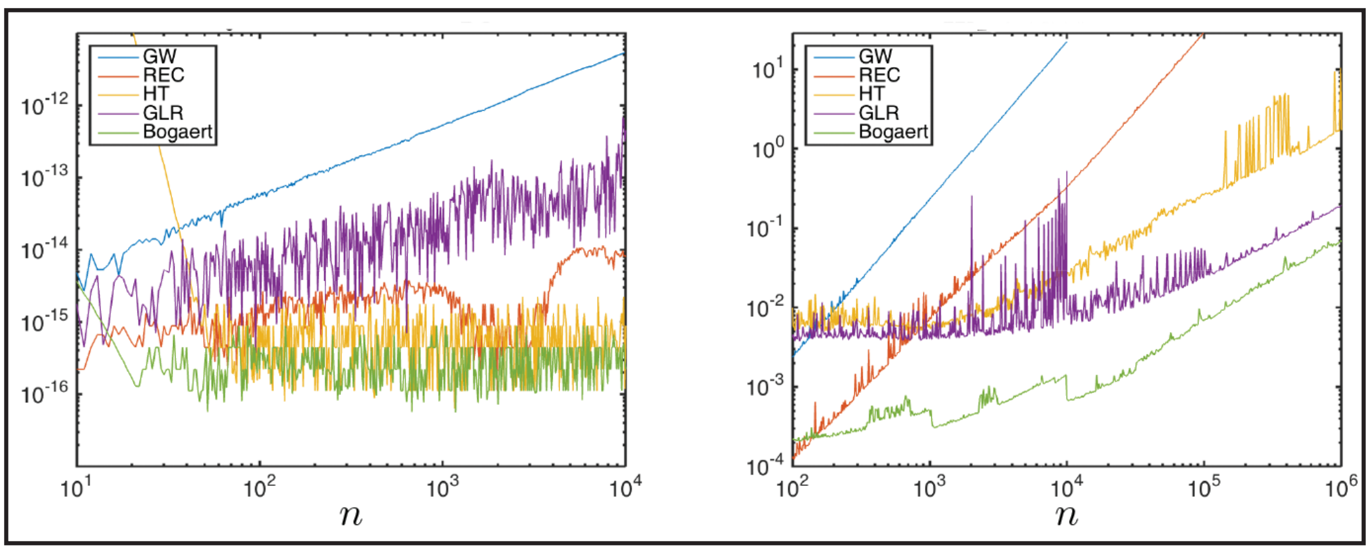

Figure 2. *Quadrature error (left) and computational time (right) for Gauss–Legendre nodes and weights computed by the Golub–Welsch algorithm [5] (GW), Newton–Raphson with three-term recurrence (REC), Newton–Raphson with asymptotic formulas [6] (HT), the Glaser–Lui–Rokhlin algorithm [4] (GLR), and Bogaert's formulas [1] (Bogaert). The timings given here are for implementations in different programming languages and cannot be used for direct comparisons.*

Acknowledgments

I thank Ignace Bogaert and Nick Hale for helpful comments and suggestions, and Nick Trefethen for encouraging me to write this article.

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Untangling the Threads of a Heroic and Complicated Life

Alan Turing: The Enigma. *By Andrew Hodges, Princeton University Press, Princeton, New Jersey, 2014, 768 pages, $16.95.*

BOOK REVIEW
By James Case

The Imitation Game opened to solid reviews on Christmas Day and, as announced on January 15, received several 2015 Oscar nominations (including for Best Picture). Loosely based on Andrew Hodges's 1983 book *Alan Turing: The Enigma*, the film stars Benedict Cumberbatch as Turing and Keira Knightley as his wartime co-worker (and one-time fiancée) Joan Clarke. To coincide with the release of the movie, Princeton University Press has reissued Hodges's book, with a new preface by the author.

Among the finest scientific biographies known to this reviewer, the book chronicles Turing's career in pure and applied mathematics, along with his many related interests, to his death, apparently by suicide, in 1954. Hodges devotes several chapters to Turing's early life, in an effort to identify the roots of his adult behavior. His task is facilitated by the numerous letters Turing wrote throughout his life.

■ ■ ■

Mere months after his birth, in London, in 1912, Turing's mother and father—the latter a career officer in the Indian Civil Service—returned to the subcontinent, leaving Alan and his elder brother in the care of an English couple, Colonel and Mrs. Ward. From then until they entered university, the boys' time was divided between stays with the Wards, a succession of boarding schools, and occasional family vacations when the parents were on leave in England.

