MODELING across the CURRICULUM

Report on a SIAM-NSF Workshop*
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Contents

Executive Summary: Where do we go from here?

The Modeling Across the Curriculum Workshop had three themes for discussion by professionals who are immersed in the areas:

1. develop STEM high school courses based on math modeling,
2. increase math modeling in undergraduate curricula, and
3. assess/improve college STEM readiness.

There are many overlaps in the issues and the possible ways to address these challenges. Not surprisingly, the deeper we dig into these issues, the more there is a need to expand the view and take more aspects into account when proposing solutions. Education, with the resulting ability to think, process, use, and understand information, forms a multi-layered set of issues. The complete summaries of workshop discussions are included in the report.

In this executive summary, we attempt to prioritize some specific objectives. The focus is on what SIAM, and its membership, can do in collaboration with other applied mathematicians and computational scientists.

This report fulfills one of the primary objectives of the workshop. It details plans for advancing all of the theme areas in a coordinated manner. This report will inform decisions on the use of existing NSF programs, and perhaps the development of new ones, to help achieve improvements across the broad STEM educational front. Setting priorities for STEM education efforts over the next 1, 5, and 10 years is important to maximize the benefits of planned investments.

The **top level goal** of proposed efforts is to:

ENGAGE AND KEEP YOUNG PEOPLE IN STEM DISCIPLINES, FROM K-12 THROUGH UNDERGRADUATE (AND GRADUATE) STUDIES, AND INTO THE WORKFORCE.

Recommended Actions:

1. Expand modeling in K-12:

A. Identify one or two strong approaches to modeling curriculum for K-12 (possibly from schools with demonstrated strength based on success in SIAM's M^3 Challenge, a national math modeling contest).

B. Develop content and teacher training material—how to do modeling; how to use models.

C. Engage a network of experts for mentoring and inquiries. Involve high school teachers and judges with successful experience in SIAM's M^3 Challenge as team coaches and evaluators.

D. Build an awareness campaign for teachers, math curriculum supervisors, and academic counselors.

E. Measure outcomes. Establish benchmarks of undergrad STEM retention from graduates of schools with strong modeling background vs those with standard course sequence in math. Monitor schools with pilot programs for change.

2. Develop a high school one-semester or one-year modeling course (with stratified content):

A. Should be multi-disciplinary drawing together mathematical and scientific experiences.

B. Should be influenced heavily by successful programs already offered.

C. Develop a professional development training component for teachers to instruct and engage students on the modeling approach to education.

D. An aspirational goal: make such a course mandatory for high school graduation in order to demonstrate the usefulness and relevance of other math courses, laying the foundation for success in college STEM majors.

3. Develop modeling-based undergraduate curricula

A. Concentrate initially on the first year of undergraduate STEM experience.

B. Investigate two models:
1. Using modeling and applications as a skeleton on which the calculus sequence is built, "Modeling across the Curriculum", or

2. A first year modeling/applied mathematics course that precedes and motivates the study of calculus and other fundamental mathematics for STEM majors.

C. Provide seed grants for faculty to develop, implement and evaluate new approaches to the high school-college math transition for STEM majors.

4. Develop a repository of materials for all aspects and levels of math modeling instruction and understanding. To include but not limited to: course lesson plans, articles, books, web sites, videos, contests, problems and solutions.

This process should be started immediately with a follow-up workshop to address some of these specifics as early as possible, perhaps even during the summer of 2013. If that timing is feasible, then an aggressive timetable of important targets for the K-12 program would be:

Summer 2013—Follow up workshop to zero in on the specific actions and related tasks. Lay groundwork for principal staff to move forward with actions.

Fall 2013—Identify and begin discussions and capturing of information from high performing schools in modeling. Identify possible schools to host pilot programs — public and private, urban, suburban, rural.

Spring 2014—Develop pilot curriculum in detail.

Summer 2014—Train pilot school teachers to teach.

Fall 2015—Pilot schools offer course. Mentors are available for help. Circle of feedback and encouragement. Establish relationship with all students for longitudinal study.

Spring 2016—Gather feedback, adjust curriculum, and iterate the loop expanding the range of participating schools over the next several years.

