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Grass, Trees, and Fire: Elements of a Savanna Lifecycle

An artificially fire-suppressed savanna in South Africa, depicting a woodland of savanna trees. Image courtesy of Carla Staver.

In an article on page 5, Jenny Morber explores the distinct components that characterize a savanna and describes mathematical models used to examine the impact of climate change on this unique environment.

How Paradoxes Shape Mathematics

By Paul Davis

Not many SIAM lectures begin by "proving" that $64=65$. But that is how Thomas Hales of the University of Pittsburgh chose to open his talk at the 2018 SIAM Annual Meeting, which took place in Portland, Ore., this July. Hales, who is known for his leading role in computationally-based proofs of the Kepler conjecture, was delivering the I. E. Block Community Lecture — part of SIAM's effort to raise awareness of the many applications of mathematics. The $64=65$ paradox was the first stop in a tour that took his audience from lighthearted paradoxes in the popular press to those that upset the foundations of mathematics a century ago, before returning them to present day and the current state of self-verifying systems.

The $64=65$ paradox is resolved by finding an easily overlooked area in the geometric argument offered in its support: an 8×8 rectangle cannot be chopped into triangles and trapezoids to form a 5×13 rectangle without leaving uncovered on its diagonal a thin sliver of parallelogram that has unit area. Hales observed that such paradoxes enjoyed great popularity during their "golden age" in the early 20th century. During this time, much more profound paradoxes were simultaneously shaking the foundations of mathematics.

One of the most familiar paradoxes may be the barber paradox, which presents the following scenario: *In a certain village, a male barber shaves exactly those men who do not shave themselves. Does the barber shave himself?* Mathematicians quickly recognize that the inherent contradiction—the barber can neither shave himself nor not shave himself—is proof that no such barber exists. Does the contradiction represent a tectonic threat or merely an amusement?

Despite its superficial resemblance to the innocent barber conundrum, Russell's paradox is powerfully consequential. Devised by British mathematician and philosopher Bertrand Russell, it states: *X is the set of all sets that do not have themselves as members. Does X have itself as a member?* Following the logic that resolved the barber paradox, we immediately conclude that no such set can exist. Then we pause in consternation; what became of our ability to define any set we wanted?

See **Paradoxes** *on page 4*

Recovering Lost Information in the Digital World

By Yonina Eldar

We live in an increasingly digital world where computers and microprocessors perform data processing and storage. Digital devices are programmed to quickly and efficiently process sequences of bits. A computer operating on these bits then programs mathematical algorithms translated from signal processing. An analog-to-digital converter converts the continuous time signal into samples; the transition from the physical world to a sequence of bits causes information loss in both time (sampling phase) and amplitude (the quantization step). Is it possible to restore information that is lost in transition to the digital domain?

The answer depends on what we know about the signal. One way to ensure a signal's recovery from its samples is to limit its speed of change. This idea forms the basis of the famous Nyquist theorem, developed in parallel by mathematicians Edmund Taylor Whittaker and Vladimir Kotelnikov [5]. The theorem states that we can recover a signal from its samples as long as the sampling rate (the number of samples per unit time) is at least twice the highest frequency in the signal. This result is the cornerstone of all current digital applications, which sample at the Nyquist rate or higher.

Despite the theorem's tremendous influence on the digital revolution, satisfying the Nyquist requirement in modern applications often necessitates complicated and expensive hardware that consumes considerable power, time, and space. Many applications use signals with sizable bandwidth to deliver a high rate of information and obtain good resolution in various imaging applications, such as radar and medical imaging. Large bandwidth translates into high sampling rates that are challenging to execute in practice. Thus, an important question arises: Do we really have to sample at the Nyquist rate, or can we restore information when sampling at a lower rate?

A related concern is the problem of super resolution. Any physical device is limited in bandwidth or resolution, meaning that it cannot obtain infinite precision in time, frequency, and space. For example, the resolution of an optical microscope is limited by the Abbe diffraction limit, which is half the wavelength used for illumination. We can thus view large objects like bacteria in the optical regime, but proteins and small molecules are not visible with sufficient resolution. Is it possible to use sampling-related ideas to recover information lost due to physical principles?

We consider two methods to recover lost information. The first utilizes structure that often exists in signals, and the second accounts for the ultimate processing task. Together they form the basis for the Xampling framework, which proposes practical, sub-Nyquist sampling and processing techniques that result in faster and more efficient scanning, processing of wideband signals, use of smaller devices, improved resolution, and lower radiation doses [5].

The union-of-subspaces model is a popular choice for describing signal structure [7, 9]. As a special case it comprises sparse vectors — vectors with a small number of nonzero values in an appropriate representation, which is the model underlying compressed sensing [6]. It also includes some popular examples of finite-rate-of-innovation signals, such as streams of pulses [11]. An example of this signal arises naturally in a radar system, where a pulse moves towards the targets, which reflect it back to the receiver. The received pulse hence consists of a stream of pulses, where each pulse's time of arrival is proportional to the distance to the target, and the amplitude conveys information about the target's velocity

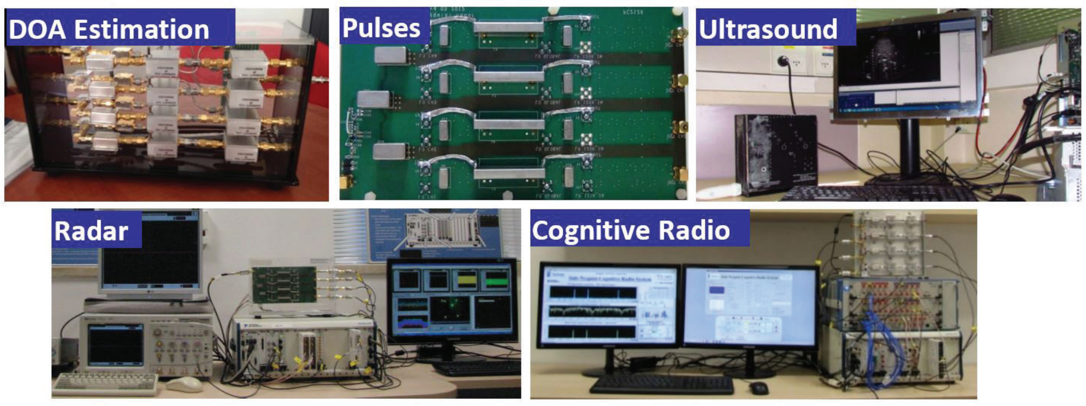

Figure 1. *Sub-Nyquist prototypes for different applications developed in the Signal Acquisition Modeling and Processing Lab at Technion – Israel Institute of Technology. Image courtesy of Yonina Eldar Lab.*

See **Digital World** *on page 3*

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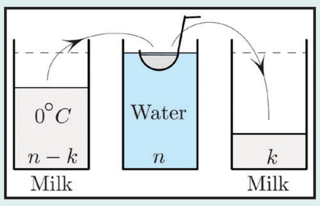
5 The Perils of Beautiful Mathematics
James Case reviews Sabine Hossenfelder's *Lost in Math: How Beauty Leads Physics Astray*, in which Hossenfelder cautions against exaggerated emphasis on mathematical beauty in theoretical physics. Citing the failure of supersymmetry validation and the lack of testable hypotheses in string theory as examples, she suggests that theoretical evaluation in the field is more aesthetic than scientific and leads to bad science.

6 Heat Exchange and Some Frivolous Aspects of *e*
Is it possible to heat a glass of milk at 0°C to over 50°C using only the heat from a 100°C glass of water? In this month's column, Mark Levi demonstrates mathematically that such a reversal of the second law of thermodynamics is indeed feasible.



6 Mathematicians and Ethical Engagement
Maurice Chiodo and Dennis Müller explore ethical issues of particular relevance to mathematicians. While math is an incredibly effective tool, powering things like smartphones and climate models, it has also engendered pitfalls — from inaccurate financial modeling and mass surveillance to targeted advertising. The authors call on mathematicians everywhere to acknowledge their social responsibilities and be more mindful of the impact of their work.

9 How a Chance Internship Inspired My Career in the Oil Industry
Anusha Sekar discusses her fortuitous encounter with industrial mathematics, which led to her successful career at Chevron. From learning new and varied topics to solving real-world problems and enjoying an informal work environment, Sekar recounts the various benefits of working in the field, asserting that a good work-life balance is indeed possible for mathematicians in industry.

10 Professional Opportunities and Announcements

A Timely Focus on Data Science

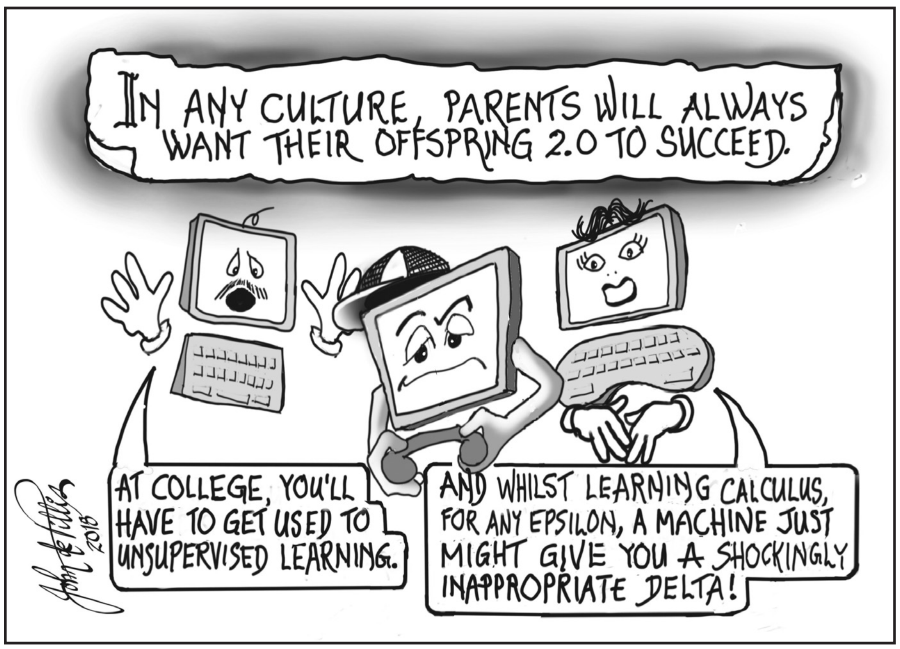
FROM THE SIAM PRESIDENT

By Nicholas Higham

It is generally agreed that data science involves mathematics, statistics, computer science, data, and applications. Attempts to characterize it have led to an assortment of Venn diagrams with variously defined sets (see Figure 1). Regardless of data science's meaning, the SIAM community—with expertise in all the component areas—is uniquely positioned to contribute to it.

It is therefore both timely and appropriate that in 2018, SIAM introduced the *SIAM Journal on Mathematics of Data Science* (with editor-in-chief Tammy Kolda of Sandia National Laboratories), launched its *Data Science* book series (with editor-in-chief Ilse Ipsen of North Carolina State University), and is planning a data science conference for 2020. Moreover, data science boasts its own research area page on the new SIAM website.[1]

The most hyped aspect of data science is undoubtedly machine learning, especially deep learning. Michael Elad examined the role of deep learning in imaging science in the May 2017 issue of *SIAM News* [3]. His analysis inspired discussion.[2] He noted that "In most cases, deep learning-based solutions lack mathematical elegance and offer very little interpretability of the found solution or understanding of the underlying phenomena." Peter Warden, data science book author and member of Google's TensorFlow team, has written that "Most of machine learning is the software equivalent of banging on the side of the TV set until it works, so don't be discouraged if you have trouble seeing an underlying theory behind all your tweaking!"[3] Advances in the theoretical underpinning of machine learning and deep learning are clearly needed. The SIAM community can contribute novel understanding and breakthroughs; in fact, it is already doing so [2]. The new journal, book series, and upcoming conference provide the perfect platforms to report such contributions.



Cartoon created by mathematician John de Pillis.

The SIAM community is well versed in the aspect of data science that concerns the accuracy of results. Mathematicians are accustomed to handling uncertainties introduced by data and rounding errors. Here, the trend towards processors that support low-precision arithmetic is impinging on data science. The NVIDIA V100 graphics processing unit (GPU) supports half-precision floating-point arithmetic and—using its tensor cores—can execute it at a rate of up to 112 teraflops, compared with seven teraflops for double precision. It is therefore tempting to run data-intensive computations in half precision. But a half-precision number has only the equivalent of around four-decimal-digit precision; will the computed results have any correct digits?

Researchers have reported success with low precision in machine learning. For example, at the 2018 SIAM Conference on Parallel Processing for Scientific Computing, held this March in Tokyo, Takuya Akiba (Preferred Networks, Inc., Tokyo) explained how he and his colleagues trained ResNet-50 on ImageNet for 90-epochs (a standard benchmark) on 1024 NVIDIA P100 GPUs in 15 minutes, thus halving the previous record time. Among the key ideas was the selective use of half precision [1].

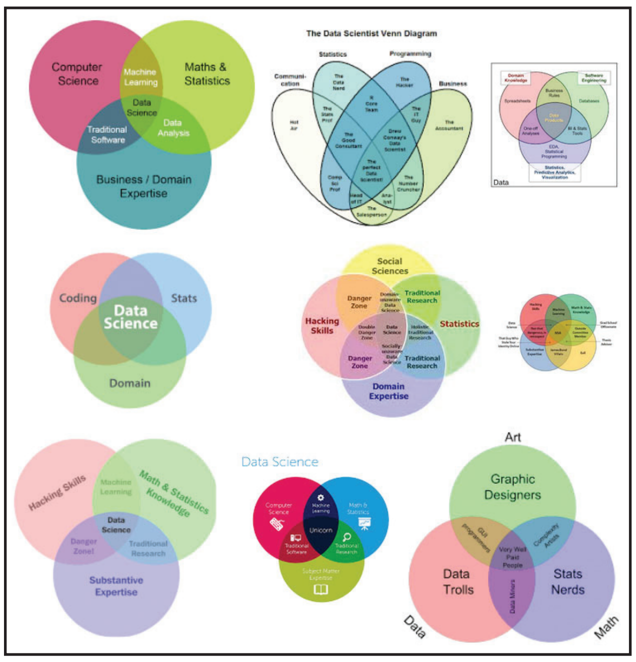


Figure 1. *A sampling of Venn diagrams that researchers have proposed to define data science. Image courtesy of Google Images.*

Of course, data science presents challenges not only in research but also in teaching — another area that benefits from SIAM community engagement. At the 2018 SIAM Conference on Applied Mathematics Education, held in Portland, Ore., this July, Gil Strang (Massachusetts Institute of Technology) organized a minisymposium on "Deep Learning and Deep Teaching." In his talk, Gil presented some of the ideas contained in his forthcoming book on the subject [4] and posed the tantalizing question of whether deep learning can learn calculus; much discussion ensued.

