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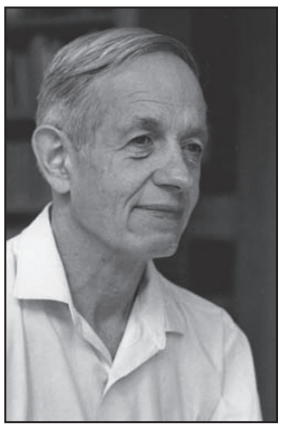
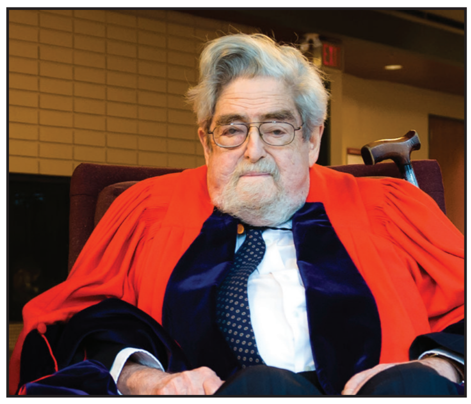
*2015 Abel Prize*

## Groundbreaking Contributions to the Theory of Nonlinear PDEs



*John Nash in 1994. Photo courtesy of Princeton University, Office of Communications.*



*Louis Nirenberg was awarded an honorary doctorate by the University of British Columbia in 2010. Photo by Don Erhardt.*

In Oslo on May 19, His Majesty King Harald of Norway will present the 2015 Abel Prize to John F. Nash, Jr., of Princeton University and Louis Nirenberg of the Courant Institute of Mathematical Sciences, New York University. In announcing the 2015 awards, the Norwegian Academy of Science and Letters cited the laureates jointly "for striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis."

*See* **Abel Prize** *on page 5*

# Mathematical Challenges and Opportunities in Optics and Photonics

*By Gang Bao and Michael I. Weinstein*

The field of optics and photonics encompasses the fundamental science of light and its applications. The long history of the field's interactions with mathematics covers topics ranging from ray optics to the classic theory of the electromagnetic spectrum to the quantum nature of light (quantum optics, light–matter interactions). A multitude of technologies that are central to our lives have emerged from these interactions.

The past several decades have witnessed major advances in applications, both realized and potential, that take advantage of the novel properties of light propagation and interactions in complex structures. Examples include super-resolution, optical resonances, near-field imaging, Raman scattering, nonlinear optics, imaging, cloaking, and nano–optics and photonics. These have been brought about by parallel advances in such areas as material fabrication at the micro- and nanoscales, scientific computation (from hardware to algorithms), and a deepening of our understanding of the "multiphysics" of such systems.

This article outlines some of the challenges and opportunities for the applied and computational mathematics community, and discusses recent Optics and Photonics initiatives of the National Science Foundation and the upcoming (2016–17) program in Mathematics and Optics at the Institute for Mathematics and its Applications.

### Challenges and Opportunities

As they increase the range of potential applications of optics and photonics, the recent advances present great opportunities and challenges for the applied and computational mathematics community. New modeling tools and techniques are needed, as

*See* **Optics & Photonics** *on page 7*

# Computational Surgery

## *A Transdisciplinary Approach to Improved Patient Care*

*By Thierry Colin, Marc Garbey, and Olivier Saut*

Computational surgery is a new discipline that focuses on the use of medical imaging, medical robotics, simulation, and information technology in surgery [3]. The fifth International Conference in Computational Surgery and Dual Training took place on the main campus of the National Institutes of Health, in Bethesda, Maryland, January 19–21. A particular emphasis at this year's conference was the clinical and translational impact of multiscale methods, as promoted by the Interagency Modeling and Analysis Group (www.imagwiki.org).

The conference operates under a few simple rules that make it unique: It is hosted by a major medical center so as to stay close to the application field, and the slate of plenary speakers is divided equally between computational scientists (broadly defined) and surgeons; the conference program includes a panel discussion in which key players from the medical industry, academia, and federal/international agencies discuss ways to foster collaborations that will advance the field. The criterion for acceptance of a paper presentation is simple: Does it consider computational work that is translational—i.e., work that could have an impact on patient care?

More than 90 mathematicians, surgeons, oncologists, anesthesiologists, and engineers of all types have attended this meeting, which attracts more people each year. What accounts for the increasing success? It is probably a combination of the true transdisciplinary nature of the conference, the depth of the scientific program, and the involvement of surgeons.

For a long time, statistical techniques were the only mathematical tools used in the clinical framework. Statistics can provide information on populations of patients and can be used to quantify risk—and, in this sense, is essential. At the same time, the development of imaging techniques, coupled to increasingly powerful signal and image processing methods, has led to the availability of dramatic amounts of very precise data for each particular patient. Whatever the disease, moreover, patients benefit from longitudinal follow-up; the data are available at several time points and are also multimodal.

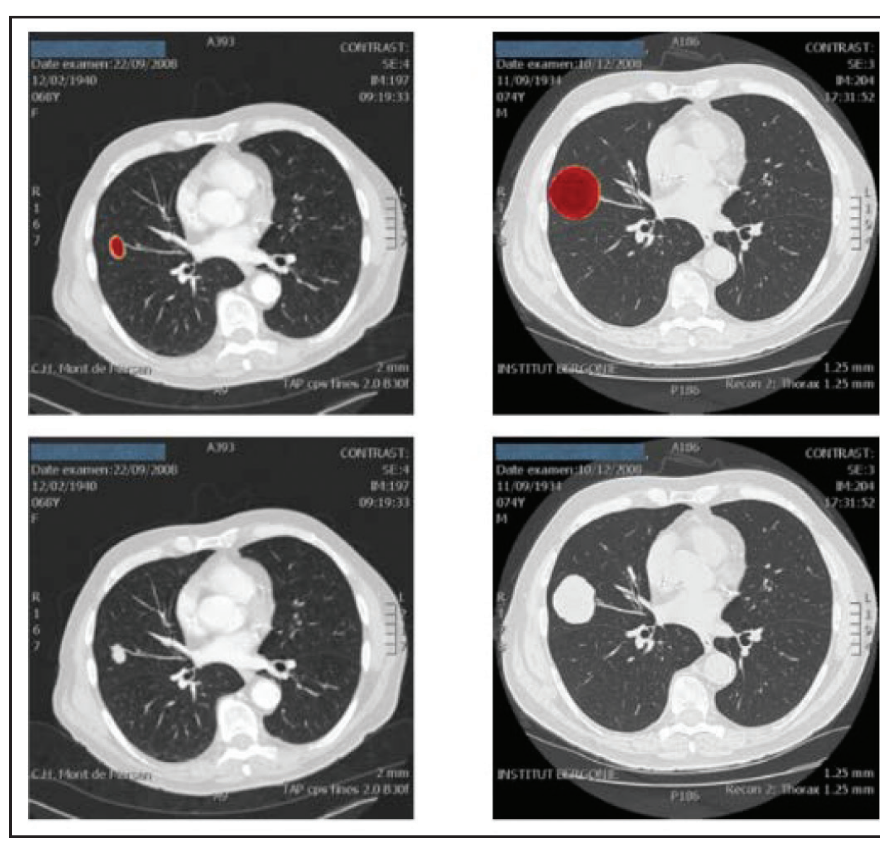


**Figure 1.** *Computation of a metastasis to the lung. The images in the top row are from simulations, those at the bottom from CT scans.*

A question that arises naturally is how multimodal, longitudinal, personalized data can be used to improve diagnostic accuracy, the prognosis for patients, and management of procedures in the operating room. As presented on the computational surgery network (computationalsurgery.org), we believe that we can tackle these problems by integrating different facets of computational science with image processing, PDE methods, machine learning, and agent-based methods. Working with real clinical data and medical doctors is the very first step of a long and challenging endeavor that will take scientists outside their comfort zones. We describe this process with the two following examples.

