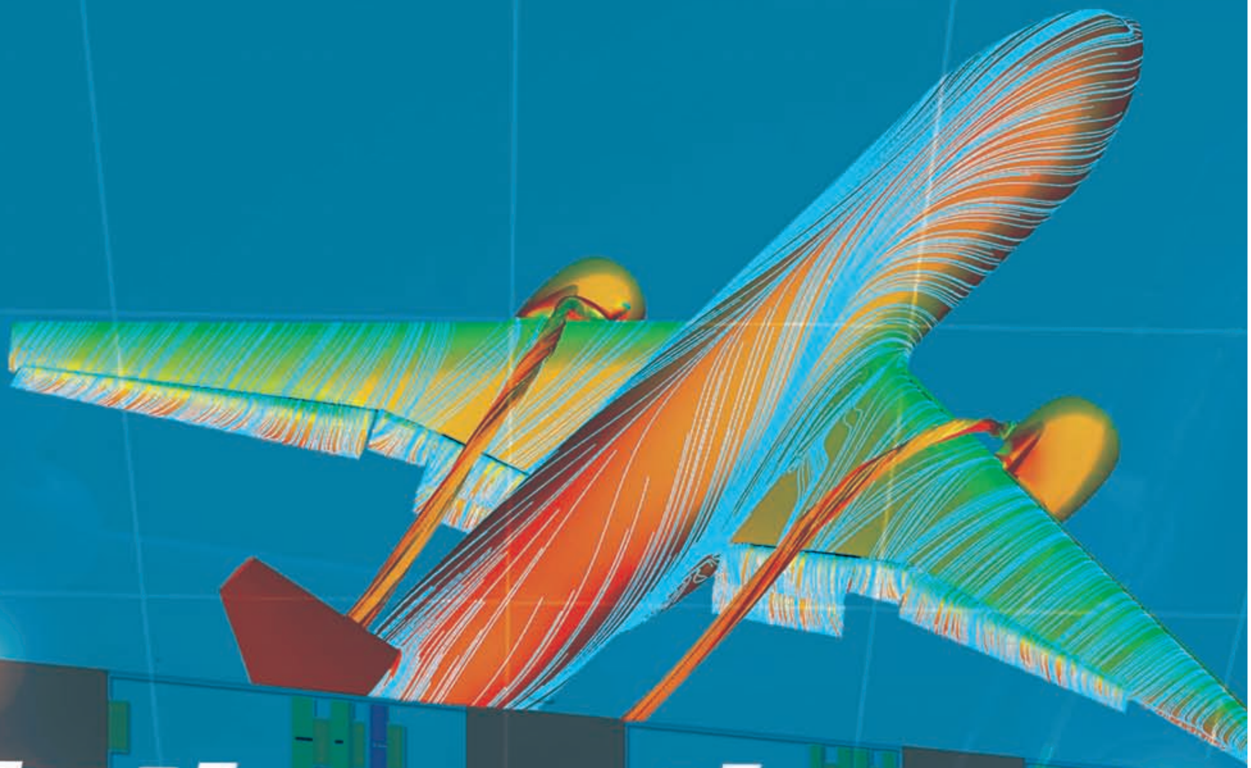

Mathematics in Industry

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The cover image illustrates the results of a fluid flow calculation over an airplane. For this design, there is little flow separation occurring on the wing except near the wingtips and near the side of the body. In addition, the flowfield streamlines from the nacelle up over the wing show vortex shedding from “chines” which are structures mounted on the nacelles that are specifically designed and optimized to shed this vortex. Without them, installing the nacelle forward of the wing as in this design would compromise both the efficiency of the wing as well as its maximum lift capability. See [Konigs 2005]

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Executive Summary

...the US economy is moving from one that is based on manu-facturing to one that is based on services. This creates employment opportunities for mathematicians in businesses that provide consulting services in the realms of business operations, science, and engineering.

In 1996, the Society for Industrial and Applied Mathematics (SIAM) published the *SIAM Report on Mathematics in Industry*, which was based on a study supported by grants from the National Science Foundation and the National Security Administration. That report, and a set of NSF-funded regional workshops that followed it, helped raise the awareness of mathematicians in academia about the role of mathematics in industry. The study was widely cited and used to motivate curricula and programs that focused on industrial and governmental problems.

Since 1996 there have been many changes in the types and scale of challenges that industry and government are facing. For example, the deciphering of the human genome and the availability of molecular dynamics simulations are beginning to transform the pharmaceutical industry. These changes create new opportunities for graduates with backgrounds in statistics, data mining and simulation. The financial sector has experienced tremendous growth since 1996 as an employer of mathematicians. Even though the credit crisis of 2007–8 brought "quants" into some disrepute, companies are still eager to hire graduates who have true insight into both mathematics and finance. Also, the US economy is moving from one that is based on manufacturing to one that is based on services. This creates employment opportunities for mathematicians in businesses that provide consulting services in the realms of business operations, science, and engineering.

In view of these changes, we felt that it was time to update the 1996 report and look at the way mathematical sciences are used in industry today. We also wanted to hear about the experiences of recent PhD graduates who have chosen to pursue industrial careers. Accordingly, we conducted focus group meetings with industrial scientists, an online survey of recent doctoral degree recipients, and onsite interviews with 56 senior scientists and managers from 23 corporations. In total, we have interviewed or surveyed 145 mathematical and computational scientists from 14 major industrial sectors.

Our most important conclusion is that the mathematical and computational sciences continue to find many applications, both traditional and novel, in industry. Some of these applications have very dramatic effects on the bottom line of their companies, often in the tens of millions of dollars. Other applications may not have an easily measured impact on the bottom line but simply allow the company to conduct business in a 21st-century data-rich marketplace. Finally, some applications have great value as contributions to science. We want to emphasize that technology transfer, including the transfer of mathematical ideas, is not a one-way street; a technology designed for or by one company often ends up enriching science as a whole.

The centerpiece of this report is a set of case studies from a variety of applications, including business analytics and optimization, manufacturing design and virtual prototyping, quantitative drug design, financial risk analysis, production planning and supply chain management, and information retrieval and data mining. We intend these case studies to be inspiring and informative for a wide range of readers: from students wanting to know "What is mathematics used for, anyway?" to academic departments seeking to understand how to prepare students for non-academic jobs, to mathematicians in industry who would like to explain the value of mathematical methods to their managers. Often we find that mathematicians in industry do not feel respected by their colleagues in academia; we hope that the impressive range of applications discussed here will convince academic mathematicians that industrial problems can be difficult, substantive and fascinating.

Mathematical scientists in industry work in a highly interdisciplinary team environment. Their contributions tend to be attributed to the dominant discipline of the team. In 1996 we wrote, “Mathematics is alive and well, but living under different names.” This comment is still apropos. Unfortunately, it means that a new mathematical approach to an industrial problem may be difficult to sell to higher management, which might not be prepared to appreciate it. One mathematical scientist and manager whom we interviewed recalled the reaction of a senior management to a suggestion for improving production-line efficiency. The senior manager responded: “These are the five things I am evaluated on; your idea does not help me in any of them.” Because of such attitudes, mathematicians in industry need to develop communication and entrepreneurial skills that may not be as critical in academia. It is not enough to have a good idea; they need to sell it in a language management will understand.

This report explores the implications of this interdisciplinary environment for the skills and traits considered essential by employers in industry and government. Our interviewees emphasized communication skills, the ability to work effectively in a team, enthusiasm, self-direction, the ability to complete projects, and a sense of the business.

In this study we also took a closer look at the technical skills that graduates need, which tend to fall into three overlapping domains: mathematics, computation, and specific application domains. Useful mathematical skills include a broad training in the core of mathematics, statistics, mathematical modeling, and numerical simulation, as well as depth in an appropriate specialty. Computational skills include, at a minimum, experience in programming in one or more languages. Specific requirements, such as C++, a fourth-generation language such as MATLAB, or a scripting language such as Python, vary a great deal from company to company and industry to industry. Familiarity with high-performance computing (e.g. parallel computing, large-scale data mining, and visualization) is becoming more and more of an asset, and in some jobs is a requirement. The choice of an application domain to focus on depends strongly on a graduate’s career goals and the requirements of his or her potential employer. In general, a student’s level of knowledge has to be sufficient to understand the language of that domain and bridge the gap between theory and practical implementation.

We conclude the report with a range of suggestions and strategies for enhancing the graduate curriculum and creating mechanisms for connecting academic, government, and industrial scientists. Some suggestions are intended for students; for the most part they are not new, and yet some of them are still ignored surprisingly often. For example, we highlight the great importance of industrial internships or direct work with a mentor from industry during the graduate years. Other recommendations are directed at academic departments and their industrial or governmental partners. Some are straightforward and easily implemented on a local scale, while others involve collaboration on an institutional, national, or global scale.

…mathematicians in industry need to develop communication and entrepreneurial skills that may not be as critical in academia. It is not enough to have a good idea; they need to sell it in a language that management will understand.

1 Introduction

In 1996, manufacturing accounted for 15.4% of US GDP, while the combination of finance, insurance, and scientific and technical services jointly contributed 12.5%. By 2010, the order had reversed, with manufacturing accounting for 11.7 % and finance, insurance, and scientific and technical services accounting for 15.9%.

Industrial mathematics is a specialty with a curious case of double invisibility. In the academic world, it is invisible because so few academic mathematicians actively engage in work on industrial problems. Research in industrial mathematics may not find its way into standard research journals, often because the companies where it is conducted do not want it to. (Some companies encourage publication and others do not; policies vary widely.) And advisors of graduates who go into industry may not keep track of them as closely as they keep track of their students who stay in academia.

In the business world, industrial mathematics is invisible because it is often not called "mathematics." It is called "analytics," "modeling," or simply generic "research." Credit for mathematical advances may go to "information technology" when it should really go to the people who use the technology and figure out how to employ it effectively.

Of course, for members of the Society for Industrial and Applied Mathematics (SIAM), it will be no revelation to read that mathematics can make a huge difference for private enterprises and, through them, for society as a whole. But we hope that this report will find its way to people who are not members of SIAM, and who perhaps have not been exposed to the kind of work that mathematicians in industry do. In particular we hope that this document will be useful, in different ways, to students who want to learn about industrial careers; academic mathematicians who advise and teach those students; university administrators who want to encourage partnerships with industry; and business managers who want to find out how mathematics can benefit their companies.

For readers who want to know what industrial mathematics is, we recommend skipping the rest of this introduction and going straight to part two, where you will learn about eight general areas of application of the mathematical sciences (broadly defined to include computing and statistics) to industry. You will also find 18 case studies that illustrate the excitement, vitality, and importance of mathematics in industry.

1.1 The SIAM 2011 Mathematics in Industry Study: Background

In 1996, SIAM published the first *SIAM Report on Mathematics in Industry* [MII 1996]. This report, and a set of regional workshops that followed its release, helped to clarify the perception in academia of the role of mathematics and mathematicians in industry. It provided an overview of employment opportunities for graduate students, and mathematics departments still refer to it in their career information for students. It has also been cited in subsequent reports by the Smith Institute, the Organization for Economic Development, and the European Science Foundation on the subject of mathematics in industry, (See [Smith 2004], [OECD 2008], and [ESF 2010].) Anecdotally, the report has been used to motivate the need for (and predict the success of) courses and programs in industrial mathematics and computational science.

Many of the recommendations and insights in the 1996 report remain valid today. However, the landscape of mathematical and computer sciences in industry has changed. Organizations now collect orders of magnitude more data than they used to, and face the challenge of extracting useful information from it. Computing technology has continued to advance rapidly, and companies are making more and more aggressive use of high-performance parallel computing.