Permanent influences on Alan included a book titled *Natural Wonders Every Child Should Know*. Meant to explain biological growth in a way that young children could understand, the book opened Alan's eyes to the nature and allure of scientific knowledge, while stressing that the human body and mind are machines adapted to certain basic tasks, but capable of untold others.

Also influential were the story of Snow White and the poisoned apple, and the death of his school friend and first romantic (though apparently Platonic) love Christopher Morcom, in February 1930. Already friends, the two had bonded while rooming together for a week at Trinity College, Cambridge, where they had gone to compete for university scholarships. Morcom was immediately successful, but Turing had to return the following year to secure a scholarship to King's, his second choice among the Cambridge colleges.

Turing's performance on the scholarship exams was commendable, but by no means without precedent. Others had won more lucrative scholarships, at earlier ages. Not until the summer of 1931, when he actually entered King's—a bastion of free-thinkers heavily influenced by John Maynard Keynes and the ageing Bloomsbury set—did he truly begin to blossom. In April 1935, at the age of 22, he was elected to a fellowship in King's. His first mathematical paper, on group theory, was published a month later. The fellowship carried a stipend of £300 a year, ordinarily renewable for an additional three years, with no specific duties. Beyond the stipend, it entitled him to room, board, and a seat at High Table whenever he chose to reside in Cambridge. The boys at Sherborne School—where he had prepped for university—were inspired by his success to intone that

> Turing
> Must have been alluring
> To get made a don
> So early on.

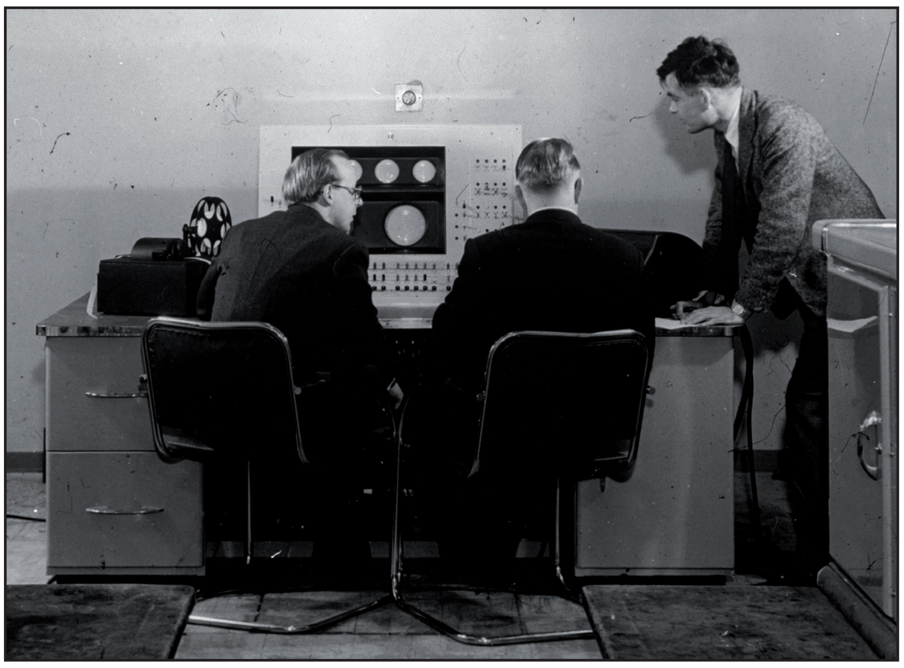
The terms of the fellowship left him free to travel, and he elected to spend time in Princeton, then replacing Göttingen as the center of the mathematical universe. Turing arrived in Princeton in September 1936, having just completed his magnum opus "On Computable Numbers, with an Application to the Entscheidungsproblem." Page proofs reached him in October, and the paper was published the following spring.

Formulated by Hilbert, the entscheidungsproblem had already been solved by Alonso Church, Turing's host in Princeton, using his powerful λ-calculus. But the "Turing machine" proof, suggesting the feasibility of a universal computer capable of performing any possible computation, was far more memorable and portentous.

■ ■ ■

Hodges identifies five key events in Turing's professional career, spaced roughly five years apart. The first two were Christopher Morcom's death in 1930 and the conception of the Turing machine in 1935. The others were Turing's conquest of the German navy's version of the Enigma device in 1940, his design for the ACE (Automatic Computing Engine) in 1945, and his formulation of the morphogenetic principle in 1950. Hodges further insists that Turing was still at the top of his game at the time of his death, having been able at last to gain hands-on experience with the Manchester computer.