### Noninvasive Imaging in Modeling Tumor Growth

Mathematical models of cancer have been extensively developed with the aim of understanding and predicting tumor growth and the effects of treatment. In vivo modeling of tumors is limited by the amount of information available. In the last few years, however, we have seen dramatic increases in the range and quality of information provided by non-invasive imaging methods [1, 2]; as a result, several potentially valuable imaging measurements can now be used to determine tumor growth quantitatively, and to assess tumor status, as well as anatomical and functional details. Using different modalities, such as CT, MRI, and positron emission tomography (PET), it is possible to evaluate and define tumor status at different levels: physiological, molecular, and cellular.

In this context, the present project aims to support the decision process of oncologists as they define therapeutic protocols via quantitative methods. The idea is

*See* **Computational Surgery** *on page 6*

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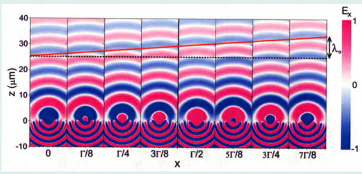
1 **Groundbreaking Contributions to the Theory of Nonlinear PDEs**

1 **Mathematical Challenges and Opportunities in Optics and Photonics**



1 **Computational Surgery: A Transdisciplinary Approach to Improved Patient Care**

2 **Play Takes Aim at the "Two Cultures" Divide**
Mathematician, novelist, and playwright Manil Suri crosses the "two cultures divide" with ease, as do others featured on these pages—including mathematicians/writers Ian Stewart and Steve Strogatz, and Mark Green, whose curatorial assistance helped build a bridge with art history.

3 **Riemann Mapping by Steepest Descent**

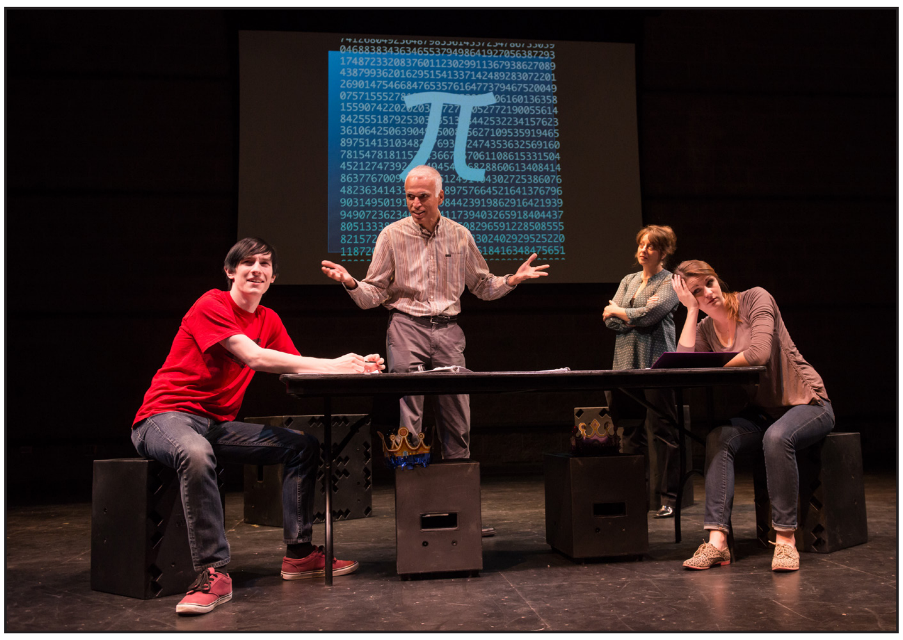
4 **Peter Lax, an Inspired and Inspiring Life in Mathematics**
Reviewing a book about Peter Lax by his former student Reuben Hersh, Gilbert Strang sketches the main contributions of Lax on "just three topics, one linear and two nonlinear," before highlighting an important property of any paper by Lax: "He didn't close out problems, he opened them."

4 **Singular Perturbations Expert Brings the Theory to Life**
Bob O'Malley's new book on singular perturbation theory could have been written, in the words of reviewer Ferdinand Verhulst, "only by an author who has devoted most of his working life to this topic, always with an eye on human interactions and the background of people in the field."

7 **Professional Opportunities**

# Play Takes Aim at the "Two Cultures" Divide

**The Mathematics of Being Human.** *Written and performed by Michele Osherow and Manil Suri, directed by Alan Kreizenbeck, and featuring performances of Chaz Atkinson and Savannah Jo Chamberlain, November 4, 2014.*



*Student performers Chaz Atkinson and Savannah Jo Chamberlain (foreground) "take over when their professors have battled themselves to an impasse." Manil Suri and Michele Osterow, the professors, play themselves and are also the playwrights.*

PLAY REVIEW
*By Katherine Socha*

A rigid, lock-on-truth mathematics professor and a poetic, metaphor-loving English professor are forced by their university to team-teach an interdisciplinary freshman course. What could possibly go wrong?

Inspired by a true story, Professor Mike Pearson (the character is not related to MAA Executive Director Michael Pearson) and Professor Naomi Kessler battle their way through the development of a syllabus and choice of texts. In the course of the play and the class, they survive fierce arguments over whose subject has "precision," use metaphor to understand mathematical and literary ideas, and by turns terrify, bore, and inspire the students who take their class. The eponymous course, "The Mathematics of Being Human," takes the starring role in this play written by Michele Osherow (an associate professor of English, University of Maryland Baltimore County, and dramaturg at the Folger Theatre in Washington, DC) and Manil Suri (a professor of mathematics and statistics at UMBC and best-selling novelist). I was lucky enough to get a seat at a performance held at UMBC, along with apparently a dozen or more of the students who had taken the real-life freshman humanities seminar that inspired this play. Judging by their appreciative laughter at pivotal plot points, I think that much of the performance rang true to their experience as humanities students suckered into a mathematics class.

The play explored major themes in often humorous ways—communicating to people outside one's own discipline, juxtaposing humanities and mathematics as approaches to understanding the same world, getting into (and out of) one's own way, and transcending ego (the confident insularity of the mathematician character) and transcending fear (the slightly math-phobic vulnerability of the humanities character).

Wonderful selections of mathematics connections in literature and art highlight the rich opportunities for the cross-cultural battles led by these sharply defined faculty characters. Shakespeare's *King Lear* and Howard Moss's poem "Particular Beauties" appear, along with more stereotypical works, such as Lewis Carroll's *Alice in Wonderland* and Edna St. Vincent Millay's "Euclid alone has looked on Beauty bare," all discussed with energy and self-deprecating humor. The play's Mike Pearson came across to me as the rigid and prejudiced embodiment of some of the worst stereotypes of mathematicians to be found in popular culture—in fact, it seemed a bit heavy-handed to set up such a straw man (however funny) for the sake of showing growth and transcending the disciplinary silo. The play's Naomi Kessler came across as defensive, resigned, and slightly insecure, as well as hilariously "literary," while serving as the wry, common-person foil for her colleague's flights into stratospheric mathematics.

In real life, Osherow and Suri first taught the course in the fall of 2011. As Osherow reported in *The Chronicle of Higher Education*,*

"I held a June 2011 orientation for the 13 incoming freshmen chosen for our Humanities Scholars program. They would take the seminar in the fall. When I revealed the title and focus of the seminar, I noted their horror. 'Don't worry,' I told them. 'You can't find this more frightening than I do.' And it was true. Ever since the word problems my father forced on us at dinner, I've always been terrified of math."

While working on the 2009 Folger

*"Mathematics and What It Means to Be Human," a three-part series, framed as a dialog between Osherow and Suri about their team-teaching experience; Part 1: http://chronicle.com/article/MathematicsWhat-It-Means/134850/.