A similar schedule would be applied to the undergraduate curriculum development. The evaluation would start with baseline data collection and then assess the changes as they are introduced, continuing with longitudinal studies.

The workshop plan predated the President's Council of Advisors on Science and Technology *Engage to Excel* report by several months but clearly begins to address some of the issues raised in a very constructive manner.

Introduction and Background

The Society for Industrial and Applied Mathematics (SIAM) was awarded a National Science Foundation grant for an initiative to increase mathematical modeling and computational mathematics in high school and college curricula. The idea to conduct a workshop was born out of discussions between SIAM and NSF Education and Human Resources representatives early in 2011 on undergraduate and K-12 courses and programs, college readiness and career preparation. Held in late August 2012 and aptly titles 'Modeling Across the Curriculum,' the workshop's three main themes were to begin to develop STEM high school courses based on modeling and computation, to investigate ways to increase mathematical modeling across undergraduate curricula, and to assess college STEM readiness within this context.

While the workshop is relevant at a time of growing concern about America's standards in math and science education, it was especially timely in the wake of the undergraduate STEM education report *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics* [6] released by the President's Council of Advisors on Science and Technology (PCAST) in February 2012. The widespread adoption of the Common Core State Standards in Mathematics [5] adds further urgency to these deliberations.

The objectives of the workshop addressed several key issues raised in the PCAST report, such as increasing student preparedness for STEM majors and overall enhancement of STEM education in the first two years of college. The results of the discussions should also help in responding to the CCSS recommendations to increase modeling and application-based learning in school curricula.

For the undergraduate component, the workshop—and this report on its activities and recommendations—proposes ideas for

multidisciplinary STEM education, taking into consideration previous work undertaken to integrate various STEM elements, such as the University of New Hampshire's Calculus-Physics program, and various programs around the country at the undergraduate and graduate level that promote expansion of computational science and engineering. Computational and applied mathematics form the cornerstone of STEM learning, and SIAM is uniquely positioned to engender such expansion, given its longstanding commitment to promoting research and education in CSE.

A suitably designed high school program could give students an introduction to STEM through courses in modeling and computational and applied mathematics, including project-based modules. These would complement existing courses, with the main goal of integrating STEM subjects without including additional basic content and, at the same time, increasing real-world focus in math and science education in high schools and earlier.

Assessment and evaluation of college readiness—the third goal of the workshop—is critical to addressing the "math gap" identified in the PCAST report. Careful evaluation of curricula is needed in order to better prepare high school students for STEM majors, at the same time adapting early college education to narrow this gap.

Based on the PCAST report, CCSSI recommendations, and anecdotal information from high school and college educators, there is a clear need to emphasize interactions and interconnections between various STEM areas and approach STEM education in a more coordinated fashion.

Carefully evaluating and developing material that enhances the STEM educational spectrum in a coordinated manner will go a long way in better preparing our students for STEM college majors and careers, thus increasing the pipeline of scientific and technical talent in America.

A workshop objective was to develop several of these ideas to a level where they can be the subjects of more specific proposals and plans. The eventual coverage should be broad in terms of both topical content and audience. Applied and Computational Mathematics including Statistics (ACMS) is a natural topical center for coordinated STEM programs both feeding and gaining from all other STEM fields. Distance learning has potential as a delivery tool especially in both rural and inner city communities, especially for any K-12 course or project implementation and for teacher professional development. A related idea is the development of a web-based resource center that should be quality controlled and well maintained and organized. All of these additional aspects were explored within the context of the primary themes.

The following quotes from high school coaches for teams in the Moody's Mega Math Challenge, a modeling contest for teams of high school students operated by SIAM on behalf of the sponsors, clearly demonstrate the need and potential impact for enhanced applied mathematics content in the K-12 curriculum and in teacher preparation—even for some of the most highly motivated high school teachers.

> *"As a teacher, I felt unprepared to coach the students. Some of the papers were difficult for me, and I didn't know really how to teach the students to develop models in such an open ended situation. I would like to see you guys do a workshop for coaches. I come from a theory based math background and want to coach a team, but feel inadequate when it comes to*

the modeling scenarios. I've even considered going back to take some applied math classes."