Another important trend is the transition of the US economy from one led by the manufacturing sector to one in which services are more important. In 1996, manufacturing accounted for 15.4% of US GDP, while the combination of finance, insurance, and scientific and technical services jointly contributed 12.5%. By 2010, the order had reversed, with manufacturing accounting for 11.7% and finance, insurance, and professional and technical services accounting for 15.9% [BEA 2011].

Since 1996 the US government and private foundations have funded several programs that have addressed the sharing of knowledge among scientists in academia, government, and industry. For example, the National Science Foundation established its GOALI (Grant Opportunities for Academic Liaison with Industry) program. The US Department of Energy expanded its Computational Science Graduate Fellowship (CSGF) program and Scientific Discovery through Advanced Computing (SciDAC) program. It also launched the Innovative and Novel Computational Impact on Theory and Experiment (INCITE) program, which enables businesses to obtain access to supercomputers and, just as importantly, access to expertise in using them. The Sloan Foundation funded efforts to develop a Professional Science Master's (PSM) degree, including degrees in mathematical and computational sciences. Several universities and colleges began building centers and programs in mathematics and computational science with a real-world focus.

Finally, the business press has discovered the importance of mathematics, statistics, and computer science to innovation. See, for example, [Baker 2006], [Baker 2008], [Lohr 2009], [Baldwin 2010], [Cohen, N. 2010], [Cohen, P. 2010] and [Hardy 2010]. During the recent mortgage credit crisis, credit swap models and quantitative models in general received criticism from inside and outside the business press ([Patterson 2010], [Taleb 2007], and [Triana 2007]). Nevertheless, this does not seem to have dimmed the enthusiasm within the business press. Corporate management and their shareholders read these articles and books, and we expect that this will create an environment in which they will be receptive to the potential value of mathematical and computer sciences.

In view of all of these changes since 1996, SIAM felt that it was time to update the old report to reflect the new business and economic environment and the new opportunities available. We are also taking the opportunity to include something that was not provided in the original report, a suite of detailed case studies illustrating the variety of ways in which mathematics is being used in industry today.

1.2 Scope and Methodology

We began by conducting five small focus groups of industrial scientists. In total, the session participants included 21 mathematical and computational scientists from nine industrial sectors. The goal was to get a broad overview of the current state of mathematics in industry and to inform our questions for the next two phases of the project. We then developed, tested, and administered an online survey of recent PhDs in mathematics and statistics who took jobs in industry. Our questions probed their backgrounds, their group's tasks, and to what extent their degrees are utilized on the job. We also requested their suggestions for current students. There were 550 PhD graduates from June 2004 through July 2007 who took jobs in industry and for whom we could identify an employer. Of these we were able to find 200 valid e-mail addresses. The survey received a typical online survey response rate of 30 percent.

Finally, and separately from the online survey, we conducted in-depth interviews with 56 senior mathematical or computational scientists (of which 21 were senior managers) in industry from 23 corporations. In total, through the focus groups, the online survey, and the in-depth interviews, we solicited the opinions of 145 industrial mathematicians from 14 industrial sectors.

There are four significant changes to the scope of this report as compared to the 1996 report. First, we included PhD graduates from statistics departments in our survey. The previous study included such graduates only if they received their degrees from a joint mathematics and statistics department. Second, we did not survey the immediate supervisors of the graduates, as we did in 1996. However, in our in-depth interviews and site visits, we interviewed senior-level managers and addressed the same questions as we did in 1996. Third, our interviews and site visits did not deal exclusively with mathematicians and statisticians, but included all mathematical and computational scientists no matter what department they received their PhD from. Finally, we did not interview or survey Master's graduates for this report. With funding from the Sloan Foundation, SIAM undertook a survey of applied mathematics Master's programs in 2002, [Crowley and Seitelman 2003], and we have not felt it necessary to repeat that effort here.

In this report, however, we do discuss the rise and development of the Professional Science Masters (PSM) degree. SIAM was involved in the early stages of the PSM movement. The SIAM Education Committee produced guidelines for a professional Master's degree in applied and industrial mathematics, ["SIAM Guidelines" 1998], using the 1996 Mathematics in Industry report [MII 1996] as a reference.

Finally, we will mention that many of the case studies in section 2 of this report were suggested by or grew out of the onsite interviews. In all cases, the interview information has been supplemented by publicly available information from published articles and company press releases. We include six (out of 18) case studies from companies with which we did not have direct contact. Those case studies are based entirely on published articles and company press releases.

2 Role of Mathematics

Trends and Case Studies

In this section of the report, we give a broad but not exhaustive survey of the business applications of mathematics. We hope that the 18 case studies presented below will provide some answers to students who want to know what mathematics is used for “in the real world.” Most of the case studies are applications we heard about on our site visits, supplemented by published articles.

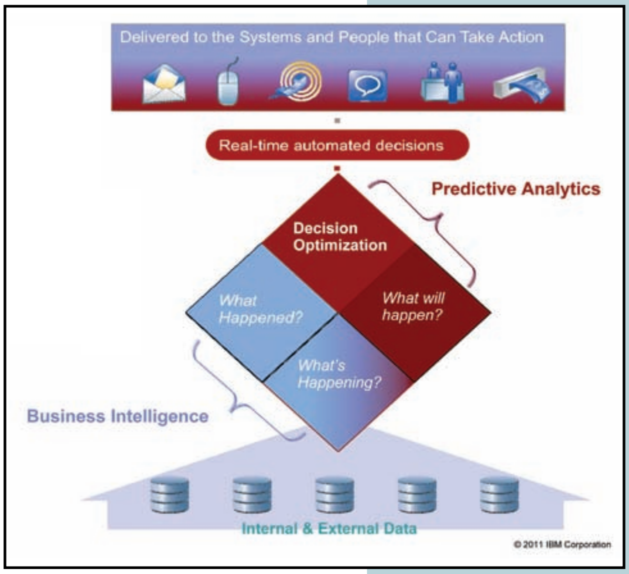
2.1 Business Analytics

The software industry is making a big bet that data-driven decision making…is the wave of the future. The drive to help companies find meaningful patterns in the data that engulfs them has created a fast-growing industry in what is known as “business intelligence” or “analytics” software and services. Major technology companies—IBM, Oracle, SAP, and Microsoft—have collectively spent more than $25 billion buying up specialist companies in the field.
[Lohr, 2011-a]

“Business analytics” has become a new catchall phrase that includes well-established fields of applied mathematics such as operations research and management science. At the same time, however, the term also has a flavor of something new: the application of the immense databases that are becoming more and more readily available to business executives.

Mathematical approaches to logistics, warehousing, and facility location have been practiced at least since the 1950s. Early results in optimization by George Dantzig, William Karush, Harold Kuhn, and Albert Tucker were encouraged and utilized by the US Air Force and the US Office of Naval Research for their logistics programs. These optimization techniques, such as linear programming and its variations, are still highly relevant to industry today.

The new opportunity, both for businesses and for students hoping to enter industry, lies in the development of algorithms and techniques to handle large amounts of structured and unstructured data at low cost. Corporations are adopting business intelligence (i.e., data) and analytics (i.e., quantitative methods) across the enterprise, including such areas as marketing, human resources, finance, supply chain management, facility location, risk management, and product and process design.



Automating decisions in business processes, IBM Corporation

Case Study 1: Predictive Analytics

In 2009 and 2010, IBM helped the New York State Division of Taxation and Finance (DTF) install a new predictive analytics system, modeled in part on IBM’s successful chess-playing program Deep Blue and its Jeopardy!-playing engine, Watson. The Tax Collection Optimization Solution (TACOS) collects a variety of data, including actions by the tax bureau (e.g., phone calls, visits, warrants, levies, and seizure of assets) and taxpayer responses to the actions (e.g., payments, filing protests, and declaration of bankruptcy). The actions may be subject to certain constraints, such as limitations on the manpower or departmental budget for a calling center. The model also includes dependencies between the actions. TACOS predicts the outcome of various collection strategies, such as the timing of phone calls and visits. The mathematical technique used is called a Markov decision process, which associates to each taxpayer a current state and predicts the likely reward for a given action, given the taxpayer's state. The output is a plan or strategy that maximizes the department's expected return not just from an individual taxpayer, but from the entire taxpaying population.

In 2009 and 2010, TACOS enabled the DTF to increase its revenue by $83 million (an 8% increase) with no increase in expenses. The results included a 22% increase in the dollars collected per warrant (or tax lien), an 11% increase in dollars collected per levy (or garnishment), and a 9.3% reduction in the time it took cases to be assigned to a field office [Apte, 2011]. Similar methods, though different in detail, could clearly be applied by other businesses in areas like collections and accounts receivable.

Case Study 2: Image Analysis and Data Mining

SAIC is a company that develops intelligence, surveillance, and reconnaissance (ISR) systems for military applications. These automated systems have been heavily exploited during the war in Afghanistan: in 2009, unmanned aerial vehicles (UAV) captured 24 years of full-motion video, and in 2011 they were expected to capture thirty times that amount.

This poses an obvious problem: How can the information in the videos be organized in a useful way? Clearly the army cannot deploy thousands of soldiers in front of computer screens to watch all of those years of video. Even if they could, humans are fallible and easily fatigued. In hours of surveillance video it is easy to miss the one moment when something isn't right—say, a car that has previously been associated with bomb deliveries drives up to a particular house.

SAIC developed a "metadata" system called AIMES that is designed to alert humans to the possible needles in the haystack of data. First, AIMES processes the video to compensate for the motion of the UAV—itself an interesting mathematical challenge. Then it searches for objects in the field of vision and stores them in a searchable database. It also "fuses" other kinds of data with the video data—for example, if the operators of the UAV say, "Zoom in on that truck!" the program knows that the object in the field of view is a truck and it may be of interest. Finally, AIMES is portable enough to be deployed in the field; it requires only a server and two or three monitors. See ["SAIC AIMES" 2010].

While stateside industry may not have quite as many concerns about terrorists or roadside bombs, audio and video surveillance are very important for the security of factories or other buildings. Cameras and microphones can be used for other purposes as well; for example, a microphone might be able to tell when a machine isn't working right before human operators can. Surveillance devices can also help first responders locate victims of a fire or an accident. See ["SAIC Superhero Hearing" 2010].

Case Study 3: Operations Research

In 2002, Virginia Concrete, the seventh-largest concrete company in the nation, began using optimization software to schedule deliveries for its drivers. The company owns 120 trucks, which had been assigned to 10 concrete plants. A significant constraint is that a cement truck has roughly two hours to deliver its load before it starts hardening inside the truck. Also, the construction business is very unpredictable; typically, 95 percent of a company's orders will be changed in the course of a day.

Virginia Concrete brought in mathematicians from George Mason University and Decisive Analytics Corporation to develop tools to automate truck dispatching. Among other changes, the mathematicians found that the company could improve delivery times significantly by moving away from the model in which individual trucks were assigned to a “home” plant. Instead, they recommended that trucks should be able to go to whichever plant is closest. Also, in overnight planning it turned out to be useful to include “phantom” trucks, representing orders that were likely to be canceled. If the order was not canceled, it could be reassigned to a real truck.

For testing purposes, the company used the software to make all of the scheduling decisions; however, since the system’s installation, dispatchers have been allowed to override the computer. The system has enabled Virginia Concrete to increase the amount of concrete delivered per driver by 26%. [Cipra 2004]

2.2 Mathematical Finance

…there is likely to be less emphasis on exotic derivatives and more trading will take place on exchanges.

In the future, models will have to have realistic dynamics, consistent with observation. Control of execution costs will also be critical, and for that, a good understanding of market microstructure and trade data will be essential. [From interviews.]

Quantitative methods in finance got a black eye from the credit crisis of 2007 and 2008, which in many circles was interpreted as a failure of quantitative models to account for dependencies in market data. Risk models assumed that real estate defaults in, say, Miami and Las Vegas were independent of one another; or at least that the correlations were small. But in a panic situation, all of the correlations went to one.