*See* **Cultures** *on page 3*

# The Lewis Thomas Prize for Writing about Science

*Alan Alda, Rockefeller University President Marc Tessier-Lavigne, Ian Stewart, and Steven Strogatz at Rockefeller's March 30 reception for the prize recipients. Photos by Scott Rudd/ scottruddevents.com.*

*Ian Stewart, Avril Stewart, and Barbara Keyfitz. In 1999, Stewart received the JPBM Communications Award "for communicating the excitement of science and mathematics to millions of people around the world for more than twenty years." Strogatz received the award in 2007.*

Since 1993, Rockefeller University has conferred the Lewis Thomas Prize each year on "the rare individual" who succeeds in bridging the worlds of writers and scientists. Ian Stewart of Warwick University, in the UK, and Steven Strogatz of Cornell University, the first mathematicians to receive the prize, are well known to the SIAM community, both for their research and for their general-audience books, articles, broadcasts. Both have made conveying the nature, interest, and uses of mathematics an integral part of their careers.

*Marc Tessier-Lavigne and Steve Strogatz, who precedes Manil Suri as a* New York Times *columnist (see facing page) and also appears frequently in Radiolab shows.*

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## Cultures

*continued from page 2*

Theatre production of Tom Stoppard's *Arcadia*, Osherow invited Suri to join her as the play's mathematical consultant. Through their work on the richly mathematical *Arcadia*, Osherow overcame her fear of mathematics and developed the productive collaboration with Suri that led to their team-teaching and their work writing their own play.

I freely acknowledge that I am not the target audience for this play: I am familiar with cross-disciplinary studies and have worked with many liberal arts mathematics faculty who respect and appreciate the humanities. (See, for example, the online *Journal of Humanistic Mathematics*, housed at the Claremont Center for the Mathematical Sciences.) Some of us from this multidisciplinary community can even be a bit jaded about seemingly tired Humanities versus Mathematics dichotomies played up for dramatic effect. However, in the end, my skepticism about a show that promised to play up this constructed divide was unnecessary, and I learned some connections between literature and mathematics that were new to me. For example, the characters spend a good amount of time discussing the idea of "nothing" as explored in *King Lear*. As pointed out in the play notes, "scholars have commented on Shakespeare's extended musing on 'nothing' in the play, connecting it with the mathematics of zero and nihilist movements in the sixteenth century." Further discussion of the mathematical exploration of *Lear* can be found in Osherow and Suri's *Chronicle* series. In this series, the interested reader will learn Suri's solution to a classroom discussion that wandered far from mathematics: offer up to the students "a mathematical version of King Lear's division problem to compensate." That is, ask the class to trisect the angle.

The post-performance talkback session brought out many additional intriguing details, including students' reactions to taking the course. While they were clearly self-selected, the students at the performance seemed very positive about their freshman experience, perhaps surprisingly so because it was an experience they did not choose for themselves. We also learned that Osherow and Suri had very different responses to their original student course evaluations; Osherow considered them among the harshest she'd seen, but Suri thought they were better than expected as his classroom experiments often were "punished" on student evaluations.

Osherow and Suri's comments in the *Chronicle* reinforced many of the ideas discussed during the play's talkback session. I learned a lot from these columns, which highlighted some of the challenges involved in teaching the course that inspired the play. Oddly, these great pieces appear online as "Do-Your-Job-Better" Columns. In Part 2 of the series,† Osherow writes,

> "I think the students liked those days when Manil and I went at it. I liked those moments, too, because I not only had to confront another perspective head on but also had to challenge my own. I loved seeing the baffled expressions of the students while their instructors disagreed; the things we were asking them to consider were perplexing and appropriate."

Suri writes of challenges of a different nature:

> "I found it difficult to sit still as meticulous mathematical principles were divested of their integrity in the service of poetic cleverness. Mathematics is so poorly understood as it is, by poets and readers alike. To further confuse its meaning in swirls of willful metaphor seemed a dubious pursuit. (I suppose I must have come off as a grumpy party-pooper—probably still do.)"

No exploration of the great Humanities–Mathematics divide would be complete without at least a mention of C.P. Snow's famous "Two Cultures" lecture and essay and the subsequent and continuing popular fascination with whether one is a "science person" or a "humanities person." In the *Chronicle*, Suri suggests that one of the course themes was indeed "the two cultures" and describes his initial enthusiasm for the humanities against the sciences battle as an exciting way to launch the course. The challenge of communicating across the two cultures and learning to respect the insights and expertise of colleagues across the divide forms the backdrop of this thoughtful and witty play.

I found the play by turns funny, annoying, fascinating, educational, and at all steps highly engaging; the high quality of the work is exactly what one would expect from a collaboration between two such distinguished and successful professionals. In this review, as often in universities, the professors have garnered all the attention. But the role of the students, both in the play and in real life, is critical to the development of both professors' ideas. Through all sections of the play (helpfully captioned at the back of the stage—for instance, 1. General Teaching Requirements; 7. King of Nothing), conversations and arguments between the two students help the audience reflect on and understand the literary and scientific ideas discussed, challenged, or even rejected by the professors. In the play, the students take over when the professors have battled themselves to an impasse—and their efforts lead to the play's denouement.

At the audience talkback, Suri said he wanted to create a "cultural profile" for mathematics, to bring people together at night, after work, on weekends to appreciate and engage with mathematics much as they do when they come together to appreciate and engage with art in galleries, or with poetry and literature in book readings. Performances of this play seem like a great step toward that goal.

*"The Mathematics of Being Human" will be performed at Bridges Baltimore 2015: The Conference on Mathematics, Music, Art, Architecture, Culture, which will be held at The University of Baltimore, July 29–August 1, 2015. The performance, the featured Public Event, is scheduled for The Wright Theater, July 29 at 8:00PM.*

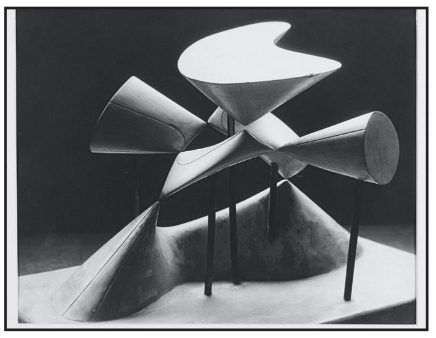
*Katherine Socha teaches mathematics at Park School of Baltimore.*

†http://chronicle.com/article/MathematicsWhat-It-Means/135114.

*As another part of his efforts to encourage people to engage with mathematics, Manil Suri recently accepted an invitation from* The New York Times *to become a contributing opinion writer. His first piece, about Pi Day, appeared in print on March 14, 2015, titled "Don't Expect Math to Make Sense." (Titles, he learned, are not shared with the author in advance.) He expects to contribute several more articles on mathematical topics in coming months.*

## Man Ray–Human Equations

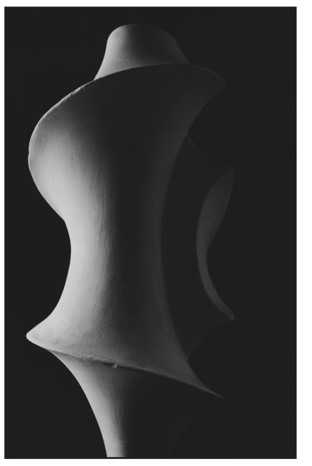
### A Journey from Mathematics to Shakespeare



*Man Ray,* Mathematical Object, *1934–35. Gelatin silver print, 9 × 11 in. Collection L. Malle, Paris. © Man Ray Trust / Artists Rights Society (ARS), NY / ADAGP, Paris 2015.*

*Man Ray,* Shakespearean Equation, King Lear, *1948. Oil on canvas, 18 1/8 x 24 1/8 in. Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington, DC. Gift of Joseph H. Hirshhorn, 1972. © Man Ray Trust / Artists Rights Society (ARS), NY / ADAGP, Paris 2015. Photography by Cathy Carver.*

