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Interpreting Deep Learning: The Machine Learning Rorschach Test?

By Adam S. Charles

Theoretical understanding of deep learning is one of the most important tasks facing the statistics and machine learning communities. While multilayer—or deep—neural networks (DNNs) originated as engineering methods and models of biological networks in neuroscience and psychology, they have quickly become a centerpiece of the machine learning toolbox and are simultaneously one of the simplest and most complex methods. DNNs consist of many interconnected nodes that are grouped into layers (see Figure 1a) with stunningly simple operations. The n^{th} node of the network at a given layer i, $x_i(n)$ is merely a nonlinear function $f(\cdot)$ (e.g., a saturating nonlinearity) applied to an affine function of the previous layer

$$x_i(n) = f\left(\boldsymbol{w}_i(n)\boldsymbol{x}_{i-1} + b_i(n)\right),$$

where $\boldsymbol{x}_{i-1} \in \mathbb{R}^{N_i}$ represents the previous layer's node values, $\boldsymbol{w}_i(n) \in \mathbb{R}^{N_i}$ are the weights that project onto the n^{th} node of the current layer, and $b_i(n)$ is an offset. However, these simple operations introduce complexity due to two factors. First, the sheer number of nodes creates an explosion of parameters ($\boldsymbol{w}_i(n)$ and $b_i(n)$), amplifying the effects of nonlinearities. Moreover, the weights and offsets are learned by optimization of a cost function via iterative methods, such as back-propagation. Despite the resulting complexity, researchers have utilized DNNs to great effect in many important applications.

A "perfect storm" of large, labeled datasets; improved hardware; clever parameter constraints; advancements in optimization algorithms; and more open sharing of stable, reliable code contributed to the relatively recent success of DNNs in machine learning. DNNs originally provided state-of-the-art results in image classification, i.e., the now-classic task of handwritten digit classification that powers devices like ATMs. While DNN applications have since spread to many other areas, their

See Deep Learning on page 4

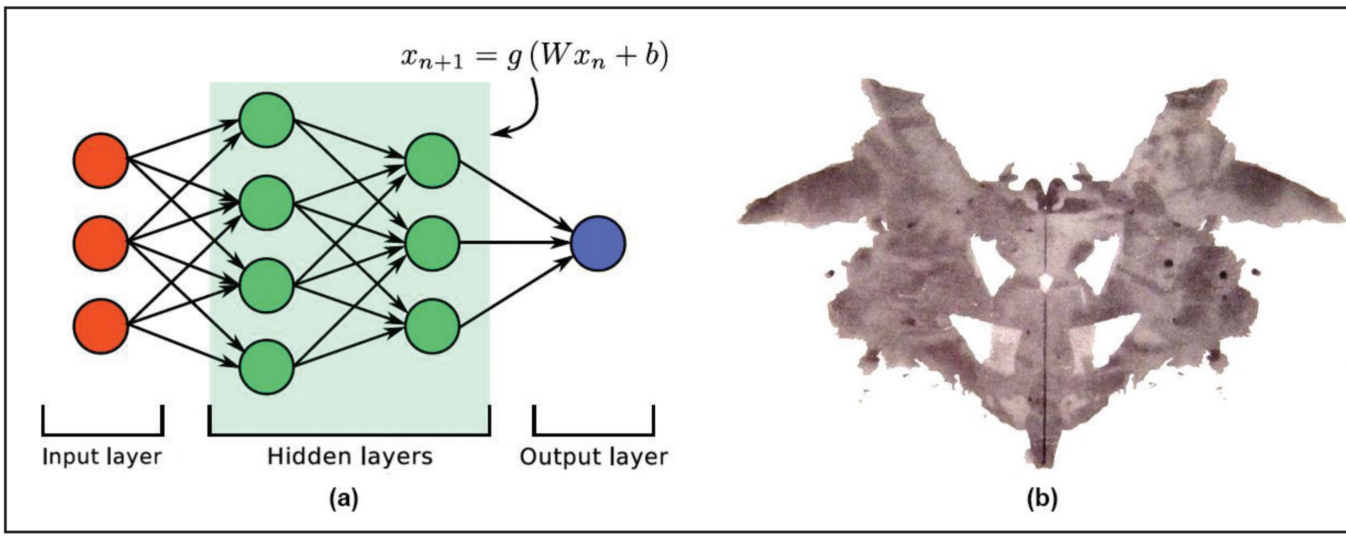

Figure 1. *What do you see? We can view deep neural networks (DNNs) in many ways.* **1a.** *Stylistic example of a DNN with an input layer (red), output layer (blue), and two hidden layers (green). This is a sample "ink blot" for DNN theory. Figure courtesy of Adam Charles.* **1b.** *Example of a normalized ink blot from the Rorschach test. Public domain image.*

Detecting Gerrymandering with Mathematics

By Lakshmi Chandrasekaran

Earlier this year, federal judges struck down the North Carolina state map as unconstitutional because it had been partisan gerrymandered. A few weeks later, Pennsylvania district maps met the same fate on similar grounds. While the Supreme Court has upheld the unconstitutionality of the Pennsylvania maps, it recently sidestepped its decision on partisan gerrymandering in Wisconsin and Maryland, letting the maps stand for the upcoming fall elections.

Gerrymandering comes into play every ten years after completion of the census. The political party in power in state legislatures uses census information to alter congressional districts in its favor via a process called redistricting. Such fudging of maps has occurred since 1812, and has been the target of numerous lawsuits. Although the Supreme Court has ruled racial gerrymandering unconstitutional, it has so far declined to overturn gerrymandering on partisan grounds.

Judiciable Standard to Curb Gerrymandering

Partisan gerrymandering involves packing vast swathes of the opponent's supporters into fewer districts, or cracking areas of opposition majorities across many districts — thereby diluting the majority. These actions reap benefits over several elections. While the Supreme Court's recent ruling declared extreme partisan gerrymandering unconstitutional, a judicially manageable standard measuring the "extremeness" of a given map is still lacking.

"The Supreme Court signed up for mathematics by ruling that a partisan gerrymander is unconstitutional if it is extreme," Eric Lander, founding director of the Eli and Edythe L. Broad Institute of MIT and Harvard, said. "There's a constitutional right to recognizing what is too far — and that is mathematical." Lander wrote a court document[1] last summer supporting the use of a statistical outlier standard. Jonathan Mattingly, professor of mathematics at Duke University, served as a consultant to the document. Mattingly has spent five years mathematically dissecting the structure of a typical redistricting to identify gerrymandering.

His interest was inspired by the 2012 elections for the North Carolina House of Representatives. "Republicans won the majority with nine out of 13 seats," Mattingly said. "I was at a meeting where someone said that Democrats won the majority of the votes. That was shocking, since they should have had at least seven

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[1] http://www.campaignlegalcenter.org/document/gill-v-whitford-us-supreme-court-amicus-brief-eric-s-lander

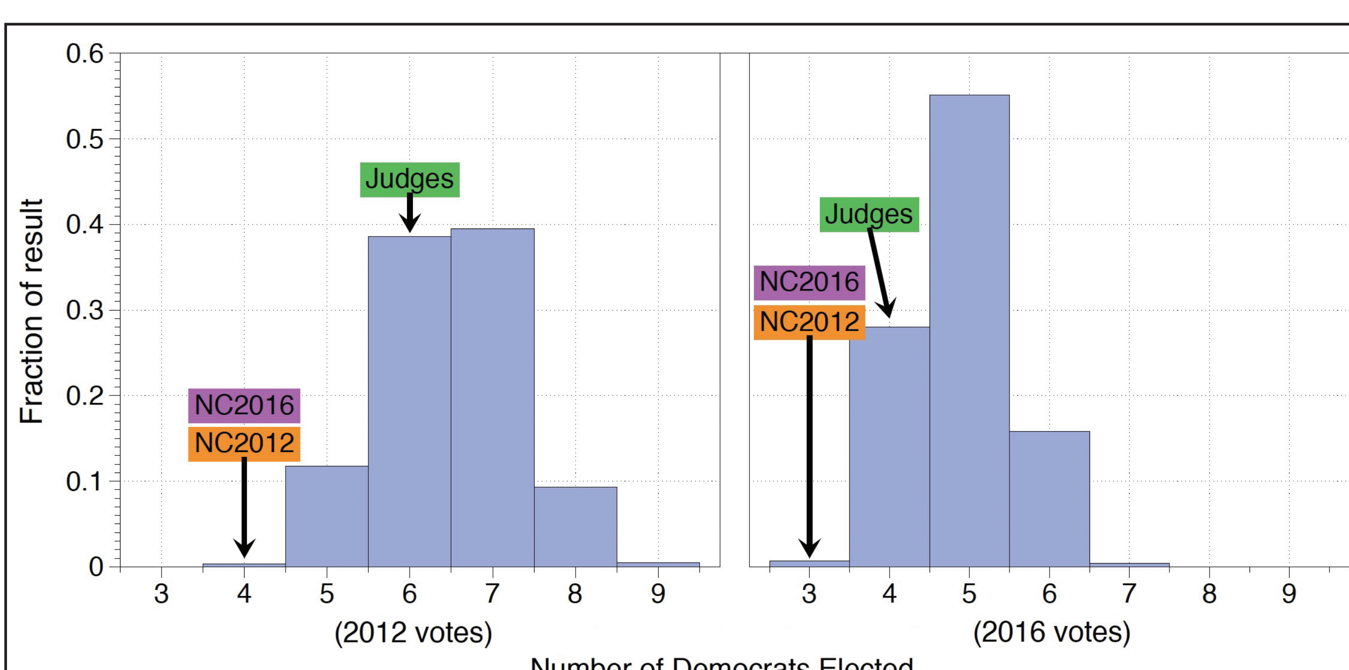

Figure 1. *Probability distribution of the congressional delegation's composition for the 2012 and 2016 North Carolina congressional elections. Based on the sample of redistricting plans, Democrats could secure four to nine and three to seven seats for the 2012 and 2016 congressional elections respectively. The plan used by the judges from a bipartisan commission shows that Democrats would win six seats in 2012 and four in 2016. In comparison, the 2012 and 2016 North Carolina congressional elections (in orange and purple) show a heavy bias towards Republicans. Image courtesy of [1].*

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Advancing SIAM

FROM THE SIAM PRESIDENT
By Nicholas Higham

At the end of April, SIAM held a two-day strategic planning workshop called ADVANCE. The 25 attendees included SIAM officers and staff, as well as other members of the SIAM community selected to bring a diverse range of viewpoints. The event, which took place just outside of Philadelphia, Pa., was facilitated by consultant and creativity expert Dennis Sherwood (see accompanying sidebar).