However, in the fallout of the crisis and subsequent recession, financial managers learned some very worthwhile lessons. They have learned that mathematical models are not just plug-and-play; you have to seriously examine the assumptions behind them. The failure of certain simplistic models does not mean all mathematical models are bad; it means that the models have to become more realistic. Above all, it is important for students to realize that the financial industry is not fleeing from quantitative analysis. Mathematicians and applied mathematicians are still in great demand; their skills will become even more valued as quantitative models become more sophisticated and as managers try to understand their limitations. It may be the case, though, that students’ mathematical skills should be backed up by a greater knowledge of the financial industry than was needed in the past.

Case Study 4: Algorithmic Trading

In 2009, Christian Hauff and Robert Almgren left Bank of America, the world’s top firm in algorithmic trading of stocks and derivatives, to form a new company called Quantitative Brokers. They saw an opportunity to apply the same principles of high-frequency trading to a class of assets that had not yet become highly automated: interest-rate futures.

Automated trading has become commonplace in the options market, in part because the tools of mathematical finance require it. Large banks want to hold their assets in a risk-neutral way, which allows

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Quantitative Brokers' STROBE algorithm finds the trajectory that optimizes the client's utility function, and it generates an envelope around the optimum that summarizes the range of acceptable deviations.

them to make money (or at least avoid losing money) no matter which direction the market moves. In the early 1970s, Fischer Black and Myron Scholes discovered how to do this with a strategy called dynamic hedging, which requires constant small trades.

The main emphasis of Black and Scholes' work was the pricing of options. It took another two decades for financial engineers to start taking into account the process of execution of a trade. There are many reasons for not executing a trade all at once. You may want to wait until trading partners, who are willing to give you a good price, arrive, or until the market moves toward your target price. If your trade represents a significant fraction of the market for an asset in a given day, you may want to move slowly to avoid unduly influencing the market price.

Trade execution is Quantitative Brokers' main business. The company uses computer algorithms to plan a strategy for a path that leads from a client's position at the beginning of the day to the desired position (say, buying X lots of Eurodollar futures at a price less than Y) at the end of the day. Each client has a certain degree of risk aversion, so the client's utility function will be a linear combination of expected profit and expected risk. Quantitative Brokers' STROBE algorithm finds the trajectory that optimizes the client's utility function, and it generates an envelope around the optimum that summarizes the range of acceptable deviations. Mathematical tools include differential equations and the calculus of variations. See ["Anatomy of an Algo" 2011.]

2.3 Systems Biology

Pharmaceutical researchers have undertaken many initiatives and technologies to stem the rising costs of drug discovery and development. Biomarkers, adaptive trial designs, modeling, trial simulations, predictive metabolism, data mining, and disease models have reshaped the way in which researchers approach discovery and development. Quantitative pharmacology, which leverages model-based approaches, operates at both cultural and technical levels to integrate data and scientific disciplines, … [Allerheiligen 2010]

The completion of the Human Genome Project in 2000 was supposed to usher in a new era of individualized medicine and targeted drug discovery. However, it turned out that only a few uncommon diseases or disease variants result directly from individual mutations in the human genome. Most common disorders—such as diabetes and the number one target of drug research, cancer—arise from the malfunctioning of complicated networks of genes. The idea of treating such diseases by fixing one gene is beginning to look as naïve as the idea of fixing an engine by replacing one screw. Instead, doctors may need a whole sequence of interventions, in targeted amounts, at particular times and in particular places in the gene network. As the complexity of gene networks becomes more apparent, mathematical methods for their analysis will become more important.

Some of the focus of research in biotechnology has shifted away from genomics to other "omics," such as proteomics, which seeks to understand the shape and folding of proteins that might become targets for drugs. Molecular dynamics simulations start at the most fundamental level, using the principles of quantum mechanics. Recent advances in algorithms, software, and hardware have made it possible to simulate molecules containing tens of thousands of atoms for up to a millisecond—the time scale at which many important biological processes happen.

Other mathematical models go in the opposite direction and operate on the level of the whole organism. These models are used, for example, to predict how a population of patients—each one with his or her unique physiology—might respond to a proposed public health intervention.

Eventually, whole-patient models may become integrated with genomic data to make truly individualized medicine possible.

The mathematical and computational techniques behind these models include network science, deterministic and stochastic differential equations, Bayesian networks and hidden Markov models, optimization, statistics, control, simulation, and uncertainty quantification.

Case Study 5: Molecular Dynamics

In 2001, David Shaw, a computer scientist who had previously been the CEO of a hedge fund that used computer-based trading strategies, launched a new private research laboratory that would be devoted to the problem of protein folding. Shaw's lab developed a supercomputer named Anton with 512 chips that were custom-built to accelerate the computation of atomic interactions. Even with such powerful hardware, though, a brute-force simulation of a protein molecule is not feasible in any reasonable amount of time. The other key ingredient is the Shaw lab's molecular dynamics software, called Desmond, which uses judicious approximations to simplify the calculation of the force fields and also employs novel parallel algorithms that reduce the amount of communication required between Anton's processors.

There was no guarantee at the outset that Anton would work better than other approaches, such as algorithms that divide the calculation up and parcel it out to many different computers. (This was the approach taken by Stanford University's Folding @ Home project.) However, D. E. Shaw Research announced in 2010 that it had simulated the folding and unfolding of a protein called FiP35, which contains 13,564 atoms, over a period of 100 microseconds. This was a tenfold increase over the length of time simulated by the best previous programs. The simulation took about three weeks to run.

The investigators chose FiP35 because its folded and unfolded structures were well understood experimentally. Even so, the simulation produced new scientific insight, by showing that the pathway from the folded to unfolded states was essentially the same each time. *Science* magazine named Shaw's simulation one of the top ten breakthroughs of the year across all fields of science. In the future, such simulations may make it possible to study drug-protein interactions that occur too rapidly to be studied in a traditional laboratory. See [D'Azevedo 2008].

Recent advances in algorithms, software, and hardware have made it possible to simulate molecules containing tens of thousands of atoms for up to a millisecond—the time scale at which many important biological processes happen.

Case Study 6: Whole-Patient Models

By 2020, virtual cells, organs, and animals will be widely employed in pharmaceutical research.
– PricewaterhouseCoopers, Pharma 2020: Virtual R&D

Two San Francisco Bay area companies, Entelos and Archimedes, Inc., are pioneering the field of computer modeling of the whole body. Although neither is close to a comprehensive simulation of human biology, both of them do model major subsystems, such as the cardiovascular system and the metabolic networks involved in diabetes.

Entelos' model, called PhysioLab, and the Archimedes Model can be used to predict adverse reactions as well as the outcome of clinical drug trials. Clearly, drug companies could save a great deal of time and money by screening out ineffective or harmful compounds before going to the expense of setting up a clinical trial. In addition, simulations can explore the effects of multiple-drug therapy, which is very difficult to do in clinical trials. While 20 different combinations of drugs might require 20 different clinical trials, a simulation can quickly hone in on the one combination that is most likely to be effective.

For example, Archimedes, Inc. was asked by a client (a large HMO) to evaluate the effectiveness of a new preventative treatment regimen called A-L-L (aspirin, lovastatin, and lisonopril) for patients with diabetes or heart disease. The Archimedes Model predicted that the combination therapy should reduce heart attacks and strokes in the target population by 71%. A subsequent clinical study confirmed the model, finding about a 60% reduction. The HMO subsequently recommended to all of its participating doctors that they prescribe the new regimen to their patients who matched the criteria for treatment.

The fields of mathematics and computer science used by Entelos and Archimedes include nonlinear dynamics, control theory, differential equations, and object-oriented programming.

See [“Virtual Patients 2010] and [“The Archimedes Model” 2010].

The fields of mathematics and computer science used by Entelos and Archimedes include nonlinear dynamics, control theory, differential equations, and object-oriented programming.

2.4 Oil Discovery and Extraction

For the oil production business, now is a time of risk and opportunity. Despite worldwide concerns about climate change and pressures to reduce our carbon footprint, our society remains heavily dependent on oil and natural gas for the near future. Dire prognostications about “peak oil” have so far not come to pass—in large part because they underestimated the ability of the oil industry to innovate and unlock new, “unconventional” sources of oil.

Enhanced production techniques—injecting carbon dioxide into the ground—make it possible to recover more oil from existing wells, and also sequester carbon that would otherwise be released into the atmosphere. Heavy oil deposits once considered too expensive to develop, such as the tar sands of Alberta and the oil shales of Colorado and Wyoming, have become more attractive as the price of oil has gone up. Deepwater drilling has also picked up momentum, bringing new risks that became apparent with the 2010 BP oil spill in the Gulf of Mexico.

As oil becomes more difficult to find and more expensive to extract, mathematical algorithms and simulations play an ever more important role in both aspects of the business. Inversion of seismic data (using seismic traces to map subsurface rock formations) has long been an important ingredient in oil prospecting. Advances in algorithms and computer hardware and software have brought three- and four-dimensional simulations within reach. Large-scale basin models help companies decide whether a rock formation is a promising candidate for drilling. Smaller-scale reservoir models are used while a field is in active production to predict the flow of oil within the field, to devise strategies for optimizing the rate of extraction, and to anticipate problems such as the reactivation of a geological fault due to changes in stress within the reservoir rocks.

Dynamic simulations also enable oil companies to analyze and minimize the risk of accidents before a facility is certified for production. However, the need for better risk analysis and modeling was brought home by the BP oil spill. Clearly, faster models using real-time data are needed to monitor conditions in a well and assess damage in the case of an unexpected event.

Case Study 7: Basin Modeling

An oil reservoir requires very special geologic circumstances to develop. There must be a source rock (sedimentary rock containing organic matter), a reservoir rock into which the oil migrates (usually not the same as the source rock), a trap (impermeable rock) that keeps the oil from escaping to the surface, and overburden rock that forces the source rock far underground, so that temperature and pressure will "cook" the organic material and create oil. Even if all four ingredients are present, there still may not be any oil, because timing is crucial. If the geological trap forms too late, then the oil will be long gone.

Basin models simulate all of these processes from basic physical principles. For instance, Schlumberger's PetroMod software starts with information about the ages and properties of each layer of rock. It computes the pressure and temperature of each layer through geologic time, and models the resulting effect on the rock's porosity, density, and other properties. This information is fed in turn into chemical models that simulate the generation of petroleum and its breakdown into gas and oil of different molecular weights. Fluid-flow models track the migration of the hydrocarbons, taking into account whether they are in liquid or gaseous form, how permeable the rock is and whether there are faults. The results of the model are validated against current measurements from trial boreholes. In many cases, the simulations are run multiple times with different parameters to ascertain the effect of uncertainty in the data.

All in all, basin models are a very interdisciplinary production that combines fluid flow, heat flow, chemical kinetics, geology, differential equations, and stochastic analysis, as well as some of the most intense supercomputing on the planet. Billions of dollars can be at stake.

Two examples illustrate the upside of basin modeling and the downside of not doing it. Near the Prudhoe Bay oil field in Alaska lies another prospect called Mukluk, where oil companies spent $1.5 billion for lease rights in the early 1980s. It was called "the most expensive dry hole in history." Although the geologic formation closely resembled Prudhoe Bay, either the time sequence was wrong or the trap rock was ineffective, and there was no oil to be found. For a more positive example, Mobil and Unocal purchased rights to a deepwater site off Indonesia called the Makassar Straits, which according to conventional wisdom was a poor candidate for drilling because the source rock was "postmature." However, Mobil's computer models indicated that oil was still being generated in the source rock. A test well in 1998 proved that the computer models were right, and Unocal began production at the site in 2003. It was the first deepwater oil field in Indonesia, and its peak production was about 20,000 barrels per day. See [Al-Hajeri 2009].