As mathematicians, we tend to forget how extraordinary are the objects we study. This exhibit, on view at the Phillips Collection, Washington, DC, through May 10, gives a glimpse of the surrealist photographer and painter Man Ray's reaction when he encountered models of some of these objects at the Institut Henri Poincaré in Paris. The exhibit brings together for the first time, as with the examples shown here, actual models, Man Ray's photographs of them, and a series of paintings he based on the photographs. A section of the catalogue is devoted to explanations of the mathematical importance of the models, written by mathematicians, who were involved in conceptualizing the exhibit from its early stages. The exhibit is supplemented by a wonderful exhibit by MoMath, in which museum-goers can manipulate the coefficients of various algebraic surfaces and see on a screen how their geometry changes. This is an extraordinary opportunity for outreach to the general public, with excellent scholarship on both the mathematical and art history axes.—*Mark Green, Professor Emeritus, UCLA Department of Mathematics, and consultant for the Phillips exhibit.*



*Also on view at the Phillips are photographs and sculptures by the Japanese artist Hiroshi Sugimoto. Here,* Kuen's Surface: A Surface with Constant Negative Curvature (Conceptual Form 0006)*, 2004. Gelatin-silver print, 58 3/4 x 47 in. Collection of the Artist, New York.*

# Riemann Mapping by Steepest Descent

MATHEMATICAL CURIOSITIES

*By Mark Levi*

In this issue's column we outline a quick constructive proof of the Riemann mapping theorem.

Here's a statement of the theorem: Any simply connected open set $D$ in the plane that is not the entire plane can be mapped conformally and one-to-one onto an open unit disk, taking any point $p \in D$ to the disk's center, with the derivative at $p$ being real and positive.

Our map is produced via a physically motivated argument: We think of a heat-conducting plate $D$, insulated everywhere except for the boundary, as shown in Figure 1. We place a heat sink at $z = 0$, drawing $2\pi$ calories per second, and maintain $u = 0$ on $\partial D$. Figure 2 shows the graph of the resulting stationary temperature distribution. Formally, we define

$$u(z) = \ln|z| + u_0(z), \tag{1}$$

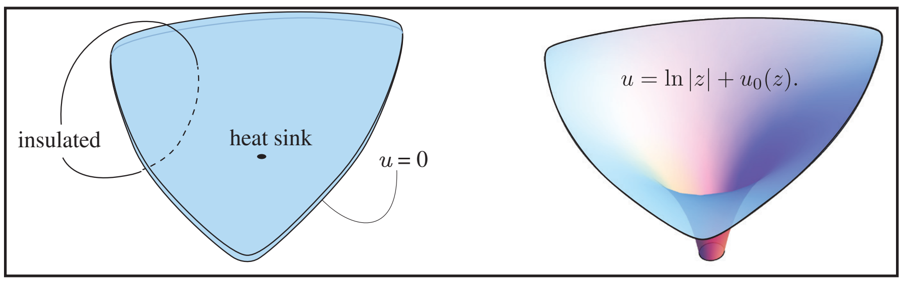



**Figure 1.** *Heat-conducting plate, insulated everywhere but the boundary.*

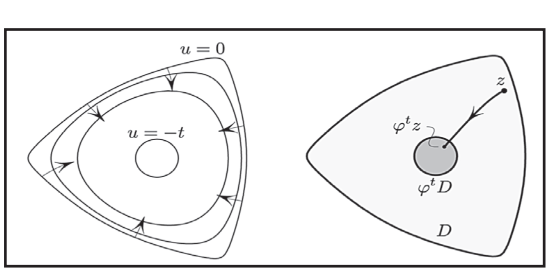



**Figure 2.** *Constructing the conformal map from D to a disk. Left, the modified gradient flow preserves level curves; right, mapping by the modified gradient flow.*

where $u_0(z)$ is a *harmonic function* with boundary conditions chosen to cancel the logarithm on $\partial D$.*

*Level curves of (1) are approximately circles near* $z = 0$; more precisely, the set $D_t = \{u \le -t\}$ for large $t$ is approximately a small disk $|z| \le e^{-t-u_0(0)}$ (see Figure 2).

*Here we use the existence of solutions of the Dirichlet problem, which limits some generality on $D$. Details can be found in F. John, *Partial Differential Equations*, 4th ed., Springer, New York, 1991.

Remarkably, the flow $\varphi^t$ of the modified gradient field

$$\dot{z} = -\frac{1}{|\nabla u|^2}\nabla u \tag{2}$$

shrinks $D$ into $D_t$ and does so conformally—i.e., $\varphi^t$ is almost the desired map! Indeed, $du/dt = -1$ along (2), and thus $\varphi^t D = D_t$. And because the right-hand side of (2) is analytic,† $\varphi^t$ is conformal. By dilating $D_t$ we obtain the desired map $f$ in the limit:

$$f(z) = \lim_{t\to\infty} e^t \phi^t z. \tag{3}$$

The missing details of the proof, which are routine, can be found in [1].

### References


[1] M. Levi, *Riemann mapping by steepest descent*, Amer. Math. Monthly, 114:3 (2007), 246–251.


†Since it can be written as $1\sqrt{\nabla u}$ = $1/(u_x - iu_y)$, where $U = u_x$ and $V = -u_y$ satisfy the Cauchy–Riemann equations because $u$ is harmonic.

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# Peter Lax, an Inspired and Inspiring Life in Mathematics

**Peter Lax, Mathematician: An Illustrated Memoir.** *By Reuben Hersh, American Mathematical Society, Providence, Rhode Island, 2015, 253 pages, $35.00.*

BOOK REVIEW
*By Gilbert Strang*

There are hundreds of biographies of Churchill and Roosevelt. They are great men on a world scale. They imposed their vision on the rest of us, or perhaps sometimes it was just their will. Either way, millions were affected.

Closer to us, there are biographies of von Neumann, Turing, Courant, and Erdős—and now we have a biography of Peter Lax. These are great men on our scale. Their vision has affected every reader of *SIAM News*. To some degree, we can understand what they did and how they did it. In Peter's case we could even ask him—but magic remains.

Peter Lax's life story is remarkable in itself; he was special from the start. Here are four parts, three of them very early:

- Fleeing Budapest with his family in 1941
- Meeting Courant and von Neumann as a New York schoolboy
- Working on the Manhattan Project in Los Alamos (as a corporal in the Army)
- Receiving the Wolf and Abel Prizes

Between the third and the fourth came Lax's life work. That barely fits into Reuben Hersh's book, and it certainly won't fit into this review. Perhaps from just three topics, one linear and two nonlinear, readers can extrapolate. My goal is to show the person behind this work—we all learn by example. Peter is an inspiration to his students and friends (as von Neumann was to him).

The first topic is stability; everyone knows its importance—computations make that clear. Lax's equivalence theorem (convergence ↔ stability) demonstrated that stability is not just desirable but indispensable. When an approximation method is unstable, a complete space will contain the limit of functions that grow faster and faster. The opposite is uniform boundedness (stability). That key point is part of pure mathematics, in the service of numerical analysis.

The equivalence theorem gave us work to do, precise results to prove, something to accomplish with our analysis and our lives. Those words sound personal and I suppose they are—by random chance Peter Henrici assigned me the Lax–Richtmyer paper to present in his graduate student seminar. And the stability requirement led Heinz-Otto Kreiss in 1962 to one of the great achievements of all time in matrix analysis. At the International Congress in Stockholm that year, Peter changed his invited address to present the Kreiss matrix theorem.

The second topic is CFD, computational fluid dynamics. Los Alamos was an enormous impetus; scientific computing came to life. An essential question was to understand shock waves and how to compute their movement. This took Peter (and von Neumann before him) into a different world, beyond the smaller and safer domain where talent in pure analysis wins.