SIAM is already a great source of expertise and information on many aspects of data science. I look forward to our community playing a growing role in the area and attracting new members with an interest in the subject.

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[1] https://www.siam.org/Research-Areas/Detail/data-science

[2] See comments from readers at https://sinews.siam.org/Details-Page/deep-deep-trouble

[3] https://petewarden.com/2016/04/18/how-to-break-into-machine-learning/

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Digital World

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through the Doppler effect. Several samplers based on union of subspace modeling appear in Figure 1 (see page 1).

Researchers have also recently explored the actual processing task. We consider three such examples: (i) scenarios in which the relevant information is embedded in the signal's second-order statistics [3], (ii) cases where the signal is quantized to a low number of bits [8], and (iii) settings in which multiple antennas form an image [1, 2].

An interesting sampling question is as follows: What is the rate at which we must sample a stationary ergodic signal to recover its power spectrum? The rate can be arbitrarily low using appropriate nonuniform sampling methods. If we consider practical sampling approaches—such as periodic nonuniform sampling with N samplers, each operating at an Nth of the Nyquist rate—then only on the order of $\sqrt{N}$ samplers are needed to recover the signal's second-order statistics. This leads to a sampling rate reduction on the order of $\sqrt{N}$. Next, suppose that we quantize our signal after sampling with a finite-resolution quantizer. Researchers traditionally consider sampling and quantization separately. However, the signal introduced by the quantizer is distorted, which begs the following question: Must we still sample at the Nyquist rate — the rate required for perfect recovery assuming no distortion? It turns out that we can achieve the minimal possible distortion by sampling below the signal's Nyquist rate without assuming any particular structure of the input analog signal. We attain this result by extending Claude Shannon's rate-distortion function to describe digital encoding of continuous-time signals with a constraint on both the sampling rate and the system's bit rate [8].

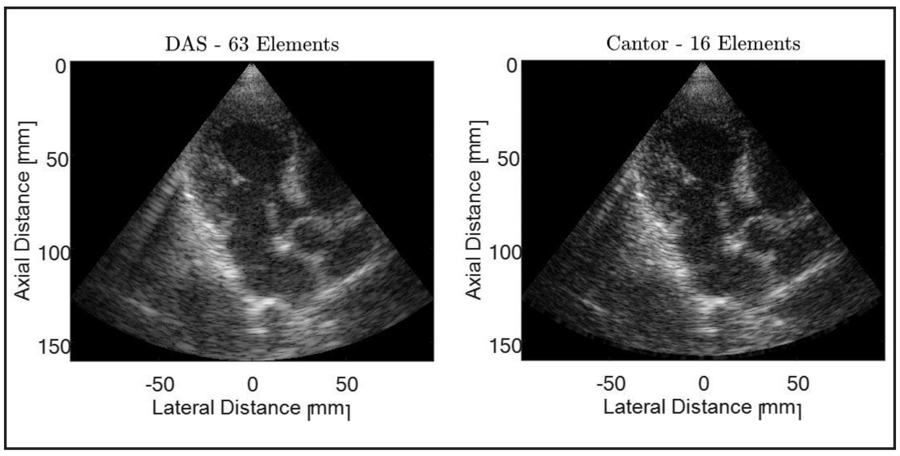

Figure 2. *The same cardiac image obtained with delay-and-sum beamforming using a uniform linear array of 63 elements (left) and convolutional beamforming using a sparse array of 16 elements (right). Image courtesy of [4].*

As a final example of task-based sampling, consider a radar or ultrasound image created by beamforming. An antenna array receives multiple signals reflected off the target; these signals are delayed and summed to form a beamformed output that can often be modeled as a stream of pulses. However, the individual signals typically lack significant structure and are often buried in noise. Nonetheless, by exploiting the beamforming process we can form the final beamformed output from samples of the individual signals at very low rates, despite the signals' structure scarcity. In addition, we can preserve the beampattern of a uniform linear array by using far fewer elements (a sparse array) and modifying the beamforming process. By applying convolutional beamforming, we can achieve the beampattern associated with a uniform linear array of N elements using only on the order of $\sqrt{N}$ elements (see Figure 2).

Combining the aforementioned ideas allows us to create images in a variety of contexts at higher resolution using far fewer samples. For example, we can recover an ultrasound image from only three percent of the Nyquist rate without degrading image quality (see Figure 3). This ability allows for multiple technology developments with broad clinical significance, such as fast cardiac and three-dimensional imaging, which is currently limited by high data rate. Moreover, the low sampling rate enables the replacement of large standard ultrasound devices and their cumbersome cables with wireless transducers and simple processing devices, such as tablets or phones. The sampled data's low rate facilitates its transmission over a standard WiFi channel, allowing a physician to recover the image with a handheld device. In parallel, the data may be transmitted to the cloud for remote health and further, more elaborate processing.

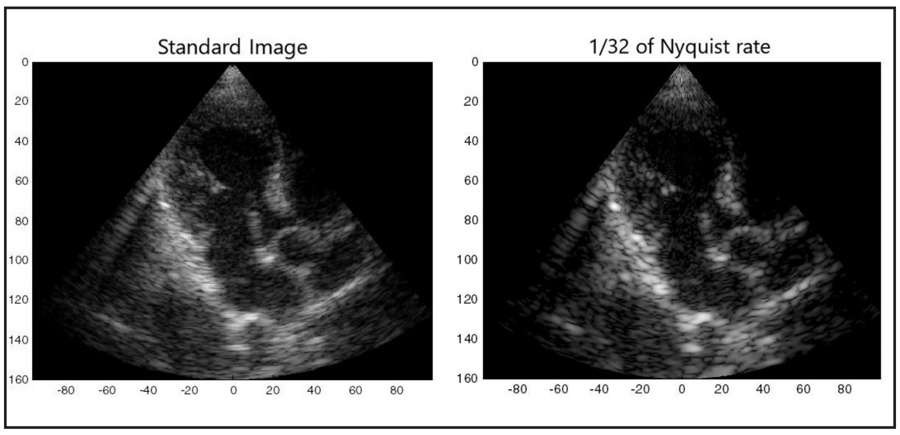

Figure 3. *Ultrasound imaging at three percent of the Nyquist rate (right), as compared to a standard image (left). Image courtesy of [1].*

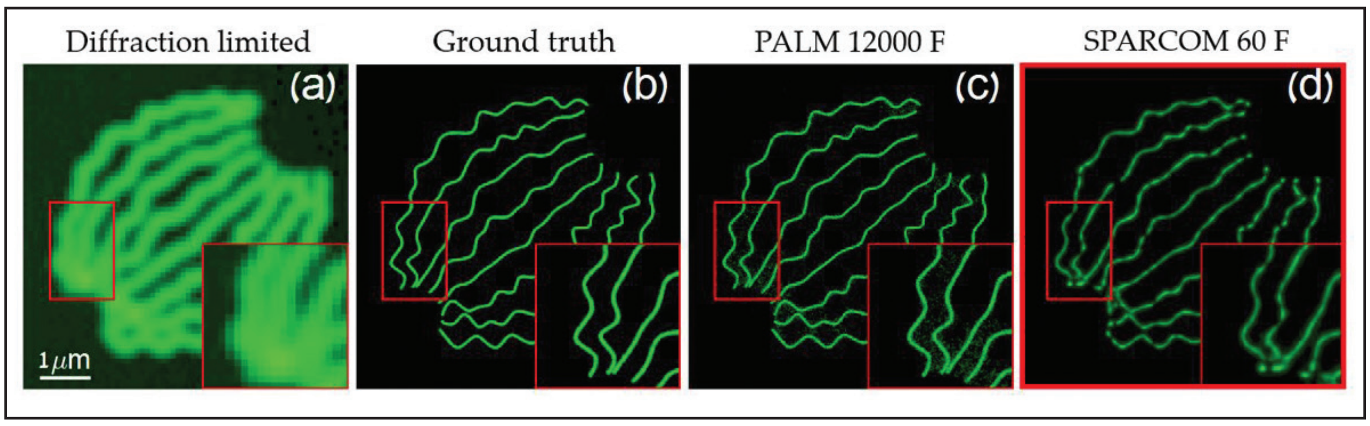

Figure 4. *Super-resolution in optical microscopy.* **4a.** *The image obtained with a standard microscope.* **4b.** *The original image at high resolution.* **4c.** *The image obtained using 12,000 frames via photo-activated localization microscopy (PALM).* **4d.** *The image obtained using only 60 frames via sparsity-based super-resolution correlation microscopy (SPARCOM). Figure courtesy of [10].*

Our approaches can also help increase resolution in fluorescence microscopy [10]. In 2014, William Moerner, Eric Betzig, and Stefan Hell received the Nobel Prize in Chemistry for breaking the diffraction limit with fluorescence imaging. They sought to obtain a high-resolution image by using thousands of images, each containing only a small number of fluorescing molecules. This method—referred to as photo-activated localization microscopy (PALM)—allows researchers to localize and average the molecules in each frame to obtain one high-resolution image. This leads to high spatial resolution but low temporal resolution. Since estimating each pixel's variance can form a brightness image, we can exploit our ability to perform power spectrum recovery from fewer samples to dramatically reduce the number of samples needed to form a super-resolved image. This approach is called sparsity-based super-resolution correlation microscopy (SPARCOM). Due to the small number of required frames, SPARCOM paves the way for live cell imaging. Figure 4 compares SPARCOM with 60 images and PALM with 12,000 images. Both approaches generate similar spatial resolution, but SPARCOM requires two-orders-of-magnitude fewer samples.

The same idea is applicable to contrast-enhanced ultrasound imaging. We may treat the contrast agents flowing through blood similarly to the blinking of the fluorescent molecules; in this way, we perform ultrasound imaging with high spatial and temporal resolution. This distinguishes between close blood vessels and facilitates the observation of capillary blood flow.

In summary, to recover information with higher precision and minimal data we must exploit all of the information we have; here we focused on exploiting structure and the processing task. This yields new mathematical theories that provide bounds on sampling and resolution, and new engineering developments that produce novel technologies to overcome current barriers. In the future, the combination of mathematics and engineering—seeing information with a precision that is presently unavailable and tracking effects faster than is currently possible—can pave the way for innovative scientific breakthroughs.

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Paradoxes

Continued from page 1

This startling outcome toppled the work of German philosopher Gottlob Frege, a luminary of mathematical logic. In a 1962 *Scientific American* article entitled "Paradox" [3]—which Hales rightly recommended to his listeners as essential homework—logician W.V. Quine demonstrated the way in which Russell's paradox compels us to give up the fundamental principle of the existence of sets.

Hales summarized this critical period with a frame from the graphic novel *Logicomix*—the second of his essential reading assignments—that depicts the construction of the logical foundations of mathematics. Russell appears in the panel, famously working on *Principia Mathematica* with Alfred North Whitehead (see Figure 1).

Ernst Zermelo used a different approach to resolve Russell's paradox. He proposed a theory of sets that dodged the paradox by prohibiting sets that are too large, such as the set of all sets. Russell's theory of types, in contast, forbade types that overlap; the barber paradox is grammatically unsound because barbers and customers are distinct types (see Figure 2). Hales observed that set theory offers a foundation for much of mathematics, while type theory establishes a foundation for computer science.

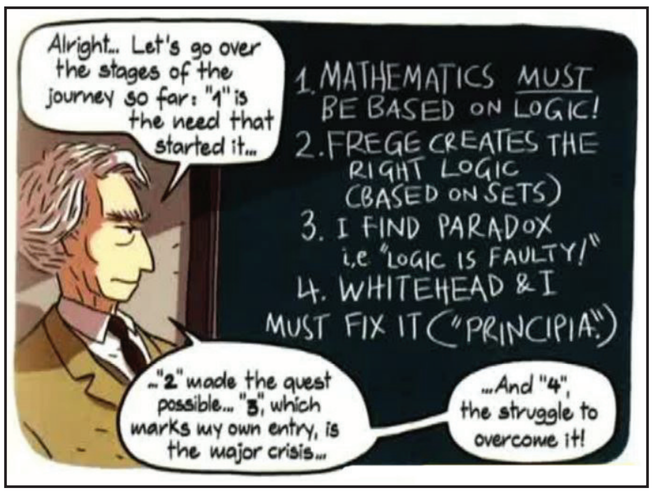

Figure 1. *One strand of the foundational work arising from Russell's paradox. Logicomix: An Epic Search for Truth, by Apostolos Doxiadis and Christos Papadimitriou, is published by Bloomsbury Publishing (RRP $25) [1].*

The rush of ideas continued. Kurt Gödel's incompleteness theorem appeared in 1931: every sufficiently powerful deductive system contains a sentence that is true if and only if it is unprovable in that system. David Hilbert's 1928 decision problem, which asks if there exists an algorithm that can determine the universal validity of a given logical statement, received independent negative answers from both Alonzo Church and Alan Turing in 1936. Turing showed that the halting problem—which asks whether a program will run forever or eventually halt, given a description of that program and an input to it—reduces to Hilbert's decision problem. He then solved the halting problem by what Hales called a "paradoxical construction — an algorithm that takes as input a data-encoding of itself."