… basin models are a very interdisciplinary production that combines fluid flow, heat flow, chemical kinetics, geology, differential equations, and stochastic analysis, as well as some of the most intense supercomputing on the planet.

2.5 Manufacturing

Applied mathematics continues to be an integral part of manufacturing in many different ways: designing prototypes, optimizing designs, planning production and inventory, and managing supply chains.

Multidisciplinary design optimization (MDO) provides procedures and analytic and computational tools for coordinating efforts of design teams from multiple disciplines. Simulation-based design of complex systems in aerospace and automotive systems, for example, relies on computer analysis (including computational fluid dynamics and finite-element analysis). One of the major challenges still facing the computer-aided design (CAD) industry is to unify design, analysis, and verification into one seamless process. Too often, design engineers and verification engineers use different algorithms, different software and different file types. This creates a bottleneck, as the CAD files have to be converted from one form to another. Isogeometric analysis is a promising new technique used to create three-dimensional virtual models that can be plugged directly into physical differential equations.

The goal of production planning is to deliver a build schedule that makes efficient use of capital resources while satisfying as much demand as possible. The build schedule needs to take into account the flexibility of production resources, the stochastic nature of supply and demand within the supply chain, and the timing of new product releases and production facility improvements. Planning processes that rely on heuristic, manual decision-making are not adequate in industries with complex mixtures of products and manufacturing processes. Better decision algorithms, improved data management, and an automated and integrated planning process are needed.

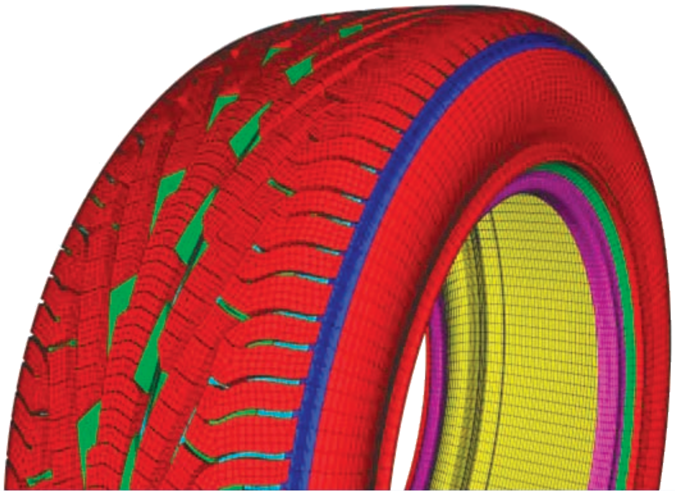
Applied mathematics continues to be an integral part of manufacturing in many different ways: designing prototypes, optimizing designs, verifying designs, planning production and inventory, and managing supply chains.

Case Study 8: Virtual Prototyping

In 1992, design and performance prediction of tires at Goodyear Tire & Rubber took months of computer time using finite-element analysis. Although tires appear simple from the outside, they have a very complex geometry with 18 or more components blended into a single tire, each made of different materials such as rubber, polyester, steel, and nylon. Rubber itself is one of the most complicated materials known to engineering. And because Goodyear's competitive edge is in the design of all-season tires, the tire's performance has to be evaluated under every driving condition.

Even though Goodyear had supercomputers, they recognized that the way they were setting up the models made them completely impractical. In 1994, Goodyear entered into a Cooperative Research and Development Agreement (CRADA) with Sandia National Laboratories, which gave them access to Sandia's physical modeling and simulation expertise. Over the next decade, Goodyear and Sandia developed new software that compressed the solution time for complex models. As a result, Goodyear could for the first time do computer simulations in advance of road tests. The "innovation engine" that came out of the project reduced development times from three years to one and costs for prototypes by 62%.

Best of all from the company's point of view, the partnership resulted in new, award-winning products, such as the Assurance tire with TripleTred Technology, which contains separate zones for traction on water, ice, and dry roads. The TripleTred Technology won an R&D100 award from R&D Magazine. See ["A New Approach", 2005] and [Sandia 2009].



The Goodyear Tire and Rubber Company uses Sandia's geometry and meshing technology to improve the performance of its tires.

Case Study 9: Molecular Dynamics

Molecular dynamics is not only used in biotechnology or pharmaceutical research. Procter and Gamble (P&G), like many other companies, is under market pressure to replace the petroleum-based materials in its products with so-called "green" materials. At the same time, the company is not willing to sacrifice the performance that its customers have come to expect. For example, a variety of factors go into customers' expectations for a dishwashing detergent: its thickness, its "feel," its foaming characteristics and mixing properties, and its separation over the product's lifetime. To develop new chemicals with the desired properties requires fundamental research into surfactants and polymers at the molecular level.

Unfortunately, lab experiments alone cannot do the job. The self-assembling structures that produce a foam are too small to be seen through a microscope. In order to visualize the foaming process, P&G turned to computer molecular dynamics simulations.

However, the company's supercomputers were to a large extent booked for other research projects as well as for routine production tasks. At most, they could have simulated a few thousand atoms rather than the billions that were required. P&G applied for access to Argonne National Laboratory's high-performance computers through the Department of Energy's INCITE program. Working with researchers from the University of Pennsylvania, company scientists reduced their simulation times from months to hours and improved the formulation of the company's products. In the future, the company hopes to use molecular dynamics simulations to create new "designer" molecules. See ["Procter and Gamble's Story" 2009].

…lab experiments alone cannot do the job. The self-assembling structures that produce a foam are too small to be seen through a microscope. In order to visualize the foaming process, P&G turned to computer molecular dynamics simulations.

Case Study 10: Multidisciplinary Design Optimization and CAD

In October 2011, the Boeing 787 Dreamliner made its first commercial flight, from Tokyo to Hong Kong. Built in response to the increasing price of jet fuel, the 787 is the first commercial plane to be predominantly made of composite materials (carbon-fiber reinforced plastic) rather than aluminum. These materials have a higher strength-to-weight ratio than aluminum, which allows the plane to be lighter and use 20 percent less fuel than any comparably-sized airplane. The 787 also has larger windows and can withstand higher interior pressures, thus giving passengers a more comfortable environment and possibly reducing jet lag.

There were many engineering challenges involved in designing a plastic airplane. For instance, the wings of the 787 flex upwards by three meters during flight. Traditional rigid-body models, which describe the wing's shape correctly in the factory or on the ground, do not describe it correctly during flight. To an aerodynamic engineer and a structural engineer, it looks like two different wings—and yet both engineers have to work from the same computer model. The computer has to "know" how the wing will bend in flight.

Boeing 787 landing at the end of its maiden flight, illustrating the flexibility of its wings.

The entire plane took shape, from beginning to end, on computers; there was not a single drawing board or physical prototype. Each of the more than 10,000 parts, made by 40-plus contractors, was designed in the same virtual environment. The contractors are not just suppliers but are actually co-designers. The virtual environment also facilitates "direct design." If a customer (i.e., an airline) wants a particular feature, whether it is different doorknobs or different floor plans, an engineer can draw it up on the computer and then it can be built. The days of the assembly line, when every product was the same as every other, are coming to an end.

The mathematical tools involved in the design process include computational linear algebra, differential equations, operations research, computational geometry, optimization, optimal control, data management, and a variety of statistical techniques. See [Grandine 2009] and [Stackpole 2007].

Case Study 11: Robotics

In industry, it isn't just the final product design that has to be right. The process for making that product also impacts the bottom line. Automated Precision Inc. (API) of Rockville, Maryland, recently introduced a technology that combines laser tracking with polynomial-based kinematic equations to improve the accuracy of machine tools. Typically, robotic machine tools have arms with three axes of rotation. Each link in the arm is controlled separately, leading to cumulative errors in three different coordinate systems and 21 error parameters overall. In API's Volumetric Error Compensation (VEC) system, the entire machining space is expressed in one coordinate system, with only six error parameters. Using algorithms based on Chebyshev polynomials, the VEC software can then compute the proper tool path in any other coordinate system.

One of API's aerospace customers stated that VEC reduced the time required to calibrate its machine tools from "one week of 12–14 hour days to one eight-hour shift." Another customer estimated that the process would reduce its assembly and fitting costs by $100 million per year. R&D Magazine cited VEC as one of its 100 technological breakthroughs of the year in 2010. See ["Precision Machining" 2010].

Case Study 12: Supply Chain Management (Biotechnology Industry)

Once you've designed the product and built it, you still have to get it to market. This seemingly elementary step can actually be very complex. An instructive example of automated supply chain management took place at Dow AgroSciences, an international company that makes pesticides and other biotechnology products.

The pesticide market is highly regulated and taxed, and the route a product takes from country to country can strongly affect the amount of duties that have to be paid. In addition, certain countries will not allow importation of certain chemicals from certain other countries. Thus, the source of every ingredient in every product has to be tracked.

At first Dow tried using an external vendor to automate its supply chain, but the unique characteristics of their business eventually forced them to model the supply chain in-house. The model represents the supply chain as a directed graph or network, with arrows indicating feasible routes from suppliers to factories to other factories to customers. Decision variables include inventories and quantities sold and produced; parameters include tax rates, shipping and material costs. In all, the network includes half a dozen suppliers, three dozen factories and more than 100 customers (each country counts as one customer). The most cost-effective route for every product can be found by solving a mixed-integer linear programming problem.

The problem is actually harder than the above figures suggest because every path through the network requires a separate set of decision variables. With about 2,100 pathways and 350 final products, the linear program contains about 750,000 variables and half a million equations. Even so, it is generally possible to find the profit-maximizing solution for a single business scenario on a quad-core workstation in two hours. See [Bassett and Gardner, 2010].

Case Study 13: Supply Chain Management (Automotive Industry)

In 2006, Ford Motor Company was on the brink of a "gruesome supply chain failure". Its major supplier of interior parts, a company called Automotive Component Holdings (ACH) that was owned by Ford but operated as an independent business, was losing money. ACH manufactured its parts in two underutilized plants in Saline and Utica, Michigan. The company faced an urgent decision: whether to close both plants and outsource all the production to other suppliers (including relocating much of the production machinery), consolidate both operations into one plant, or pursue a mixed strategy of outsourcing and consolidation.

Ford's management quickly realized that the number of possibilities to evaluate—involving the disposition of more than 40 product lines, requiring 26 manufacturing processes, among more than 50 potential production sites—was far beyond "traditional business analysis." Over a two-month period, Ford's research department constructed a model of the constraints and costs for every phase of production. Unfortunately, the model had 359,385 variables and 1,662,554 constraints. Even worse, the problem was nonlinear (primarily because of the effects of capacity utilization). A mixed integer linear program of this size can be solved (cf. Case Study 12), but a nonlinear program, in general, cannot.

The researchers came up with an ingenious workaround. They split the large module up into a facility capacity model and a facility utilization model, each of which was linear. By passing the solutions back and forth between the two models in iterative fashion, they were able to converge on optimal solutions for both. These solutions provided a crucial tool for management, because they weighed numerous scenarios. The model identified a mixed strategy that saved Ford $40 million compared to the originally preferred strategy of complete outsourcing. In the end, 39 of the model's 42 sourcing decisions were approved by Ford's senior management. The Saline plant remained open, and its restructured business improved to the point where Ford was able to find a qualified buyer. See [Klampfl, 2009].

2.6 Communications and Transportation

Both the communications and transportation industries have long been active users of mathematics. Some of the earliest applications of operations research were to the scheduling of supply networks, and that continues to be the case today. Algorithms to direct traffic on the Internet and codes that enable everybody's cell phone to share the same bandwidth have been crucial to the commercial success of the Internet and wireless communication industries, respectively.