Turing's work on the naval Enigma machine was truly heroic. Great Britain relied on 35 million tons of imports a year; from July 1940, when German U-boats began operating from French ports, through October 1, they sank a million tons of British shipping. By the early months of 1941, the flow of imports had been reduced to an annualized rate of only 28 million tons. Churchill acknowledged to Roosevelt that, unless the war in the Atlantic took an abrupt turn for the better, Britain would be forced to sue for peace within the year. Only by reading the German navy's Enigma communications could British codebreakers hope to inform convoys at sea of the whereabouts of German submarines in time to avoid contact.


Alan Turing with two colleagues and the Ferranti Mark I computer, January 1951. Photo courtesy of the University of Manchester.

During the 1920s, Polish intelligence had obtained an early version of the Enigma machine and constructed a device ("Bombe") to decode its messages. By November 1939 Turing—who had volunteered for code-breaking duties the previous year—could refer in internal documents to "the machine ['superbombe'] now being made at Letchworth, resembling, but far larger than, the Bombe of the Poles. . . ." Hodges offers a detailed description of the improvements made by Turing and his Bletchley colleague Gordon Welchman to the superbombe. In time, those improvements enabled Bletchley to read virtually every message sent to and from the U-boat fleet between June 1941 and February 1942, when Germany added an additional rotor to the naval Enigma.

The resulting decryption blackout enabled the U-boats to reassert control of the sea lanes. Only the capture of U-559 off Port Said in October 1942 provided the clue that—some months later—enabled Bletchley to resume its "same day" decryption of most U-boat messages. In the mean-

See **Turing** *on page 5*

Last Bastion of Purity in a Corrupt World?

Mathematics without Apologies: Portrait of a Problematic Vocation. *By Michael Harris, Princeton University Press, Princeton, New Jersey, 2014, 464 pages, $29.95.*

BOOK REVIEW
By Ernest Davis

According to the dust jacket, "*Mathematics without Apologies* takes the reader on an unapologetic guided tour of the mathematical life, from the philosophy and sociology of mathematics, to its reflection in film and popular music, with detours through the mathematical and mystical traditions of Russia, India, medieval Islam, the Bronx, and beyond." In addition, the author is a major figure in the Langlands program, and he makes a valiant effort to communicate to the lay reader both the goals of the program and what it's like to be a participant. That will do fine as a summary of the book's contents.

Readers will find many fascinating and insightful nuggets in the book. Among them is an admirable comparison of Erdős and Grothendieck, the great exemplars—with Grigori Perelman—of the unworldly mathematical genius:

> "Erdős had more than a few things in common with Grothendieck. Both men were extraordinarily devoted to their mothers. Both were Central European Jews displaced, irreversibly, by World War II: Erdős left Hungary and just kept traveling, while Grothendieck remained stateless for many years by choice. While Grothendieck's premonition of the avatar ladder reaches ceaselessly skyward, Erdős built a no less tangled horizontal network of collaborations."

Overall, however, I can't remember when I last read a book that was as "unapologetically" self-satisfied and self-congratulatory. A scene reported in chapter 6 exemplifies its general spirit. Reine Graves, who co-directed the Berkeley mathematician Edward Frenkel's film *Rites of Love and Math*, was asked at a reception why she decided to make a film about mathematics. Graves, Harris writes,

> "gave the best possible answer. Mathematics, she began, is *un des derniers domains où il y a une vraie passion* [one of the last areas where there is a genuine passion]. . . . Mathematics, like a very few other activities—she mentioned physics and sculpture—is practiced without complacency [*sans autosatisfaction*]; instead there is a true *exigence au travail* [demanding work ethic]. Mathematicians seek to *percer le mystère*. You can see it at once in *l'œil qui brille* [the eye that gleams]."

Harris makes similar claims throughout the book; for instance, he writes that math is "one of the few remaining human activities not driven by commercial considerations."

Look, I'm as susceptible* as the next guy to flattery from French experimental film directors making oracular pronouncements in French; but really, what he writes, *c'est n'importe quoi*. Or, at least, it bears no relation to the world as I've encountered it. Over the years, I've known a fair number of mathematicians and a few physicists. Certainly they are for the most part deeply interested in what they do, but they have no greater *vraie passion* or work ethic than (somewhat at random) the neuroscientists, historians, librarians, musicians, violin makers, writers, journalists, photographers, Quenya enthusiasts, and rock gardeners I've known. Of these, in fact, I would say that the rock gardeners take the prize for *vraie passion* and *exigence au travail*, and the journalists for desire to *percer le mystère*. Sad to say, I have never noticed *l'œil qui brille*; perhaps that requires a film director's eye. As for *autosatisfaction*, I've rarely seen it so vividly on display as in Harris's book.