Ultimately, Alexander Grothendieck introduced his concept of universes to accommodate sets larger than those permitted by Zermelo's formulation. Figure 3 offers a caricature of these universes. The nutshell in the top center recursively contains Hales' cartoon universe. He lightheartedly suggested that Hamlet foresaw these mathematical constructions when declaiming, "I could be bounded in a nutshell, and count myself a king of infinite space."

Hales followed this path of paradoxes to offer his audience a glimpse of the superstructure needed to support a proof assistant—a programming language capable of validating mathematical proofs—built within such a nutshell universe. His perspective was surely influenced by his 1998 submission of a computationally-based proof by exhaustion of Johannes Kepler's sphere-packing conjecture to *Annals of Mathematics*. (In 1611, Kepler had conjectured that a face-centered arrangement of uniform spheres packs most efficiently into a given volume; the stack of oranges in the upper right corner of Figure 3 is an example.)

The challenges of verifying Hales' and his co-author Samuel Ferguson's work overwhelmed the volunteer referees before they could definitively accept its validity. "It is very unusual to have such a large set of reviewers (working) … over a three-year period," one editor reported. "The reviewing process produced in these reviewers a strong degree of conviction of the essential correctness of this proof approach…" This was good news, but hardly the traditional unequivocal endorsement.

Another editor relayed the thinking of referee panel head Gábor Fejes Tóth, younger half of the Fejes Tóth father-son team that played important 20th-century roles in the Kepler saga. "Fejes Tóth thinks that this situation … is similar to the situation in experimental science," the editor said. "Other scientists acting as referees can't certify the correctness of an experiment, they can only subject the paper to consistency checks. [Fejes Tóth] thinks the mathematical community will have to get used to this state of affairs."

Despite his eventual publication of papers describing both the mathematical and computational portions of his proof of Kepler's conjecture, Hales pursued the possibility of a computer checking "every step of the proof back to the foundations of mathematics." In 2017, he published—with 21 collaborators—a formal proof of the Kepler conjecture in *Forum of Mathematics, Pi* [2]. That effort, known as the Flyspeck project,[1] used the proof assistants Isabelle and HOL Light.

HOL Light is the work of John Harrison; its kernel requires less than 500 lines of code. After Hales discovered a bug at an early stage, Harrison used the corrected proof assistant to confirm the absence of any others. Such proofs of self-consistency dodge Gödel's incompleteness theorem by looking back in from a slightly larger universe. Self-verification then enables mutual verification within the ecosystem of a multitude of proof assistants. A verified self-compiling compiler is accessible through the CompCert project,[2] and the proof assistant community is within sight of top-to-bottom verification tools — from high-level code down to machine language.

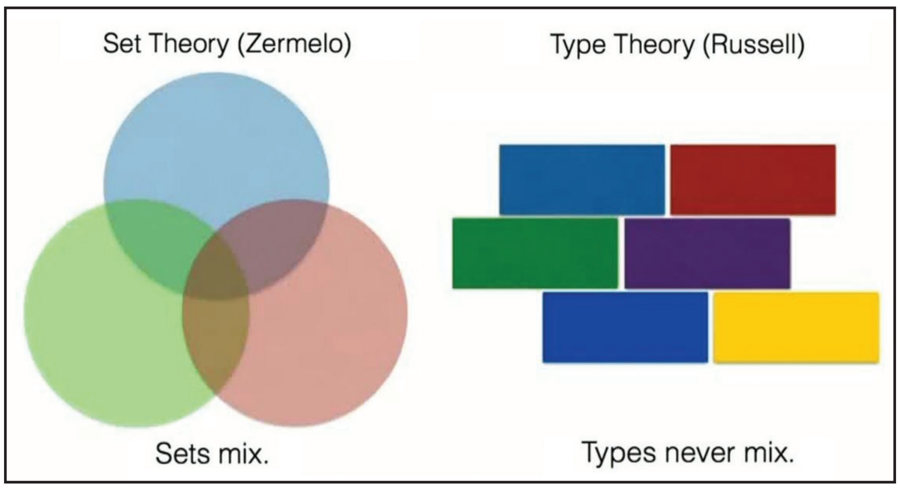

Figure 2. *Two responses to Russell's paradox. Figure courtesy of Thomas Hales.*

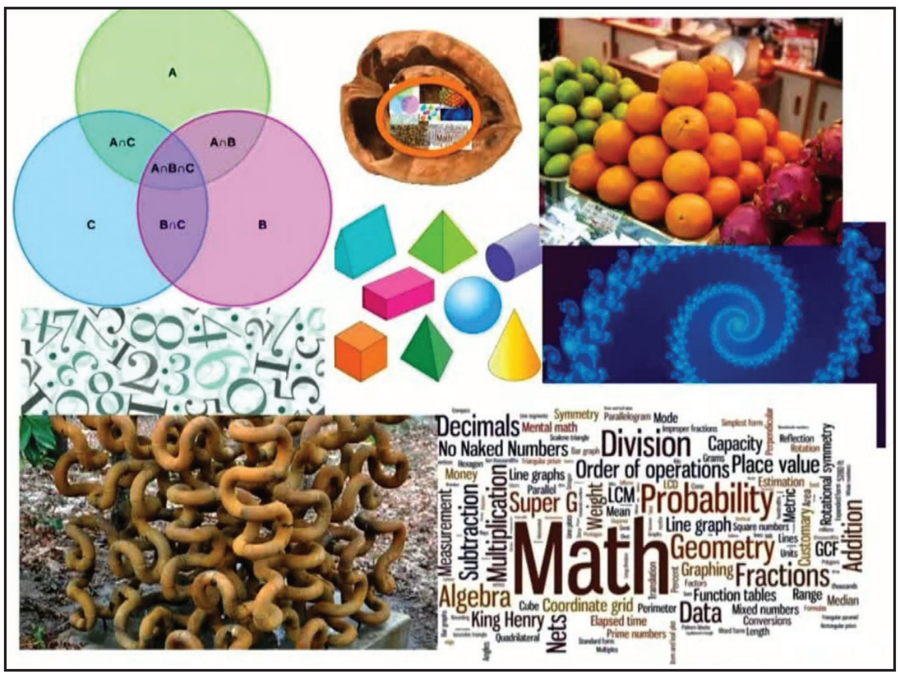

Figure 3. *Thomas Hales' caricature of a mathematical universe, enclosed recursively in the nutshell at the top center. Creative Commons and Google Images.*

Finally, Hales addressed confidence — the elephant in the room. "Do I completely trust computers?" he asked. "No. Do I completely trust human referees? No. The technologies have reached the point where I trust them as much as I trust any technical tool." Mathematics might indeed be on the threshold of a brave new world—if not universe—of formal validation.

Hales' presentation—including the "proof" that $64 = 65$—is available from SIAM either as slides with synchronized audio or a PDF of slides only.[3]

The I. E. Block Community Lecture is given annually at the SIAM Annual Meeting. It honors the vision of I. Edward Block, the co-founder and former managing director of SIAM, by encouraging public appreciation of the excitement and vitality of science. The lectures are open to students, teachers, and members of the local community, as well as SIAM members, researchers, and practitioners in fields related to applied and computational mathematics.

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[1] https://github.com/flyspeck/flyspeck

[2] http://compcert.inria.fr/

[3] https://www.pathlms.com/siam/courses/8264/sections/11775

The Perils of Beautiful Mathematics

BOOK REVIEW

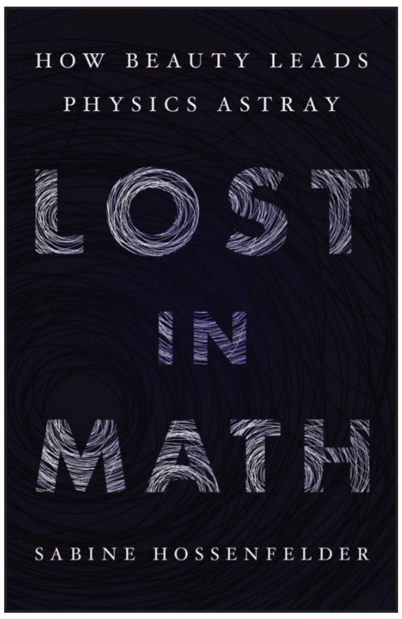
By James Case

Lost in Math: How Beauty Leads Physics Astray. *By Sabine Hossenfelder. Basic Books, New York, NY, June 2018. 304 pages, $30.00.*

Sabine Hossenfelder is inclined to blame the dearth of meaningful progress in theoretical physics since the 2012 detection of the Higgs boson on an ill-considered quest for mathematical beauty. A theoretical physicist, Hossenfelder acknowledges the aesthetic appeal of the known laws of physics in *Lost in Math: How Beauty Leads Physics Astray*, but doubts that beauty alone ever led to their discovery or is likely to inspire further innovations.

What seems to discourage her most is the failure of CERN's Large Hadron Collider (LHC) to detect even a few of the new fundamental particles predicted by the "supersymmetry" theory — "susy" for short. Post-World War II physicists, led by Murray Gell-Mann and Richard Feynman, assembled what has come to be known as the Standard Model of the subatomic world. Comprising 25 presumably fundamental particles from which all other particles may be constructed, the model explains virtually every known subatomic interaction in terms of "gauge symmetries" — symmetries of the Lagrange equations governing the quantum fields of three of nature's four fundamental forces. Only gravitational forces remain unexplored by the Standard Model.

Susy postulates the existence of a "partner" for each of the 25 particles in the Standard Model, and perhaps a few others. No such partners were found at the Large Electron-Positron Collider (LEP), which ran until 2000, or at the Tevatron, which reached higher energies than the LEP and ran until 2011. Even the powerful LHC, which reuses the LEP's tunnel and has been running off and on since 2008, has failed to divulge any evidence of the elusive susy partners. The simplest explanation is that the unseen partners are much larger than expected, requiring even higher energy colliders for detection.



Lost in Math: How Beauty Leads Physics Astray. By Sabine Hossenfelder. Courtesy of Basic Books.

Unsurprisingly, particle physicists are lobbying for such colliders. Some have proposed a Chinese Circular Collider (CCC) that would reach collision energies approaching 100 trillion electron volts (TeV). The Japanese have expressed interest in building an almost equally powerful International Linear Collider, while CERN has plans for a super-LHC with a circumference of 100 kilometers that reaches energies comparable to those expected of the CCC. But many physicists anticipate discovery of at least a few susy partners at collision energies as low as 2 TeV — easily obtainable from the LEP, Tevatron, and original LHC incarnation. Who is to say, Hossenfelder asks, that more powerful colliders will succeed where others have failed? Where do leaders in the field stand on the matter? Are they deliberately misleading their governments about the prospects of increasingly costly experiments?

String theory is another source of distress to Hossenfelder. She points out that the field has yet to generate a single testable hypothesis after 30 years of development. Worse still, it has spawned a willingness in some circles to modify—if not abandon—the scientific method itself. Hossenfelder references Austrian philosopher Richard Dawid's recommendation to amend the scientific method to allow for the evaluation of scientific hypotheses on purely theoretical grounds. In his book, *String Theory and the Scientific Method*, Dawid specifically cites three non-empirical arguments already in use by string theorists: (i) the absence of alternative explanations, (ii) the use of previously successful mathematics, and (iii) the discovery of unexpected connections. According to Hossenfelder, string theorists welcome such philosophical support while most other physicists refrain from doing so. What, she wonders, will become of everyone if climate scientists rely on non-empirical criteria to evaluate their models?

In part because of the high cost of field experiments, physicists have developed criteria to identify the proposed theories most likely to survive empirical testing. The most obvious, of course, is simplicity. A simple theory is always preferable to a complicated one that explains the same observations. So is one that extends an established theory, since it automatically explains the same observations and more. But the physics community has gone further, developing a "theory of theories" situated in something called "theory space."

To introduce this idea, Hossenfelder points out that theoretical physics is an amalgam of weakly-related theories operating on different scales. Small-scale (high-resolution) physical theories tend to imply larger-scale (lower-resolution) physical theories. For instance, Newton's laws of motion—developed at the level of an apple falling from a tree—imply Kepler's theory of planetary motion. Likewise, atomic-scale quantum mechanics suggests a theory of large-scale chemical reactions and another of fingernail-size computer chips. And so on.

See **Beautiful Mathematics** *on page 8*

Grass, Trees, and Fire: Elements of a Savanna Lifecycle

By Jenny Morber

Imagine a savanna — an undulating plain of long grass stretching towards a horizon dotted with scrubby trees. Unobscured by foliage, the sky is expansive. Winds ripple the golden grass while distant smoke curls from a recent fire, still smoldering.

Savannas are unique environments with a special ecology. Research suggests that they sometimes appear in surprising areas where we might expect to find forests or other landscapes. What makes a savanna? Will existing savannas persist for hundreds of years? How might climate change affect today's savanna ecosystems? And are changes that occur in response to outside forces? Researchers Jonathan Touboul (Brandeis University), Carla Staver (Yale University), and Simon Levin (Princeton University) explore these and other questions in their work [1].

The team defines a savanna as a combination of grasses, saplings, and adult savanna trees, with fire as the driving element. Grasses fuel fires and sparse tree covering, which encourages grass growth and instigates more fires. Fires damage saplings and prevent tree maturity. This loop sustains savannas. Adult trees, however, withstand fires and shade the ground. Robbed of sunlight, shaded grasses wither and fire decreases, thus allowing continued tree propagation. Such conditions foster woodlands.

Touboul, Staver, and Levin codify these relationships in units of aerial cover where variables G, S, and T represent the fractional cover of grass, saplings, and savanna trees. This yields a three-dimensional system in which

$$G' = \mu S + \nu T - \beta GT$$

$$S' = \beta GT - (\omega(G) + \mu)S$$

$$T' = \omega(G)S - \nu T,$$

where β is the savanna sapling birth rate and μ and ν are respectively the rates at which savanna saplings or adult trees die. Saplings grow into trees at a rate ω, a nonlinear decreasing function of grass cover that accounts for the ecological system's nonlinear response to fires. Fires readily spread in systems with sufficient grass biomass but are quickly limited when tree cover exceeds a certain threshold. Therefore, this function has a sigmoidal shape that approximates a steep drop at a crucial percolation threshold associated with fire spread probability. Because G, S, and T represent fractional cover, they are nonnegative and can be expressed as $G + S + T = 1$; this reduces the above equations to a two-dimensional system.