*Peter Lax and Burton Wendroff, in a photo by Norman Bleistein. Combining stability with second-order accuracy, the Lax–Wendroff method made it possible to compute the movement of shock waves with the accuracy required in scientific computing. From* Peter Lax, Mathematician.

Applied mathematics requires a combination of skills, not perfection of one. Lax identified and solved the crucial problem at the moment of discontinuity, to apply the jump equations (Rankine–Hugoniot) to start the solution again beyond the shock. With a little viscosity you can go suddenly but smoothly through that singularity, or you can achieve the same result with finite differences. Here the Lax–Wendroff method combined stability with second-order accuracy—the upgrade from simple but inaccurate first-order approximation that scientific computation always demands.

The third topic is Lax pairs. They are the ultimate example of an insight in algebra that rewrote the theory of nonlinear integrable PDEs. The key idea is that eigenvalues are preserved as the solution evolves and the differential operator changes. Special cases like Korteweg–deVries had been seized on, one at a time, but the pattern was unknown. Lax found it in the equation $dL/dt = BL - LB$, with solution $e^{Bt} L(0)\, e^{-Bt}$. If we can write a PDE in this form, then $L(t)$ is similar to $L(0)$. An amazing number of special nonlinear equations do have that form—they come from Lax pairs $(B,L)$ and conservation laws follow.

One more note about Lax's papers, an important one. He didn't close out problems, he opened them. After reading a paper, you had something to think about.

*See* **Peter Lax** *on page 5*

# Singular Perturbations Expert Brings the Theory to Life

**Historical Developments in Singular Perturbations.** *By Robert E. O'Malley, Springer, New York, 2014, 256 pages, $53.00.*

BOOK REVIEW
*By Ferdinand Verhulst*

There exist books on the history of mathematics that avoid mathematics and emphasize social interactions, and mathematics books that offer historical notes. The book under review is in a third category: Its extensive sketch of the development of the theory of singular perturbations is interspersed with historical considerations and anecdotes about the mathematics and the people involved. Such a book could be written only by an author who has devoted most of his working life to this topic, always with an eye on human interactions and the background of people in the field.

Bob O'Malley, a former president of SIAM (1991–92), has contacts in many different countries. He is the author of a large number of papers on singular perturbations and a number of valuable books. His book *Thinking about Ordinary Differential Equations* typifies his approach: Starting from a fascination with phenomena, he seeks deeper insight about them.

The theory of singular perturbations has a somewhat special place in mathematics. As we know, algebra has a profound and highly structured collection of theories, and work on famous basic conjectures helps to unify these theories. Analysis is different. Analysis has theorems and rules, though more exceptions than rules, a wealth of assorted theories, few basic conjectures, and many different points of view. The theory of singular perturbations, which is part of (asymptotic) analysis, represents a typical, and for some people somewhat distasteful, example of the problem with analysis. To algebraists (in the unlikely case that they have heard about singular perturbations), the theory looks like a bag of tricks—people who work on singular perturbation problems perform mental ballet and acrobatics, using mathematical tools that are sometimes concrete, sometimes abstract.

The boundary value problem

$$\varepsilon \frac{d^2\phi}{dx^2} - \phi = -1 - x^2, x \in (0,1),$$
$$\phi(0) = 2,\ \phi(1) = 0$$

can serve as a relatively simple example. Here ε is a small positive parameter. In a crude approach we take ε = 0 to obtain the function

$$\phi_0(x) = 1 + x^2$$

as a possible approximation of the solution of the boundary value problem. But how can $\phi_0(x)$ be an approximation when it does not even satisfy the boundary conditions? The answer is that $\phi_0(x)$ is a good approximation in the interior of [0, 1] but not near the boundary. Near the endpoints $x$ = 0 and $x$ = 1, we can construct good approximations from local solutions, and by smoothly connecting these "boundary layer solutions" to the interior approximation (a process called "matching") we obtain an approximation over the entire domain; see Figure 1. The term "singular" is used because the unperturbed problem (ε = 0) does not produce an acceptable approximation over the entire domain.

*The theory of singular perturbations, which is part of (asymptotic) analysis, represents a typical, and for some people somewhat distasteful, example of the problem with analysis.*

Before 1900, a few people had noticed singular perturbation phenomena, but the theory really got its start in Heidelberg in 1904, with a lecture by Ludwig Prandtl at the International Congress of Mathematicians. Felix Klein, a leading mathematician of the time who arranged a chair for Prandtl in Göttingen, praised Prandtl for "his strong power of intuition and great originality of thought with the expertise of an engineer and the mastery of the mathematical apparatus."

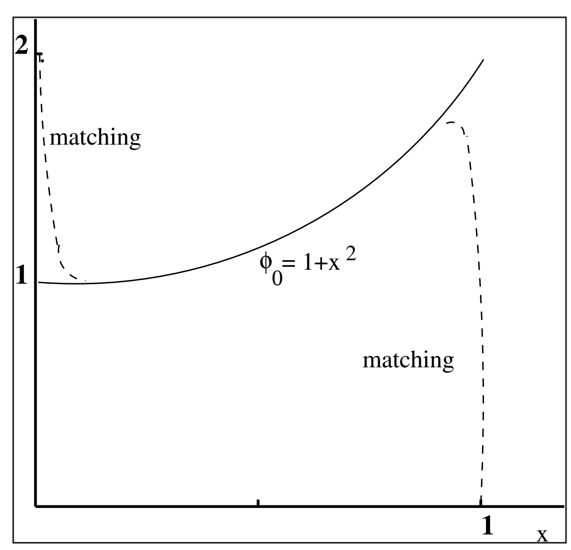


**Figure 1.** *The process of smoothly connecting interior solutions and local solutions near the boundary to obtain a global approximation is known as "matching."*

Prandtl's breakthrough was in constructing approximate solutions of the nonlinear partial differential equations of fluid dynamics without and with friction. Although the motion of an airplane can be understood only when friction is considered, the corresponding equations are extremely difficult. Prandtl introduced the notion of a boundary layer, a thin layer near the aircraft wings where friction gives rise to solutions very different from those far from the wings. The idea of constructing different types of solutions in different parts of the spatial domain and using special matching rules turned out to be exceptionally fruitful. The insight that solutions of differential equations in different time and space domains can be qualitatively very different moved research a huge step forward.

Many mathematicians and engineers subsequently started to use Prandtl's ideas. A remarkable number of them were German–Jewish mathematicians who later had to leave Nazi Germany. They went to Great Britain and the U.S., where the field gained momentum, stimulated further by work of G.I. Taylor, S. Goldstein, and N. Levinson. After the Second World War, singular perturbation theory became an important area of research again in continental Europe, especially in France, the Soviet Union, and The Netherlands.

The book under review describes many aspects of the historical development and is a didactic introduction to singular perturbation theory. The book presupposes a working knowledge of ordinary differential equations; notwithstanding the many technical details, it provides more of an introduction and description of ideas than a mathematically rigorous treatment. For proofs and technical details, the author has compiled a useful list of 538 references.

The text should be very useful for introductory lectures on singular perturbations. It starts with a discussion of the asymptotics of divergent series, followed by the intriguing topic of matching solutions of linear and nonlinear equations. Partially unsolved problems are discussed, with attention to different points of view. This part, with its strong appeals to the reader's intuition, is mathematics that is full of life. Subsequent chapters deal with turning points, canards, and multiscale methods, for oscillatory as well as boundary layer problems.

The book is a great gift to the world of mathematical analysis.

*Ferdinand Verhulst is a professor at the Mathematisch Instituut, University of Utrecht, the Netherlands.*

## Peter Lax

*continued from page 4*

I am sure this was appreciated by his students, one of whom was Reuben Hersh (who is also a celebrated writer). That connection allows this book to include many insights on Peter's life—ordinary as well as extraordinary, exciting and quiet, part happy and also part sad. His first wife Anneli and his first son Johnny are unforgettable.