Automated Precision Inc. (API) of Rockville, MD, recently introduced a technology that combines laser tracking with polynomial-based kinematic equations to improve the accuracy of machine tools.

Case Study 14: Logistics

If any company is synonymous with the word "logistics," thanks to its advertising program, that company is the United Parcel Service (UPS). The company now operates the ninth-largest airline in the world, one with no human passengers but a lot of cargo. To figure out how to get all those Christmas packages where they're going, without wasting any space on the planes, it is no surprise that the company has turned to computer algorithms and operations research.

In fact, UPS uses three layers of software, which can be described as short-term, medium-term, and long-term planning. The long-term software projects capacity 10 years into the future, and is used, for instance, to make decisions about acquiring new companies. Medium-term optimization allows the company to plan routes. The short-term optimization tool, called the Load Planning Assistant, helps each hub plan its operations up to two weeks in advance. In addition, a system-wide tool called VOLCANO plans next-day operations for the airplane network, figuring out how to match the current number of packages to the planes available, taking into account their capacities and airport constraints. Both LPA and VOLCANO were developed in collaboration with academic researchers, at Princeton and MIT respectively.

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Because UPS has used operations research for more than 50 years, it is difficult to say how much money these programs have saved, but it is fair to say that the company's ongoing reputation depends on them. See ["Analytics at UPS" 2011].

Case Study 15: Cloud Computing

When Hurricane Katrina hit New Orleans in 2005, the American Red Cross website experienced a sudden 14-fold increase in traffic. The website crashed, preventing donors from making urgently needed donations. The Red Cross contacted Akamai Technologies to deal with the crisis. Within eight hours their website was running again and the donations were flowing. The Red Cross has continued to work with Akamai ever since, coping successfully with a 15-fold increase in traffic during the California wildfires of 2009 and a 10-fold spike after the Haiti earthquake in 2010. See ["American Red Cross" 2010].

Akamai is in the business of operating high-volume websites, and its recipe for success has both hardware and software components. Much of the slowdown in online traffic occurs in the Internet's disorganized and unpredictable "middle mile." To a considerable extent, Akamai can circumvent the middle mile by assigning most of the on-the-fly computing to Internet servers that are close to the individual user. This enhances the user's perception of the responsiveness and interactivity of a website. Akamai manages more than 35,000 servers, so it has a server close to almost everyone.

Still, the servers do have to talk with each other over the "middle mile," and Akamai remains committed to using the public network rather than building a proprietary one (which would be prohibitively expensive). The company works around the limitations of the middle mile in a variety of ways. It distributes software to all of its servers that improves on clunky, standard Internet protocols. Also, load-balancing and load-managing software expects and plans for failures in parts of the network, so that alternate routes are found automatically. As a result, the network operates with little human intervention; on average, only 8 to 12 people are required to keep all 35,000 servers running.

Akamai has always depended very heavily on mathematical and computational techniques such as probabilistic algorithms, combinatorial optimization, load balancing, graph theory, discrete mathematics, and operations research. It also supports mathematical education through the Akamai Foundation.

2.7 Modeling Complex Systems

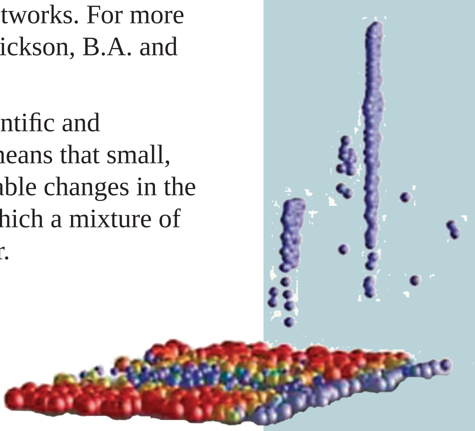
We traditionally sold components for other people's products. Now we are also selling systems. That changes the character of our business. Mathematics, analysis, simulation, and computation have become essential. [From interviews.]

Mathematical modeling is a key technology in complex systems engineering, from analyzing multi-scale systems in science to evaluating architectural tradeoffs to verifying system designs. Modeling, analysis, simulation, optimization, and control reduce the length of the product design cycle. They also help to document, visualize, and ensure the quality of the resulting system, and identify and estimate risks of large failure events. Complex distributed systems include the Next Generation Power Grid (or "smart grid") [Beyea, 2010], traffic networks, water supply systems, energy efficient buildings, and medical information networks. For more information on the mathematical challenges in complex systems. See [Hendrickson, B.A. and Wright, M.A. 2006].

Another kind of complexity is the nonlinear behavior exhibited by many scientific and engineered systems, often exhibiting such behavior at multiple scales. This means that small, incremental changes to the inputs can sometimes lead to large and unpredictable changes in the output. Nonlinear dynamical systems remain an active field of research, to which a mixture of theoretical mathematics and computational techniques can be brought to bear.

Case Study 16: Viscous Fluid Flow

Ordinarily most of us don't think about what our computer or television screens are made of—we're more interested in what we see on the screen. Nevertheless, new glass technology has been a significant contributor to the spectacular commercial success of large flat-screen TVs, computer monitors, and smart phones in recent years.



Visualization of electrons accelerated by a laser interacting with a plasma as computed by Vorpal™, TechX Corp.

As liquid crystal display (LCD) technology advances, thickness uniformity specifications, flatness specifications, and defect limits have been getting more and more stringent. More importantly, the tempo at which customers expect improvements has increased several fold. Corning, a leading manufacturer of LCD glass substrates, uses mathematical models to explore process advancements to improve the attributes of its glass. These models, like the formulation of the glass, are continually refined over time. For instance, one process, called the fusion-draw process, involves two steams of molten glass that flow down the sides of a V-shaped trough and merge into a planar sheet. Modeling the flow of this sheet, and understanding instabilities such as oscillation and buckling, requires the solution of a complex system of nonlinear differential equations.

The use of mathematical models enables Corning to introduce new products at a rapid pace and with reduced technology risk. An example is Corning Gorilla glass, which differs from LCD glass in composition and hence in its behavior during the sheet manufacturing process. The use of models enabled Corning to rapidly find the process window for manufacturing the new composition. As a result, only limited process start-up experimentation was needed, and the product introduction time was shortened from years to months. See [“Glass once used” 2012].

As the population becomes increasingly urbanized, it has become a greater challenge to manage the traffic, public safety, water, power, and health-care systems that sustain all these people. IBM has become a leading proponent of “smart cities…”

Case Study 17: Smart Cities

In 2008, for the first time, more than half of the world’s population lived in cities. In the U.S., more than four-fifths do. As the population becomes increasingly urbanized, it has become a greater challenge to manage the traffic, public safety, water, power, and health-care systems that sustain all these people. IBM has become a leading proponent of “smart cities,” a movement that will surely grow. Two examples illustrate the kinds of projects that IBM has worked on.

In 2008, the District of Columbia Water and Sewer Authority (DC Water) contracted with IBM Global Services to improve the management of its infrastructure. IBM installed a database that kept track of every asset in the system, down to the last pipe and manhole cover. As a result, DC Water could begin to anticipate problems rather than merely react to them. The authority made better-informed decisions about repairs, service-call volumes were reduced, and defective meters were replaced. All of these factors enabled DC Water to save $20 million over three years, an impressive return for a project that cost less than $1 million. See [“DC Water” 2011].

Information technology is changing the way that police departments do business. IBM helped New York install a new database, the Crime Information Warehouse, which enables analysts to detect crime patterns in real time. Memphis went one step further, using IBM statistical and predictive software to forecast which precincts will see more crime activity. While it is impossible to know the effect of these initiatives with certainty, New York’s serious crime rate has dropped by 35 percent since 2001, and Memphis’ has dropped by 30 percent since 2004. See [“Memphis PD” 2011].

In Chicago, the police department has networked an estimated 15,000 surveillance cameras around the city in a project called Operation Virtual Shield. When a crime is reported, the system can immediately call up a live video feed from the nearest camera (as well as recorded video from the time when the crime occurred). Chicago police say that the system has aided in thousands of arrests. See [Bulkeley 2009].

Areas of mathematics and computing involved in these projects include data mining, data storage, biometrics, pattern recognition, risk assessment, statistics and statistical modeling.

2.8 Computer Systems, Software, and Information Technology

Watson's advances in deep analytics and its ability to process unstructured data and interpret natural language will now be tailored to fit the requirements of new solutions in science, healthcare, financial services, and other industries. [Groenfeldt, 2011]

Many businesses are interested in high-performance computing (or "supercomputing") to address current industrial problems. As shown in some of the above case studies, simply owning a supercomputer is not enough. Businesses need programming and modeling expertise, numerical libraries, and a broad range of software tools that will work on parallel and distributed platforms. Often, businesses of small to medium size cannot afford to build their own IT structure to support high-performance computing, but they can greatly improve their modeling capability by using software on a large distributed network (i.e., cloud computing).

Other rapidly growing areas of IT are computer vision and imaging, natural language processing, information retrieval and machine learning. One of the most spectacular examples of the potential for applications of natural language processing (as well as information retrieval and machine learning) is IBM's Watson computer system, which defeated the two most successful human contestants in Jeopardy! IBM has already begun to leverage this technology for a range of other applications.

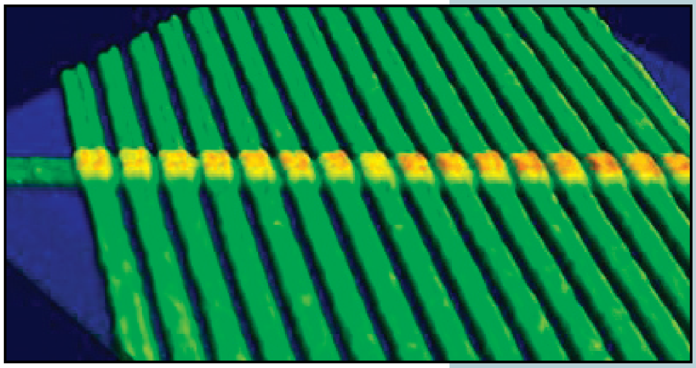
… simply owning a supercomputer is not enough. Businesses need programming and modeling expertise, numerical libraries, and a broad range of software tools that will work on parallel and distributed platforms.

Case Study 18: Serendipity

My guess is that the real killer app for memristors will be invented by a curious student who is just now deciding what EE courses to take. [Williams 2008]

In any commercial enterprise, basic research with no foreseeable bottom-line impact is always the hardest kind of R&D to justify. For this reason it is especially important to acknowledge the rare but transformative occasions when curiosity-driven research hits the jackpot. A beautiful recent example was the discovery of the memristor at HP Labs in 2008.

Stanley Williams had been hired by HP in 1995 to start a fundamental research group, in accordance with company founder David Packard's belief that HP should "return knowledge to the well of fundamental science from which HP had been withdrawing for so long." [Williams 2008]. A decade later, while studying approaches to molecular-scale memory, he accidentally created a device—a sandwich of titanium dioxide between two platinum electrodes—whose resistance changed according to the amount of charge that passed through it. In essence, its resistance preserves a memory of its past. This is the origin of the term "memristor."



An array of 17 titanium oxide memristors, HP Labs

Perhaps the most amazing thing is that memristors had been predicted, as a purely mathematical construct, in a little-noticed paper by Leon Chua of UC Berkeley in 1971 [Chua, 1971]. They are the fourth basic passive circuit element, joining resistors, capacitors, and inductors. (A passive element is one that draws no energy.) The first three elements were discovered back in the 19th century, and are the basis for all of today's electronics. Williams has stated that he would not have been able to understand what his lab had produced if he had not read Chua's paper and thought deeply about it. Indeed, other researchers had noticed similar effects without understanding why.