In this simple model, nudges from natural fluctuations, climatic events, or human activity can prompt the vegetation down a bifurcated path that may persist for many years. Landscapes are the product of a hysteresis that is heavily dependent on history, as shown in Figure 1.

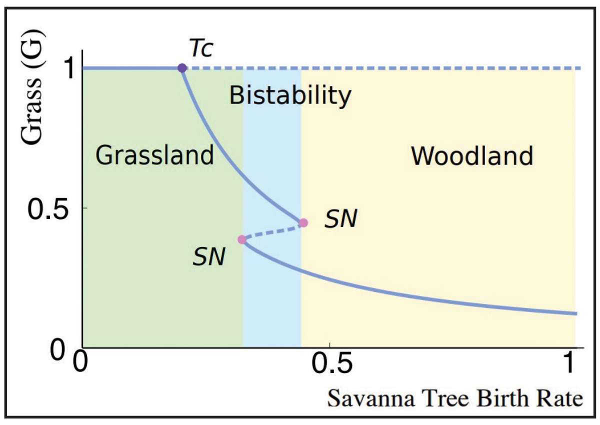


Figure 1. *Grasslands and woodlands in the absence of forests yield a bifurcation diagram with respect to the savanna tree birth rate. In ecosystems with low savanna tree birth rate, grass covers the landscape and prevents woodlands from emerging; at high rates, woodlands dominate and grasslands cannot establish themselves. For intermediate birth rates, both woodlands and savannas may emerge and remain stable over time. Figure courtesy of [1].*

This behavior conflicts with an underlying tenet of ecology. "In the traditional view in ecology, if you know something about the environment you basically know what kind of vegetation you're going to have," Staver said. "What's different about this is that…even in a very rainy place that ought to be able to be a forest, if it ends up being a savanna we should expect it to continue to be a savanna."

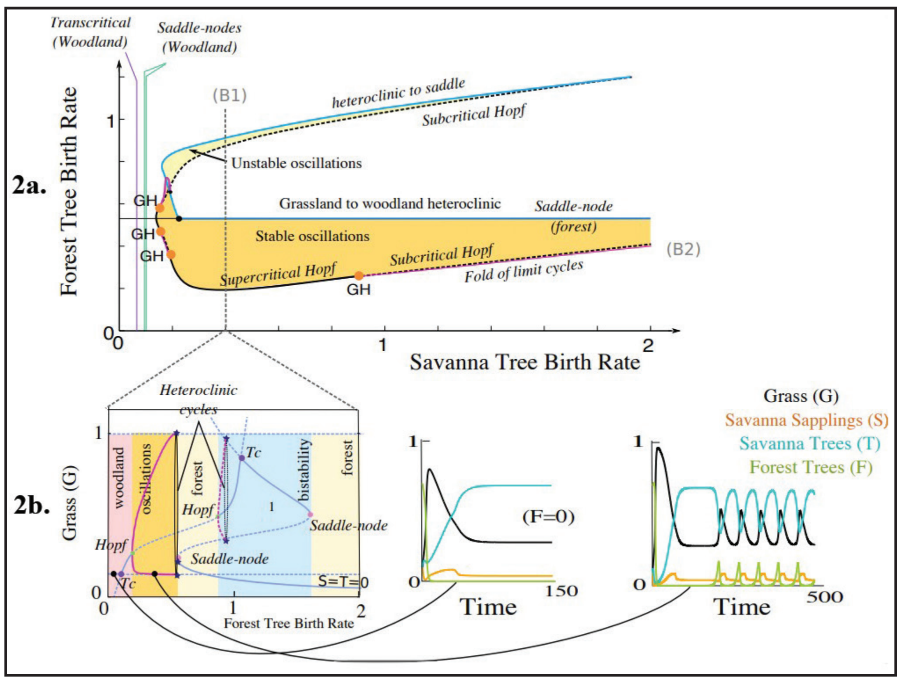


Figure 2. *Complex dynamics—including periodic orbits—emerge in the presence of a forest tree subtype.* **2a.** *Highly intricate dependences in the birth rates of forest and savanna trees organize these dynamics.* **2b.** *For a fixed savanna birth rate, increasing the forest tree birth rate drives the system from savannas (left trajectory) to oscillatory behaviors with the emergence of forest trees (right trajectory), and from bistable forest-grassland regimes to forests at high birth rates. Figure adapted from [1].*

But high densities of hardy scrub trees do not signify a real forest. Forest trees are more sensitive to fires than savanna trees; they also create denser shade. To better approximate these different landscapes, Touboul, Staver, and Levin add a variable F, a forest tree with birth rate α, and a mortality that depends on fire's ability to spread. To incorporate this element, the researchers model the mortality rate as an increasing function $\phi(G + \gamma(S+T))$, which accounts for the nonlinear dependence of fire spread on grass and tree cover and refines this dependence by considering the level at which savanna and woodland tree foliage propagate fires (parameter γ).

Using their model, the team studies this element's possible effect on landscape dynamics and identifies the system's complex co-dimensional two and three bifurcations that organize intricate multistability or oscillatory dynamics. They also account for differences between savanna and woodland tree foliage by varying the degree to which they exclude grasses. Savannas, scrub woodlands, and true forests can coexist in this more realistic model, described by the set

$$G' = \mu S + \nu T - \beta GT + \phi(G + \gamma(S+T))F - \alpha GF$$

$$S' = \beta GT - (\omega(G + \gamma(S+T)) + \mu)S - \alpha SF$$

$$T' = \omega(G + \gamma(S+T))S - \nu T - \alpha TF$$

$$F' = \Big(\alpha(1-F) - \phi(G + \gamma(S+T))\Big)F,$$

See **Savanna Lifecycle** *on page 7*

Heat Exchange and Some Frivolous Aspects of *e*

MATHEMATICAL CURIOSITIES

By Mark Levi

Is it possible to heat a glass of $0^\circ\mathrm{C}$ milk to $>50^\circ\mathrm{C}$ using only the heat from an identical glass of $100^\circ\mathrm{C}$ water, thus cooling the water to $<50^\circ\mathrm{C}$? No heat is exchanged with the outside world, extra containers are available, and heat capacities per unit volume of the water and milk are assumed to be the same.

Despite the fact that heat flows "downhill" temperature-wise (the second law of thermodynamics), one can indeed reverse the order of the two liquids' temperatures, as illustrated in Figure 1. We scoop $1/n$th of the milk into a ladle, dip the ladle in the hot water until the temperatures equalize, and dump the warmed milk into the glass on the right. After n repetitions, all of the milk ends up in the last glass. Dipping the $0^\circ\mathrm{C}$ milk ladle in the warm water reduces the water temperature by the same factor on each step:

$$T_{k+1} = \frac{n}{n+1} T_k, \quad T_0 = 100^\circ\mathrm{C}, \tag{1}$$

since the heat of n units of water spreads equally among the $n+1$ units of liquid.

After n steps,[1] with all of the milk in the third glass, the water therefore cools to

$$\frac{100}{\left(1+\frac{1}{n}\right)^n} \approx \frac{100}{e} \approx 36.8^\circ\mathrm{C};$$

coincidentally, this is the human body temperature. The milk's temperature is thus $\approx 63^\circ\mathrm{C}$, considerably above $50^\circ\mathrm{C}$. This is actually the perfect temperature for cooked salmon. To summarize,

$$T_{\text{body}} + T_{\text{salmon}} \approx 100^\circ\mathrm{C}$$

and

$$e \approx \frac{100^\circ\mathrm{C}}{T_{\text{body}}}.$$

Even more surprising than the reversal of the order of temperatures is the fact that a near-perfect temperature swap is possible in principle. Biological evolution "invented" the mechanism of such a swap, which may be described in another article.

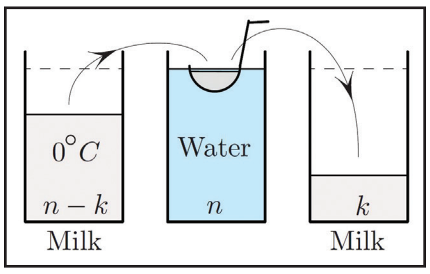


Figure 1. *The milk is colder than the water at the beginning of the process and hotter than the water at the end. Figure courtesy of Mark Levi.*

References

[1] Levi, M. (2012). *Why Cats Land on Their Feet: And 76 other Physical Paradoxes and Puzzles*. Princeton, NJ: Princeton University Press.

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[1] We assume a small ladle, hence a large n.

Mathematicians and Ethical Engagement

By Maurice Chiodo
and Dennis Müller

In the past, some mathematical societies have discussed ethical policies and issues [3, 4] and disseminated their own codes of conduct to address specific ethical concerns encountered by research mathematicians, such as those arising during publication. While ethical and behavioural issues specific to well-defined mathematical areas are of course still relevant, the last two decades have yielded many *new* ethical concerns that now affect *all* mathematicians in some way. Having taught these issues for more than two years at the University of Cambridge, we came to the realization that mathematicians can assume several different levels of ethical engagement [1]. Ethics in mathematics is not a binary process.

As the oldest consistently used scientific tool in Western thinking, mathematics carries perhaps the greatest scientific authority. It has become an extraordinarily powerful instrument ubiquitous to all of science and technology. How many hours of mathematical work underpin the technology behind smartphones, airplane flights, or models of global climate dynamics? But the applications—and therefore ethics—of mathematics go well beyond engineering. Modern mathematics is at the heart of economics and finance, and excessive trust in mathematical models contributed to the 2007 financial crisis. Even the most ardent purists in number theory or algebra can no longer claim to "just do the mathematics" and "leave the implications to ethicists", as recent revelations about global mass surveillance have underscored their work's immediate social and political impact. It is now evident that one can wield practically *all* branches of mathematics both for good and harm. Modern mathematics is a double-edged sword.

Just as physicists had to recognise the enormous ethical implications of their work after the atomic bombing of Hiroshima in August 1945, socially responsible mathematicians must also realise the existence of ethics in mathematical practise, which leads to issues far more complex and harder to characterize than publishing-related decisions. Plagiarism and the ethics of journal submission are real concerns, but hardly of the same order as these new ethical matters.

The inner workings of even areas of broad appeal—such as data science, machine learning, and optimisation—are often beyond the layman's comprehension. Lawyers and judges struggle to understand policing and sentencing algorithms, politicians stretch to comprehend the full capabilities of state surveillance agencies, and electoral commissions barely grasp the algorithms and mathematical psychometrics behind Cambridge Analytica's targeted advertising. Thus, only mathematicians can begin the process of unveiling the meaning, validity, applicability, and reliability of modern mathematics, paving the way for judges, politicians, and regulators to step in.

Even if we feel that mathematical research is beyond all ethical consideration, as academics we must ask ourselves: What do our students do after graduation? We train them in a wide range of mathematics, but do we teach them to be aware of possible ethical issues in its use? As a society, we have long agreed that the so-called Nuremberg defense—simply saying "I'm just doing my job" or "I was only following orders"—is not a valid excuse. Thus, it is imperative for us to teach ethics to our students and help them better contextualise their mathematical work. In April 2016, we began giving ethics seminars featuring guest speakers from industry, academia, and intelligence agencies to researchers and students in the Faculty of Mathematics at Cambridge. Shortly thereafter, we organized the first conference on "Ethics in Mathematics."[1] Through observation and case studies, we noticed that mathematicians can demonstrate what we term the "four levels of ethical engagement." These levels form a recurring theme throughout our seminars.

ETHICS IN MATHEMATICS

The *first* level is the fundamental understanding that the practice of mathematics is *not* ethics free, and that ethical issues can surface in any mathematical work. One always performs mathematics in a social and political context, never in value-free isolation. Thus, all mathematicians must think about their individual responsibilities, as ethical issues may emerge at any time. This diligence can be as simple as considering environmental impact rather than merely optimising over time and money during a construction project. Mathematics can pose immediate or distant consequences that generally manifest as good, sometimes as not entirely good, and occasionally as downright bad. On this individualistic level, mathematicians modify and adapt their own ethical consciousness and actions, taking the important first step towards a more robust ethical awareness.

The *second* of these four levels involves mathematicians *speaking out* to other mathematicians, raising awareness of ethical issues among their peers. Individual mathematicians may recognize ethical issues in the mathematical work of others and try to inform them. They might precipitate unified action among their colleagues and locally bring about a collective ethical awareness and approach. Or they might write an article about ethics for their community, as we have done here.

The *third* level is more complex. It teaches mathematicians to *take a seat at the tables of power*. Mathematicians often need to learn the specific skills required to work with politicians, corporate management, and other non-scientists. These include engaging in policy discussions, establishing and rationalising their mathematical work's objectives, and communicating potential limitations and possible drawbacks. Engineers and computer scientists are taught this at the undergraduate level, but mathematicians seldom receive such lessons explicitly. Many mathematicians in advancing industry careers unexpectedly find themselves in positions that require these abilities. Mathematics is becoming an increasingly powerful social tool, and seeing its creators hiding behind formulae and retrospectively apologising is not appropriate. If we want to take credit for our output's positive impact, we should also be able to defend and properly contextualise our work and engage in apparently non-mathematical debates.

Our *fourth* and final level is the responsibility of mathematicians to *call out the bad mathematics of others* by proactively seeking out, learning about, and acting upon instances where mathematics has "gone wrong" — possibly in unrelated organisations. However, bad mathematics occurs in two distinct forms. First, it can refer to the practice of claiming results that are not mathematically true. The catastrophic misuse of statistics in the trial of Sally Clark [2], which the Royal Statistical Society reprimanded through the release of a statement [5], is one such example. Bad mathematics can also refer to trained mathematicians' inappropriate use of mathematics by giving it excessive authority or directing it in ways that cause harm and exploit others. Members of any profession have the responsibility to hold their work—and the work of their colleagues—to high standards. Like statisticians, engineers, and doctors, mathematicians must adapt their own form of professional standards in academia, industry, and overall society. Some mathematicians are already questioning the validity and fairness of various decision-making algorithms or identifying the potential harms of artificial intelligence (AI), bringing such dangers into public consciousness and proposing workable solutions.