*In 2005, for "contributions to the theory and application of partial differential equations and to the computation of their solutions," Peter Lax received the Abel Prize. He is shown here in Oslo, with Norwegian H.R.H. Crown Prince Haakon, who presented the prize. From* Peter Lax, Mathematician, *courtesy of the Norwegian Academy of Science and Letters.*

As long as I am using more space than the editor expected, let me look back to other early leaders of numerical analysis, contemporaries of Peter. For a well-posed (and stable) question, let's exclude those who are still alive. Jim Wilkinson in England and Germund Dahlquist in Sweden come immediately to mind. In France the choice must be Jacques-Louis Lions; he deserves a full biography. Those three were great men, world class. In the U.S. I will name Gene Golub, I hope not too controversially. Gene was active with SIAM but disappointed in a SIAM decision about journals. Then, by a happy twist of fate, the Summer School that flourishes today became his legacy (along with the SVD).

Peter Henrici was Swiss, and before him was Stiefel. China gave us Feng Kang, a wonderful character and a creator of the finite element method. The Soviet Union was too big for a single name—Olga Ladyzhenskaya was a SIAM von Neumann lecturer on wave equations, and Olga Oleinik analyzed conservation laws. We have Lanczos from Ireland, Ron Mitchell from Scotland, Fujita from Japan. But powerhouses of research like Germany, Italy, Spain, and Australia developed a little later. I don't have a conclusion to draw, except to say how fortunate we are to work together.

And I hope that another Peter Lax comes along soon.

*Gilbert Strang is a professor of mathematics at MIT.*

## Abel Prize

*continued from page 1*

As Bob Kohn pointed out at the Courant Institute's celebration of the prize to Nirenberg, the Abel committee made the 2015 prize a joint award not because it was unable to choose between Nirenberg and Nash. Rather, it based its choice on "parallels between their mathematical styles and themes, as well as the outstanding impact of their work."

An extended citation, part of a wealth of material available on the Abel Prize website,[*] opens with a succinct overview of the laureates' contributions to the theory of nonlinear PDEs:

> "Nash and Nirenberg have played a leading role in the development of this theory, by the solution of fundamental problems and the introduction of deep ideas. Their breakthroughs have developed into versatile and robust techniques, which have become essential tools for the study of nonlinear partial differential equations. Their impact can be felt in all branches of the theory, from fundamental existence results to the qualitative study of solutions, both in smooth and non-smooth settings. Their results are also of interest for the numerical analysis of partial differential equations."

Highlights of their contributions include:

■ *Fundamental results in elliptic regularity.* Nash proved the first Hölder estimates for solutions of linear elliptic equations in general dimensions with no regularity assumptions on the coefficients (a result also proved by De Giorgi around the same time by an entirely different method). Nirenberg's many results on elliptic regularity include his systematic study (with Agmon and Douglis) of elliptic boundary value problems with $L^p$ data.

■ *Transformational results in geometric analysis.* Nash's smooth ($C^\infty$) isometric embedding theorem established the equivalence of Riemann's intrinsic point of view with the older extrinsic approach; his nonsmooth ($C^1$) isometric embedding theorem demonstrated the possiblilty of embeddings seemingly forbidden by such geometric invariants as Gauss curvature. Nirenberg's contributions to geometric analysis include his earliest work, in which he solved problems posed by Minkowski and Weyl concerning the realizability of metrics on $S^2$ with positive curvature by convex surfaces in $R^3$; of comparable importance is the Newlander–Nirenberg theorem, a fundamental result in complex differential geometry.

■ *New methods and crucial tools.* Among them in Nash's case are the Nash–DeGiorgi–Moser regularity theory and the Nash–Moser inverse function theorem; for Nirenberg they include the development (with Joseph Kohn) of the theory of pseudodifferential operators. Also noteworthy are crucial inequalities—including the the John–Nirenberg inequality (which laid the foundation for a new chapter in harmonic analysis involving duality between the space of functions of bounded mean oscillation and the Hardy space $H^1$), the Gagliardo–Nirenberg interpolation inequalities (an extremely useful generalization of the Sobolev inequalities), and Nash's inequality (first proved by Stein, important among other reasons for the study of probabilistic semigroups).

■ *And much, much more.* In Nirenberg's case, another very influential theme was development (with Gidas and Ni) of the "method of moving planes," a powerful technique for demonstrating that solutions of certain elliptic problems must share the symmetries of the problem. Another of Nash's striking contributions was the realization of manifolds as algebraic varieties.

"Towering figures" individually in the analysis of PDEs, "Nash and Nirenberg influenced each other through their contributions and interactions," the Abel Prize citation concludes. "The consequences of their fruitful dialogue, which they initiated in the 1950s at the Courant Institute of Mathematical Sciences, are felt more strongly today than ever before."

*Nirenberg was the first (2010) recipient of the Chern Medal, which is awarded every four years at the International Congress of Mathematicians; http://www.icm2010.in/prize-winners-2010/chern-medal-louis-nirenberg. In 1994 Nash was one of three recipients of the Nobel Prize in Economic Sciences for work on the theory of non-cooperative games; http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1994/.*

*For a deeper and more informal perspective on Nash's and Nirenberg's lives and work, the Simons Foundation's Science Lives interviews are a good resource. Nirenberg was interviewed in 2010, and again in 2011, by Jalal Shatah; Charles Fefferman and the late Harold Kuhn interviewed Nash in 2011.*

[*]Along with the polished and well-organized citation and biographies of the laureates, readers who visit www.abelprize.no/ will find information about Norway-wide activities of "Abel Week."

## Computational Surgery

*continued from page 1*

to build mathematically and physically sound phenomenological models that can lead to patient-specific full-scale simulations, starting with data typically collected via medical imaging technologies like CT, MRI, and PET, or by quantitative molecular biological study in the case of leukemia. Our ambition is to provide medical doctors with patient-specific tumor growth models able to estimate, on the basis of previously collected data and within the limits of phenomenological models, the evolution at subsequent times of the disease, and possibly the response to the treatment.

The final goal is to provide numerical tools that will help clinicians answer crucial questions:

- When to start a treatment?
- When to change a treatment?
- When to stop a treatment?

We also intend to incorporate real-time model information in the hope of improving the precision and effectiveness of non-invasive or micro-invasive tumor ablation techniques, such as acoustic hyperthermia, electroporation, and radiofrequency or cryo-ablation.

We specifically focus on the following tumors:

- Lung and liver metastases of distant tumors
- Low-grade and high-grade gliomas, meningiomas
- Renal cell carcinomas

These tumors have been chosen because of existing collaborations between INRIA Bordeaux, the Institut Bergonié, and the university hospital.

Our approach is deterministic and spatial: We solve an inverse problem based on imaging data. The PDE-type models are coupled with a process of data assimilation based on imaging. The patients in our test cases have been followed at Bergonié for lung metastases of thyroid tumors. These patients have slowly evolving, asymptomatic metastatic disease that has been monitored by CT scans. On two thoracic images obtained at successive times, the volume of the tumor under investigation is extracted by segmentation. To test our method, we chose patients without treatment and for whom we had at least three successive scans. We used only the first two scans and compared our results with the third or later scans. An example is shown in Figure 1.