At present, the main application envisaged for memristors, and the one that HP is betting on, is computer memory. A computer with memristor-based storage would not need to “boot up”—turn it on, and it would instantly remember where it was when you turned it off. Over the long term, as Williams’ quote above suggests, they might be used for something no one has thought of yet. For example, because memristors behave in a somewhat similar way to neurons, perhaps they would be the key to a true artificial brain. See [“Properties of memristors” 2011].

For R&D managers, the story of memristors has at least two lessons. First, basic research will pay off eventually… for somebody. And second, pay attention to mathematics.

3 Life After the PhD

Results from the MII Survey

The job market for PhD mathematicians has fortunately improved considerably since the 1996 MII survey, which was conducted at a time when unprecedented numbers of new PhDs were unable to land an academic position or any job at all. By comparison, very few people in this survey were forced into taking industrial jobs because they couldn't get a job in academia.

A detailed discussion of the findings of the survey follows below. Here are some of the highlights:

- Roughly half of all mathematical scientists hired into business and industry are statisticians. The second-largest group by academic specialty is applied mathematics.
- By far the strongest employers of mathematicians are the finance and insurance sector and the pharmaceutical/medical device sector. Pharmaceutical hires are almost exclusively statisticians, while in the financial sector the majority of the hires are mathematicians.
- As reported in 1996, almost none of the mathematicians have "mathematics" in their job title. By contrast, the title of statisticians often refers to their specialty.
- The job satisfaction of the survey respondents was quite high, with nearly 90 percent reporting satisfaction with their compensation and benefits. Median pay for the respondents was about $100,000 for both men and women.
- Compared to the 1996 survey, fewer graduates reported "modeling and simulation" as an important academic specialty for their jobs, and more reported "statistics."
- However, in a somewhat contradictory finding, the most important item evaluated in performance reviews was reported to be mathematical models.
- Programming and computer skills continue to be the most important technical skill that new hires bring to their jobs.

3.1 Background and Demographics

As mentioned in the introduction, a major difference between this survey and the one conducted in 1996 was the inclusion of graduate students from statistics or biostatistics departments. We felt that this provided a more realistic and inclusive view of the job market for students in the mathematical sciences. However, in cases where we have compared the two surveys, for consistency, we have only used data on the mathematics and applied mathematics students.

The four groups covered by this survey are the 2004-05, 2005-06, 2006-07, and 2007-08 hiring cohorts. Some basic demographic information on these cohorts is provided by the annual AMS-IMS-SIAM survey of doctoral candidates. Of all the PhD graduates in the mathematical sciences from 2004–2008, 787 (15%) took jobs in industry. Of these, 426 (54%) did their thesis work in statistics and 361 (46%) in mathematics. The areas of mathematics most frequently represented were applied mathematics (10%) and probability (9%), see Table 1. Note that these are figures for the total set of PhDs hired by industry in the AMS-IMS-SIAM data.

	All	Survey
Area of Degree	%	%
statistics	54	39
applied mathematics	10	18
probability	9	5
discrete mathematics	6	4
algebra	5	4
numerical analysis	5	5
differential equations	4	2
optimization	3	5
geometry	3	4
analysis	2	5
other	0	4

Table 1: Degrees of all hires 2004-2008 and those in the survey

Our intention was to conduct an e-mail survey of all PhDs in the above data set. However, we could find working e-mails for only 40% of the PhDs using the data provided or using search on the web. Our web-based survey attained a 30% response rate which is typical of such surveys. As a result, we make no claims of statistical significance for our results. However, the distribution of specialties for our survey respondents is similar to the population as a whole (see Table 1), and this gives us some confidence that we obtained a representative sample. Statisticians are slightly underrepresented and applied mathematicians are slightly overrepresented in our sample.

The respondents to our survey included 19 women and 37 men. With such small sample sizes it is difficult to do any serious comparative analysis between the two groups. We will simply report that a majority of the statisticians in our survey were female (12 women, 10 men), while the applied mathematicians were overwhelmingly male (one woman, nine men). We do not offer any interpretation for this disparity, but it bears watching in the future.

A large majority (81%) of the PhDs in our survey work for companies with at least 250 employees. Thirteen percent work for small companies, with fewer than 50 employees. The median salary ($100,000) was identical for men and women. The variation about the median was also extremely similar, with the first quartile at $90,000 for both genders and the third quartile at $115,000 for men and $123,000 for women.

	Total	Stat	Math	Survey
Employers	% hires	% hires	% hires	% hires
aerospace and defense	3.4	0.2	7.2	9.4
business services	14.2	15.0	13.3	5.7
engineering and scientific services	4.2	1.9	6.9	5.7
finance and insurance	30.4	21.1	41.3	28.3
pharmaceutical and medical devices	28.2	50.7	1.7	18.8
software	5.0	1.6	5.8	20.8
research and development	4.3	3.1	8.9	0.0

Table 2: Top employers 2004-2008 by broad industry classification. Statistics and mathematics are broken out separately. The last column is the employers from the survey.

Using the data from the AMS-IMS-SIAM annual survey, we classified employers by broad industrial category. The top employers were finance and insurance (30%) and pharmaceuticals and medical devices (28%). These sectors employ about twice as many recent graduates as the next leading category, business services (14%), see Table 2. Nearly every major company in the pharmaceutical industry hired several PhDs per year, and nearly all of these hires worked in statistics. Most of the major financial firms hired about two PhDs per year. The companies not in these two categories that also hired on average two PhDs per year were the SAS Institute, Google, IBM, and Microsoft Research. The last column of Table 2 compares the full population data to our survey recipients. Some discrepancies arise because we under-sampled statisticians, and also because the survey asked respondents to classify the division of the company in which they work (which may be different than the company itself).

We analyzed the cohorts 2008–2009 and 2009–2010, asking whether or not the leading employers or the number of hires changed during the recession years 2009 and 2010. The answer was essentially no change. The leading employers hired between 87% and 94% of all PhDs in the same proportions.

Although we did not survey PhDs who took jobs in government, the AMS-IMS-SIAM data for these graduates allowed us to analyze their research backgrounds and employers. The areas of research were similar to those PhDs who went into industry, with strong concentrations in statistics, applied mathematics, numerical analysis, differential equations, and discrete mathematics (in that order). The top employers were the FDA, NSA, NIH, Los Alamos National

Laboratory, Sandia National Laboratories, the other national laboratories taken as a group, and the Veterans Administration. These employers accounted for 81% of government PhD hires. The PhDs who took jobs in government from 2004-2008 accounted for 3% of all PhDs. Interestingly, while the financial industry was bulking up on PhDs from the mathematical sciences, the government hired only one such PhD into its financial agencies, in this case to the Federal Deposit Insurance Agency.

For the years 2004-2008, we analyzed the 25 departments that graduated the most PhDs who took industrial jobs. The top 25 programs in statistics sent, on average, twice as many of their graduates into industry (on a percentage basis) as the mathematics programs. The range for statistics programs is 15% to 70%, while the range for mathematics programs is 10% to 40%.

We also extracted data on PhD advisors and the number of PhD graduates they supervised who went into industry. Of the PhDs who took a job in industry, 82% of their advisors had only one such PhD over the four-year period, while 11% had two, 6% had three, and 1% had four or more. These data suggest that for a large majority of graduate students, the decision to enter industry is more of a personal choice than one resulting from pressure or expectations from the advisor. Many of the students in our survey completed internships or had some other formal interaction with industry, e.g. industrial workshops or mentoring by an industrial scientist. In such cases the advisors may have facilitated or encouraged the students' interaction with industry, even if they themselves did not collaborate with industry.

3.2 Role of Recent Graduates in Their Companies

(The job is) intellectually stimulating without all the built-in failure of academic research. Academia necessitates a "publish-or-die" mindset in early career to make tenure. Work-life balance is not respected or encouraged.

The reasons given by the PhDs in our survey for joining industry were topped by expectations of higher compensation and better opportunities for career advancement, see Table 3. Nearly half mentioned their experience with industrial internships and roughly a third already had jobs in industry; both of these figures suggest that in most cases the decision to take an industrial job was not a last-minute choice, and the students were well-prepared. Not surprisingly, an undercurrent of dissatisfaction with academia runs through the comments, as exemplified by the above quote. Only two students reported that they took a job in industry because they could not get one in academia or government.

Rationale for Taking a Job in Industry	%
higher compensation	66
opportunities for career advancement	52
experience with industrial internships or programs	48
intellectual challenge	32
had a job in industry	32

Table 3: Rationale for joining industry

A high percentage of the PhDs in our survey were very satisfied or satisfied with compensation and benefits (88%) and lifestyle (80%), even if those were not stated as a primary reason for choosing a job in industry, see Table 4. The jobs did not require them to sacrifice intellectual challenge, which was highly rated. However, not as many were satisfied with the opportunities for scientific growth. This contrast is likely due to the more project-oriented focus of industrial jobs, particularly in the first few years after graduate school.

Very Satisfied/Satisfied with	%
compensation and benefits	88
lifestyle	80
intellectual challenge	74
opportunities for career advancement	72
opportunities for scientific growth	56

Table 4: Satisfaction with aspects of the job

The survey collected information on the primary mission of the groups that employ PhD mathematical scientists, see Table 5. There are no clear leaders, but one noteworthy trend is the reduction in percentage, compared to the 1996 survey, of groups whose primary mission is

The reasons given by the PhDs in our survey for joining industry were topped by expectations of higher compensation and better opportunities for career advancement.

software (35% in 1996 versus 13% in the current survey). Except in some cases of small groups of five or fewer people, the groups were interdisciplinary, including some mixture of engineers, computer scientists, physical scientists, or graduates in finance and economics.

As we pointed out in 1996, mathematicians rarely have job titles that are specific to their degree, see Table 6a. The exception is statistics, for which titles like "senior statistician" or "biostatistician" are more common, especially in the pharmaceutical sector.

A mathematician is much more likely to be identified as an "analyst," a "modeler," or simply a "researcher." The practical significance is that mathematical scientists who are not statisticians are likely to face competition from graduates with strong quantitative skills from other disciplines, and they may need to convince their employers of the relevance of their mathematical background. In Table 6b, we also list actual job functions, as reported by the respondents. In job functions, we see an even stronger emphasis on statistics, software, programming, and computer science.

An impressive 15% of our respondents were already identified as "managers," and 21% reported management as one of their job functions. In view of the fact that many students choose industrial careers because they expect to advance more rapidly than in academia, it is encouraging to see that, for a significant fraction, these expectations are being rewarded.

Group Mission	%
engineering	9
investment/trading	11
modeling	15
analysis/finance	9
research in engineering/science	16
software development	13
statistics	16
business strategy	11

Table 5: Mission of groups from survey

Job Title	%
statistician	17
analyst/ modeler	20
researcher	21
management	15
consultant	9
engineer	7
software developer/programming	11

Table 6a: Job titles from survey

Primary Job Function	%
computer Science	7
consulting	6
engineering	10
modeling	17
operations research	1
scientificprogramming	6
software development	8
statistics	33
strategy	7
other	6

Table 6b: Primary job function for respondents ("other" includes financial trading, business analysis, technical support)

3.3 Qualifications and Skill Sets

Computation is not enough. Our recruits need to understand algorithms, mathematical modeling, and the application. Perhaps 20% is implementation on computers, but 80% of the work involves understanding the physics and engineering.

The skill set of graduates encompasses technical depth in a relevant discipline, breadth of knowledge across the mathematical and computational sciences, interest in and experience with the scientific or business focus of the employer, enthusiasm for varied challenges, the flexibility and communications skills required to work in an interdisciplinary team, the discipline to meet time constraints, and a sense for a reasonable solution.

The technical skill set has not changed dramatically since 1996, but continues to depend very much on the industrial sector and the group in which a graduate is employed, see Table 7.