The impetus for the workshop came from the July 2017 Board and Council meetings, at which attendees recognized that it was timely for SIAM to engage in some strategic planning. The Board duly authorized the expenditure necessary to run an appropriate event. Over several months, executive director Jim Crowley, chief operating officer Susan Palantino, and I developed—in consultation with Sherwood—a set of approximately 30 exercises for workshop use. Each one presented a list of questions on a particular topic and asked breakaway groups of six or so participants to individually and silently write down their thoughts, share them with each other, and then ask, "how might this be different?" The topics included journals, membership, diversity, conferences, fundraising, chapters and sections, new products, and the ways in which SIAM could make use of unlimited funds (a dream scenario designed to encourage new ideas).

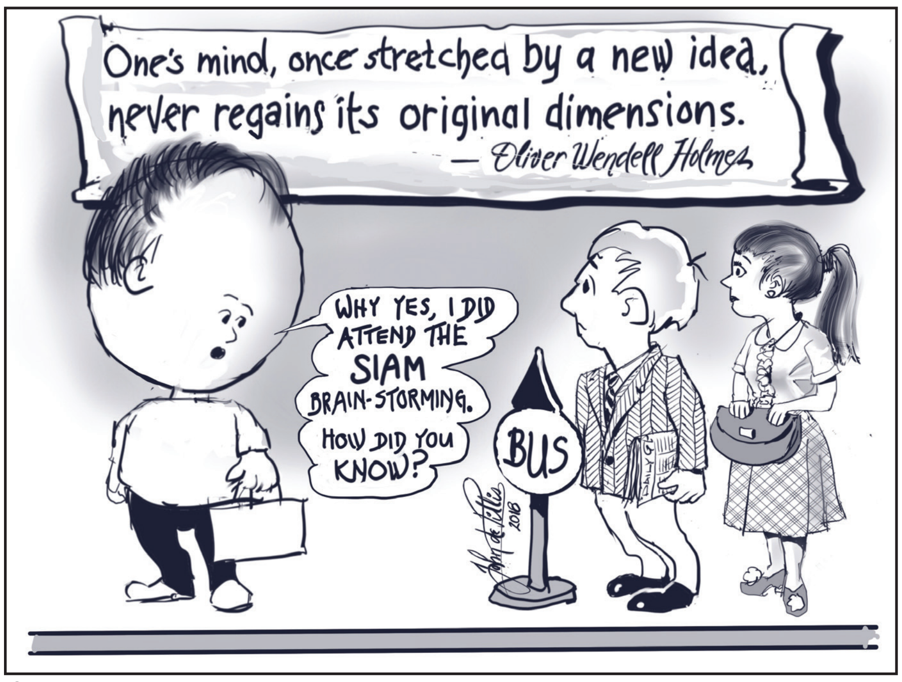
The group generated a large number of objectives that SIAM staff, officers, committees, and the Board and Council will take forward, including through discussions at the 2018 Annual Meeting in Portland, Ore., this July. One idea aims to substantially increase SIAM's fundraising efforts. Others target various aspects of SIAM's journals programme, especially the utilization of technology; the provision of a vehicle for industrial members to communicate open problems and interesting applications; the enhancement of membership benefits; and an increase in the number of SIAM sections and student chapters.

A recurring theme was the need to better exploit data when making decisions, though participants recognized that the required data is not always easy to obtain. Sherwood pointed out that inadequate data is often an excuse for organizations' lack of action, and urged that SIAM not fall into this trap.

After two long days of hard work, ADVANCE participants were exhausted but energized by the productive discussions. Several contributors observed that achieving the workshop's results would have been difficult with the (necessarily) short discussions that occur at Board and Council meetings, and that the group of attendees was even more diverse than at those bodies.

I look forward to working with SIAM staff and volunteers to take the ideas forward over the coming months.

Nicholas Higham is Royal Society Research Professor and Richardson Professor of Applied Mathematics at the University of Manchester. He is the current president of SIAM.



Cartoon created by mathematician John de Pillis.

How to Have Great Ideas

I had been thinking about creativity for some time, and was stuck. Can you have ideas "on demand"? That seems crazy; ideas just "happen," don't they?

These thoughts were on my mind as I browsed the window of a game shop, seeking a present for my son's eighth birthday. Then I saw a chess set, with all of the pieces laid out. But something was wrong — whoever had placed the pieces did not know the rules of chess, as the positions of the knights and bishops had been flipped. BANG! Creativity! That's it!

As we all know, the starting positions of chess pieces are predetermined: the rooks in the corners, the king and queen in the middle, the pawns at the front. But suppose players *could choose where to put their pieces*. Game play would then be different, and none of the conventional gambits would work.

I became aware of "chess variants" much later, but that sudden insight answered my question about creativity — how to have ideas "on demand." Start with what you know (in chess, all the starting positions are given), ask how this might be different ("suppose the rooks do not start in the corners"), and then let it be...

It's that simple, and it works every time. This philosophy also embodies two very important first principles:

• When it comes to creativity, the goal is not novelty, but rather *difference* — difference from the *status quo*, which is nicely grounded in reality, for we know what the status quo is.

• As Arthur Koestler points out in *The Act of Creation*, creativity is the discovery of a fresh pattern formed from existing elements, not a "bolt from the blue." It is all about things that are already there, simply configured in different patterns and combinations.

— Dennis Sherwood

Dennis Sherwood runs his own U.K.-based consultancy, The Silver Bullet Machine Manufacturing Company Limited.


Members of the SIAM community gather in a breakaway group to discuss membership opportunities during ADVANCE, a recent strategic planning workshop that yielded various ideas about future directions for SIAM. Photo credit: Dennis Sherwood.

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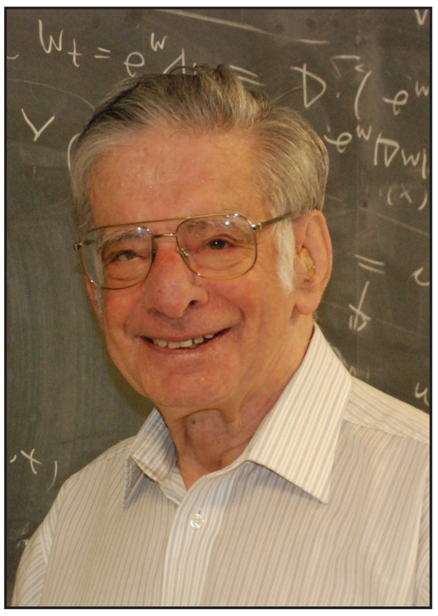
Obituary: Hans F. Weinberger

By Donald G. Aronson, Peter J. Olver, and Fadil Santosa

Austrian-born mathematician Hans Felix Weinberger passed away at the age of 88 in Durham, N.C., on September 15, 2017. He was a faculty member of the University of Minnesota's School of Mathematics for 37 years, and retired as professor emeritus in 1998. Hans played a vital role in elevating the school to its current eminence and establishing the Institute for Mathematics and its Applications (IMA) on the Minneapolis campus. He specialized in the study of various aspects of partial differential equations; notable topics include his influential work on isoperimetric inequalities and the estimation of eigenvalues, as well as contributions to and applications of the maximum principle. More recently, Hans turned his attention to mathematical biology. He was active in research throughout his academic life, a habit he maintained through retirement and until his death.

Hans was born in Vienna, Austria, on September 27, 1928. The Weinberger family emigrated to the U.S. in 1938 and eventually settled in Altoona, Pa. Hans was an excellent student and graduated from high school at the age of 16. He was very interested in science and became a finalist in the 1945 Westinghouse Science Talent Search for his design of a self-inflating life vest for the U.S. Navy, which earned him a patent. Hans enrolled as a physics major at the Carnegie Institute of Technology (now Carnegie Mellon University), and received his M.S. in physics in 1948 and his Sc.D. in mathematics in 1950 — at the age of 21. His thesis, entitled "Fourier Transform of Moebius Series," was supervised by Richard Duffin. Interestingly, the legendary John Nash was Hans's roommate for one semester; Sylvia Nasar's *A Beautiful Mind* offers a brief account of their relationship.