"I wish that there were some examples of models that are created to test situations and compare theories. Maybe some recommended modeling programs, or advice on programming Excel to model situation. Your other resources are fantastic."

"My state, Georgia, does not have junior/senior level mathematics elective for Mathematical Modeling. As M³Challenge becomes more popular in Georgia, please use your influence to advocate for such a course. Thanks for a great experience for my students!"

There is an observed need to approach STEM education in a more coordinated fashion, rather than simply developing distinct educational program streams that pay little or no attention to interactions and interconnections. ACMS represents a natural vehicle for this coordination. One possible avenue is the development of undergraduate STEM degree programs as alternatives to traditional discipline majors. These might mirror the growth of CSE programs over the past 5—10 years. Coordinating the fundamental mathematics, computation, statistics and science content to support application in a wide range of STEM fields may have strong appeal to potential students. Often high school juniors and seniors are not well informed about career opportunities in the STEM fields and have difficulty selecting appropriate majors. STEM degree programs would likely preserve options while not restricting choices for graduate, professional, or career opportunities.

In the K-12 arena, the CCSSI are described as representing around 85% of the curriculum, with potentially additional content being at state or local discretion. This may provide one opportunity for math and science education to develop material that enhances the whole STEM educational spectrum in a coordinated manner. It is also possible that clever curriculum design could allow modeling and computational mathematics to assume an apparently greater share than this suggests. At the upper level, an alternative "AP-like" curriculum in applied and computational mathematics could be a coordinated STEM educational experience that would reinforce prior experience in a way that would yield more and better prepared students for STEM college majors and careers.

Note: While planning for the workshop was underway the New York Board of Regents was considering a proposal for the introduction of such a STEM course as a potential requirement for a Regents diploma.

The full agenda for the 1.5 day workshop is included as Appendix A. The first afternoon and early evening were plenary sessions describing the "big picture" and outlining the initial focus areas for discussion. Speakers at these plenary session were members of the steering committee. All participants gave brief introductions to their particular experience and area of interest.

The morning and first part of the afternoon of the second day were devoted to three working group sessions each based on one of the theme areas. Each of these has a moderator from the steering committee, and a recorder. There was then a final session devoted to reporting out and planning the follow up activities. Inevitably in the short time available most working groups' responses are characterized by the questions they raised and recommendations

for future work. Detailed, specific recommendations were not a reasonable target for a day-and-a-half workshop.

The rest of this report will focus on the findings and recommendations of the different working groups.

Workshop Theme 1: K-12 STEM Education

The CCSSI for mathematics claims significant emphasis on modeling and applications of mathematics—but the details do not (in our view) substantiate that claim [5], [6]. However, it is also stated that the detailed curricular content may not represent the entire syllabus. (The science standards are constructed in a similar way.) SIAM and the SIAM community are ideally placed to design appropriate problem/project-based content to satisfy the desire for greater modeling and applications content.

By stressing integrated applied projects that span disciplines, there can be both economy of resource demands and improved returns in terms of students understanding—both disciplinary and interdisciplinary. There is also a call for increased "community/informal education". SIAM is currently exploring collaboration with the Museum of Math (MoMath) which can potentially help address these same issues in ways that are independent of the formal school classroom.

The working group report and recommendations below raise many questions and recommend initial work needing to be performed—in data gathering, community building, and awareness and outreach. The interconnections among the various themes also came to the fore, emphasizing the belief that all of these need to work in conjunction to solve the bigger issues that were the raison d'être for the proposal.

Workshop Theme 2: Undergraduate Curricula: Creating a Model for Undergraduate STEM Degree Programs

There have been some genuine attempts to create STEM courses and programs at the undergraduate level—the integrated Calculus-Physics studio program, developed at University of New Hampshire by Kelly Black et al a few years ago, is one example at the course level, and various programs in CSE (minors, majors, and others) represent progress at a larger level [1], [2].

However the desire for cross-disciplinary understanding and education in a broader STEM field has not yet been fully explored. SIAM has been a leader in the CSE arena at undergraduate, graduate, and research levels [14], [15], and is well placed to play a major role in extending this to a more comprehensive STEM educational movement. Such developments would certainly need close cooperation of other disciplines and professional societies, but applied and computational mathematics will necessarily be a core element of the resulting STEM education, both formal and informal.