### Interrogating a Multiscale/Multifactorial Model of Breast-Conserving Therapy

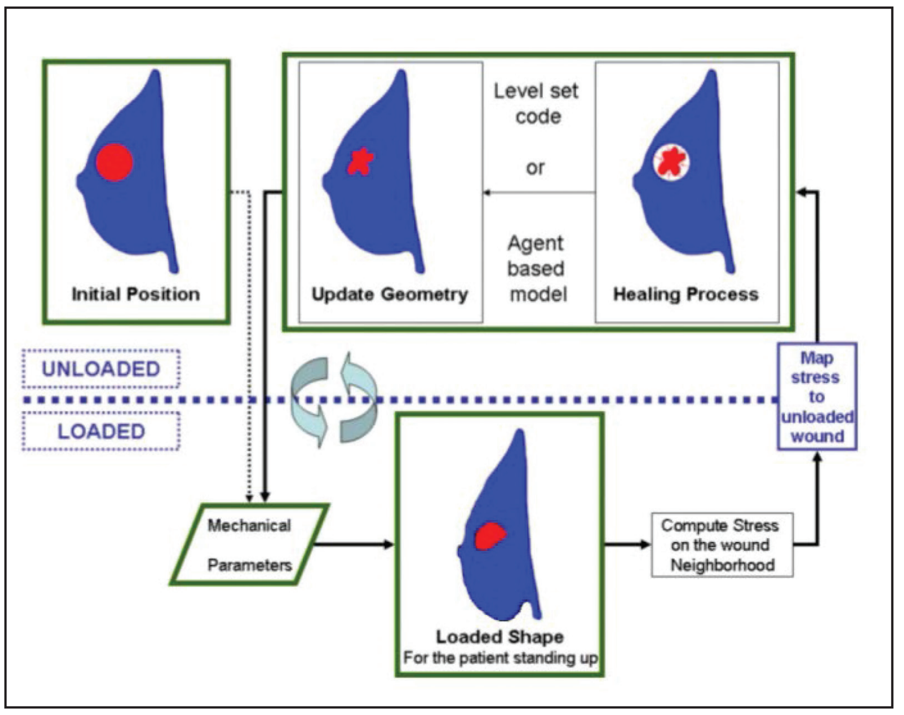


**Figure 2.** *Multiscale/multifactorial model of breast-conserving therapy.*

Treatment of most women with early-stage breast cancer does not require removal of the entire breast; up to 70% of affected women can be effectively and safely treated by breast-conserving therapy (BCT)—surgical removal of the tumor only (lumpectomy), followed by radiation of the remaining breast tissue. Unfortunately, the final contour and cosmesis of the treated breast is suboptimal in approximately 30% of patients.

The ability to accurately predict breast contour after BCT for breast cancer could significantly improve patient decision-making regarding the choice of surgery for breast cancer. Our overall hypothesis is that the complex interplay among mechanical forces—gravity, constitutive laws of breast tissue distribution, inflammation induced by radiotherapy, and internal stress generated by the healing process—plays a dominant role in determining the success or failure of lumpectomy in preserving the breast contour and cosmesis.

As shown in our initial patient study, even in the ideal situation of excellent cosmetic outcome, this problem requires multiscale modeling [4, 5]. We propose a method for deciding which component of the model works best for each phase of healing, and what parameters should be considered dominant and patient-specific. See Figure 2.

We refer interested readers to our book series and journal to provide many more examples of the promises of computational surgery that should apply in virtually every field of surgery.

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*Thierry Colin and Olivier Saut are professors in the Institut de Mathématiques de Bordeaux at the Université de Bordeaux and researchers at INRIA. Marc Garbey is a professor in the department of biology and biochemistry at the University of Houston, director of research at the Methodist Institute for Technology Innovation and Education at Houston Methodist Hospital, and a professor at the Laboratoire des Sciences de l'Ingénieur pour l'Environnement of the Université de La Rochelle, France.*

## Optics & Photonics

*continued from page 1*

are new theoretical studies of the underlying PDE-based models and dynamical systems. In addition, novel computational methods must be developed to handle more sophisticated multiscale and multiphysics models.

Optical interactions with nanoscale structures and materials are a prototypical example. To characterize the quantum effects of the interactions, the Schrödinger equation needs to be incorporated into the model; the Maxwell equations alone are no longer adequate. At the same time, because of the computational complexity of the many-body Schrödinger equation, computational techniques must be devised to render such physical processes across many spatial and temporal scales computationally tractable.

We offer the following sample of scientific problem areas in which applied and computational mathematicians have a central role to play, whether by expanding their "toolbox," by tackling novel applications, or by developing new theory and methods.

**Broadband control of waves and metamaterials.** We have unprecedented ability to fabricate materials with specified microstructures, passive and active, e.g., antennas. For a single frequency, the principles of using photonic media to control waves are fairly well understood. A striking approach to the control of light is the example of *cloaking* through "transformational optics," which is based on the coordinate invariance of Maxwell's equations. However, such control can be achieved only for preselected frequencies or narrow frequency bands. A major challenge is to control light across a wide band of wavelengths. Metamaterials, whose individual microfeatures are passive to some wavelength ranges but respond resonantly to others, are very promising tools for such control.

**Nonlinear optics.** Driven by advances in materials science and nano-fabrication, there is intense interest and activity in optical and electromagnetic wave propagation through nonlinear and inhomogeneous media. Nonlinear waves are known to self-focus and/or filament as a result of, for example, the nonlinear optical Kerr effect, in which the local refractive index depends on the light intensity. *Optical shocks* may occur due to resonances in pulse propagation that are spectrally supported near the zero dispersion point. Can we devise media that suppress, inhibit, or control such singularity formation? What are the novel coherent structures that result from the combination of nonlinearity and microstructure?

Spatial discreteness plays an important role in many linear and nonlinear optical systems; examples include coupled waveguide arrays or lattices, and tight binding in the limits of continuum problems. These models are important to our understanding of energy transport properties for device design and optimization. A new generation of materials and media with novel microstructures are making nonlinear effects accessible to study at lower intensities. The potential for the development of new technologies is huge. Finally, a foundational question concerns the effect of randomness of nonlinear wave propagation—for example, how does the *Anderson localization* paradigm change in the presence of nonlinearity?

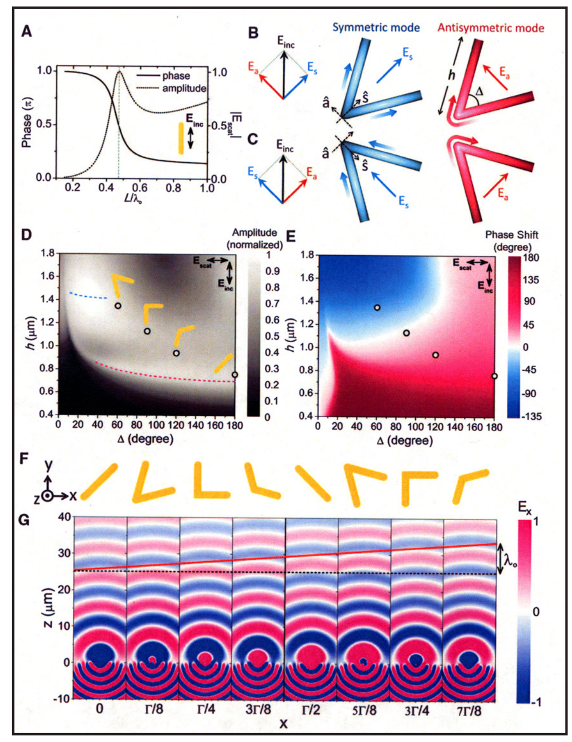
**Optimal design, inverse and control problems.** Mathematical optimization has long been a fundamental tool for the design of optical systems. Traditionally, ray tracing and asymptotic diffraction models have provided the basis for optimization and design. More recently, as optics and photonics research has focused on phenomena at smaller and smaller length scales, full PDE models have become necessary to adequately capture system behavior. In this realm, device behavior is less intuitive and computational simulations become indispensable. Modern optimal design and optimal control techniques can yield novel designs faster and more efficiently than direct experimentation. Methods of optimization, control, and inverse problems are directed at an extremely diverse range of optical applications, from astronomy to nanoscale optical materials and nanostructures.