After receiving his Sc.D., Hans worked at the University of Maryland, College Park and spent 10 years at its Institute of Fluid Dynamics and Applied Mathematics. In 1960, he joined the faculty of the University of Minnesota as a full professor, serving as department head from 1967 to 1969. He supervised nine Ph.D. students and counted David Gilbarg, Joseph Keller, Lawrence Payne, George Pólya, and Murray Protter among his collaborators. Hans also collaborated locally with Donald Aronson, Leonid Hurwicz (Nobel laureate in economics) and James Serrin, among others. He wrote or coauthored over 140 research papers, with the last appearing in 2015.



Hans Felix Weinberger, 1928-2017. Photo courtesy of the Institute for Mathematics and its Applications.

Hans published three influential books: *Variational Methods for Eigenvalue Approximation*, based on his Conference Board of the Mathematical Sciences lectures; *Maximum Principles in Differential Equations* with Protter, which was the standard reference for many years; and the widely-used textbook *A First Course in Partial Differential Equations*. In 1986, he was elected as an American Academy of Arts and Sciences member. He was also a member of the inaugural class of American Mathematical Society fellows.

In 1979, in response to the National Science Foundation's (NSF) request for proposals to establish a new national mathematics research institute, Hans—along with George Sell and Willard Miller, Jr.—submitted a proposal to establish the IMA at the University of Minnesota. They envisioned an institute that would look outward from the core of mathematics towards applications, and unite mathematicians with scientists from industry and other disciplines to work on problems of mutual interest. The proposal—though radical—was funded by the NSF, and Hans served as the institute's first director from 1982 to 1987. Under his leadership, the IMA quickly became known for its cutting-edge scientific programs; unique, collaborative atmosphere; and reputation as a training ground for postdoctoral researchers. Hans was very much a hands-on director, attending nearly all lectures and collaborating with visitors and postdocs.

As a lifelong member of SIAM, Hans was active in the organization since its inception in 1952. His earliest SIAM publication dates back to 1957, in the *Journal of the Society for Industrial and Applied Mathematics* (the only SIAM journal at the time). He later served on the editorial boards of the *SIAM Journal on Matrix Analysis and Applications* and the *SIAM Journal on Mathematical Analysis*, and was elected to the Board of Trustees in 1983.

Hans was modest and unassuming, but possessed prodigious mathematical talent. His office door was always open, welcoming students, colleagues, and visitors to come in and discuss their current work. Hans was quick to see the essence of any problem and often able to offer extremely helpful comments and suggestions. He was, in sum, an ideal colleague.

Hans is survived by his wife Laura and three children: Catherine, Sylvia, and Ralph.

Donald G. Aronson is professor emeritus in the School of Mathematics at the University of Minnesota. His research interests include analysis of partial differential equations—especially nonlinear diffusion—and applied dynamical systems. He collaborated with Hans Weinberger on a number of publications. Peter J. Olver has been at the University of Minnesota since 1980 and has headed its School of Mathematics since 2008. His research interests revolve around wide-ranging applications of symmetry and Lie groups to differential equations. He has written over 140 research papers and several books. Fadil Santosa is a professor of mathematics at the University of Minnesota, and served as director of the Institute for Mathematics and its Applications from 2008 to 2017. He has worked in several areas of applied mathematics, including inverse problems, optimal design, and optics.

A Look at the Plenary Talks at MPE18

The second SIAM Conference on Mathematics of Planet Earth (MPE18)[1] will take place this September in Philadelphia, Pa. As in the 2016 inaugural conference, researchers will use plenary talks, minisymposia, and contributed papers and posters to communicate mathematical problems and results about Earth as a physical system, a system supporting life, a system managed by humans, and a system at risk. This year's plenary talks will focus on the role of humans in managing natural resources, predicting and mitigating hazards, and shaping a livable environment. The following are capsule previews of these four talks.

Clint Dawson

Where Water Meets Land: The Mathematics of the Coastal Ocean

Coastal regions around the world are home to millions of people. These regions are economic engines that hold delicate ecosystems. However, as we have seen in the past decade, they face threats from a variety of factors, including a combination of hazardous events, climate change, and overdevelopment. In this talk, I will explore problems related to water at the coast: the interaction of ocean with land, the impacts of tropical storms and hurricanes, and our attempts to protect coastal populations while facing the realities of an uncertain future. I will describe mathematical challenges, along with model- and data-driven studies to better understand these issues.

Clint Dawson is the John J. McKetta Centennial Energy Chair in Engineering at the University of Texas at Austin. He is also a professor in the Institute for Computational Engineering and Sciences, and head of the Computational Hydraulics Group.

Suzanne Lenhart

Optimal Control Techniques Applied to Management of Natural Resource Models

Humans manage natural resources for a variety of reasons, such as to optimally harvest them with minimal ecological impact or to suppress or prevent large-scale damaging events. Through two examples—one involving fish harvesting and the other concerning fire events—my talk will demonstrate the use of techniques from optimal control theory for such purposes. In the first example, I model the optimal harvest of fishery stocks while minimizing negative effects on fish habitat with a system of partial differential equations that capture the problem's spatiotemporal dynamics. I then investigate the tradeoff between managing forests for fire prevention and monetary spending to suppress fires with a model that incorporates the stochasticity of large-scale fire events.

Suzanne Lenhart is a Chancellor's Professor of Mathematics at the University of Tennessee and the Associate Director for Education and Outreach at the National Institute for Mathematical and Biological Synthesis. She also worked as a part-time research scientist at Oak Ridge National Laboratory for 22 years.

The second SIAM Conference on Mathematics of Planet Earth (MPE18) will take place this September in Philadelphia, Pa.

Bertrand Lemasson

Insights from Studying the Interface between Sentience and Sociability in Animal Movement Patterns

Animals' sensory capabilities have important fitness consequences, but so does their sociability. Creatures large and small, from herds of grazing animals to schools of fish, rely on their senses for both navigation through the world and social interactions, which in turn influence their abilities to track resources, avoid threats, and find mates. Efforts to model and manage natural resources rarely take such social processes into account. An important complication is that the benefits of social interaction are context-dependent; organisms must rapidly decide when and whom to follow. This presentation will demonstrate how individual animal interactions arise and why their study is important. It will examine the influence of these interactions on individual capabilities like stamina, memory, and the ability of ephemeral social ties to generate collective movements that deviate drastically from our expectations. I will then review how we can integrate such information into mathematical models and put it into practice to improve ecosystem management.

Bertrand Lemasson is a behavioral ecologist at the U.S. Army Engineer Research and Development Center at the Hatfield Marine Science Center in Oregon. He is currently part of an interdisciplinary group that studies questions at the interface of cognitive ecology and ecohydraulics.

Claudia Sagastizabal

How Energy Optimization is Responding to the Challenge of Decarbonizing our Economies

Worldwide efforts to complete the transition to sustainable energy systems rely crucially on the shift from electricity generation based on fossil fuels to renewable energy sources. Unfortunately, the process is fraught with complications: such renewable sources are often intermittent, and their storage requires more distributed generation, new flexibility markets, and the facilitation of the roles of "prosumers" (producers/consumers) and aggregators in the energy value chain. These changes in technology and market structure, combined with the expected massive electrification of transportation, will make electricity supply and demand less predictable but potentially more versatile. The new paradigms for the operation and pricing of energy systems result in multiobjective and bilevel optimization models whose inherent nonconvexity poses challenges from the mathematical viewpoint — on both theoretical and numerical levels. During this talk, I will discuss the resulting opportunities for mathematical researchers and new problems of control and optimization that arise in this context.

Claudia Sagastizabal is an independent mathematical researcher based at the Instituto Nacional de Matemática Pura e Aplicada in Rio de Janeiro, Brazil. Trained in numerical optimization, she has extensive experience working with large companies in the energy and automobile sectors in Europe and South America.

[1] https://www.siam.org/conferences/CM/Main/mpe18

Integrated Catastrophic Risk Management: Robust Balance between Ex-ante and Ex-post Measures

By Yuri M. Ermoliev, Stephen M. Robinson, Elena A. Rovenskaya, and Tatiana Y. Ermolieva

Humans continually face catastrophes involving natural disasters, such as floods, droughts, hurricanes, and large-scale fires. In today's highly interconnected world, losses from such incidents have increased greatly due to growing population densities, asset concentration in disaster-prone areas, and environmental change from anthropogenic impacts.

Catastrophic natural disasters are random events that are rare but very impactful. Traditionally, most catastrophic losses are paid ex-post (adaptively) by individuals (property owners), government agencies, insurers and reinsurers, charity institutions, and international organizations, rather than through explicit ex-ante (forecast-based) arrangement via long-term strategic decisions [7].

Moreover, there is typically little or no prior agreement as to who should bear what portions of the monetary cost. In anticipation of the need to cover potentially large losses in an ad-hoc way, responsible agencies retain certain budget resources for this purpose. However, such retention reduces the options for profitable investment; in the case of large funds, it can potentially stifle economic growth.

We propose that intensification of ex-ante measures—combined with a more intelligent method for setting aside resources to build adaptive capacities for ex-post compensations, contingent credits, catastrophic bonds, monitoring, and regulation—can significantly reduce the overall burden on national economies and strike a healthy balance between economic growth and security. Integrated long-term approaches to risk management and economic development, with an explicit emphasis on the possibility of rare high-consequence catastrophes, enable effective decisions in this context. This tactic requires one to account for the dependence between decisions and risk distributions.