Workshop Theme 3: Evaluation and Assessment of College STEM Readiness and Retention

An underlying evaluation theme for all the workshop topics is student preparation for STEM college majors. This needs a careful longitudinal study using the current structures to provide a baseline against which to measure progress. In measuring readiness for college-level study in STEM, the focus will be on placement exams. We need to know more about the effectiveness of the MAA's Calculus Concept Readiness project and of the various placement exams. Colleges and universities are employing a wide assortment of such tools.

Some of the workshop participants were selected on the basis of their experience and expertise in this area. Turner and colleagues at Clarkson have an extensive local study [16], [17], [18], while Bressoud has been the lead for an MAA study of mainstream Calculus I and has familiarity with the MAA's Calculus Concept Readiness project. These are potential launch points for this topic in order to answer some of the many questions identified in the group's discussions.

The most basic of these questions are

- What is meant by readiness for college-level study in Science, Technology, Engineering, or Mathematics (STEM)?
- What is the proper role for modeling in the preparation for and implementation of undergraduate STEM education, and what do we mean by "modeling" in the context of K-12 education?
- How do we measure success?

Modeling and Applied Mathematics in the K-12 STEM Curriculum

Although this group was asked to explore the potential development of a high school course in modeling (possibly an AP course), we question whether such a narrow focus would lead to transformational, large-scale effects on pre-college teaching and learning of applied and computational mathematics. However, this questioning does not negate the working group's view that all K-12 students should have a significant modeling experience.

We also note that even our relatively small working group expressed divergent viewpoints about the fundamental role of mathematical modeling in preK-12 education. Some of us felt that mathematical modeling should be the "skeleton" of mathematics teaching and learning especially at the K-12 levels, wherein worldly questions motivate and knit together mathematical ideas and practices. Others felt that modeling should be infused in our classes, but should not be the central skeleton on which the mathematics gets built. Continuing the biological metaphor, these group members felt that perhaps modeling is some of the muscle. That is, the mathematics is central, while questions and practices of modeling support a deepening understanding of that mathematics. The ultimate result, in either framework, should be a nice blend of theory and modeling leading to a strong mathematical understanding that allows the learner to grow.

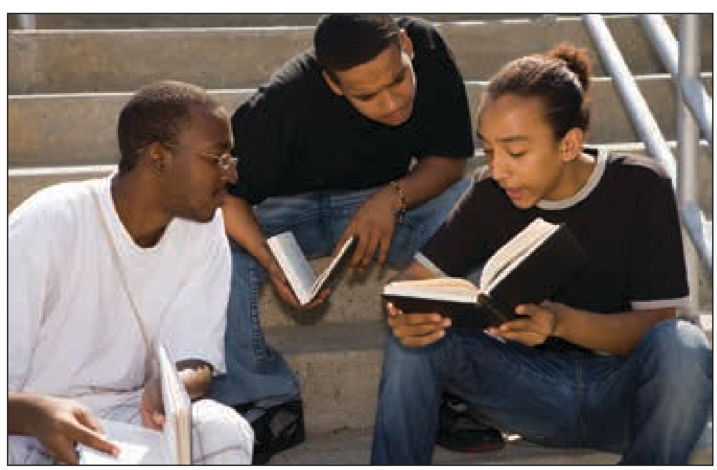
Future work on this project requires consideration of four realms:

Content **Students** **Teachers** **Public Awareness**

Future work on this project also requires some consensus on a definition (or a cluster of context-appropriate definitions) of modeling.

Structure of this section:

1. Comments on relations with the themes of the other groups (college readiness; undergraduate curriculum)
2. Questions of interest
3. Call for data
4. Recommendations for next tasks (including research topics)



1 Relations to college readiness and the undergraduate curriculum

Teaching from a modeling viewpoint enhances mathematics learning, especially when teachers are trusted with community buy-in at all levels of teaching and learning. Infusing applied and computational mathematics throughout preK-12 education does not detract from "covering" material necessary for success in future STEM studies. Infusing applied and computational mathematics throughout preK-12 in fact strengthens students' abilities to continue in all STEM studies.