Creative artists and adoring sophisticated women with a worshipful attitude toward mathematics and mathematicians are a recurring presence in Harris's book. "Is it any wonder," he muses, "that, in popular culture's serious precincts, the mathematician has become the romantic figure of choice?" Chapter 6 traces the romantic cult of the mathematician from the 18th century to the present. I am sorry to spoil Harris's idea of the 18th-century mathematics students at Cambridge as "objects of romantic interest," but the quote he cites does not mean that the Wranglers were admired by society ladies but rather that they visited brothels. In any case, the quote is from a satire and can hardly be relied on for historical accuracy.†

Harris is just as pleased with himself as with his chosen field. Early in the book, he tells us that "By granting me tenure at the age of twenty-seven, Brandeis University ratified my permanent admission to the community of mathematicians. . . . [T]he

*By which I mean that presumably I would be as susceptible, if it ever happened, which is exceedingly improbable, and if I spoke French.

†The full quote, from *The Friendly and Honest Advice of an Old Tory to the Vice-Chancellor of Cambridge* (1751), is as follows: "The Wranglers I am told on the first Day of their Exercise have usually expected that all the young Ladies of their Acquaintance (whether such as have sometimes made their Bands, or who are more genteely employed in keeping the Bar at a Tavern or a Coffee-house) should wish them Joy of their Honours. To give them an opportunity of doing so, their Manner has been to spend the Morning in going to several of their Houses." Quoted in *Social Life at the English Universities in the Eighteeth Century*, p. 398. http://archive.org/stream/cu31924100477466/cu31924100477466_djvu.txt.

See **Mathematical Life** *on page 6*

Turing

continued from page 4

time, Allied shipping suffered losses that, if continued, would surely have prevented the buildup of men and materiel needed to undertake the Normandy landings of June 1944.

Turing, in fact, had relatively little to do with the final decipherment of U-boat communications, having been dispatched to the U.S. in November 1942 with authority to disclose anything and everything known to the British about Enigma decryption. After several months spent conferring with U.S. codebreakers, he returned to England in March 1943. Thereafter, he devoted most of his time to the development of a secure telephone communication system, first at Bletchley and later at a satellite location. That project, which he completed almost single-handedly, was not finished in time to impact the war effort.

In October 1945, he joined a group at the National Physical Laboratory tasked with the construction of a working stored-program computer. Later he moved to the University of Manchester, where a group led by engineer F.C. Williams had taken the lead in hardware development. On June 21, 1948, by finding the largest factor of a given integer, the group's prototype became the first "electronic computing machine" to successfully execute a stored program. The milestone had taken longer to reach than expected because, in the absence of wartime urgency, progress in all directions had slowed considerably. Perhaps to alleviate his frustrations, Turing increased his involvement in long-distance running during these years. As a marathoner, he nearly qualified for the London Olympics of 1948.

In 1949, a contract was signed with Ferranti Ltd. to produce a commercial version of the Manchester machine, which owed more to von Neumann's design than to Turing's. Known as the Mark I, it was delivered in May 1951, well before NPL produced either its "Pilot ACE," a drastically scaled-down version of the machine proposed by Turing, or the less than full-scale version that eventually followed. Both were obsolete by the time they were completed. It was on the Manchester Mark I that Turing was finally able to launch experiments that he had long been planning. Among them were early versions of the Turing test of a machine's ability to think, and several trials related to his theory of morphogenesis.

It was at about this time that he was convicted of gross indecency—the legal euphemism for homosexuality—and given a choice between jail time and chemical castration. Choosing the latter, he endured a series of hormonal treatments that clearly affected his work and personality. The treatments were discontinued after a year, and he seemed rather quickly to resume his former lifestyle.

■■■

Unlike the book, the film makes no attempt either to convey the substance of Turing's mathematical activities, or to depict his life as a homosexual. Beginning with his arrest on suspicion of espionage, rather than gross indecency, *The Imitation Game* reduces Turing's prior history to a series of flashbacks depicting the hazing he endured at boarding school, his relationship with Morcom, his lifelong attraction to long-distance running, his arrival at Bletchley, his romance with Joan Clarke, and his many battles with the military brass.

Turing's wartime discovery process is boiled down to a single "Aha!" moment sited, like its counterpart in the movie version of *A Beautiful Mind*, in a bar. A chance remark made there by one of Bletchley's many Morse code-reading girls sends the entire Bombe crew on the run to Hut #8, where, fed the input she describes, "Alan's machine" succeeds for the first time in determining the Enigma machine's current rotor setting, thereby sealing the fate of "Fortress Europe." It is an entertaining film, with an excellent cast, which succeeds by ignoring the threads that Hodges so deftly unravels!