Practising ethics in mathematics is not binary, and mathematicians must consider various levels of engagement and ethical sensibility. Of course, our aforementioned four levels are an artificial and simplistic construct. One can refine them ad nauseum, but collectively they illustrate the depth and complexity of ethics in mathematics.

Not every mathematician will face problems pertaining to all levels, but everyone should remain aware of their social responsibilities, acknowledge the existence of ethical issues in the mathematical context, and appreciate their complexity. We teach students a broad spectrum of mathematics to prepare them for a wide variety of academic and professional eventualities. Why shouldn't we teach a broad spectrum of ethical situations in mathematics, which go beyond specialised courses such as ethics in AI? Lawyers, medics, biologists, engineers, physicists, and computer scientists learn subject-specific ethics because they will encounter these questions as professionals. Comprehending the seemingly-limitless uses of mathematics is difficult, and the ethical implications of modern mathematics depend on subtleties that only the mathematically-trained can understand. We are the only ones who can see behind the formulae. Thus, we should no longer leave these issues to professional ethicists and philosophers. No one else can address them, so we must.

References

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Maurice Chiodo is a research mathematician at the University of Cambridge, where he runs a seminar course on ethics in mathematics and is principal investigator of the University of Cambridge Ethics in Mathematics Project.[2] Dennis Müller is the president of the Cambridge University Ethics in Mathematics Society.[3] He was originally trained in computer science and currently studies mathematics at the University of Cambridge.

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[1] http://www.ethics.maths.cam.ac.uk/EiM1/

[2] www.ethics.maths.cam.ac.uk

[3] https://cueims.soc.srcf.net/

Savanna Lifecycle

Continued from page 5

where α and ϕ represent forest tree birth and death rates respectively. This system is reduced by $G+S+T+F=1$ to

$$\begin{aligned} G' &= \mu(1-G-T-F) \\ &\quad +\nu T - \beta GT + \\ &\quad \phi(G+\gamma(1-G-F))F - \alpha GF \end{aligned}$$

$$\begin{aligned} T' &= \omega\big(G+\gamma(1-G-F)\big) \\ &\quad (1-G-T-F) - \nu T - \alpha TF \end{aligned}$$

$$\begin{aligned} F' &= \big(\alpha(1-F) - \\ &\quad \phi(G+\gamma(1-G-F))\big)F. \end{aligned}$$

The result is a complex system that is quite sensitive to the parameters (see Figure 2, on page 5). A variety of equilibria appear, and families of periodic orbits undergo complex transformations.

Levin and Staver were surprised to find heteroclinic orbits—trajectories that connect and cycle between multiple states—during which the system flipped from one state to another. At the saddle node bifurcation, the orbit expands and approaches three coexisting unstable equilibria, which results in sharp, slow oscillations (see Figure 3).

These oscillations describe what the researchers call a "winnerless competition." Hardy savanna trees invade grasslands, shade the ground, and reduce grass growth and fire risk. Diminished fires promote tree growth, and soon forests dominate the landscape. But forest trees are more sensitive to fires than savanna trees, and they give way to grassy plains as fire again sweeps across the land.

Strangely, these patterns were not well-behaved. "They didn't have many of the features that we expected of heteroclinic orbits," Levin said. "Namely, as they approach the equilibrium they ought to be slowing down. That's what we didn't understand."

Levin and Staver consulted Touboul, who was familiar with multiple timescales thanks to his neuroscience experience. However, unlike in the brain, all savanna species evolve at comparable timescales. "It's the dynamics and the nonlinearity itself that create these heteroclinic cycles," Touboul realized. "The cycles may have branches that are very fast and branches that are very slow." Contrary to normal ecological assumptions, the model demonstrated that the landscape could change independently of any external process.

In this case, change—even rapid change—might just be a cycle from one state to the next rather than an indication that something or someone is pushing it. If something does drive it, like climate change, the corresponding effects would manifest in even the simplest model — as increased fire risk from less precipitation or faster tree growth due to warmer temperatures, for example.

However, a real savanna is more than simplified trees and grass with constant rates of initiation and growth. Fires are sporadic. Tree growth depends on water, nutrients, sunlight, and the voracity of predators. Researchers often assume that this extra "noise" averages out, but Touboul, Staver, and Levin realized that this was not the case upon examining these stochastic effects to test the model's relevance to real systems.

Rather than averaging out, relatively small noise perturbations exacerbated bifurcations and caused the model to veer off. For noise perturbations away from bifurcations, the system deviated only slightly from deterministic trajectories. But for ecological systems near the heteroclinic cycle, these perturbations triggered large amplitude periodic oscillations between grassland, savanna woodland, and forest where the noiseless system stabilized on a fixed landscape.

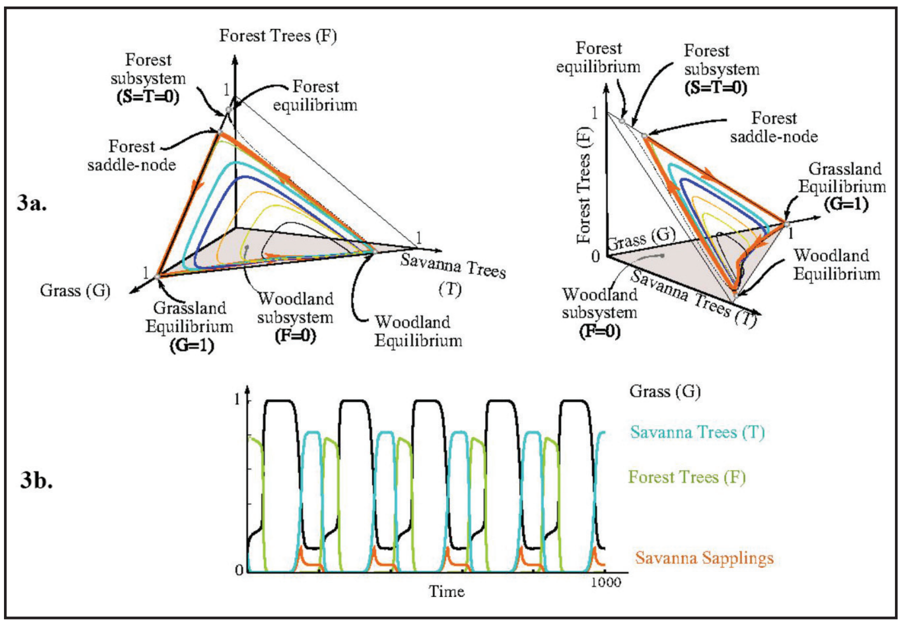

Figure 3. *Heteroclinic loop and nearby periodic orbits.* **3a.** *The heteroclinic loop (orange) joins a grassland, forest, and woodland equilibrium — all of which are unstable for those parameters.* **3b.** *For smaller values of the forest tree birth rate, periodic orbits emerge and progressively deviate from this cycle. The forest equilibrium is stabilized for larger values. Figure courtesy of [1].*

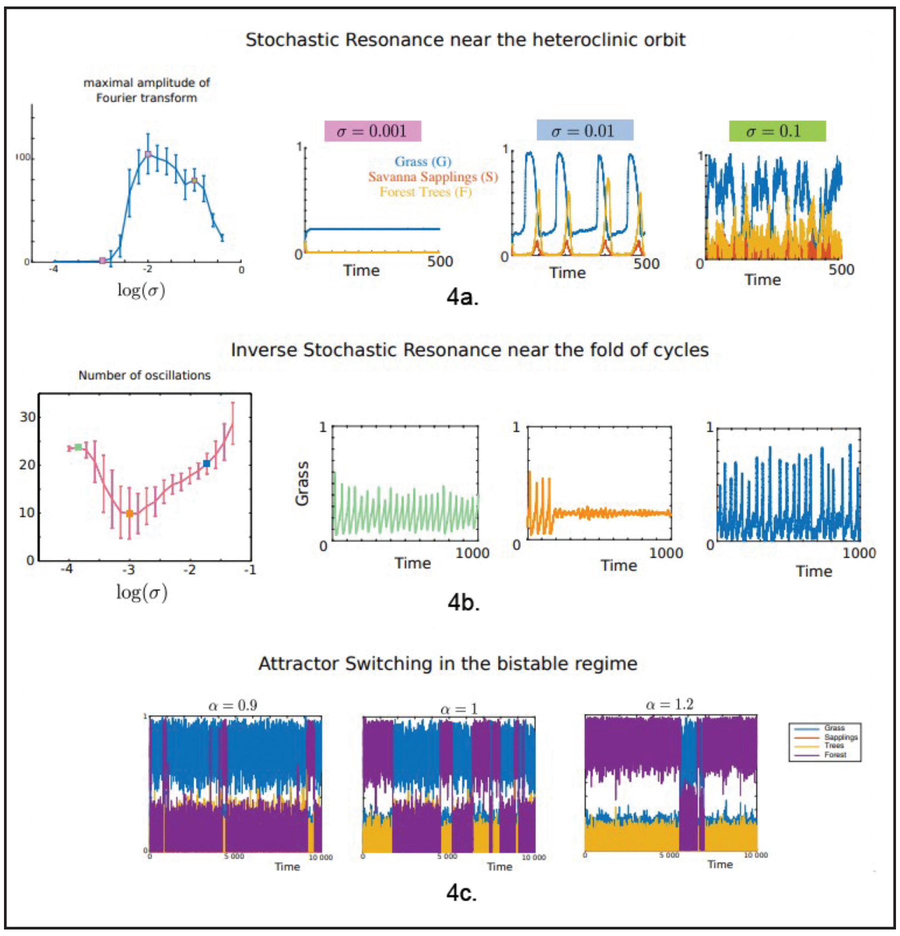

Figure 4. *Complex responses to noise in various parameter regimes.* **4a.** *Near the heteroclinic loop, an optimal level of noise triggers highly periodic responses, where the noiseless system stabilizes in a forest (stochastic resonance).* **4b.** *An optimal level of noise cancels the oscillations in the noiseless system (inverse stochastic resonance) near the fold of limit cycles.* **4c.** *Noise induces irregular switches between the different attractors in the bistable regime. Figure courtesy of Jonathan Touboul, adapted from [1].*

In the vicinity of another transition called fold of limit cycles, these perturbations had the opposite effect. They dampened the emergence of oscillations in the model where such stochastic effects are ignored. "When I applied noise to the system, I was very surprised to see that noise could actually act both ways, either in creating regular oscillations or destroying existing oscillations in the noiseless system," Touboul said. Figure 4 illustrates these novel results.

The team asserts that one should not view this model as predictive; an ecologist cannot simply tabulate all parameters, feed them into the model, and expect to know the landscape's vegetation type in coming centuries. Furthermore, parameters are difficult to estimate and check. "I can say with some confidence that when the system is pushed far enough, you will obtain these transitions," Levin said. "But in ecological systems, one must be very cautious about using models for prediction."

Touboul, Staver, and Levin are currently using paleoecological data to test the model's ability to describe past ecosystems. Ancient pollen grains embedded in layers of lake sediment provide clues about vegetation types that existed as early as 20,000 years ago. "It's complicated because the vegetation dynamics are often on the same timescale as climate change," Staver said. "So it's really difficult to determine if those cycles are occurring independently."

The team is also working to refine the model to account for spatial dynamics. The current model assumes the system is well-mixed, but in reality, one patch of vegetation influences another even if they are identical.

The researchers' work demonstrates an elegant and practical example of how simple starting conditions can produce highly complex non-equilibrium phenomena. It is also a cautionary tale against ignoring the sometimes-substantial effects of ecological noise and ascribing change to external drivers. Their model provides scientists with an improved understanding of this complex ecological system — with expandable techniques to help understand global vegetation patterns and potential changes due to climate change. And it is not limited to savannas. Other multi-layered systems—such as coral reefs, brain activity, and financial markets—would benefit from similar modeling.

References

[1] Touboul, J.D., Staver, A.C., & Levin, S.A. (2018). On the complex dynamics of savanna landscapes. *PNAS, 115*(7), E1336-E1345.

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Beautiful Mathematics

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Hossenfelder likens theory space to a box (see Figure 1). Each point in the box represents a different theory, and curves depict chains of theories related by implication, with high-resolution theories implying low-resolution ones. The totality of such curves, presumably capable of branching and/or merging, is called the "flow" of theories.

If the curves emanating from different versions of a particular high-resolution theory appear to converge on a low-resolution theory of interest—marked by X in Figure 1—the latter is termed "natural," since all versions of the high-resolution theory yield essentially the same low-resolution conclusions. Such conclusions "follow naturally" from the assumptions. But if the curves *diverge* and only a few pass near the low-resolution theory of interest, the latter is "fine-tuned" because it follows from only a handful of versions of the high-resolution theory. These must be adjusted just right to suggest the low-resolution theory. In this case, one must know every detail of the high-resolution theory with precision to justify confidence in the low-resolution one.

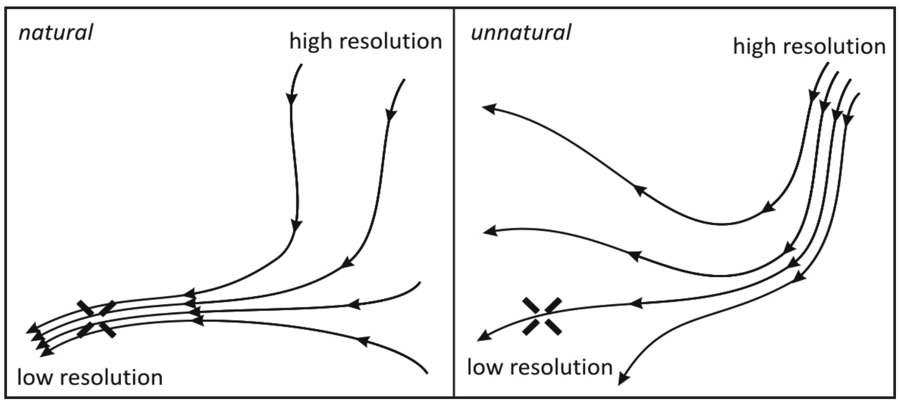

Figure 1. *Sabine Hossenfelder depicts theory space as a box where chains of related theories form curves, with high-resolution theories implying low-resolution ones. Image courtesy of Basic Books.*

The greater part of *Lost in Math* consists of interviews with leading physicists regarding their opinions on significant issues facing the discipline. Why do they find the Standard Model unsatisfactory? Is susy the only viable alternative? Why is *naturalness* beautiful but *fine-tunedness* unattractive? Will the quest for beauty produce something better? Is the long-sought "theory of everything" within reach?