It is impossible to implement a serious program at the preK-12 levels without close collaboration with the college readiness and the undergraduate curriculum groups. We are reminded that every course a student takes K-16 is a course for future high school (and other) teachers.

2 Questions of interest

Eight basic questions to decide how to identify critical prerequisites are given below. The primary motivation is to determine the necessary components for a program to support pre- and in-service teachers with respect to modeling:

A. What would it take to weave modeling into the K-12 (or even preK-12) curriculum, not as an add-on but as an integral part of understanding mathematics and experiencing research? In the CCSSM, modeling is one of eight mathematical practices — how can we use this as our foot in the door to K-12 education?

B. Modeling is not equal to story problems. How can we keep modeling in K-12 from being reduced to test-prep strategies? Further, we wonder whether modeling has not traditionally been part of math class because curriculum developers and coordinators may presume that modeling is "done" in science labs.

C. While there are not as many service courses at the high school level as there are in post-secondary programs, could a course in applied and computational mathematics be framed as a service course that integrates high school science and mathematics learning? What is valuable and what is risky about making this case?

D. What differences (if any) are there between modeling for the STEM student and modeling for all?

E. The ASA 2007 report, *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) gives a framework for a statistics curriculum for grades preK-12. What would a SIAM equivalent of the ASA GAISE report look like? What advantages would be garnered from having such a report? Would there be any disadvantages?

F. Would adding a laboratory component to existing high school and earlier courses be feasible? Unlikely? Effective? What about introducing a high school capstone course?

G. How should we support teachers who want to change their practice—teaching modeling as a life skill?

H. What are the links to assessments corresponding to CCSSM? No matter what we recommend, as long as teachers are measured by their students' performances on "the test" most teachers will not risk a new approach without lots of support from all sorts of stakeholders.

3 Call for data

A. On The System:

1. Examine the implications of NAEP data concerning mathematics and science course-taking at the 8th grade level for insight into variations of student academic backgrounds in STEM preparation at entry to high school, including consideration of gender, racial/ethnic, and economic disparities where they exist.

2. Examine the publicly available PARCC and Smarter Balance sample test items for their treatment of modeling.

3. Investigate the sequencing of CCSSM to determine where real modeling tasks naturally fit and would support the classroom work.

4. Examine what has been and is being done with modeling in other countries, such as in the Netherlands in modeling education and China in modeling institutes; examine ICMI resources.

5. In recent decades, college level calculus, computer science, and statistics courses have become commonplace in high schools as AP, accelerated, general academic, and introductory courses. What factors contributed to this trend and might these factors apply also to the consideration of promoting a college level modeling course at the high school level? A first step is to examine the ASA GAISE Report and to examine the development and expansion of AP Statistics in high schools nationwide.

B. Beyond considering collegiate level content taught in high schools, investigate curricula over the past two decades, looking into resources such as *www.mathmodels.org*, courses such as the W. M. Keck Curriculum Project (four courses for students in grades 9-12 in which mathematical modeling provides the skeleton for all content), and exploring the possibility of supporting educators and students e-collaborating with expert modelers; ensure appropriate consideration of existing resources such as COMAP and the Moody's Mega Math Challenge, or through the MAA Special Interest Group on Teaching Advanced High School Mathematics. We urge care to avoid reinventing existing resources: collect and categorize existing resources as much as possible (starting with the work of COMAP). Potential categories: traditional existing courses at K-12, traditional existing courses for teachers, online or alternative existing courses at K-12, online or alternative existing courses for teachers, lesson plans, laboratory and activity designs, workshops and professional development materials. Can we identify exemplary high school mathematical modeling courses currently being taught, for example at North Carolina School of Science and Mathematics, Academy of Science at Dominion High School in Virginia, High Technology High School in New Jersey, Brooklyn Tech or Bronx Science in New York City?

C On Branding for Public Awareness, e.g. “This Is Math!”:

1. What, if any, are the public definitions of modeling? Can modeling be defined satisfactorily and concisely, including needed topics and uses in a way that can be somewhat standardized in public conversations about curriculum in schools?