James Case writes from Baltimore, Maryland.

Geometry, Invariants, and the Search for Elusive Complexity Lower Bounds

Jan Draisma, an associate professor in the Department of Mathematics and Computer Science at Technische Universiteit Eindhoven, is chair of the SIAM Activity Group on Algebraic Geometry. To give SIAM News *readers an idea of one current research theme in the field, he dropped in on the semester-long (fall 2014) program in algebraic geometry at the Simons Institute for the Theory of Computing.*

In a Simons Institute Open Lecture [2] that marked the beginning of the programme *Algorithms and Complexity in Algebraic Geometry*, Peter Bürgisser of TU Berlin gave an overview of recent developments in geometric complexity theory. This article is loosely based on Bürgisser's lecture and on lectures by others in the programme's boot camp one week earlier.

To set the stage, Bürgisser introduced three families of polynomials:

$$\mathrm{esym}_{k,n} := \sum_{1 \le i_1 < \ldots < i_k \le n} X_{i_1} \cdots X_{i_k},$$

$$\det_n := \sum_{\pi \in S_n} \mathrm{sgn}(\pi) X_{1\pi(1)} \cdots X_{n\pi(n)},$$

$$\text{and } \mathrm{perm}_n := \sum_{\pi \in S_n} X_{1\pi(1)} \cdots X_{n\pi(n)},$$

known as the kth elementary symmetric polynomial, the determinant, and the permanent. If k is roughly $n/2$, then these polynomials look very similar in that their degrees grow linearly in n, while they have super-exponentially many terms. But how efficiently can these polynomials be evaluated at given values x_i or x_{ij} for the variables?

Using Gaussian elimination, we can evaluate $\det_n$ in $\mathcal{O}(n^3)$ arithmetic operations. This is not optimal—and I return to this issue below—but at least it is polynomial in n. To evaluate $\mathrm{esym}_{k,n}$, we can first evaluate the polynomial $p_n(T) := (T+x_1) \cdots (T+x_n)$ at n values for T, interpolate, and extract the coefficient of T^{n-k}. Again, the complexity is $\mathcal{O}(n^3)$, and we can do even better by using the discrete Fourier transform.

Now how about perm_n? We can do better than evaluating the $n!$ terms individually and adding them up; one way to reduce the complexity to exponential is depicted in Figure 1. But no polynomial-time algorithm is known for evaluating the permanent. Indeed, probably none exists: A theorem of Leslie Valiant states that the sequence $(\mathrm{perm}_n)_n$ is complete in the complexity class VNP [17]. This class can be thought of as an arithmetic analogue of NP, and the common belief that P ≠ NP would imply that not all elements of VNP can be evaluated in polynomial time. Yet how would it be possible to prove lower bounds on the complexity of the sequence $(\mathrm{perm}_n)_n$?

Valiant argued that a major step in this direction would be to prove that if perm_n is expressed as $\det_N(A)$ for some $N \times N$ matrix A of affine-linear functions in the x_{ij}, as in Figure 1, then N must grow super-polynomially in n. In 2004, using geometric properties of the hypersurfaces defined by $\det_N = 0$ and by $\mathrm{perm}_n = 0$, Mignon and Ressayre proved the best-known lower bound to date on the *determinantal complexity* of the permanent: $N \geq n^2/2$ [11].

The currently most active route toward lower bounds is the geometric complexity theory (GCT) programme [12–14]. At a basic level, this approach involves two key ideas. The first is to think of $\det_N$ and $Z^{N-n}\mathrm{perm}_n$ (the *padded permanent*}, where Z is a homogenising variable that can be taken equal to X_{NN}) as points in the same vector space V_N of homogeneous polynomials of degree N in N^2 variables, where the padded determinant just happens to use only n^2+1 of the variables. The group GL_{N^2} of linear transformations among the variables acts on this space, and a stronger version of Valiant's conjecture states that if the orbit $\mathrm{GL}_{N^2} \cdot Z^{N-n}\mathrm{perm}_n$ of the padded permanent lies in the topological closure of the orbit $\mathrm{GL}_{N^2} \cdot \det_N$, then N must be super-polynomial in n. Consequently, if N is too small, then there must be a *witness polynomial function* $F : V_N \to \mathbb{C}$ that vanishes on the orbit $\mathrm{GL}_{N^2} \cdot \det_N$, but not on the orbit of the padded determinant. Such a witness is a needle in a haystack that would not be found by random search.