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Compared to the earlier survey, fewer respondents cited "modeling and simulation" as essential or important. However, this last finding seems to contradict the answers we received to a question on what metrics were important to the respondents' annual performance reviews. The leading answer to that question was "mathematical models," at 67%, followed closely by "presentations to management" at 64%, (see Table 8). Perhaps the apparent contradiction means that mathematical models remain crucial for job performance, but the pedagogy of mathematical modeling is in some way not keeping current.

In the 1996 survey, advanced computation was rated essential or important by 83% of graduates. In the current survey, we explored this in more detail. Programming (86%), computational science (57%), data mining (40%) and software engineering (34%) were the computational disciplines rated as essential or important. Overall, 65% of the respondents rated the mathematical or computational sciences as highly important to the success of their group.

In our 1996 report, we clearly emphasized the importance of communication skills. Table 8 shows that outputs related to communication skills ("presentations to management," "preparation of internal reports") are at least as important for advancement as technical accomplishments ("mathematical models," "software development").

Likewise, our on-site interviews returned again and again to the theme of soft skills such as communication, teamwork, flexibility, and willingness to listen. Some of the comments we received are given below.

"Soft skills make the difference."

"You often can't guess who can't make the transition [to industry]. After the fact, you can see that the successful ones are good listeners who are tolerant of other people's ideas and willing to make incremental improvements."

"You have to be able to explain projects to non-experts. PhDs often have more flexibility than MSs, because they have been required to think outside the box."

"Must be willing to 'get hands dirty' to help the firm meet its business imperatives."

"Must be able to attack and solve unstructured problems."

Rated essential/important	Survey	MII96
	%	%
statistics	61	51
probability	60	50
applied mathematics	56	-
modeling and simulation	49	73
numerical analysis	42	65
optimization	38	38
discrete mathematics	30	26
differential equations	29	50

Table 7: Percent of mathematical specialty rated as an essential or important requirement for their job. Multiple answers allowed. Comparison to MII96 included.

Essential/important to annual review	
	%
mathematical models	67
software development	43
presentations to management	64
preparation of internal reports	59
presentations to customers	53
presentations at conferences	39
publication in the open literature	29

Table 8: Percent of respondents rating task outcomes as essential or important for their review

Our survey also asked the recent graduates for advice that they would give to graduate students who are now considering careers in industry. We received 21 responses, such as these:

"In banking, the current challenge is to sieve through huge amounts of customer data. Any training in mass data manipulation would be a plus."

"Be open-minded, attend practitioners' seminars, learn to program, and you'll be fine."

"Pursue internships. Follow your interests. Acquire database skills."

"PhDs tend to underestimate the quality of science done in industry. You will get to solve challenging problems in industry, too."

"Get internships and learn to program a computer. From my perspective, programming is essential for almost any industrial mathematician."

4 Perspectives on Graduate Education

Encourage graduate degrees that involve dual mentoring by mathematics or computer science and another department, government laboratory, or industry.

"Seek internships in industries you are interested in. Do not focus so much on the degree. Think outside the academia box. Work on people skills. It does you no good to be smart if you can't communicate or work in teams."

4.1 PhD Education

The 1996 report made several recommendations for improving graduate education for students interested in taking a job in industry, including coursework in an application, experience with formulating and solving real-world problems, and coursework in computer or computational science.

In the current survey, we asked three related questions to gauge to what extent these recommendations have become a part of graduate education. First, we asked if the graduates had been involved in industrial-related programs. Of the respondents, 28% participated in industrial internships, 7% had an industrial mentor, 7% participated in problem sessions, and 5% participated in an industrial workshop. Multiple answers were allowed. However, 59% either did not respond or indicated that the question was not applicable.

On the other side of the coin, many of the companies we visited in on-site interviews stressed the advantage of internships, and many of them offer such internships: for example, Boeing, D.E. Shaw, Cray, IBM, GM, AT&T, Intel, Akamai, HP Labs, Google, and Solidworks. Likewise, workshops have proven to be a successful platform transferring knowledge from academia to industry and giving students experience at solving industrial problems. Leaders include the mathematical problems in industry workshops in the United States, the industrial problem solving workshops of the Pacific Institute for the Mathematical Sciences in Canada, and the European Study Groups with Industry.

In view of the appreciation that companies express for internships and workshops, and the variety of opportunities, it is disappointing that participation in such programs has not become more universal, at least among students considering an industrial career.

"I think it is important to bridge the gap between theory and practical applications—most people do not realize this."

"Programming experience (preferably on team software project) is very important and frequently omitted in a mathematically oriented curriculum."

Training in another discipline

	%
programming	65
scientific computing	35
other computer science	28
scientific discipline	58
business discipline	29
engineering discipline	9
other discipline	4

Table 9: Percent of respondents with training in another discipline

We asked graduates about graduate-level training outside of their major. In this case, 79% of the graduates had at least one such experience, see Table 9. Often this took the form of training in programming, scientific computing, and other computer sciences. Finally, we asked how valuable this training was for obtaining a job in industry, and 65% of graduates considered it very valuable or valuable. We also asked how important they found the experience of working in a team, and 70% rated this as very valuable or valuable.

We contacted one graduate program that ranks very high in the percent of graduates entering industry to see if that department has any special insights into the preparation of students for industrial jobs, particularly ideas that might be portable to other institutions. The Department of Computational and Applied Mathematics at Rice University sent eight of its twenty graduates (40%) into industry from 2004 to 2008, and six out of eleven from 2009 to 2010. Some points made by department chair Matthias Heinkenschoss were:

- The CAAM department enjoys strong contacts with local companies, such as BP, Shell, ExxonMobil, and Chevron. While the specific companies will differ, the concept is certainly generalizable to other departments.

- Students do not, however, only do local internships. About half of the internships in which students participate are outside of Houston.
- It is important to keep in touch with alumni to build "pipelines" into specific companies and industries.
- Students going into industry take the same courses as other students. However, the required courses include topics like numerical methods and high-performance computing, which might not be required in a traditional pure-math department.
- All students have to write a Master's thesis at the end of their second year and take a thesis-writing course (which also covers other communication skills, such as the art of making a 5-minute presentation).
- Finally, CAAM is an applied-math department, rather than a program or a group within a department.

Of course the last-mentioned feature is very far from portable, and we do not advocate that other institutions should emulate this model. However, traditional math departments that have applied-math programs or groups should consider ways to make those programs sensitive to the needs of graduates going into industry.

You can't learn everything in school—a lot of the schoolwork is theoretical.

4.2 The Professional Master's Degree

In the 1996 report, we surveyed Master's graduates in mathematics and their supervisors. For this report we did not survey Master's graduates. We focused instead on an emerging trend in Master's education.

Shortly after the 1996 report was published, the Alfred P. Sloan Foundation launched its Professional Science Masters (PSM) program to establish an innovative Master's degree in the sciences and mathematics that would equip graduates for work outside academia. PSMs are rigorous, interdisciplinary programs that give students advanced training in science and mathematics, while emphasizing the professional skills that are highly valued by employers in a wide range of fields.

In 2005, the Council of Graduate Schools (CGS) assumed the task of making the PSM degree an accepted academic offering. In 2007, the National Professional Masters Association (NPSMA) was formed as a voice for program directors, faculty, administrators, alumni, and students. Its objective was to support PSM initiatives through conferences, workshops, gathering of data, and development of best practices. Both of these organizations received initial funding from the Sloan Foundation.

There are now more than 230 programs at nearly 110 colleges and universities in 30 states and the District of Columbia, as well as in Canada, the United Kingdom, and Australia. These include 23 programs in the mathematical sciences and several closely related programs in the computational sciences and bioinformatics. See ["PSM Programs" 2012].

A recent survey of the alumni of PSM programs, [NPSMA 2009], provides some insight into the jobs taken by graduates. Of the 281 respondents to an online survey, 80% held non-academic jobs: 62% were in industry, 9% in the nonprofit sector and 9% in government. Most jobs were in large organizations. The median salary was approximately $63,000, and the first and third quartiles were about $43,000 and $74,000 respectively. However, the mode (19%) was in the over-$90,000 category. Of the respondents in the NPSMA survey, 19% came from programs in the mathematical or computational sciences.

The PSM curriculum has a scientific component that includes depth in mathematics and breadth in science, engineering, or business, as well as a skills-based component in management, business and professional skills. PSM programs emphasize writing and communication skills, and require a final project or team experience. They provide opportunities for a structured internship. In addition to an innovative, targeted curriculum, a PSM program is expected to have an active advisory board that includes leaders from industry, business, and government. It is also required to collect and publish data on enrollment, degree completion, and employment history.

A recent National Research Council report, Science Professionals: Master's Education for a Competitive World, emphasized the importance of PSM degrees:

Policymakers, universities, and employers should work together to speed the development of professionally-oriented Master's degree programs in the natural sciences. Graduates of these programs—which build both scientific knowledge and practical workplace skills—can make a strong contribution to the nation's competitiveness. [NRC 2008]

Of the Master's programs in the mathematical and computational sciences that are not affiliated with the PSM program, some are very traditional, but others are very much focused on preparing students for jobs in particular industries. Perhaps the most prominent of these are in finance, where they go by variants on the names "mathematical finance," "computational finance," "financial engineering," or "quantitative finance." A few mathematical finance programs are affiliated with PSM, but the majority of them are not. For example, all seven PSM programs in financial mathematics participate in the National Financial Mathematics Career Fair, held at the Courant Institute every fall, but 42 other Master's programs that are not affiliated with PSM also participate.

So, if the curriculum expectations are met, why hire a Master's graduate? In an article in *Advanced Trading*, [Gibbs 2008], Emanuel Derman, the director of the Master's program in financial engineering at Columbia University and head of risk management at Prisma Capital Partners, says he "looks to hire quants who have learned the basics in areas such as modeling but who also understand why the model behaves as it does and which factors are driving the market. … You can't learn everything in school—a lot of the schoolwork is theoretical. So students go through the program and get experience in the real financial world." This rationale applies not just to "quants" but also to most Master's graduates in the mathematical and computational sciences who intend to follow a non-academic career. A balance has to be struck in a Master's curriculum between understanding of theory, understanding of business (or of a particular business), and developing practical experience.

5 Suggestions & Strategies

The suggestions that follow below are based on detailed comments from the focus sessions, survey, interviews with mathematical and computational scientists and managers on site visits, individual interviews at meetings, and finally on the experience of the committee members. They are also informed by the recommendations made in the 1996 report. In fact, we believe that the suggestions made in 1996 are still valid. The suggestions we make here are variants on those in the previous report, and the differences are a matter of emphasis rather than radically new proposals.

The principles guiding the educational suggestions here are knowledge of a relevant application, real-world problem-solving experience, facility with computation, communication skills, the ability to work in and provide leadership for an interdisciplinary team, and the desire to develop a sense of a business and its objectives.

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5.1 Global Strategies

In July 2009, the Organization for Economic Co-operation and Development issued a report on mechanisms for promoting mathematics in industry [OECD 2009] that contains an impressive but not necessarily comprehensive list of programs. The list includes:

Academic Initiatives

- interdisciplinary research centers within academia
- academic positions with an industrial focus
- student degrees (PhD and Master's) programs
- modeling weeks

Academic-Industrial Collaborations

- workshops
- study groups
- internships
- team exercises
- networks of academic and non-academic institutions
- facilitators (e.g., Mitacs business development officers)

The United States, Canada, and most countries in Europe have developed programs that fall into the most of these categories. As the report points out, many of these programs are organized by academia, and governments provide the main resources via academic channels.