2. What kinds of branding strategies would work or have been studied?

3. Would linking to ‘citizen math’ be attractive? (Think: “citizen science” or “everyday modeling” as examples.)

D. Investigate existing research (e.g. the work of Lynn English in Australia, Kathy Heid in the United States) and existing curricula that incorporate computer algebra systems in Australia, Austria, the United States, and other countries to study the potential opportunities and difficulties in discussing modeling and computational mathematics at the high school level. We note that this is a huge task and that there is much to consider about the roles of technology.

E. Take advantage of our community members who have the most experience and knowledge of the past couple of decades in education—what do they advise? What do they envision the ‘first failure’ will be and how can/should we plan long-term in any of the strategic tasks we choose to move forward on?

F. Investigate existing and historical professional development for in-service teachers, such as perhaps the existing Vermont Mathematics Initiative for elementary school teachers or the now-vanished NSF workshops for teachers.

4 Recommendations for next tasks and research topics.

We need to take the very long view, laying the groundwork for infusing modeling throughout the system. All K-12 students by graduation should have significant modeling experiences, possibly in part through a well-designed course supported or endorsed by SIAM, AMS, ASA, ICM, IEEE, MAA, and ACM. Content accessibility, whether through digitally delivered modeling content, through explicit modeling courses, or through modeling laboratories, is critical for teachers (pre- and in-service), students, and school leaders.

A. We should explore the possibilities of the ‘Trojan mice’ approach: infusing modeling into the full K-12 curriculum through many entry points, in many small ways, which strengthen what is already being taught. Certainly a high school course or capstone experience in modeling should not be the only place in K-12 where students experience and know they’re experiencing mathematical modeling.

B. We should explore existing resources (and the creation of new resources) such as handbooks, courses, effective professional development, and other electronic materials for teachers (pre- and in-service) at the various certification levels, generally elementary, middle, and high school.

1. Explore developing a course and a curriculum for teachers at the master’s level (both pre- and in-service). Explore developing a modeling course online that teachers and students can take, together or in parallel.

2. Explore ways to tie existing or new PD materials for teachers to the new standards.

C. Investigate attaching modeling to ‘career-readiness’ issues similarly to the way statistics has been attached to quantitative literacy. Find ways to expand modeling: it’s not just for STEM anymore.

D. Make an effort to add one or more experts in the realm of technology to this working group. A fundamental part of modeling across the curriculum should be meaningful integration of computation and its enabling technologies.

E. Explore how to document and formulate a research-based recommendation that all teacher education programs include a course in which actual mathematical modeling takes place.

F. Pay careful attention to the roles of K-12 guidance counselors, especially as relating to their understanding of the value of pre-college modeling as part of a STEM experience.

G. Investigate developing a rigorous “seal of approval” from the major mathematical sciences societies for validating a high school modeling course. Concurrently, investigate partnering with organizations such as NCSM as well as other science groups in formal and informal education.

H. Develop a STEM(+) modeling mentor network (school leaders/boards/supervisors) to collaborate (share data), support, and implement modeling throughout the preK-12 curriculum. (See the draft outline below for a potential deliverable from the next stage of this committee’s work.)

A final point: the glut of popular news stories about K-12 ‘education reformers’ strongly indicates we adopt a degree of caution in urging K-12 teachers and leaders to try yet another new approach. Our recommendations should be grounded in serious, reputable research and include results of pilot efforts in suitable school environments, so that we may ascertain the effects of our recommendations on students’ and teachers’ attitudes about STEM fields.

5 Outline of a STEM(+) modeling mentor network (including school leaders, boards, supervisors)

Purpose:

The basic idea and a list of the important aspects of a mentor network are given. The primary attributes include collaboration among faculty, support and infrastructure, and the plans to implement any recommendations.

The proposed STEM modeling mentor network will support K-12 educators interested in implementing mathematical modeling concepts into their classrooms. The regionalized mentor-future mentor approach will encourage growth and future collaboration among educators.

Components of the project model include:

- similar to Math Circles, Project NExT, AWM different mentoring programs
- workshop introduction to each other

- need for personal interaction
- facilitate regional workshops around the country
- Initial participants become mentors
- one week? two weeks?
- Initial workshops targeted to current "users" (see below)
- Year two "new" mentors run local workshops

Supporting materials should include enough projects to fill a 2—3 day workshop.