Existing observations demonstrate the increasing magnitude and variability of risks, indicating that one cannot assume catastrophic risk distribution to be Gaussian; in fact, they are skewed and have fat tails. Their focus on tails makes quantile-based risk measures—e.g., value at risk (VaR) and conditional value at risk (CVaR)—more appropriate than variance-based measures applicable only to Gaussian distributions. We have developed and applied a new approach to stochastic optimization in a number of case studies. Our strategy allows us to include quantile-based performance functions in decision support models for integrated catastrophic risk management. These models are characterized by complex nested distributions shaped by the decisions of policymakers. Here we briefly outline this approach, its

See **Risk Management** *on page 6*

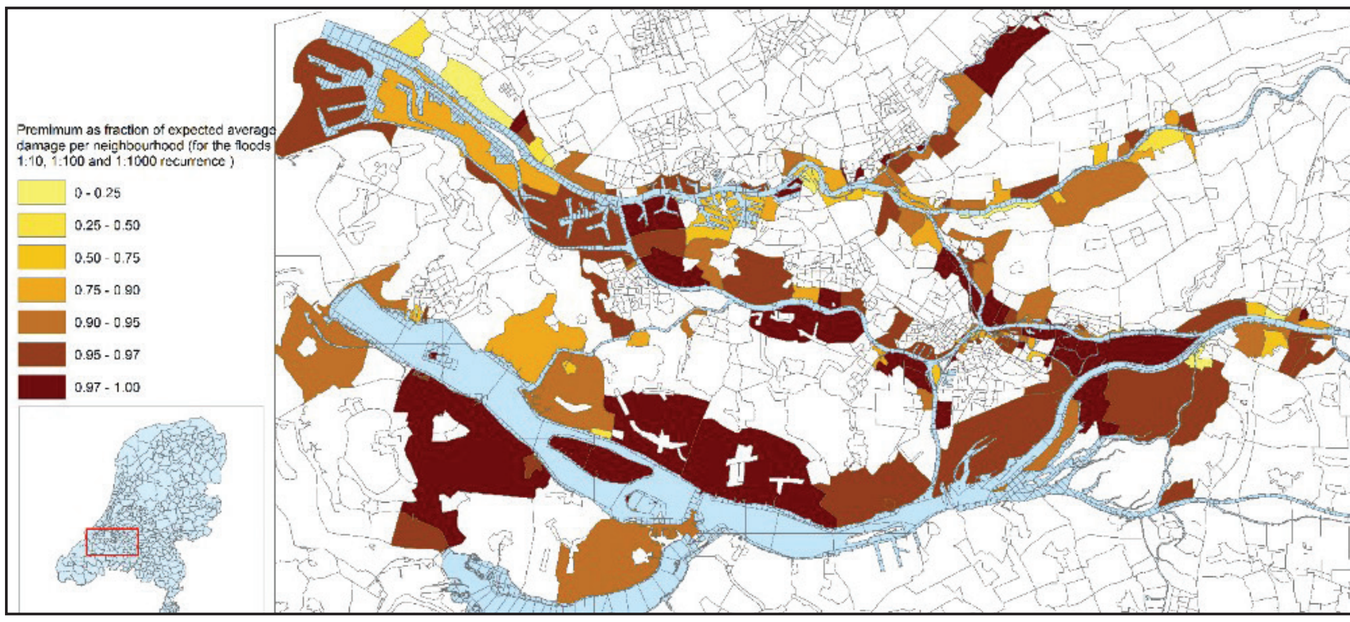


Figure 1. *Geographical distribution of robust premiums as percentage of the 100-year flood damages. Figure courtesy of [6].*

Deep Learning

Continued from page 1

well-publicized success in image classification has encouraged continued work and produced other amazing technologies, such as real-time text translation.

Unfortunately, DNN adoption powered by these successes—combined with the open-source nature of the machine learning community—has outpaced our theoretical understanding. We cannot reliably identify when and why DNNs will make mistakes. Though this does admittedly provide comic relief and fun fodder in research talks about applications like text translation, a single error can be very costly in tasks such as medical imaging. Additionally, DNNs have shown susceptibility to so-called adversarial examples, or data specifically designed to fool a DNN. We can generate such examples with imperceptible deviations from an image, causing the system to misclassify an image that is nearly identical to one that is correctly classified. Adversarial examples in audio applications can also exert control over popular systems like Amazon's Alexa or Apple's Siri, allowing malicious access to devices containing personal information. As we utilize DNNs in increasingly sensitive applications, a better understanding of their properties thus becomes imperative.

Early DNN theory employed learning and function approximation theory to analyze quantities like the Vapnik-Chervonenkis dimension. Although such quantities characterize DNN complexity with respect to training data, many important questions pertaining to generalization, expressibility, learning rule efficiency, intuition, and adversarial example susceptibility remain. More recent interpretations begin to address these questions and fall into three main analysis styles. First are methods to understand the explicit mathematical functions of DNNs by demonstrating the ways in which specific combinations of nonlinearities and weights recover well-known functions on the data. The second approach analyzes theoretical capabilities and limitations of the sequence of functions present in all DNNs — again, given assumptions on the nonlinearities and weights. These analyses include quantifications of the data-dependent cost-function landscape. Finally, a third class of techniques focuses on learning algorithms that solve the high-dimensional, nonlinear optimization programs required to fit DNNs, and attempts to characterize the way in which these algorithms interact with specific DNN architectures.

Advances in DNN theory include many different sources of intuition, such as learning theory, sparse signal analysis, physics, chemistry, and psychology. For example, researchers have related the iterative affine-plus-threshold structure to algorithms that find sparse representations of data [3]. A generalization of this result temporally unrolls the algorithmic iterations that solve regularized least-squares optimization programs

$$\arg\min_{\boldsymbol{x}} \left[\left\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\right\|_2^2 + \lambda R(\boldsymbol{x}) \right], \quad (1)$$

via a proximal projection method that iteratively calculates

$$\widehat{\boldsymbol{x}}_{t+1} = P_\lambda\left(\widehat{\boldsymbol{x}}_t + \boldsymbol{A}^T\left((\boldsymbol{y} - \boldsymbol{A}\widehat{\boldsymbol{x}}_t)\right)\right), \quad (2)$$

where $P_\lambda(\boldsymbol{z})$ is the nonlinear proximal projection

$$\min_\beta \left\|\boldsymbol{z} - \boldsymbol{x}\right\|_2^2 + \lambda R(\boldsymbol{x}).$$

When the regularization function $R(\cdot)$ is separable, $R(\boldsymbol{z}) = \sum_k R(z_k)$, the proximal projection is a pointwise nonlinearity that mimics DNN architectures. Treating $\widehat{\beta}_t$ as different vectors at each algorithmic iteration, these variables can map to the node values at subsequent DNN layers, with weights $\boldsymbol{w} = \boldsymbol{A}^T\boldsymbol{A} + \boldsymbol{I}$ between layers, a bias $\boldsymbol{b} = \boldsymbol{A}^T\boldsymbol{y}$, and nonlinearity defined by the proximal projection. This example offers a sense of the intuitions gleaned by mapping the network operations onto well-known algorithms. And this single interpretation is just the tip of the iceberg; a larger, non-exhaustive list of additional explanations is available in [1].

The sheer quantity of recent publications on DNN theory demonstrates just how relentless the search for meaning has become. An interesting pattern begins to emerge in the breadth of possible interpretations. The seemingly limitless approaches are mostly constrained by the lens with which we view the mathematical operations. Physics-based interpretations stem from researchers with a physics background. Connections to sparsity and wavelets come from well-known scientists in those fields. Ultimately, the interpretation of DNNs appears to mimic a type of Rorschach test — a psychological test wherein subjects interpret a series of seemingly ambiguous ink-blots (see Figure 1b, on page 1). Rorschach tests depend not only on *what* (the result) a subject sees in the ink-blots but also on the *reasoning* (methods used) behind the subject's perception, thus making the analogy particularly apropos.

On the one hand, these diverse perspectives are unsurprising, given DNNs' status as arbitrary function approximators. Specific network weights and nonlinearities allow DNNs to easily adapt to various narratives. On the other hand, they are not unique in permitting multiple interpretations. We can likewise view standard, simpler algorithms through many lenses. For example, we can derive the Kalman filter—a time-tested algorithm that tracks a vector over time—from at least three interpretations: the orthogonality principle, Bayesian maximum *a-priori* estimation, and low-rank updates for least-squares optimization. These three derivations allow people with different mathematical mindsets (i.e., linear algebra versus probability theory) to understand the algorithm. Yet compared to DNNs, the Kalman filter is simple; it consists of only a handful of linear-algebraic operations. Its function is completely understood, allowing for validation of each viewpoint despite the different underlying philosophies.

Similar validation for DNN theory requires a convergence of the literature. We must distinguish between universal results that are invariant to the analysis perspective and those that are specific to a particular network configuration. A healthy debate is already underway, with respect to the information bottleneck interpretation of DNNs [4, 5]. We should also work to better understand the interactions between functions that DNNs perform, their mathematical properties, and the impact of optimization methods. Unfortunately, DNN complexity introduces numerous challenges. Many standard tools, such as those that attempt to comprehend a model's generalization from training data [6] or empirically assess important network features [2], are difficult to apply to DNNs. Luckily, there is no shortage of excitement, and we continue to enhance our understanding of DNNs with time. The community is also beginning to coalesce, and dedicated meetings—like workshops at the Conference on Neural Information Processing Systems and the recent Mathematical Theory of Deep Neural Network symposium at Princeton University—will further accelerate our pace.