Hossenfelder is a skilled interviewer, with a talent for drawing her subjects out on topics of mutual interest and an admirable distaste for trivial gossip. She is also humorous at times. Better still, she seems well-schooled in the subject's history, including the disputes that have disrupted it over the years. Hossenfelder's take on these is refreshing, as it rebuts the prevailing view whereby "progress in the sciences is made at the funerals of scientists."

In hindsight, many historic scientific disputes—such as those surrounding heliocentricity and/or the reality of atoms and molecules—seem foolishly one-sided. Yet, says Hossenfelder, this was not always the case. In almost every instance, good arguments seemed to exist on both sides of the disputed issue for many years. In time, the preponderance of evidence came to rest on one side or the other. But until scientists gathered decisive proof, the eventual outcome remained unpredictable.

The argument surrounding heliocentrism is a prime example. Copernicus' contemporaries found it hard to accept the model because the planets' rotation around the sun should imply movement of the fixed stars in the sky as Earth travels from its nearest approach to the farthest remove of a given star. The magnitude of this movement, known as "parallax," depends on the average distance between Earth and the star — a greater distance correlates with a smaller apparent change in position.

The stars do indeed change position slightly during the course of a year. But because astronomers could not detect such miniscule changes until the 19th century, generations of people were forced to conclude that either Earth remained stationary or the fixed stars were exceedingly far. Moreover, since a beam of light from a distant star that passes through a circular aperture—such as an eye or telescope—will "smear out" and undergo magnification, those distant stars seem gigantic in comparison with other celestial bodies, including the sun. Scientists also did not understand this magnification until the 19th century, so earlier generations of astronomers had to either agree with Ptolemy that the stars remain fixed in the "celestial sphere" or conclude that they were unimaginably large *and* distant. How surprising is it that many found the more familiar teaching both simpler and easier to accept? After all, simplicity has long been regarded as a reliable indicator of beauty.

Hossenfelder quotes Paul Krugman to the effect that "The economics profession went astray because economists, as a group, mistook beauty, clad in impressive-looking mathematics, for truth" [1]. Her interviews are meant to discover the extent to which physicists are in danger of making the same mistake. After failing to identify a consensus beyond "time will tell" among her impressive roster of respondents, she elects to close on a positive note. "The next breakthrough in physics will occur in this century," Hossenfelder writes. "It will be beautiful."

References

[1] Krugman, P. (2009). How Did Economists Get It So Wrong? *The New York Times Magazine*. Retrieved from https://www.nytimes.com/2009/09/06/magazine/06Economic-t.html.

James Case writes from Baltimore, Maryland.

How a Chance Internship Inspired My Career in the Oil Industry

By Anusha Sekar

The story of my career begins at the midpoint of my graduate studies. After earning my master's in mathematics from the Indian Institute of Science in Bangalore, India, I moved to the University of Washington (UW) to complete my Ph.D. I was working on my thesis in 2006 when my advisor, Kenneth Bube, hosted John Washbourne, a scientist from Chevron Corporation. Ken had a long history of collaboration with researchers at Chevron, but I had only a nebulous idea of the content of this research. John offered me an internship, but I was hesitant at first. At the time, my goal was to finish my thesis and work in math education. I wasn't completely sure if I wanted to pursue a career in industry.

However, Ken and my husband convinced me to give it a try, and I completed my first internship in the summer of 2007 with John as my mentor. We worked on what appears at first to be a very simple problem: averaging certain properties of rocks from well logs. A single MATLAB function can seemingly achieve this, until you realize that you must combine these averages to yield the coefficients of a hyperbolic partial differential equation (PDE). The objective is to average the coefficients for consistency; solving the PDE with both the original (finer grid) and averaged (coarser grid) coefficients must produce matching wavefields, i.e., the correct effective medium.

Geophysicists had already derived a formula for this in one dimension [3]. Using their work, we numerically confirmed that the effective medium formulas produced the correct wavefields. The amount of mathematics, physics, and computer engineering necessary to understand this small problem was impressive. I was hooked! I also enjoyed my time at Chevron and met several interesting people. The atmosphere was very collegial; colleagues shouted concepts across the hallway to one another and went out of their way to help each other. There were many fun lunches and even a hike on Mount Diablo. John convinced me to participate in a second internship the following summer, which solidified my desire to work in industry. I approached one of the team leads about a job, and thanks to good reviews from John, received an offer. John continues to mentor me to this day.

CAREERS IN MATHEMATICAL SCIENCES

I returned to UW to complete my thesis. About two months before I was to join Chevron, I discovered that I was pregnant. I had heard about so many unpleasant incidents involving pregnant women and corporations that I was convinced my offer would be rescinded — or at least delayed. I called my team lead with the news and will never forget his response. "Don't even think about waiting, Chevron has very good medical benefits!" he said. He assured me that I could start as planned and proceeded to preview the project on which I would be working. Six months into my pregnancy, I began my career at Chevron. With help and mentoring from coworkers and support from family, I had a successful first year despite taking time off to have a baby. My manager later confessed that his only concern was whether I would decide to stay home after the baby's birth.

Chevron's culture is very conducive to a good work-life balance. I have met driven women and men who dedicate the same amount of energy to both work and raising a family, and there is broad diversity in terms of gender and race. Employees also have a range of technical backgrounds, which allowed me—a mathematician with minimal geophysics knowledge—to contribute.

My biggest struggle—and I believe this is true of all industry positions—was understanding the jargon. Sometimes my colleagues use words that mean one thing in mathematics but something entirely different in the world of oil. Furthermore, geologists and geophysicists have different interpretations of the same terms. Exploration geophysicists and whole-earth geophysicists (seismologists) do not agree on some definitions. Acronyms abound. Even the Fourier transform is defined with a different sign on the exponent in certain instances! I could not digest the fact that some algorithms use adjoint operators of a non-unitary operator as their approximate inverse and still produce reasonable answers (some interesting mathematics validates this use [9]).

On the flip side, I have picked up a host of new topics; I learned more optics than I did in physics classes, gained much knowledge of signal processing, and practiced designing and writing code that others can use. It was great to see algorithms validating theory and even better when results matched field measurements. I am constantly surprised and pleased that I can still use parts of my thesis to solve real-world problems.

The oil industry utilizes a wide variety of mathematics. Researchers solve Navier-Stokes equations to study historical sedimentation [2], employ porous media flows to understand reservoir flow [5], and use data analytics to improve production [6]. The science that we develop is also applicable to other areas, like medicine.[1] I now work on inverse problems where an expensive hyperbolic PDE represents the forward engine [1]. Rich mathematical theory underlies the existence of solutions [4]. Local minima are a big headache, but there are ways to get around it [8]. However, unanswered questions pertaining to whether artificial intelligence can completely replace the physics remain [7].

You can easily lose your identity as an applied mathematician in industry. It is also quite tempting to stay within the cocoon of your particular industry and ignore other fields. In my role, I could choose to confine myself to academic work on geophysical problems. But mathematicians can see patterns and understand a problem's essence, extracting it out of the business in which it is embedded. It is important for us to capitalize on and develop this ability. Hence, networking and collaborating with other applied mathematicians within or outside your specific industry becomes crucial. This is where SIAM came in for me and inspired a few of us to create a SIAM Texas-Louisiana Section. The section provides a platform for applied mathematicians to establish or reestablish links with their peers. Thus far we have organized two successful work-

See **Oil Industry** *on page 12*

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[1] http://www.uh.edu/nsm/math/seminars-and-events/Houston%20Imaging%20Sciences%20Symposium/

Data Science at the IMA: An Industry-Supported Initiative

By Benjamin Brubaker, Fadil Santosa, and Daniel Spirn

The Institute for Mathematics and its Applications (IMA) recently established an agreement with Target and Cargill, launching a new era in industrial collaboration at the institute. The two Minnesota-based companies are placing a substantial investment in the mathematical sciences — more than $2 million over a two-year period, likely representing the largest research grant from the industrial sector to a U.S.-based mathematical sciences institute. “The partnership is a win-win for everyone: the IMA, the University of Minnesota (UMN), the two companies, and the region,” Mostafa Kaveh, dean of the College of Science and Engineering at UMN, said. “We are completely behind this initiative and have provided significant institutional support.”

The new initiative fits squarely within the central mission of the IMA: to connect mathematics with its applications. The IMA’s history of collaboration with industry dates back to 1981. Its founders articulated a vision for an interdisciplinary institute with substantial representation from industry and government scientists in the initial proposal submitted to the National Science Foundation (NSF). Since then, the IMA has become the go-to place for collaboration between industry and academia, with the following activities serving as mainstays of the IMA model: (i) industry outreach, (ii) Industrial Problems Seminars, (iii) industrial math modeling workshops, and (iv) industrial postdoctoral fellowships.

After the NSF’s 2015 decision to only partially fund the IMA’s renewal proposal, the institute sought to preserve institutional activities through diversified funding sources. In addition to mitigating the inevitable ups and downs of government funding cycles, this approach aligns the IMA’s incentive structure to better represent its constituents. So the institute devised a new model that makes a compelling case for organizations to support it.

It seemed natural to partner with mathematically sophisticated companies located in the IMA’s own backyard. One of the first companies that the IMA approached was Target. Target’s Chief Data and Analytics Officer, Paritosh Desai—trained in operations research at Stanford University—has a vision for mathematics’ ability to provide companies with a competitive advantage. He was instrumental in shaping the new Data Science Consortium at the IMA and enlisting the participation of Cargill as a second founding member.

Although data science is an inherently interdisciplinary field, its underpinnings are mathematical and algorithmic. “While there are many spectacular successes in machine learning and artificial intelligence, fundamental understanding of their behavior is still quite open,” Desai noted. “This is where we need more mathematics.” Such collaboration provides a meaningful source of interesting and important problems that expand the discipline’s boundaries.

The IMA has previously run programs on the mathematical aspects of data science and is well positioned to address outstanding challenges in the field. Three distinct parts—the industrial postdocs program, the research program, and the training program—reflect the earlier mainstay activities of the industrial portfolio and form the consortium’s core. The industrial postdocs spend half of their time working on directed company research projects. “The three IMA postdoctoral fellows in the Cargill data science group are already making an impact on our business by bringing in new technologies,” James Weed, Vice President of Analytics and Digital Economy at Cargill, said. “We hope that they will consider remaining with Cargill when they are done.” One postdoctoral researcher started at Target last year, and two others will join the company this year. See accompanying sidebar for descriptions of the postdocs’ research projects.

The research program offers topics for thematic semesters, which are determined by consortium members in consultation with the advisory board. The board consists of Desai, Weed, Katherine Ensor (Rice University), Peter Glynn (Stanford), Piotr Indyk (Massachusetts Institute of Technology), Gilad Lerman (UMN), and Joel Tropp (California Institute of Technology). A semester-long program on spatiotemporal forecasting took place in the spring of 2017, and a program on applications of machine learning to supply chains will be held this fall. The training program trains both students and company-based data scientists, providing the companies with both a talent pipeline and cutting-edge tools from academic research. These endeavors enhance the existing core activities at the IMA.

The IMA is using its collaboration with Target and Cargill to lay the foundation for a national data science resource. The initiative has many benefits that extend beyond the partner companies. With its workshops and scientists-in-residence program, the institute continues to function as a center where mathematical scientists can congregate and collaborate. In addition, it acts as a training ground for mathematical scientists at various stages of their careers. The unique industrial postdoc program—admired and imitated around the world—offers industry work experience to recent Ph.D. graduates while retaining connections with academia. It opens career paths in industry while enhancing participants’ academic job prospects. The IMA also sponsors training programs for graduate students interested in transitioning to data science; these programs include an opportunity to work on industry problems — an experience that aids the students’ job search.

IMA leadership has always believed that the most exciting and transformative research arises from problems directly confronting industry, government, and society. This new collaborative program presents yet another avenue to explore these questions and provides exciting possibilities for the future of academic-industrial partnerships in the mathematical sciences.

Benjamin Brubaker is a professor in the School of Mathematics at the University of Minnesota (UMN) and currently serves as the deputy director of the Institute for Mathematics and its Applications (IMA). His research interests lie in analytic number theory and representation theory. Fadil Santosa served as IMA director from 2008 to 2017 and is a professor of mathematics at UMN. He works in inverse problems, optimal design, and photonics. Daniel Spirn is the present director of the IMA, having previously served as deputy director from 2015-2017. He is a professor in the School of Mathematics at UMN and works in analysis of nonlinear partial differential equations.

Industrial Postdoctoral Projects

Hossein Keshavarz received his Ph.D. in statistics from the University of Michigan in 2017. He worked on a natural language processing project with Spencer Schaber at Cargill. His current Cargill project—in collaboration with Hartmut Durchschlag—focuses on algorithmic trading strategies derived from high-dimensional time series analysis, with applications in commodity futures. Keshavarz’s research at the IMA centers on change point detection in high-dimensional, sparse graphical models. In this work, he actively collaborates with George Michailidis (University of Florida) and Yves Atchadé (Boston University).

Stuart Rogers earned his Ph.D. in 2017 from the University of Alberta, where he studied dynamics and control. He worked at Lockheed Martin before attending graduate school. At Target, Stuart works with Kaveh Khodjasteh on recommender systems for shipment packaging based on mixed-integer linear programming. In terms of academia, he studies the dynamics and optimal control of rolling ball robots and falling cats. Stuart’s postdoctoral mentor is Peter Olver (University of Minnesota).