Resources to support recruiting may be obtained from sources including:

- mentors from Project NExT
- Moody's Mega Math Challenge/HiMCM participants can be identified. Contact that group.
- Knowles Science Teaching Fellows

The continued growth depends on developing an infrastructure based on

- centralized locations
- database (website) management searchable by mentors, materials, and meetings.

Use existing resources (i.e., initial MAA PREP workshop with follow up work).

Undergraduate STEM Education: The Role of Computational and Applied Mathematics

1 Introduction

Our group of 15 scholars from various institutions across the U.S. were tasked with examining the opportunities and challenges with modeling across the undergraduate college curriculum, with an eye toward informing the various STEM players, e.g., universities, funding agencies, policy analysts, and professional organizations, of our findings and the steps that our collective efforts can take to further mathematical modeling as part of STEM Education in the college curriculum. With the PCAST report *Engage to Excel* [6], and its urgent recommendation to increase STEM majors by 34% in the next 10 years, all of the above players need to coordinate our respective efforts to achieve this goal. As applied and computational mathematics is among the most cost-effective and flexible STEM majors, we recommend an "all-hands" approach to fostering, creating, and sustaining new and innovative courses, minors, and majors throughout higher education to achieve this important goal. We hope that this report will help facilitate discussion and action.

The group had some initial wrangling around several issues. Part of our early discussion dealt with a broad spectrum of semantic and existential issues. For example, we considered several viewpoints on what modeling across the curriculum means. We also had a diverse set of perspectives due in part to our group members coming from a wide range of institutions and thus having a variety of cultures, constraints, and needs. As we came to a consensus and our ideas converged, some interesting realizations were revealed. We were then able to break up our findings into five categories, together with a list of recommendations.

One of our first realizations was that there is a big difference between modeling across the curriculum and the models across the curriculum. Mathematical modeling is an abstract and/or computational approach to the scientific method, where hypotheses are made in the form of mathematical statements (or mathematical models), which are then used to make predictions and/or decisions. The quality of these models is then examined as part of the verification process, and the entire cycle repeats as improvements and adjustments to the model are made. The teaching of models, by contrast, is simply a presentation of the final product, and does not provide many insights into the process or the understanding gleaned from it. One also misses the creative genius, the art of problem solving, and the long hours that went into the process. It's like the difference between painting a picture and looking at paintings in a museum. Therefore, we focused our discussion on how to educate students and develop programs around modeling instead of just models.

Another important realization in developing modeling programs is the challenge of sustainability and buy-in. As we shared our experiences and insights, we quickly learned of the large number of past efforts that were started only to later die out due to a lack of buy-in, resources, institutional support, and/or infrastructure that is necessary to keep these programs running long term. Thus, we discussed at length different strategies for sustainability.

Ultimately, we broke down our study into the following five areas:

A. Scale and Resources: Different institutions have different cultures, constraints, and needs. For example, a large university needs to scale its faculty resources in a way that reaches many students, and so courses and programs may be a better direction. A four-year institution may not be able to have the same depth of course offerings, but they may have smaller class sizes with more flexible content where modeling can be injected into the curriculum.

B. Teaching Concepts and Methods: There is a broad array of teaching methods for modeling. While we do not take a position on pedagogy, we do attempt to provide a wide range of ideas and examples from successful programs.

C. Transitions and Audiences: Which groups of students should be targeted in the curriculum? Math majors or non-majors? Upper or lower-division students?

D. Communication and Buy-In: What are some effective ways of getting colleagues invested and unified in advancing modeling efforts across the curriculum? This is an essential consideration to achieve sustainability.

E. Best Practices and Recommendations: What are some of the hazards and lessons learned from successful programs already in play?

2 Mathematical Modeling

Models are a simplification of reality, and can come in many forms. Some models are physical devices, such as a scaled-down model airplane. Mice are often used as human substitutes in biomedicine. Sometimes a model is a closed system such as an aquarium or insect farm. Other models are expressly quantitative in that they are phrased in the symbolic language of mathematics. We refer to these as mathematical models, and they can take the form of equations, algorithms, graphical relations, and sometimes even paragraphs.