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Gerrymandering

Continued from page 1

of the 13 seats." He successfully testified about these numbers in October 2017 during *Common Cause v. Rucho* — the North Carolina partisan gerrymandering case.

Mattingly began to ponder the significance of seven as the magic number. "Maybe it's not fair that Republicans won nine seats, but it could be seven or eight," he said, highlighting the difficulty of discerning whether the number of seats won by any party is fair, given an election outcome. He also investigated the number of Democrats or Republicans that a district should have when affiliated with a particular party. Essentially, how much is too much?

Evaluating Partisan Gerrymandered Maps

Along with his postdoctoral fellow Gregory Herschlag and a team of students, Mattingly employs sampling methods to estimate the entire population of admissible redistricting plans. They accomplish this by sampling a probability measure placed on compliant redistricting plans. Mattingly's goal is to characterize the level of gerrymandering in a district plan by identifying ways in which a plan deviates from what is typical. The team also utilizes sampling methods to estimate the population of redistricting's characteristic and label outliers.

Districts are required to comply with certain federal and state criteria in order to be viable. To construct his model, Mattingly considers the district standards proposed by North Carolina legislation. The first of these is *compactness*, which enables the use of geometry to quantify a district. Mattingly defines compactness with the isoparametric score (popular in legal literature) — the ratio between the square of the perimeter and district area. Compared to other measures, the isoparametric measure is less forgiving to undulating district boundaries.

Since North Carolina has 13 districts, Mattingly's model defines the score as

$$J_I(\xi) = \Sigma_{i=1}^{13} \frac{\left[boundary\left(\partial D_i(\xi)\right)\right]^2}{\left[area\left(D_i(\xi)\right)\right]}, \quad (1)$$

where $D_i(\xi)$ is the i[th] district and $\partial D_i(\xi)$ denotes the corresponding boundary. The function $\xi : V \rightarrow \{1, 2 \ldots 13\}$ represents the redistricting plan and covers the 13 districts.

The second criterion ensures that the *state population is evenly distributed* across districts, as mandated by legislation. One defines it as

$$J_P(\xi) = \sqrt{\Sigma\left(\frac{pop(D_i(\xi))}{pop_{ideal}} - 1\right)^2}, \quad (2)$$

where an ideal population $pop_{ideal} = \frac{N_{pop}}{13}$.

The third stipulation ensures *minimal splitting of counties* across districts to maintain communities of interest. A single county becomes a split county if it is broken into two districts. "We want to penalize whenever you split the county," Mattingly said. "In North Carolina, the Wake and Mecklenburg counties are split where Raleigh and Charlotte are respectively located. Both counties have too many people for one congressional district. The score penalizes whenever the county is further split, and we wanted to use the score to limit it to two splits utmost — hence the soft penalization." The metric Mattingly thus described is called the county score function, and is given by

$$J_C(\xi) = \{\#\, of\ counties\ split\ between\ two\ districts\}.\ W_2(\xi) + M_c.\ \{\#\, of\ countries\ split\ between\ three\ or\ more\ districts\}.\ W_3(\xi),$$

with $W_2(\xi)$ and $W_3(\xi)$ as the weight functions and defined as

$$W_2(\xi) = \Sigma(Fraction\ of\ county\ VTDs\ in\ second{-}largest\ intersection\ of\ district\ with\ county)^{1/2}$$

$$W_3(\xi) = \Sigma(Fraction\ of\ county\ VTDs\ not\ in\ first\ or\ second{-}largest\ intersection\ of\ district\ with\ county)^{1/2}. \quad (3)$$

$W_2(\xi)$ and $W_3(\xi)$ are summed over counties split between two and three districts respectively. But what does "second-largest intersection of district with county" entail? "Splitting the county into two uneven chunks of one large and one small, such as 90-10, is better than 50-50," Mattingly said. In the case of a 90-10 split, "10" is used. When the county is split in three or more different ways, M_c—a large constant—reflects the heavy penalty.

The Voting Rights Act (VRA) of 1965, which ensures that minorities elect a fair number of representatives that accurately mirrors their population, is the *final* criterion. African Americans make up 20 percent of North Carolina's population. Thus, the 2016 interpretation of VRA stipulation warrants that they elect leaders from at least two districts, defined by

$$J_m(\xi) = \sqrt{H(44.48\% - m1)} + \sqrt{H(36.2\% - m2)}, \quad (4)$$

with $m1$ and $m2$ representing the current percentage of the African American minority population living in districts with first- and second-highest percentage of the community, determined by the 2016 North Carolina redistricting plan to be 44.48 and 36.2 percent respectively. H is defined as $H(x) = 0, x \leq 0$ and $H(x) = x, x > 0$. If $m1$ and $m2$ underrepresent the current percentage of African Americans, a positive value for $J_m(\xi)$ results, thus converting the score into a penalty.

Mattingly calls these mathematical models of conditions "soft versions of the constraints," referring to smoothing terms such as county-splitting constants—W_2 and W_3 in (3), and a square root function in (4)—to avoid discrete jumps and instead provide a smooth (continuous) ramping of values.

The researchers use a score function to add these subscore functions:

$$J(\xi) = w_p J_p(\xi) + w_I J_I(\xi) + w_c J_c(\xi) + w_m J_m(\xi).$$

The weights given by w are all positive constants.

Redistricting plans define a probability distribution function $P_\beta(\xi) = \frac{e^{-\beta J(\xi)}}{Z_\beta}$. $\beta > 0$ is characterized as the "inverse temperature," analogous to the constant used in thermodynamics with an exponential distribution — a standard technique in Bayesian sampling. Thermodynamically speaking, low "energy"—represented by $\beta J(\xi)$—would imply higher probability $P_\beta(\xi)$. Because exploring the entire state space of the gerrymandering model comes at a large computational cost, Mattingly uses a Metropolis-Hastings algorithm—a Markov chain Monte Carlo method—to produce a set of random samples from the distribution.

He and his collaborators create a sample of 24,000 possible redistricting plans. They tally the votes for each fictional district and compare the outcomes with those of actual districts. Using the sample of redistricting plans for the 2012 and 2016 North Carolina congressional elections, Democrats could secure four to nine and three to seven seats respectively (see Figure 1, on page 1). The results tally with those from the redistricting plan used by a bipartisan commission as part of the "Beyond Gerrymandering" project.[2] Figure 1 indicates that when compared to the bipartisan plan, the 2012 and 2016 North Carolina congressional elections show a bias towards Republicans. Results were calculated using fixed vote counts and changing district boundaries.

Utilizing their sample of redistricting plans, Mattingly's group represents the Democratic vote share distribution as a marginal box plot ordered from the most Republican to the most Democratic district, as shown in Figure 2 for the voting data from the 2012 (left) and 2016 (right) elections. They compare it with actual maps used in the 2012 and 2016 North Carolina elections, and the map generated from the judges' bipartisan plan. The judges' map almost follows a linear trend, very similar to the median map in Mattingly's simulation set in the box and whisker plot. However, the actual election outcomes are quite different and resemble an "S" curve, with Democratic voters "packed" into overwhelmingly few districts with a Democratic majority (see upper right of Figure 2; the orange and purple dots occur as outliers). Similarly, the third- to sixth-most Democratic districts (eighth- to tenth-most Republican districts) seem to be "cracked," i.e., underrepresented, with the election outcomes not reflective of the Democratic vote fraction, which is equal to or more than 50 percent.

When considering the impossibility of defining a universal score function across all states, Mattingly indicates that one must recognize each state's different geopolitical properties and every election's varied geopolitical makeup.