**Near-field thermal radiation.** Near-field thermal radiation plays a significant role in a variety of emerging applications, including energy conversion, imaging, and precision heating and cooling. The theory for near-field thermal radiation is well described within the framework of fluctuational electrodynamics, which was established half a century ago. Only recently, however, has computational implementation of fluctuational electrodynamics become possible for non-planar geometries. Meanwhile, the rapid development of nanophotonic and metamaterials has introduced a number of new concepts to the area of near-field thermal transfer.

**Nano–optics and photonics.** Nano-optics refers to the study of optical phenomena at length scales larger than atomic or molecular, yet smaller than a micron. Exciting opportunities arise from interactions of light and matter at such length scales for the development of nanoscale devices with applications to electronics, optoelectronics, biology, and medicine. The goal is to address systems in which different electromagnetic entities interact—e.g., waveguide modes with surface charges, photons with excitons, plasmons with plane waves. Possible applications include single-molecule spectroscopy, novel microlasers and optical switches, plasmonic materials, optical nanocircuits, Raman scattering, and novel nonlinear photodetectors.

The proper theoretical and computational description of the interaction of optical fields with nanostructures presents significant challenges because of coupled effects over a vast range of scales, giving rise to phenomena not encountered in conventional optics. Important directions include scattering problems for highly localized wavefields, and the roles of plasmonic materials, nonlinearity, anomalous diffraction, and dissipation in photonic phenomena.

**Multiscale problems in optics and electromagnetics.** Micro- and nano-structured optical media have huge potential for application to communication and computational devices. Through variations in the materials and their microstructure, it is possible to manipulate a variety of optical properties: dispersion, nonlinear optical effects, resonances through material composition, material contrast, defect geometry, and defect distribution. Common to these problems is the interaction of fields at different spatial and frequency scales in the propagation of light. Novel geometries, broadband frequency ranges, and physically realistic dispersion relations pose important questions




*(A) Calculated phase and amplitude of scattered light from a straight rod antenna made of a perfect electric conductor. (B) A V-antenna supports symmetric and antisymmetric modes, which are excited, respectively, by components of the incident field along ŝ and â axes. (C) V-antennas corresponding to mirror images of those in (B). (D and E) Analytically calculated amplitude and phase shift of the scattered light from the V-antennas. (F) V-antenna array that introduces a constant phase gradient along the array. Optical "metasurfaces" constructed by tiling a surface using the antenna array allow demonstration of the generalized laws of reflection and refraction (i.e., arbitrary bending of light beams as they reflect from or pass through the metasurface). (G) Full-wave simulations of the scattered electric field for the individual antennas composing the array in (F), showing that a wave that impinges on the array in the normal direction is deflected into a new direction. From N. Yu et al.,* Light propagation with phase discontinuities: Generalized laws of reflection and refraction, *Science, 334:6054 (2011), doi: 10.1126/science.1210713. Reprinted with permission from AAAS.*

*See* **Optics & Photonics** *on page 8*

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# Optics & Photonics

*continued from page 7*

for fundamental and applied mathematics.

Classical homogenization theory yields effective properties for the regime of wavelengths much larger than the scale of spatial variations in the microfeatures of the material; the theory of geometric optics/semi-classical analysis, by contrast, is valid in the regime of very short wavelengths, on the scale of the medium's spatial variations. Many optical and electromagnetic media with novel optical properties fall in the regime in which scales interact. Furthermore, these optical properties are not necessarily valid for a wide frequency spectrum. Central research themes are the relation between time domain equations (which have all frequencies) and the equations that govern single-frequency waves (the Helmholtz and Maxwell equations), and bridging the considerable gap between classical homogenization theory and geometric optics.

**Integration of optics in communications and computation.** Many performance-limiting issues in communication networks are related to optical–electrical interfaces. An important technological goal is to develop approaches that seamlessly integrate the two, for example, using recent innovations in silicon nanophotonics. All-optical interconnects will play a central role. Model-based control and simulation techniques need to be developed to optimize the performance.

## Initiatives in Optics and Photonics

**Government Initiatives.** The growing importance of optics and photonics has been highlighted in a series of reports from the U.S. National Academy of Sciences [2, 3, 4].

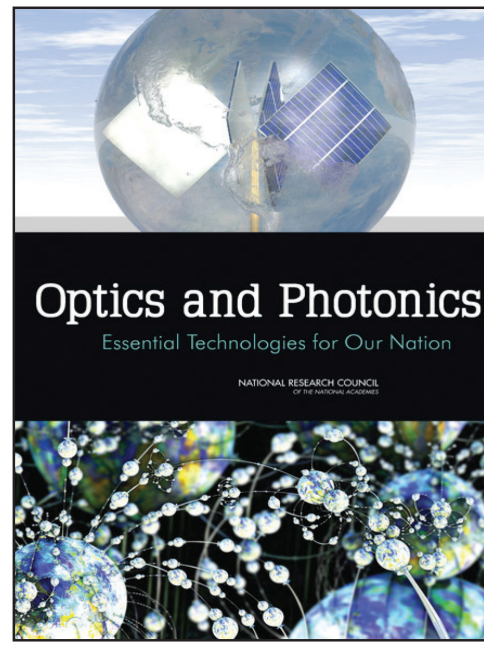
Several initiatives of the National Science Foundation, now under way, provide a range of opportunities for the mathematical sciences community. Most recently, NSF announced a broad new Optics and Photonics initiative that crosses three directorates: Mathematical and Physical Sciences (with the Division of Mathematical Sciences taking the lead); Engineering; and Computer and Information Science and Engineering [5, 6]. NSF will release a program solicitation in fiscal year 2016.

**IMA Annual Program on Mathematics and Optics.** During the 2016–17 academic year, the Institute for Mathematics and its Applications at the University of Minnesota will hold the year-long thematic Program in Mathematics and Optics [1]. The program will address the very wide range of questions that arise in the study of optical phenomena and the relevant areas of applied and computational mathematics. The goal is to connect mathematical and computational scientists working on deep problems of optics and photonics with the dynamic interdisciplinary community in the field. The program will also seek to prepare the next generation of applied and computational mathematicians for collaborations with fundamental and applied scientists and engineers.

The IMA Program in Mathematics and Optics will be built around multiple week-long tutorials and workshops, and will have a visitors program. Broad areas slated for coverage include the interaction of light with novel photonic micro- and nano-structures and resonant metamaterials, optimal design and control, and nonlinear phenomena. An overarching theme of the workshops is the role of classical and developing ideas in multiscale analysis and computation in optics and photonics.

Among the topics of proposed workshops are:

- Control of waves in resonant materials
- Optimal design and control in optics and photonics
- Nonlinear optics and photonics
- Multiscale problems in micro- and nano-structures
- Photonic band-gap structures and "topological photonics"
- Near-field thermal radiation
- Multiscale computational methods in optics and photonics
- Imaging and sensing




## Conclusion

The search for an understanding of optical phenomena and strategies for their control has been a catalyst of pure and applied mathematics in many areas: dynamical systems, ordinary and partial differential equations, harmonic analysis, calculus of variations, geometry, and functional analysis. Conversely, developments in the mathematical and computational sciences have provided foundational tools for the discovery of new phenomena, and for the development of deeper physical theories and their application.

Optical phenomena span vast temporal and spatial scales: macroscales, governed by Maxwell's equations with macroscopic constitutive relations; intermediate (mesoscopic) wavelength-scales, e.g., where wavelength and micro-feature dimensions are of the same order; and nano- and atomistic scales, requiring quantum descriptions governed by the Schrödinger equation. Different mathematical concepts and techniques have long played a role in the analysis of each of these regimes—for example, homogenization at the macroscale, coupled-mode theory at the mesoscale (e.g., Bragg resonance), semi-classical analysis in the quantum regime. Many challenges, applied and fundamental, arise from the coupling of these regimes, and modern applied and computational mathematics has a very important role to play.

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