Guanglin Xu completed his Ph.D. in operations research at the University of Iowa in 2017. His IMA mentor is Shuzhong Zhang (University of Minnesota). With collaborator Andrés Merchán, Xu is developing optimal supply chains for animal nutrition for Cargill’s feed business. His research interest at the IMA is optimization under uncertainty, including multistage and distributionally-robust optimization and their applications in healthcare and operations management.

Dongmian Zou received his Ph.D. in 2017 from the University of Maryland under the direction of Radu Balan, who was incidentally an IMA industrial postdoc in 2000. At Cargill, Zou works with Hartmut Durchschlag to forecast commodity prices and understand factors that drive their movement. His research mentor at the IMA is Gilad Lerman (University of Minnesota), with whom he is using the scattering transform to examine fundamental issues in graph convolutional neural networks.

Professional Opportunities and Announcements

*Send copy for classified advertisements and announcements to **marketing@siam.org**.*
*For rates, deadlines, and ad specifications, visit **www.siam.org/advertising**.*

*Students (and others) in search of information about careers in the mathematical sciences can click on “Careers and Jobs” at the SIAM website (**www.siam.org**) or proceed directly to **www.siam.org/careers**.*

Williams College

Department of Mathematics and Statistics

The Department of Mathematics and Statistics at Williams College invites applications for a new **tenure-track position in statistics**, beginning fall 2019, at the rank of assistant professor. A more senior appointment is also possible for a qualified candidate at a later stage in their career. The candidate should have a Ph.D. in statistics or a closely-related field by the time of appointment. We are seeking candidates who show evidence and/or promise of excellence in teaching and a strong research program that can engage undergraduate students. The candidate will become the seventh tenure-track statistician in the department, joining a vibrant and innovative group of statisticians within an established statistics major. For more information on the Department of Mathematics and Statistics, visit **http://math.williams.edu/**.

Candidates may apply via **https://apply.interfolio.com/50978** by uploading a cover letter addressed to Professor **Richard De Veaux**, a curriculum vitae, a teaching statement, a description of research plans, and three letters of recommendation on teaching and research. The department is committed to building a diverse and inclusive community. In your application materials, we also ask you to address how your teaching, scholarship, mentorship, and/or community service might support Williams’ commitment to diversity and inclusion.

Expectations: The teaching load is two courses per 12-week semester and a winter term course every other January. The candidate will be expected to teach introductory statistics, core courses for the statistics major, and elective courses in their areas of interest. The successful candidate will establish an independent research program that results in scholarly publications. Williams College provides broad support for start-up funds, funding for student research assistants, faculty professional development funds, and a shared computer cluster for parallel computation.

Review of applications will begin on or after **October 1st** and will continue until the position is filled. All offers of employment are contingent upon completion of a background check. Further information is available at **https://faculty.williams.edu/prospective-faculty/background-check-policy/**.

Williams College is a coeducational liberal arts institution located in the Berkshire Hills of western Massachusetts. The college has built its reputation on outstanding teaching and scholarship, and on the academic excellence of its approximately 2,000 students. Please visit the Williams College website at **http://www.williams.edu**. Beyond fully meeting its legal obligations for nondiscrimination, Williams College is committed to building a diverse and inclusive community where members from all backgrounds can live, learn, and thrive.

California Institute of Technology

Department of Computing + Mathematical Sciences

The Computing + Mathematical Sciences (CMS) Department at the California Institute of Technology (Caltech) invites applications for a tenure-track faculty position in the fundamental mathematics and theory that underpins application domains within the CMS Department, the Division of Engineering and Applied Science (EAS), or the institute as a whole. Areas of interest include (but are not limited to) algorithms, data assimilation and inverse problems, dynamical systems and control, geometry, machine learning, mathematics of data science, networks and graphs, numerical linear algebra, optimization, partial differential equations, probability, scientific computing, statistics, stochastic modeling, and uncertainty quantification.

CMS is a unique environment where research in applied and computational mathematics, computer science, and control and dynamical systems is conducted in a collegial atmosphere; application foci include distributed systems, economics, graphics, neuroscience, quantum computing, and robotics and autonomous systems. The CMS Department is part of the broader EAS Division, comprising researchers working in—and at intersections of—the fields of aerospace, civil, electrical, mechanical, and medical engineering, as well as in environmental science and engineering plus materials science and applied physics. The institute as a whole represents the full range of research in biology, chemistry, engineering, physics, and the social sciences.

A commitment to world-class research, as well as high-quality teaching and mentoring, is expected. The initial appointment at the assistant professor level is for four years and is contingent upon the completion of a Ph.D. degree in applied mathematics, computer science, statistics, or a related field in engineering or the sciences.

Applications will be reviewed beginning **November 7, 2018**, and applicants are encouraged to have all of their application materials, including letters of recommendation, on file by this date. For a list of required documents and full instructions on how to apply online, please visit **https://applications.caltech.edu/jobs/cms**.

Questions about the application process may be directed to **search@cms.caltech.edu**.

We are an equal opportunity employer and all qualified applicants will receive consideration for employment without regard to race, color, religion, sex, sexual orientation, gender identity, national origin, disability status, protected veteran status, or any other characteristic protected by law.

Boston University

Department of Mathematics and Statistics

The Department of Mathematics and Statistics at Boston University (BU) invites applications for a tenure-track assistant professor position in dynamical systems. BU is committed to building a culturally diverse faculty and strongly encourages applications from female and minority candidates. All candidates are encouraged to describe in their application previous activities mentoring minorities, women, or members of other underrepresented groups, and how they plan continual engagement with related issues. Ph.D. required. Begins July 2019, pending budgetary approval. Commitment to research and teaching at the undergraduate and graduate levels is essential.

Submit cover letter, CV, research statement, teaching statement, and four recommendation letters (one of which addresses teaching) online to **mathjobs.org**. Alternatively, submit materials to: Dynamical Systems T-T Search, Department of Mathematics and Statistics, Boston University, 111 Cummington Mall, Boston, MA, 02215. Application deadline: **December 15, 2018**. We are an equal opportunity employer and all qualified applicants will receive consideration for employment without regard to race, color, religion, sex, sexual orientation, gender identity, national origin, disability status, protected veteran status, or any other characteristic protected by law. We are a VEVRAA Federal Contractor.

A Visual Way to Teach the Fast Fourier Transform

By Jithin D. George

The algorithm behind the fast Fourier transform (FFT) has a simple yet beautiful geometric interpretation that is frequently lost in translation in a classroom. Here I provide a visual perspective that aims to capture the algorithm's essence.

Students are often confused when they encounter the FFT for the first time. This confusion likely stems from two sources:

1. The belief that one needs to completely understand the Fourier transform to comprehend the FFT. This is not true; the FFT is simply an efficient way to compute sums of a special form, and the terms in the discrete Fourier transform (DFT) just happen to be in that form:

$$A_k = \sum_{n=0}^{N-1} a_n e^{-i\frac{2\pi n}{N}k}. \tag{1}$$

2. The standard presentation of the Cooley-Tukey algorithm [1]. This is the heart of the FFT, and indicates that it is possible to decompose the DFT of a sequence of terms into a DFT of even terms and a DFT of odd terms. When applied recursively, it results in a computational cost of $O(N\log N)$. Researchers generally use the following decomposition of A_k into odd and even terms to illustrate the idea:

$$\sum_{n=0}^{N-1} a_n e^{-i\frac{2\pi n}{N}k} = \sum_{n=0}^{N/2-1} a_{2n} e^{-i\frac{2\pi(2n)}{N}k} + \sum_{n=0}^{N/2-1} a_{2n+1} e^{-i\frac{2\pi(2n+1)}{N}k}. \tag{2}$$

Let us take a simplified look at the terms in a DFT:

$$A_k = \sum_{n=0}^{N-1} a_n e^{-in\frac{2\pi}{N}k} = \sum_{n=0}^{N-1} a_n e^{ik\theta_n}. \tag{3}$$

One can visualize $a_n e^{ik\theta_n}$ as the value a_n located at angle $k\theta_n$ on a unit circle in the complex plane. As n goes from 0 to N-1, the θ_ns divide the circle into N arcs of angle $\frac{2\pi}{N}$. Each term in the summation in (3) is a multiple of a point on the unit circle in the complex plane (see Figure 1). With this geometric view, the Cooley-Tukey algorithm in (2) becomes obvious through Figure 2.

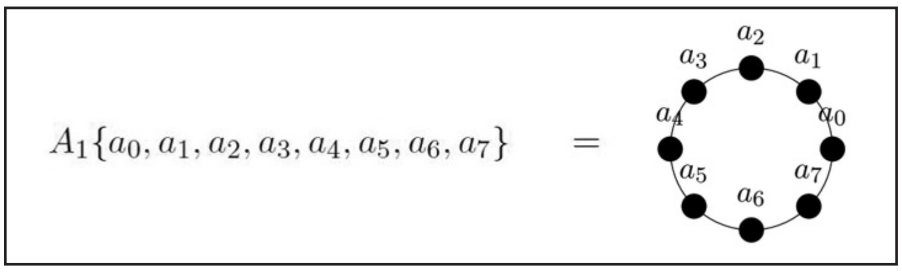

Figure 1. *The "n"th term in a discrete Fourier transform can be expressed as the summation of points that lie on a circle separated by angle θ_n.*

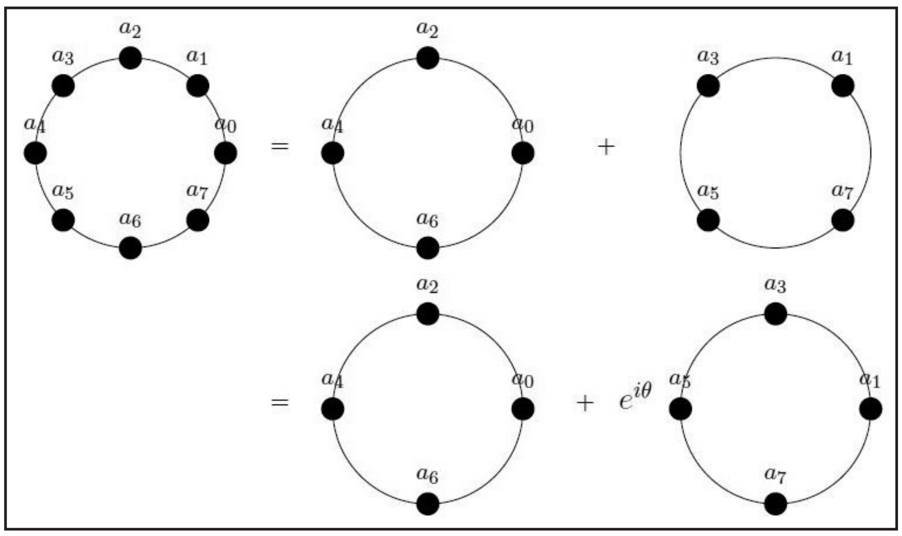

Figure 2. *The circular representation of a discrete Fourier transform term can be split into the circular representations of its even and odd components (with a rotation).*

We can compute sums like the FFT in this way because the odd terms are a "rotation" away from the even terms. This is quite elegant, but does not provide any new computational efficiency in itself. We are able to decompose a sum into two smaller sums of half the size, but still must calculate all of the sums. The smaller sums' ability to be "recycled" into new sums gives the FFT its computational efficiency. We can recycle the two terms that when added yield A_1 by subtracting them to produce A_5 (see Figure 3, on page 12). This certainly saves some computational cost, but how much? To obtain the finer details, we must work out a simple example.

To that end, let us examine the DFT of the vector $\{a_0, a_1, a_2, a_3\}$ (see Figure 4, on page 12). We can obtain the FFT of $\{a_0, a_1, a_2, a_3\}$ using the terms in Figure 5 (on page 12). The first two terms are the FFT of $\{a_0, a_2\}$, and the last two form the FFT of $\{a_1, a_3\}$. Thus, one can decompose an FFT into an FFT of even terms and an FFT of odd terms. This saves a lot of computational cost — almost half, since computing the DFT naively yields $\mathcal{O}(N^2)$. It also varies from the decomposition of sums with a cost of $\mathcal{O}(N)$, where decomposition did not help conserve computational cost.

FFT$\{a_1, a_2, a_3, a_4\}$ has the combined computational cost of FFT$\{a_1, a_3\}$, FFT $\{a_2, a_4\}$, and $4c_2$, where c_2 represents the cost of multiplication and addition for each A_n.

When expanding the FFTs recursively, FFT $\{a_1, a_2, a_3, a_4\}$ has the combined computational cost of FFT$\{a_1\}$, FFT$\{a_3\}$, FFT$\{a_2\}$, FFT$\{a_4\}$, and $8c_2$. The cost $8c_2$ comes from the $4c_2$ cost of operations required to combine the one-point DFTs to form each of the four terms in Figure 5 (on page 12), plus the $4c_2$ cost from the previous step.

Naively computing the DFT with N points requires c_1N^2 work, while comput-

See **Fast Fourier Transform** *on page 12*

Faculty Position - Operations Research and Information Engineering (ORIE)

A faculty position in Operations Research and Information Engineering (ORIE) is available at the Cornell Tech campus in New York City. The position is part of the Jacobs Technion-Cornell Institute, and we particularly encourage candidates whose work fits into Jacobs Institute application-domain emphases in the areas of urban technology, especially related to the intersection of digital and physical systems, and digital health technologies.

The position is within Cornell University's School of ORIE, and applicants with research interests represented within Cornell ORIE are welcome at all levels, including tenured and tenure-track. The School consists of a diverse group of high-quality researchers and educators interested in probability, optimization, statistics, simulation, and a wide array of applications such as e-commerce, supply chains, scheduling, manufacturing, transportation systems, health care, financial engineering, service systems and network science. Cornell ORIE spans both the Ithaca and New York City campuses, but the successful candidate's teaching and research will be based in New York City. (Interested candidates can apply for a Cornell Tech in NYC position, a Cornell Ithaca ORIE position, or both, but the two campuses have different application sites; please see the Cornell Ithaca ad for the Ithaca application URL).