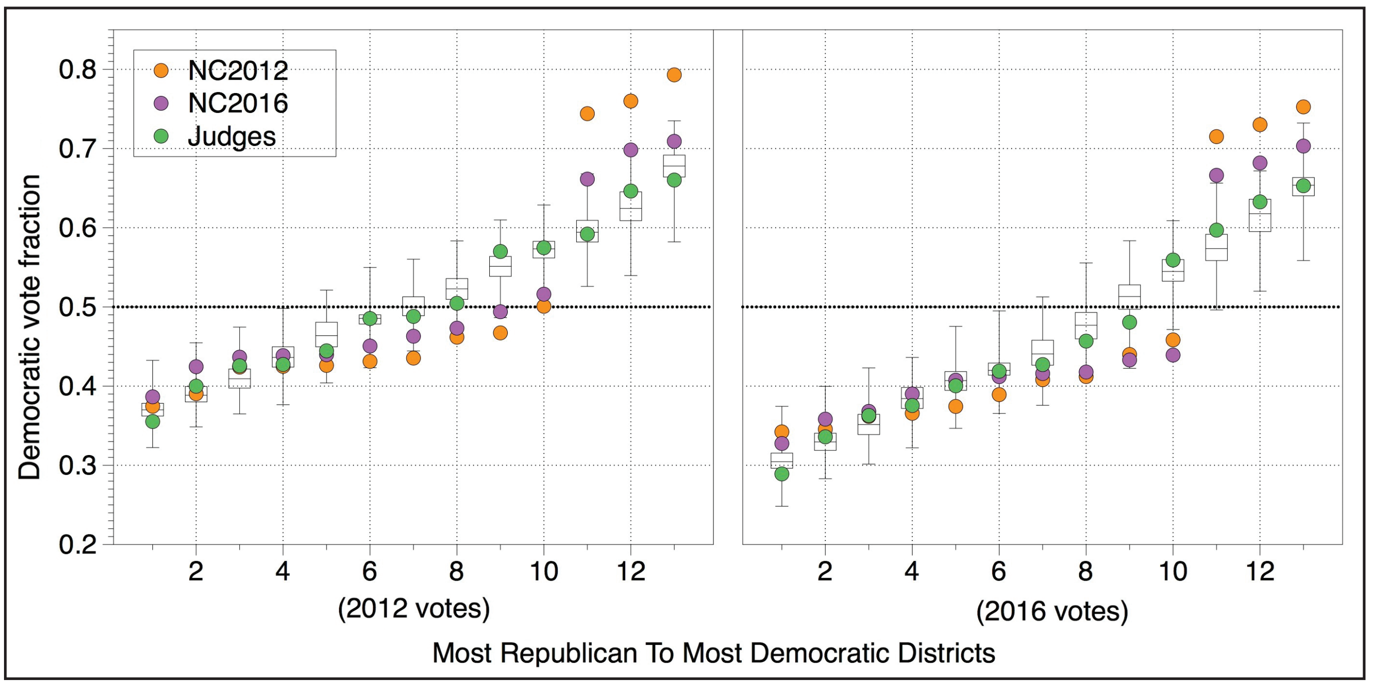

Figure 2. *Statistical result summary of districts ordered from most Republicans to most Democrats for 2012 and 2016 election data, shown as box plots. This data is compared with three redistricting plans (maps) — "NC2012" (orange dots) and "NC2016" (purple dots) congressional elections, as well as the judges' map (green dots). Image courtesy of [1].*

Nevertheless, mathematics is now at the forefront of the gerrymandering debate, with more states requiring mathematicians to perform fair evaluations of redistricted maps. Pennsylvania Gov. Tom Wolf recently enlisted mathematician Moon Duchin, who leads the Metric Geometry and Gerrymandering Group[3] at Tufts University, to determine if the state's maps were gerrymandered with a partisan bias. As Duchin succinctly put it, "This math is at the center of what seems to be a promising breakthrough in developing a legal framework to identify gerrymanders."

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Further Reading

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[2] https://sites.duke.edu/polis/projects/beyond-gerrymandering/

[3] https://sites.tufts.edu/gerrymandr/resources/

Anomalous Localized Resonance and Associated Cloaking

By Graeme W. Milton and Ross C. McPhedran

When you ring a bell, strike a drum, pluck a violin string, or excite a molecule, the length scale of oscillations in the associated eigenfunction (or eigenfunctions, when several modes are excited) dictates the length scale of the observed oscillations in the system. As the loss in the system moves towards zero, you approach a pole of the associated linear response function. By contrast, anomalous localized resonance (ALR) is associated with the approach to an essential singularity. It has the following three distinguishing features:

(1) As the loss goes to zero, finer and finer scale oscillations develop as modes increasingly close to the essential singularity become excited.

(2) As the loss goes to zero, the oscillations blow up in the region of anomalous resonance, but the fields outside of this region converge to a smooth field.

(3) The boundary of the region of anomalous resonance depends on the source position.

We first discovered ALR when exploring a seeming paradox [10]. While analyzing quasistatic dielectric equations, formal calculations showed that a coated disk—with a core of radius r_c and dielectric constant ϵ_c, a shell with outer radius r_s and dielectric constant $-\epsilon_0$, and outer radius r_s—surrounded by a medium with dielectric constant ϵ_0 would respond to any applied multipolar field in the same way as a solid disk of dielectric constant ϵ_c and radius $r_0 = r_s^2/r_c$, embedded in the same medium of dielectric constant ϵ_0. We were solving $\nabla \cdot \epsilon \nabla V = 0$ for the possibly complex potential V, with $\epsilon(x)$ taking the values ϵ_c, $-\epsilon_0$, and ϵ_0 in the core, shell, and surrounding material. If the equivalence held, a dipole source at distance a from the center of the coated disk would be identical to a dipole source at distance a from the center of the solid disk. In this case, the method of images implies that the actual dipole source—plus an image source at distance $a_I = r_0^2/a = r_s^4/(r_c^2 a)$ from the center—represents the exterior field. But if this is greater than r_s, then the image source is in the physical region outside the coated disk, which contradicts both the rules of the method of images and the maximum principle.

To make things mathematically and physically kosher, you must add a small imaginary part $i\delta$ to the dielectric constant $-\epsilon_s$ of the shell and take the limit as $\delta \to 0$. The analysis and numerics show that the field converges to the expected field outside radius a_I while developing enormous fine-scale oscillations blowing up as $\delta \to 0$ inside radius a_I. From outside radius a_I, it thus looks almost as if an actual singularity exists at the expected position of the image charge, which we term a ghost source (see Figure 1, on page 8). The underlying theory and connection with essential singularities was developed in [1]. Mathematically understanding ghost sources is simple. Take the Taylor series expansion of $f(z) = 1/(1-z)$ and truncate the sum after $1/\eta$ terms to obtain function $f_n(z)$. The series converges to $f(z)$ inside the radius of convergence $|z| < 1$ as $\eta \to 0$, and for small η it appears that $f_n(z)$ has a ghost source at $z = 1$. For $|z| > 1$, the series diverges and $f_n(z)$ develops enormous oscillations as $\eta \to 0$, corresponding to the anomalous resonance. Though the explanation is simple, it is difficult to find a physical system where the truncation parameter η is tied to the system's loss and the ghost source moves when the actual source moves.

Scientists later rediscovered ALR and ghost sources while theoretically and numerically investigating John Pendry's assertion [12] that a slab of material with thickness d, dielectric constant ε_0, and magnetic permeability $-\mu_0$—surrounded by a medium of dielectric constant ϵ_0 and magnetic permeability μ_0—would behave like a perfect lens, capable of producing a point-like image of a point source and unconstrained by the conventional diffraction limit. The image is not an exact reproduction of the source, as that would correspond to a singularity in the field; rather, it is a ghost source at the boundary of an anomalous resonance region, similar to what we found outside the coated disk. To further elucidate the connection, you can view the slab as approximately a coated cylinder of enormous radius and shell thickness d. The quasistatic approximation remains valid in the anomalous resonance regions—even when considering the time-harmonic Maxwell equations—because the field gradients are so high. The essential role of anomalous resonance is evident as it sets the length scale of resolution.

Alexei Efros remarked that the slab lens did not make sense in the presence of a constant amplitude source positioned at

See **Cloaking** *on page 8*

Risk Management

Continued from page 4

advantages, and problems to which one can effectively apply it.

Optimization under Chance Constraints

We consider maximization of a prescribed objective function—such as an insurer's expected profit or a country's social welfare—defined in a feasible set under chance constraints. These constraints can specify the desired or accepted probability of a system's default, or the violation of certain security constraints (e.g., exceeding a prescribed emission level). The initial problem of maximizing an expected utility under chance constraints is equivalent to including the expected utility combined with a nonsmooth function penalizing constraint violation.

The solution to such an augmented problem is often called a robust solution, as it is "reasonably good" for most realizations of the random input. The equivalence between the two problems holds true for a rather general class of problems [3]. Specifically, the penalty term in the equivalent problem emerging from the problem's transformation with chance constraints is essentially the expected shortfall, or CVaR risk measure.

The robust solutions derived by this approach combine ex-ante and ex-post decisions, where ex-ante measures are typically long-term investments in preventive actions (e.g., dams to inhibit flooding, earthquake-resistant buildings, or water and energy infrastructure). Ex-post practices are flexible short-term actions in response to random events (e.g., reconstruction of damaged infrastructure). Design of a robust mix of ex-ante and ex-post policies aims to invest in long-term precautionary procedures enabling optimal adaptive capacity. Application of robust solutions affords security for large quantities of resources, as we observed in our case studies.

Transformation of the maximization problem (with discontinuous "hit-or-miss" type chance constraints) into one with the expected shortfall as penalty in the objective function renders the resulting optimization problem nonsmooth. Standard gradient-based solution methods are thus inapplicable. Another fundamental complexity arises from catastrophic events' dependence on agent decisions, eliminating conventional independent scenario simulations and optimization procedures. Brute-force approaches quickly become computationally infeasible, even for problems of realistic dimension. For example, straightforward joint evaluations of $n = 10$ location-specific decisions and $m = 10$ independent scenarios for each location with only one second per evaluation could require 10^{10} seconds — more than 317 years.

A numerical method that solves this problem efficiently combines a Monte Carlo-based catastrophe generator that produces realizations of random inputs/variables (e.g., insolvency of an insurance system or damages to critical infrastructure) and a specific iterative stochastic optimization quasi-gradient procedure [1] with random stopping time moments. Such moments define catastrophe arrivals and induce long-term catastrophe-related social discounting [2].

Applications: The Value of an Integrated Catastrophic Risk Management Approach

Researchers have applied the aforementioned strategy in case studies of floods, earthquakes, windstorms, energy and information infrastructure networks, and homeland security [5]. For example, Ermolieva et al. considered the flood risks in a flood-prone area around Rotterdam in the Netherlands [6]. Due to the elevated risk, insurers set high premiums that many firms and households could not afford; this left a large number of assets uninsured. One derives high insurance premiums using the traditional actuarial average annual loss approach, which ignores heterogeneous exposures and sets the same premium for all regional contracts. Most importantly, it averages the expected losses over a period of years. Ermolieva et al. compared this technique to robust premiums computed using the proposed approach to integrated catastrophic risk management, which explicitly accounts for the fat-tailed distribution of flood losses over the years and the regional differences in exposure (see Figure 1, on page 4). Numerical results demonstrated that the high premiums computed and applied in the region are unwarranted; insurers are essentially overpaid. Quantile-based stochastic optimization suggests lower premiums, which can ensure the insurers' solvency under all flood scenarios relevant to national flood safety standards (see Figure 2). This optimizes the balance between the interests of insurers and the insured.