The second key idea is to exploit the fact that the group GL_{N^2} acts on the space of polynomial functions $V_N \to \mathbb{C}$ and preserves the set of witnesses. Under this action, the space of polynomial functions decomposes into a sum of irreducible building blocks, and the search for F can be narrowed down to some of these building blocks. This leads to exciting questions in the representation theory of GL_{N^2} that are currently generating a lot of research activity.

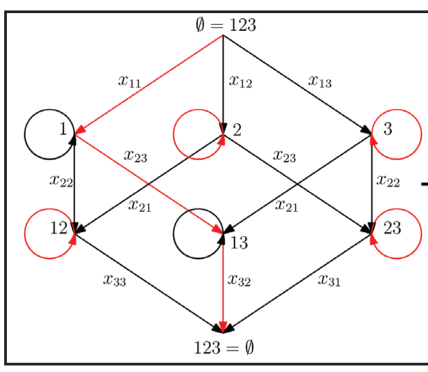
So far, this approach has not led to better lower bounds on the determinantal complexity of the permanent. But the method is universal, and it applies in particular to another notorious question in complexity theory—namely, the complexity of matrix multiplication, which governs the complexity of other linear algebra operations, such as the evaluation of $\det_n$.

Volker Strassen discovered that 2×2 matrices can be multiplied with seven instead of eight scalar multiplications, and that applying this multiplication scheme recursively decreases the complexity of $n \times n$-matrix multiplication from $\mathcal{O}(n^3)$ to $\mathcal{O}(n^{\log_2 7}) = \mathcal{O}(n^{2.81})$ [15]. Don Coppersmith and Shmuel Winograd improved Strassen's result to $\mathcal{O}(n^{2.376})$ [4]. In the last few years, several researchers have followed the Coppersmith–Winograd approach to improve the exponent [5, 6, 18]; the current record of $\mathcal{O}(n^{2.3728639})$ was obtained by François Le Gall. While many researchers believe that the real complexity should be $\mathcal{O}(n^{2+\varepsilon})$ for any positive ε, it was recently proved by Andris Ambainis and Yuval Filmus that the Coppersmith–Winograd approach cannot possibly prove an upper bound of $\mathcal{O}(n^{2.3078})$ [1].

But how could *unconditional* lower bounds on the complexity of matrix multiplication be proved? The idea is to see $n \times n$*-matrix multiplication* as a point M_n in a space $V_n := \mathbb{C}^{n^2} \otimes \mathbb{C}^{n^2} \otimes \mathbb{C}^{n^2}$ of tensors, and—as in the determinant-versus-permanent question—to find a witness polynomial function $F : V_N \to \mathbb{C}$ that gives a lower bound for the *border rank* of this tensor. Again, representation theory serves as a guide in the search for such witnesses. To illustrate, Jonathan Hauenstein, Christian Ikenmeyer, and J.M. Landsberg found a degree-20 polynomial F on the space V_2 that up to scalars is preserved by the group $\mathrm{GL}_4 \times \mathrm{GL}_4 \times \mathrm{GL}_4$ and vanishes on all tensors of border rank at most 6 but not on M_2 [8]. This gives a new, computational proof that the border rank of M_2 is at least 7; Strassen's result then implies that the border rank is exactly 7. Straightforward combinatorics shows that the space of degree-20 polynomials on the 64-dimensional space V_2 is $C(63+20, 20) = 8{,}179{,}808{,}679{,}272{,}664{,}720$-dimensional—it is striking how representation theory helps us to find F and evaluate it at M_2!

Bürgisser and Ikenmeyer found a sequence of explicit witnesses that show the border rank of M_n to be at least $3n^2/2 - 2$ [3]. While this is slightly weaker than the longstanding bounds of [10, 16], it is the first non-trivial bound proved along the

See **Geometric Complexity** on page 8



	∅ = 123	1	2	3	12	13	23
∅ = 123	0	0	0	0	x_{33}	x_{32}	x_{31}
1	x_{11}	1	0	0	0	0	0
2	x_{12}	0	1	0	0	0	0
3	x_{13}	0	0	1	0	0	0
12	0	x_{22}	x_{21}	0	1	0	0
13	0	x_{23}	0	x_{21}	0	1	0
23	0	0	x_{23}	x_{22}	0	0	1

Figure 1. *The matrix on the right is the weighted adjacency matrix of the directed graph on the left, with vertices ∅ and 123 identified, variables along arrows as indicated, and 1's along loops. Its determinant equals $(-1)^{3-1}$ perm_3; one of the terms in the expansion is shown in red. This construction, from Bruno Grenet, generalises to show that the determinantal complexity of perm_n is at most $2^n - 1$ [7].*

Mathematical Life

continued from page 4

privileges befitting my charismatic status . . . included and still include regular invitations to research centers like the IAS [Institute for Advanced Study], . . . the IHES, or the Tata Institute of Fundamental Research . . . the Fields Institute in Toronto, the Mathematical Sciences Research Institute (MSRI) in Berkeley, or the Institut Henri Poincaré in Paris," and so on and on, for two pages. Presumably this is somewhat tongue-in-cheek, or at least Harris has convinced himself that it is. But the line between this and the garden-variety arrogance of a person whose idea of conversation is to recite his CV is fainter than Harris may realize.