Candidates must hold a Ph.D. in operations research, mathematics, statistics, or a related field by the start of the appointment, and have demonstrated an ability to conduct outstanding research at the level of tenure-track or tenured faculty in Cornell ORIE. They must also have a strong commitment to engagement outside of academia in ways that foster significant commercial or societal impact, as aligned with the mission of the Cornell Tech campus. The Institute seeks candidates with demonstrated transdisciplinary interests and a track record of translational science. The successful candidate will be expected to pursue an active research program, to teach Master's and Ph.D.-level graduate courses, and to supervise graduate students.

All applications completed by November 16, 2018 will receive full consideration, but we urge candidates to submit all required material as soon as possible. We will accept applications until we fill the positions. Applicants should submit a curriculum vitae, brief statements of research and teaching interests, and the names and contact information of at least three references. They should also identify one or two top publications to which they have made significant contributions. A distinguishing characteristic of research at Cornell Tech, in addition to world-class academic work, is that it engages deeply with external communities, organizations, K-12 education, and industry to address real-world problems and contexts that amplify the direct commercial and societal impact of our research. Accordingly, within a clearly identified subsection of the research statement, the candidate should address prior accomplishments and future plans related to this kind of direct commercial and/or societal impact of their research. Applications are on-line at

https://academicjobsonline.org/ajo/jobs/12018

Inquiries about your application may be directed to Sheri Minarski at **slm339@cornell.edu**.

Cornell University is an innovative Ivy League university and a great place to work. Our inclusive community of scholars, students and staff impart an uncommon sense of larger purpose and contribute creative ideas to further the university's mission of teaching, discovery and engagement. With our main campus located in Ithaca, NY Cornell's far-flung global presence includes the medical college's campuses in Manhattan and Doha, Qatar, as well as the new Cornell Tech campus located on Roosevelt Island in the heart of New York City.

Diversity and Inclusion are a part of Cornell University's heritage.
We are a recognized employer and educator valuing AA/EEO,
Protected Veterans, and Individuals with Disabilities.

Tenured/Tenured-Track Faculty Position(s)

Cornell University's School of Operations Research and Information Engineering (ORIE) seeks to fill multiple tenured/tenured-track faculty positions for its Ithaca campus. We will primarily consider applicants with research interests in the areas of discrete optimization and financial engineering, especially those individuals who do computation, who work with data, or whose work intersects with machine learning. Nevertheless, we welcome strong applicants from all research areas represented within ORIE, especially those in resonance with the College of Engineering Strategic Areas: **https://www.engineering.cornell.edu/research-and-faculty/strategic-areas-research**.

Requisite is a strong interest in the broad mission of the School, exceptional potential for leadership in research and education, an ability and willingness to teach at all levels of the program, and a Ph.D. in operations research, mathematics, statistics, or a related field by the start of the appointment. Salary will be appropriate to qualifications and engineering school norms.

Cornell ORIE is a diverse group of high-quality researchers and educators interested in probability, optimization, statistics, machine learning, simulation, and a wide array of applications such as e-commerce, supply chains, scheduling, manufacturing, transportation systems, health care, financial engineering, service systems and network science. We value mathematical and technical depth and innovation, and experience with applications and practice. Ideal candidates will have correspondingly broad training and interests.

Please apply online at **https://academicjobsonline.org/ajo/jobs/11870** with a cover letter, CV, statements of teaching and research interests, sample publications, at least three reference letters and, for junior applicants, a doctoral transcript. All applications completed by November 16, 2018 will receive full consideration, but we urge candidates to submit all required material as soon as possible. We will accept applications until we fill the positions.

ORIE and the College of Engineering at Cornell embrace diversity and seek candidates who can contribute to a welcoming climate for students of all races and genders. Cornell University seeks to meet the needs of dual career couples, has a Dual Career program, and is a member of the Upstate New York Higher Education Recruitment Consortium to assist with dual career searches. Visit **www.unyherc.org/home** to see positions available in higher education in the upstate New York area. Diversity and Inclusion are a part of Cornell's heritage.

Cornell University is an innovative Ivy League university and a great place to work. Our inclusive community of scholars, students and staff impart an uncommon sense of larger purpose and contribute creative ideas to further the university's mission of teaching, discovery and engagement. With our main campus located in Ithaca, NY Cornell's far-flung global presence includes the medical college's campuses in Manhattan and Doha, Qatar, as well as the new Cornell Tech campus located on Roosevelt Island in the heart of New York City.

We are a recognized employer and educator valuing AA/EEO, Protected Veterans, and Individuals with Disabilities. We strongly encourage qualified women and minority candidates to apply.

Fast Fourier Transform

Continued from page 11

ing two DFTs of size $\frac{N}{2}$ requires only half as much work: $2c_1\left(\frac{N}{2}\right)^2 = \frac{1}{2}c_1N^2$, plus c_2N work to combine the two results.

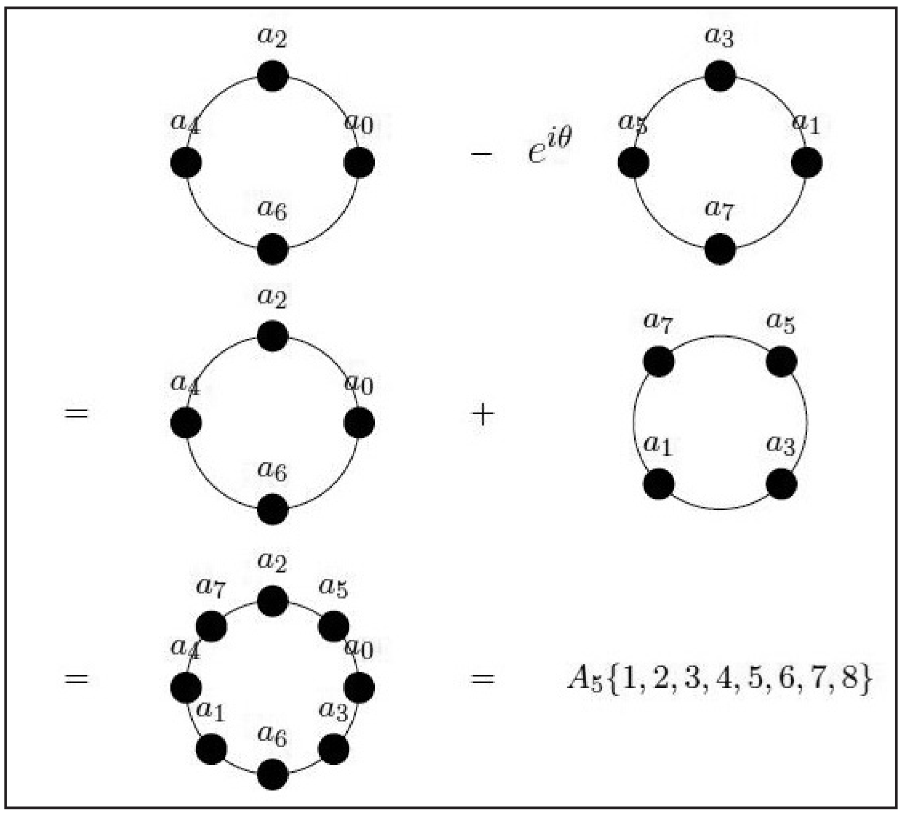

Figure 3. *The even and odd circles from Figure 2 can be recycled to yield a completely different term of the discrete Fourier transform.*

We can apply this recursively $\log_2 N$ times to completely eliminate the quadratic cost, leaving only the cost of N one-point DFTs plus $\log_2 N$ combinations—each requiring $\mathcal{O}(N)$ work—for a total of $\mathcal{O}(N)\log_2 N$ work.

Thus, the total cost in general is $\mathcal{O}(N) + cN\log_2(N) \approx \mathcal{O}(N\log_2(N))$.

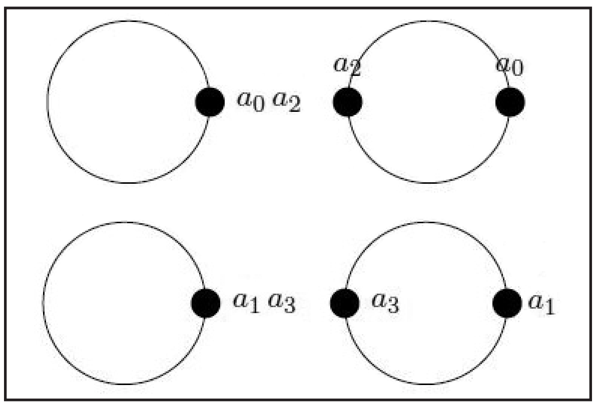

Figure 5. *The top two terms are the discrete Fourier transform (DFT) of* $\{a_0, a_2\}$ *and the bottom two are the DFT of* $\{a_1, a_3\}$*. Together they give the DFT of* $\{a_0, a_1, a_2, a_3\}$*, as shown in Figure 4.*

An earlier version of this description is available in [2].

The figures in this article were provided by the author.

Are you a graduate student? Have you discovered a unique way to learn, teach, or understand a complicated mathematical concept? Write to us at sinews@siam.org! We may publish your insights in an upcoming issue of SIAM News.

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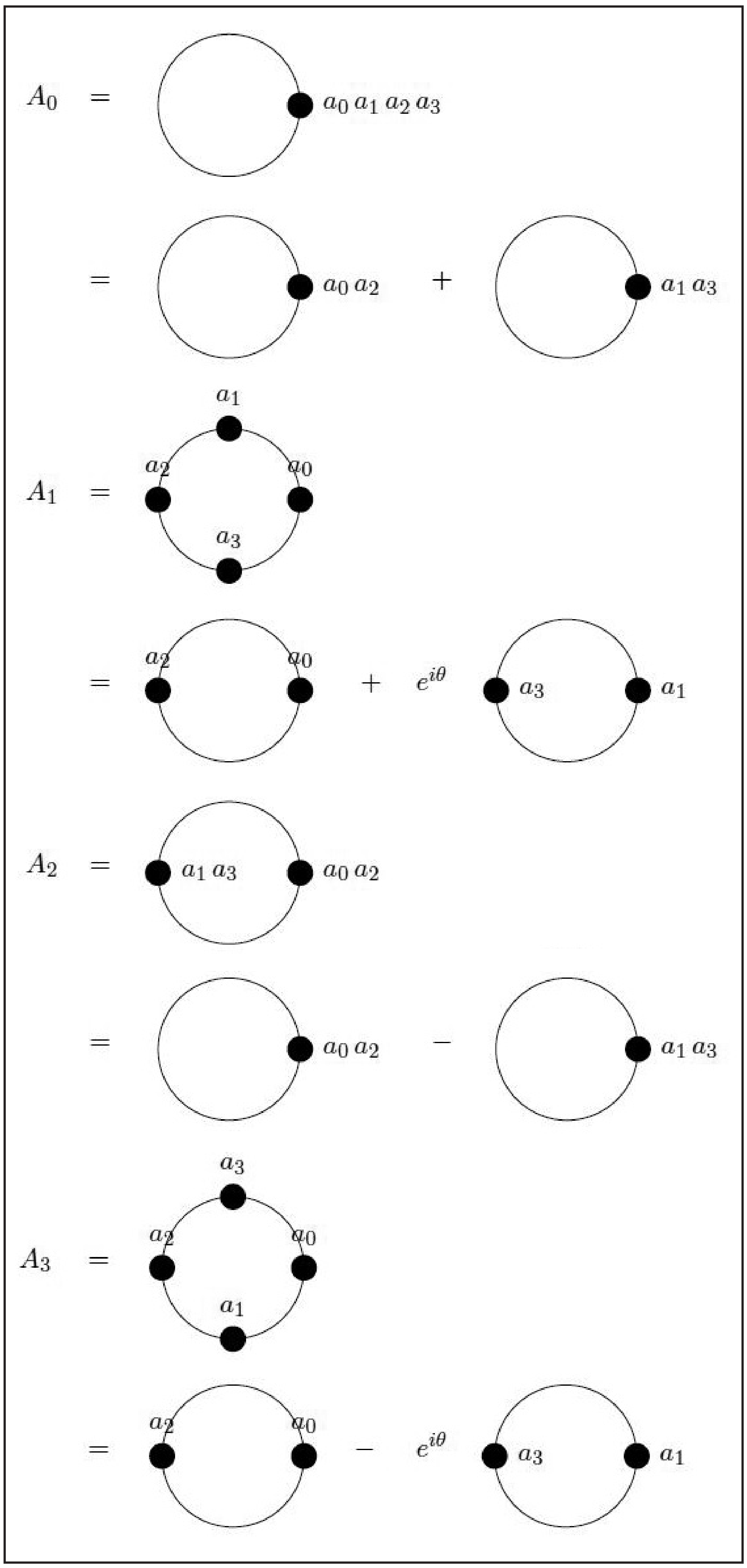

Figure 4. *All of the terms in the discrete Fourier transform of* $\{a_0, a_1, a_2, a_3\}$*.*

Oil Industry

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shops on data analytics and imaging and our first annual section meeting.[2]

People often ask me if I expect to continue in this line of work for the next few decades. I find it difficult to predict what the future will hold. Until non-hydrocarbon technology matures to a point where it reduces the need for hydrocarbons, the latter will remain a commodity required to power many things (including the medium on which you are reading this article). I am happy to be working on a small part of this fascinating real-world problem. While I don't always get to work on problems that catch my fancy, those with business value are interesting enough. Above all, my colleagues are the main reason I continue to work in this field.

It often feels like I am juggling several hats — wife, mom, geophysics researcher, mathematician, programmer, SIAM workshop organizer, Girl Scout troop leader, and so on. Though I am sometimes afraid that they are all going to come crashing down, every day I get a little better at managing my time and prioritizing. The hats are still in the air and I am having fun for sure!

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Anusha Sekar partakes in a hike with her family. Chevron's flexible work culture encourages a healthy work-life balance. Image credit: Bharad Anjur.

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[2] https://www.math.lsu.edu/siam-texas-louisiana-section/