Many researchers have adopted and used the quantile-based approach. Our method is novel in that it integrates geographically-explicit modeling of dependent catastrophic risks with quantile-based stochastic optimization for robust ex-ante and ex-post disaster risk management. It complements the standard risk-pooling concepts, extreme value theory, and mean-variance approach, all of which are valid and useful for independent, frequent, low-consequence risks like car accidents. Due to the skewness of natural disasters' loss distribution, application of variance-based risk measures, for instance, would result in an underestimation of high-magnitude risks, which can lead to disastrous societal consequences [4]. The approach we present is capable of handling non-Gaussian, decision-dependent risks that are interdependent in space and time; such features are applicable to a variety of applications, from floods and other natural disasters to terrorist attacks.

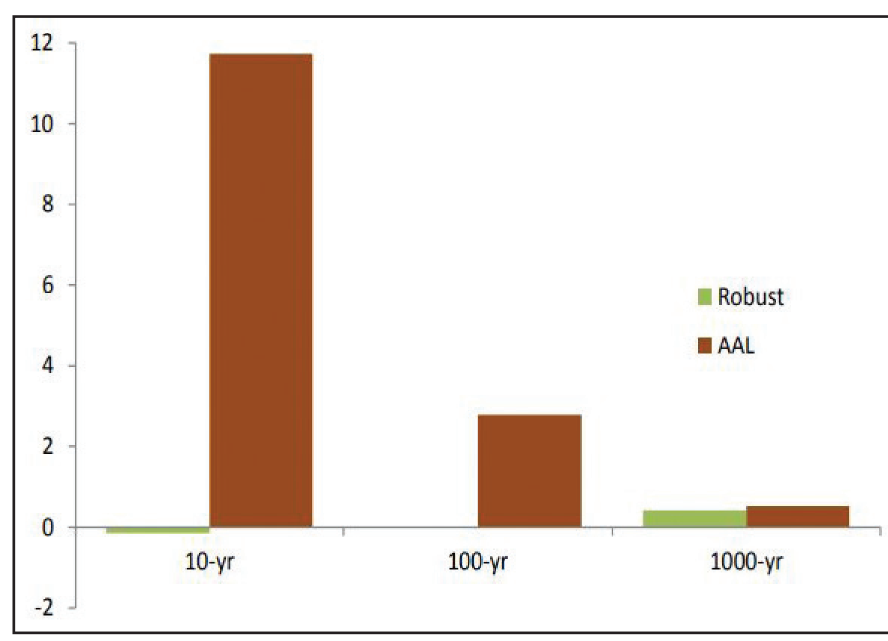

Figure 2. *Insurers' balance between premiums and coverage (in millions of euros) for 10-, 100-, and 1000-year floods for robust and conventional—average annual loss (AAL)—premiums. Large positive numbers in AAL cases indicate the level of overpayment. Figure courtesy of [6].*

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NIH Releases Strategic Plan for Data Science

In early June, the National Institutes of Health (NIH) Office of Science Policy released its new Strategic Plan for Data Science. To account for the rapidly increasing supply of data spread across a broad number of researchers in a variety of formats, the NIH seeks to mobilize advancements in storage, communication, and processing using tools—such as artificial intelligence, machine learning, and deep learning—that can revolutionize the way in which data is stored and maintained. Furthermore, the NIH recognizes the importance of developing robust information security approaches to preserve public trust and patient protection. This strategic plan offers the external community further insight into the organization's future priorities and needs in data creation and maintenance.

Many members of the SIAM community responded to the NIH's initial draft with feedback related to data management, analytics, tools, and workforce development. Thanks to SIAM involvement, the finalized plan now recognizes the importance of mathematics when advancing biomedical science and references the National Science Foundation's (NSF) Division of Mathematical Sciences/National Institute of General Medical Sciences' Mathematical Biology Program as a model for the promotion of research at the intersection of these two fields.

The Strategic Plan for Data Science was created in response to specific challenges identified by the NIH:

• The growing cost of data management could diminish the NIH's ability to enable scientists to generate data for understanding biology and improving health.

• The current data-resource ecosystem tends to be "siloed," and is not optimally integrated or interconnected.

• Important datasets exist in many different formats and are often not easily shareable, findable, or interoperable.

• The NIH has historically often supported data resources using funding approaches designed for research projects, which has resulted in a misalignment of objectives and review expectations.

• Funding for tool development and data resources has become entangled, making it difficult for one to independently assess the utility of each and optimize value and efficiency.

• No general system currently exists to transform innovative algorithms and tools created by academic scientists into enterprise-ready resources that meet industry standards of ease of use and efficiency of operation.

With the overarching principle that data should be Findable, Accessible, Interoperable, and Reusable (FAIR), the NIH has outlined five specific goals for its strategic plan, with objectives and a progress evaluation method under each goal:

1. Support a Highly Efficient and Effective Biomedical Research Data Infrastructure

1-1. *Optimize Data Storage and Security*

1-2. *Connect NIH Data Systems*

2. Promote Modernization of the Data-Resources Ecosystem

2-1. *Modernize the Data Repository Ecosystem*

2-2. *Support the Storage and Sharing of Individual Datasets*

2-3. *Leverage Ongoing Initiatives to Better Integrate Clinical and Observational Data into Biomedical Data Science*

3. Support the Development and Dissemination of Advanced Data Management, Analytics, and Visualization Tools

3-1. *Support Useful, Generalizable, and Accessible Tools and Workflows*

3-2. *Broaden Utility, Usability, and Accessibility of Specialized Tools*

3-3. *Improve Discovery and Cataloging Resources*

4. Enhance Workforce Development for Biomedical Data Science

4-1. *Enhance the NIH Data-Science Workforce*

4-2. *Expand the National Research Workforce*

4-3. *Engage a Broader Community*

5. Enact Appropriate Policies to Promote Stewardship and Sustainability

5-1. *Develop Policies for a FAIR Data Ecosystem*

5-2. *Enhance Stewardship*

The NIH lists its implementation tactics under each objective in further detail. Several of the tactics under "Enhance Workforce Development for Biomedical Data Science" may be of interest to the research community. Relevant provisions include the following:

• The NIH states that the NSF is at the "forefront of supporting disciplines that contribute to data science," and that it intends to work with the NSF on joint initiatives related to the training and education of researchers at different stages of their careers.

• To train its internal workforce, the NIH will recruit data scientists and others from industry and academia for one- to three-year sabbaticals for "NIH Data Fellows," who will be embedded in a range of high-profile, transformative projects like the Cancer Moonshot, the *All of Us* Research Program, and the Brain Research through Advancing Innovative Neurotechnologies Initiative to provide expertise not internally available.

The Strategic Plan for Data Science is available on the NIH website.[1]

— Lewis-Burke Associates LLC

[1] https://datascience.nih.gov/sites/default/files/NIH_Strategic_Plan_for_Data_Science_Final_508.pdf

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Institute for Pure and Applied Mathematics

Call for Proposals

The Institute for Pure and Applied Mathematics (IPAM) seeks program proposals from the mathematical, statistical, and scientific communities for long programs and workshops, to be reviewed at IPAM's Science Advisory Board meeting in November. Long programs (three months) bring together researchers from mathematics and other disciplines—or multiple areas of mathematics—with the goal of facilitating collaborative, cross-disciplinary research. Winter workshops are typically five days in length. Exploratory workshops, which address an emerging problem or new application of math, are typically three days. Proposals for workshops on multiscale physics will be considered for inclusion in a series of workshops made possible by an endowment from the Julian Schwinger Foundation for Physics Research (JSF). For more information, go to **www.ipam.ucla.edu/propose-a-program/** or contact the IPAM director at **director@ipam.ucla.edu**. For all proposals, the inclusion of women and members of underrepresented minorities as speakers, organizers, or participants is required.

A Perspective on Altitudes

MATHEMATICAL CURIOSITIES
By Mark Levi

In my "geometry for teachers" class a few years ago, I was trying to explain why the altitudes in a triangle are concurrent. The (perhaps) most common proof, which identifies the concurrency point as the orthocenter of another larger triangle, still felt insufficiently direct to me. I also wondered whether a more direct geometrical characterization of the concurrency point exists.

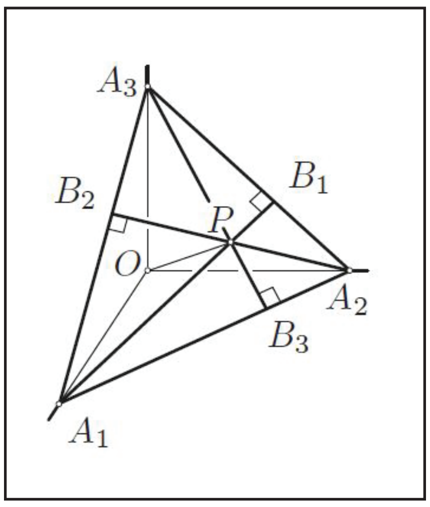

Figure 1. *Pushing the triangle into a corner. B_i is the point at which the line A_iP intersects with the opposite side.*

As it turns out, embedding the problem in three dimensions yields an additional insight. To begin, we shove an arbitrary acute[1] triangle $A_1A_2A_3$ into the corner of a rectangular quadrant, as shown in Figure 1; each vertex now lies on a coordinate axis.