Harris pats himself on the back vigorously for being too pure of soul to *grok* anything so vulgar and grasping as finance:

> "It's not the equations that make it difficult for a mathematician like me to grasp quantitative finance. My problem is with adopting the psychology, the motivations, the *persona* of *Investor*. . . Someone who . . . has never aspired to playing *Investor*, a figure whose cardinal virtue is maximizing returns, is at a distinct disadvantage."

I'm not buying it. I don't understand much about finance myself, for the simple reason that I find it boring. I'd much rather spend my time thinking about other things, and my income allows me to live comfortably without being clever about investing. Moreover, whether or not Eugene Fama is right that no one can ever beat the market, it would unquestionably require a lot of work—I would need to outsmart a lot of people who are pretty much as smart as I am and are working hard at it. Presumably, I am just as well off with my savings in an index fund. Dollars to donuts Harris's actual motivations are the same. In any case, ignorance is never a matter for self-congratulation; it is too easy to attain.

Harris is disgusted with the philistines in government who dare question that mathematical research should be funded at taxpayer expense. In an extensive historical survey and deep analysis of the various justifications for doing mathematics, he primarily sets the argument that mathematics is beautiful or that it is art (Hardy's justification) against the argument that mathematics has practical benefits (the "golden goose" justification). He argues, obviously correctly, that the golden goose argument has very little to do with the practice or motivations of most pure mathematicians; he is not content with the art argument, as mathematics is in many ways actually not similar to the mainstream arts. The position he ends up with is that mathematics, and other abstract intellectual studies, are forms of creative play and deserve support on that basis. He writes,[‡]

> [W]hy is it a matter of general interest . . . to have a small group of people working at the limit of their creative powers on something they enjoy? . . . [I]f the question is taken at face value, it answers itself. Indeed, if the notion of general (or public) interest means anything at all, it should be a matter of general interest that work should be a source of pleasure for as many people as possible."

The idea that funding for philosophers or mathematicians is the most central category of the public interest does seem rather parochial and self-serving. Nowhere in his long discussion does Harris raise or acknowledge the obvious question here: If the goal is to maximize the pleasure that people get from their work, is the best use of finite government funding actually to support research on algebraic number theory? Might the net gain of utility be greater if the funds were spent in alleviating the working conditions of migrant workers, people who pack things in Amazon.com warehouses, and so on? Harris asks, What's the right way to think about mathematics? The question he doesn't ask—What's the right way to spend government funds?—is the problem the philistines are obliged to face.

In practical terms, arguing that mathematics without clear direct practical applications (essentially all of pure math and much of so-called applied math) should be funded on the basis that it is a creative pleasure rather than a golden goose of practical applications is pretty much tantamount to saying that it should be funded at the level of the National Endowment for the Humanities, rather than at the level of the National Science Foundation.[§] Whether Harris would be content with this outcome is not clear to me. To make the case that government funding for math should continue to be greater than that for history, comparative literature, philosophy, and so on, it's necessary to argue that mathematics serves the general interest in some ways that these other fields do not. The "creative pleasure" justification does not distinguish math from these other fields, and the claim that these fields are more corrupted by commercialism than math is hogwash.

The idea of math as the last bastion of purity in a corrupt world is a destructive delusion, as is the image of mathematicians as an elect group of noble souls, deserving of being placed on pedestals by glamorous women. "We had fed the heart on fantasies / The heart's grown brutal from the fare," Yeats wrote. I am not sure that "brutal" is the right word here, but certainly "arrogant" applies.

Ernest Davis is a professor of computer science at the Courant Institute of Mathematical Sciences, NYU.

‡The immediate subject here is philosophy; but clearly he intends it to apply to mathematics as well.

§The 2015 budget for NSF's Division of Mathematical Sciences is $224 million. The entire NEH budget is $167.5 million, that of the National Endowment for the Arts, $158 million.