For example, the Canadian center, Mitacs, has a business development team that consists of members who have a strong scientific background, business development abilities, and a commitment to facilitating the development of multi-disciplinary research collaborations with non-academic partners. The members report to the Executive VP, Business Development and work closely with the Mitacs scientific staff. They are distributed in regions of Canada and funding is provided by government and industry partners. The major tasks of the team members include introducing companies to collaborations with academia for advanced research, finding internship opportunities for students by connecting university researchers with industry on research projects, (see [“Mitacs Accelerate” 2012]), creating and maintaining a network of partners in the private and public sectors, formulating new partnerships for Mitacs, and managing relationships with key stakeholders.

A second example of a center with a strong industrial focus is the DFG Research Center, Matheon, in Berlin, Germany. Matheon develops mathematics for key technologies and supports partners in industry, the economy, and science. It also interacts with schools and the general public to increase the visibility of applied mathematics.

The center fosters awareness in industry in several ways. It maintains a website that explains the expertise of Matheon's members with pointers to industrial projects, references, and success stories. See ["Matheon Services" 2012]. It has a transfer office that acts as a knowledge broker and matches requests from industry with the appropriate group. It offers industry internships to graduate students. Students work on R&D problems of industrial partners under the supervision of Matheon scientists for four months. The financing of students and scientists is provided through a partnership of Matheon and the industrial partner. This has turned out to be a win-win situation for industry, students and academia. And it carries out industry-funded projects in the areas of discrete mathematics, optimization, numerical analysis, scientific computing, and stochastic analysis.

The report concludes that "the creation of national and international networks can both stimulate mathematical awareness and creativity concerning industrial problems and avoid duplication of intellectual effort." We support the OECD initiative and suggest that the United States should strongly support it. One outcome of the initiative should be the development of a best practices document and a discussion of metrics for evaluation of knowledge transfer along the lines of those developed in [Holli 2008].

Perhaps because the report above focuses on mathematics and not specifically on computational science and engineering, it does not address the collaboration between industry and government laboratories on high-performance computing. We discussed this earlier and include recommendations below.

5.2 Graduate Education

Graduate education in the mathematical and computational sciences provides graduates with specific technical expertise, the ability to think analytically, formulate problems, and develop mathematical models. All of these are key demands in industry. However, these technical skills are not by themselves sufficient for a graduate to succeed in an industrial career. The graduates in our survey and the industrial scientists and managers in our on-site interviews emphasized a set of additional skills and experiences that are needed within industry. We mentioned these skills above in §3.3 and discuss them in more detail here.

Exposure to a relevant application and real-world problem solving

The majority of the industrial scientists we interviewed work in companies that have internships and use them to find potential employees. Both the respondents to our survey and the industrial scientists strongly suggest industrial or government internships as the best course for gaining experience on real-world problems. Obtaining an internship presupposes coursework in a relevant application discipline. Math departments should maintain a database with links to national summer positions and internships in industry or government laboratories. Faculty and administrators should provide encouragement and support applications. But an even more effective way to connect graduate students with internships is to routinely invite your PhD and Master's graduates who have taken jobs in industry or government to come back to the department and give a colloquium or workshop. Ask your alumni to speak about their jobs and the opportunities in their organizations.

Expertise in programming

We hired a bright candidate with a PhD in topology. However, because he was too humble about his expertise in programming, he almost lost a job offer.

A story like this one should never happen. Students need to know that programming is often an essential tool in industry, and if they have these skills they should certainly not hide them or be afraid to mention them.

The programming requirement varies by industry, by company, by department, and by work group. In a small to medium-size company, everyone may be expected to contribute to the IT effort. The programming languages in which expertise is expected also vary. In some cases, a fourth-generation language, such as MATLAB, R, SAS, or SPSS is sufficient. In other cases, a programming language such as C++ or Java and a high-level scripting language such as Python are required. (Some interviewees even spoke disdainfully of job applicants who know "only" MATLAB.) It is therefore very important for potential job applicants, and their mentors, to find out as early as possible what are the current expectations in their desired industry concerning programming expertise.

High-Performance computing

A faculty is doing a disservice to their students if it does not offer a course in parallel computing.

A 2008 white paper sponsored by DARPA, the DOE, and the Council on Competitiveness [Council on Competitiveness 2008] concluded that "American industry is in the midst of a new 21st-century industrial revolution driven by the application of computer technology to industrial and business problems. HPC plays a key role in designing and improving many industrial products … as well as industrial-business processes." Our study provided many examples where high-performance computing was helpful or even critical in solving business problems. The lack of qualified personnel was mentioned, in one form or another, by several of the individuals we interviewed.

For students, it is a distinct advantage to develop skills in modeling and computation in a particular application. It is also an advantage to develop skills in high-performance computing. A combination of these skills is certainly in great demand.

Communication and teamwork

It is important that during their education, students work on team projects with more than two members.

Communication skills remain as essential today as they were in 1996. But these skills go beyond writing and presenting well. In business, an individual's success is most often determined by the success of the team(s) he or she was a member of. To be an effective communicator on a team,

"…the creation of national and international networks can both stimulate mathematical awareness and creativity concerning industrial problems and avoid duplication of intellectual effort."

- Matheon

you need broad enough technical skills to understand what other members are saying. The ability to listen to and learn from other team members is just as important as the ability to generate your own ideas. You need leadership and presentation skills to get your ideas across, a strategic sense of the team's goals, and the drive, discipline, and energy to meet project deadlines. One industrial scientist summarized this list as the "get-things-done factor."

Participate in or initiate programs in computational science that stress a combination of mathematics, computer science, and an application.

Summary of suggestions for students

Depending on your department's policies and experience with preparing students for industrial careers, you may need to be pro-active and go beyond the minimum requirements for your degree. Seek out opportunities to speak with alumni of your department who pursued a successful career outside academia. Develop multidisciplinary skills including an application area, computing, and mathematical modeling. As you progress in your chosen discipline, don't forget to develop a broad understanding of the mathematical sciences. Employers like applicants with a "T-shaped" profile, with depth in one area and breadth of understanding.

Pursue opportunities for summer jobs, cooperative employment, and especially internships in industry or government laboratories. These activities are likely to be helpful in finding a permanent job, as well as helping to decide whether an industrial career is right for you.

When it comes time to select an advisor, look for an advisor or co-advisor who has experience in research collaborations with industry or government scientists. You may want to find your co-advisor outside of your department.

Finally, be on the lookout for talks by industrial or government scientists in departments outside of yours or at meetings of scientific societies.

5.3 Suggestions for Faculty and Administrators

The following suggestions are designed to enhance connections among faculty in the mathematical and computational sciences, faculty members in the physical, social, and economic sciences, and nonacademic colleagues working in related disciplines. We note, however, that none of these suggestions are intended to substitute for the requirement to teach the core courses in mathematics or computer science.

- Encourage graduate degrees that involve dual mentoring by mathematics or computer science and another department, government laboratory, or industry.
- Encourage undergraduate and graduate students to take advantage of the summer programs at the NSF Mathematics Institutes or other NSF programs, such as Research Experiences for Undergraduates, especially those with a focus on industrial or business problems.
- Create new mathematics courses focusing on the techniques most needed in certain applications, and include substantial course material on those applications. It is natural to share teaching responsibilities for such courses with faculty from other departments or with nonacademic scientists.

- Participate in or initiate programs in computational science that stress a combination of mathematics, computer science, and an application. Again, try to involve scientists from other academic departments and industrial or government scientists.
- Invite speakers from other disciplines, including scientists from industry, to speak in research seminars or colloquia. Such interdisciplinary gatherings can also be organized in cooperation with other departments.
- Invite industrial or government scientists as visiting faculty for a one- or two-week period. Encourage them to conduct seminars with practical emphases, suggest changes in current courses, and participate in student-sponsored colloquia. Provide travel funds if needed.
- Build and maintain a board of advisors for the department and include faculty from computer science, from science or business disciplines, and from industry and government, to serve as co-advisors and mentors for students. Schedule a meeting of the board for undergraduate and graduate students with the theme of student preparation for jobs in industry and government.

A recurring issue in our interviews with industrial scientists and managers was the difficulty of negotiating intellectual property rights with universities. One interviewee said, “A few universities handle IP issues well, others make it very difficult.” In cases of substantial collaborative research projects, faculty and administrators should be willing to negotiate a way around IP roadblocks. Administrators especially should be warned that an over-zealous approach to intellectual property is a good way to discourage industrial partners and impair the educational mission of the university.

5.4 Suggestions for Scientists and Decision Makers in Industry and Government

Collaborations among industrial groups, government laboratories, and academic programs in the mathematical and computational sciences can be mutually reinforcing approaches to achieving organizational goals. This is, in the end, what technology transfer is about. It is not a one-way street. Therefore, we provide a list of suggestions that we believe can help industry, government, and academic organizations use the available mathematical and computational resources for mutual benefit.

- Identify and become acquainted with academic and computational scientists and establish an industry-academic program of exchanges of personnel. These can be formal or informal, short term or medium term (a week or two). This may lead to collaborative research projects, and it is a good way to identify student candidates for internships.
- Take advantage of government programs that foster interaction between academia, government, and industry. These include NSF’s GOALI program, Research Experiences for Undergraduates (REU), the industrial programs at the NSF Mathematical Sciences Institutes, and the Department of Energy’s INCITE and SciDAC programs.
- Encourage industrial and governmental employees to share applications and success stories with their academic counterparts. In many cases, academic mathematicians derive considerable satisfaction from seeing the technologies they know and love being applied in productive and interesting ways to real-world problems. Sharing these successes enables considerable bridge building between the different communities.

Encourage industrial and governmental employees to share applications and success stories with their academic counterparts. In many cases, academic mathematicians derive considerable satisfaction from seeing the technologies they know and love being applied in productive and interesting ways to real-world problems.

- Be willing to serve on advisory boards for government programs on research and education.
- Become active in scientific societies. Volunteer to serve on committees or interest groups in these societies, particularly organizing committees of meetings, and suggest topics for short courses at national meetings.

For policy makers, our interviews highlighted two areas of government policy that adversely affect the environment for industrial research. Both of these areas extend far beyond the mathematical sciences, but we mention them here for completeness.

First, restrictions on visas for foreign students and nationals continue to be a burden. For example, computer-aided geometric modeling is an area with fewer American students compared to Europe; thus American aerospace and automotive companies that rely on geometric modeling struggle to find enough qualified candidates. While visa policies may depend on many other factors, scientists need to make their voices heard and emphasize that the mobility of qualified scientists can only benefit American competitiveness.

Second, our interviewees at several companies pointed out that intellectual property policies at universities involving government-funded work, since the Bayh-Dole Act of 1980, have tended to discourage cooperation with industry. Although there are examples of successful policies, there are also examples of policies that make it difficult for industry to cooperate. University decision makers should review their policies to make sure that they encourage cooperation.

6 Conclusion

This report updates the 1996 *SIAM Report on Mathematics in Industry* in light of advances in the mathematical and computation sciences, changes in the economy, and the evolution of applications in industry and government.

Our economy and that of the developed world is in the midst of a transition from a product-based economy to a knowledge-based economy, in which innovation and advances are driven by the expertise of individuals and organizations. We are convinced that the mathematical and computational sciences have contributed and will continue to contribute to the nation's economy by providing new knowledge and new ways of doing business. Universities will continue to play a key role as the source of talented individuals with the desire and ability to apply mathematical knowledge to real-world problems. But this will not happen by itself; university faculty must actively encourage students to consider careers in industry and prepare those students for the very different world they will encounter upon graduation.

We reaffirm the last conclusion of the 1996 report in the current context. Ideas and inspirations have flowed strongly in all directions between the mathematical sciences, computational sciences, and applications. Nonacademic applications enrich and deepen the mathematical and computational sciences as well as a wide variety of other fields, including science, engineering, medicine, and business.

We are convinced that the mathematical and computational sciences have contributed and will continue to contribute to the nation's economy by providing new knowledge and new ways of doing business.

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