I claim that the concurrency point of the altitudes is precisely the foot P of the perpendicular from the origin onto the plane of the triangle.

Proof

With P defined as in the previous sentence, let B_1 be the point at which the line A_1P intersects with side A_2A_3.

[1] Unfortunately, this approach does not seem to extend to obtuse triangles; or perhaps I am not acute enough to find an extension.

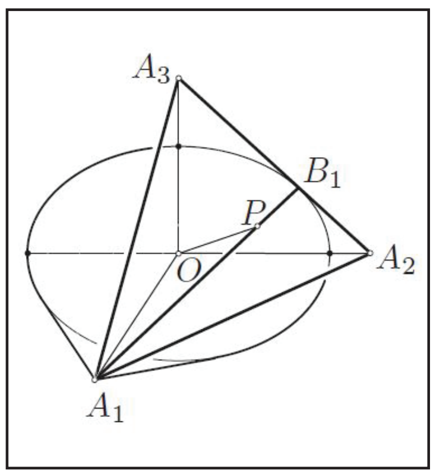

Figure 2. *The tangent cone.*

I claim that A_1B_1 is an altitude of the triangle. Indeed, $A_2A_3 \perp OP$ (since $A_2A_3 \in \text{plane}(A_1A_2A_3) \perp OP$) and $A_2A_3 \perp OA_1$ (since $A_2A_3 \in \text{plane}(OA_2A_3) \perp OA_1$). In summary, because A_2A_3 is normal to two lines (OP and OA_1) in the plane POA_1, it is normal to every line in that plane and thus to A_1B_1. So A_1B_1 is indeed an altitude. The same argument applies to A_iB_i for $i=2,3$, meaning that all altitudes pass through P. Q.E.D.

Proof 2

Here is a slightly different way to express essentially the same idea. Referring to Figure 2, construct the circular cone tangent to the plane of the triangle, with vertex A_1 and axis A_1O. Define B_1 as the point at which the line of tangency intersects with side A_2A_3. Now $A_1B_1 \perp A_2A_3$, according to Figure 3a, and A_1B_1 passes through P, according to Figure 3b (with P defined as above). This shows that the altitude from an arbitrarily chosen vertex passes through P. Q.E.D.

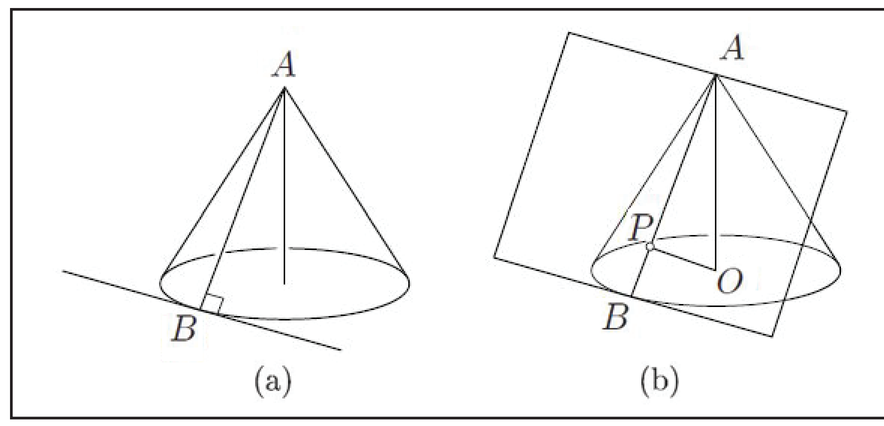

Figure 3. *In* **(a)**, *the generator is orthogonal to the base for a right circular cone. In* **(b)**, *the foot P of the perpendicular to a tangent plane from a point O on the axis lies on the line of tangency.*

The figures in this article were provided by the author.

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Cloaking

Continued from page 6

less than distance $d/2$ from a lens with dielectric constant $-\varepsilon_0 + i\delta$ and magnetic permeability $-\mu_0 + i\delta$ because the power absorbed by the lens blows up to infinity as $\delta \to 0$. Further exploration showed that realistic sources—such as polarizable dipole sources with a strength proportional to the field acting on them, or those producing constant power—would become cloaked as the loss $\delta \to 0$ [5]. These sources would create a region of anomalous resonance but essentially fail to influence the field outside of this region.

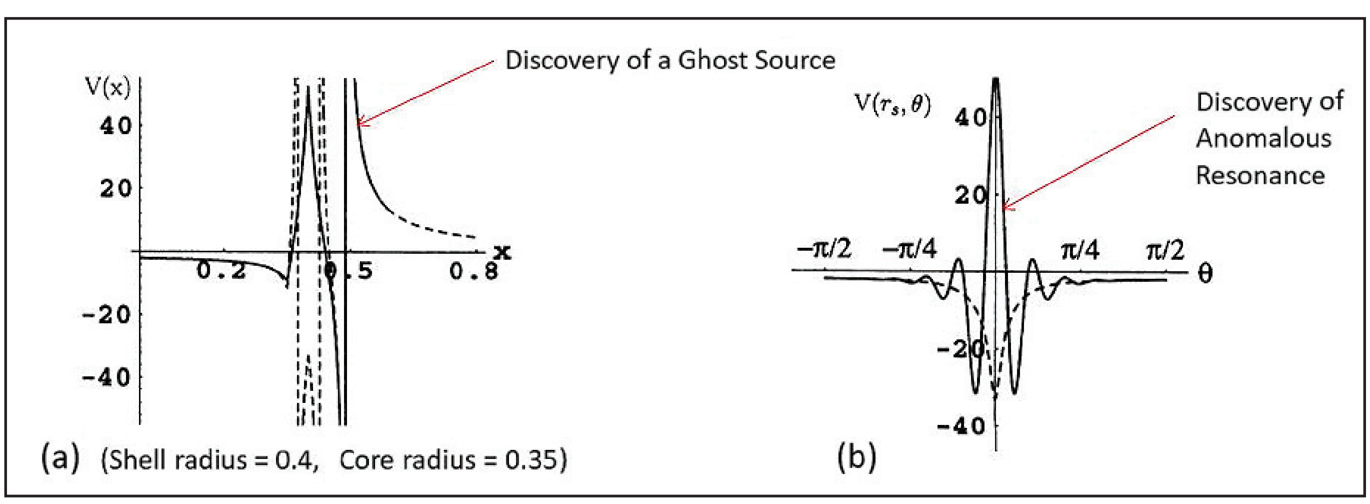

Figure 1. *Discovery of ghost sources and anomalous resonance.* **1a.** *The apparent divergence in the potential at a radius of 0.52, which is outside the shell radius of 0.40.* **1b.** *The large oscillations of the potential show the anomalous resonance. Image courtesy of [10].*

Multiple media sources covered our discovery, which marked the beginning of an avalanche of news articles about cloaking. This led to some amusing situations: A crew planning a film about how James Bond changed the world wanted to interview us, and a South American show asked if we could appear invisible on stage. Our follow-up paper [11] was downloaded over 13,000 times — a good example of how beautiful animations (made by Nicolae-Alexandru P. Nicorovici) can attract an audience.

Many illuminating developments have followed. Worthy of special mention is Hoài-Minh Nguyên's proof of cloaking due to ALR for a wide variety of coated inclusion shapes [8], and proof that the annular cloak cloaks a nearby small dielectric object [9]. Several mathematical questions remain. For instance, how do the anomalously resonant fields change if the source amplitude varies in time? Rather than being perfect, the lossless slab lens (with $\delta = 0$) cloaks a dipole source less than distance $d/2$ from the lens when one turns on the source exponentially slowly [6], but what about other time dependencies? Furthermore, in what classes of equations can you see ALR and cloaking due to ALR? An exact correspondence shows that it holds for static coupled equations of magnetoelectricity [7], and recent discoveries indicate that cloaking due to ALR holds for quasistatic elastodynamics [2, 3]. Can this type of cloaking feature multiple overlapping cloaking regions? Initial studies suggest that it cannot [4]. It will be fascinating to see how our understanding of this intriguing subject continues to evolve.

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The National Academies is currently collecting responses from the scientific community for its **2018 Global Survey of Mathematical, Computing, and Natural Scientists**.[1] The survey is part of an international interdisciplinary project called "**A Global Approach to the Gender Gap in Mathematical, Computing, and Natural Sciences: How to Measure It, How to Reduce It?**" 11 partners, supported by the International Council for Science, seek to better understand the problems faced by mathematical, computing, and natural science academics and practitioners around the world.

The Gender Gap project homepage[2] offers the following description: *Currently, existing data on participation of women in the mathematical and natural sciences is scattered, outdated, and inconsistent across regions and research fields. The project will provide evidence to support the making of informed decisions on science policy. Temporal trends will be included, as the situation of women in science is constantly evolving, sometimes with some negative developments. Data will be collected[3] via both a joint global survey and a bibliographic study of publication patterns. The survey is planned to reach 45,000 respondents in more than 130 countries using at least 10 languages, while the study of publication patterns will analyze comprehensive metadata sources corresponding to publications of more than 500,000 scientists since 1970. Contrasts and common ground across regions and cultures, less developed and highly developed countries, men and women, mathematical and natural sciences, will be highlighted.*

1 http://statisticalresearchcenter.org/global18
2 https://icsugendergapinscience.org
3 The American Institute of Physics' Statistical Research Center is collecting the data.