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Call for Nominations: 2016 Hans Schneider Prize of the International Linear Algebra Society

The International Linear Algebra Society (ILAS) is seeking nominations for its 2016 Hans Schneider Prize. The prize is awarded every three years for research, contributions, and achievements at the highest level of linear algebra. The prize, which may be awarded for an outstanding scientific achievement or for a lifetime's contributions, consists of a plaque and a certificate containing the citation, along with an invitation to give a lecture at the ILAS meeting in Leuven, Belgium, July 11–15, 2016.

Nominations should include a brief biographical sketch and a statement explaining why the nominee is considered worthy of the prize, including references to publications or other contributions of the nominee that are considered most significant in making this assessment.

The chair of the prize committee is Richard A. Brualdi of the University of Wisconsin–Madison. Nominations should be sent to brualdi@math.wisc.edu by December 1, 2015. For more information see the ILAS homepage: http://ilasic.org/.

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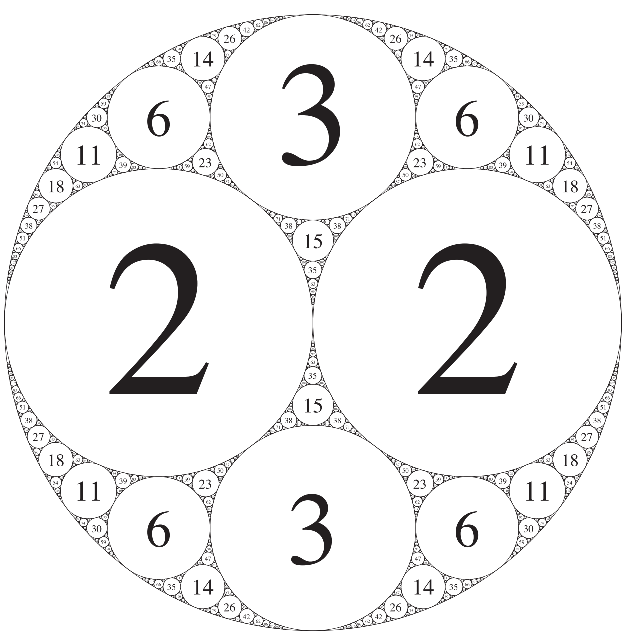
MAA Chauvenet Prize Awarded to Dana Mackenzie

At *SIAM News* we're always happy (but not particularly surprised) to have outside confirmation of the talents of our writers. A case in point is the awarding of the MAA's Chauvenet Prize to Dana Mackenzie at the Joint Meetings in San Antonio in January.

The illustration shown here is from "A Tisket, a Tasket, an Apollonian Gasket" (*American Scientist*, Vol. 98, January–February 2010), the paper for which Mackenzie was honored. Here's his summary of the paper: "Start with three mutually tangent circles, insert a fourth circle tangent to all three, and iterate. If the four circles have curvatures that are integers, then all subsequent curvatures are integers too, and the construction yields an integral Apollonian circle packing."

An earlier (2012) recipient of the Joint Policy Board for Mathematics Communication Award, Mackenzie nonetheless points out that gratification for writers can be long delayed or nonexistent. For *SIAM News* he often chooses the topics he'd like to write about; what usually captures his attention is some new mathematical approach to a physical problem. A few recent examples of his articles are: "Synthetic Biology, Real Mathematics" (December 2014), "Kadison–Singer Problem Solved" (January/February 2014), and "Smells Like a Traffic Jam" (November 2013).

A compliment to a writer is never amiss. Suggested topics matched to the interests of our writers are always welcome too—at siamnews@siam.org.



The numbers represent the curvatures (1/radius) of each circle, with the boundary circle having curvature 1 and the two largest circles having curvature 2 (radius 1/2).

Geometric Complexity

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general lines of the GCT programme. The lower bound record, however, is currently held by Landsberg and Giorgio Ottaviani: In a 2011 paper [9], guided by algebraic geometry tailored to the border rank of M_n, they found witnesses that prove a lower bound of $2n^2-n$ on the border rank of M_n.

The geometric and representation-theoretic approach to complexity lower bounds, as Bürgisser concluded in his lecture, is starting to pay off. By attracting many of the world's experts on geometric complexity theory to its current programme, the Simons Institute for the Theory of Computing has created ideal conditions for further progress.

Applications of algebraic geometry to complexity theory will also be one of the many topics on the programme of the upcoming SIAM Conference on Applied Algebraic Geometry (http://camp.nims.re.kr/activities/eventpages/?id=200&action=overview), August 3–7, 2015, at the National Institute for Mathematical Sciences and the Korea Advanced Institute of Science and Technology in Daejeon, South Korea.

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