There are diverse reasons for using a model. These include:

- to summarize succinctly a collection of observations
- to infer the implications of assumptions about interactions between components of a system
- to predict the response of a system under conditions not yet observed
- to control the response of a system and to inform decision-making when there are alternative policies under consideration.

The type of model used depends intimately upon the purpose for which the model is to be developed. No one model can do everything—there are trade-offs in any model including the generality, realism and precision of the model [11]. For example, a model airplane may be constructed to illustrate the variety of detailed external features of the much larger actual plane, such as a World War II vintage fighter plane, or it might be a scale-model constructed to actually fly, but without all the details of the vintage plane. A model that meets both of these construction criteria would be close to the plane itself, and therefore not much of a simplification. However, sometimes models are cost effective versions of the real thing, and therefore still worthwhile.

Modeling is the creative process by which a model is developed. This includes determining the objectives for which the model is to be developed, the choice and construction of the model, the evaluation criteria to be used to determine if the model is useful for the purposes for which it is being constructed and an iterative procedure for model modification (or elimination if the approach is deemed unsuitable) to meet the objectives. Modeling is often inherent in the scientific process of observation, identifying patterns, hypothesis formulation, setting criteria to evaluate the hypotheses, abstracting the key features of the system under consideration, carrying out further observation or experiment, and evaluating the hypotheses based upon the chosen criteria. The abstraction carried out in the scientific process typically involves a model or set of models that are applied to suggest appropriate experiments or observations and to infer the implications of the assumptions inherent in the abstraction. The modeling process can point out the need for more data in order to create a useful model, and as is true of science, models are modified regularly to incorporate new features and account for new data.

While models are included at essentially every level of K-16 quantitative education, teaching about models is not equivalent to teaching about modeling. Models often appear as examples in mathematics courses, in an attempt to motivate the utility of the mathematics being discussed. In the vast majority of situations, a model is presented, mathematical techniques are applied to allow some inferences to be made concerning the model, and then sometimes this is related back to observation. Examples include the use of exponential functions to mimic population growth or decay of drug concentration, descriptions of vector fields and conservation laws used to illustrate phenomena in physical systems, and regression models to summarize a data set and extrapolate to situations for which the data are not available. In these cases, the models are presented as a fait-accompli in that there is generally no discussion of how they were derived, the assumptions incorporated within them, or any presentation of the modeling process described above. Without exposure to the modeling process, students see these and other models included in their science and math courses as "revealed" and are not provided the context to understand that they arise from careful thought that the students can themselves carry out.

If exposure to and practice with the modeling process occurs anywhere in the curriculum, it is typically in upper-division courses focused on mathematical or computational modeling, which do not reach the majority of students with quantitative interests. For the objective of providing an entrée to the modeling process for the diverse array of students who would benefit from exposure and practice in this, there are multiple routes to success to incorporate the conceptual approaches and skills useful in modeling, dependent upon institutional cultures, constraints, and needs. We provide some examples here, but a good compendium of approaches from over 20 different institutions is available, for the area of biological modeling, in Ledder et al., [10]. A theme that runs throughout our suggestions is to encourage learning goals based on modeling as inquiry, rather than "revealed knowledge."

3 Primary Considerations

For each topic below, we raised several questions and offered a number of recommendations toward developing modeling in the undergraduate curriculum.

3.1 Scale and Resources

Given the wide range of institutions, cultures, and resources available, it is very clear that there is no one-size-fits-all solution. What is optimal for a large institution with hundreds of math majors and tens of thousands of students is unlikely to be as effective for a small liberal arts college, and vice versa. Different institutions have financial constraints, teaching loads, research expectations, etc. Therefore, the approach taken to advance the cause of mathematical modeling at one's institution must be understood and thought about deeply. The strategies and tactics employed should also be revisited repeatedly. The following are some more specific questions to ask:

- **What resources are available? What constraints are present? What is the necessary scale for such efforts?** For example, large universities will often find that resources are constrained by student-faculty ratios, whereas smaller institutions may have greater constraints on time and personnel as a result of teaching loads and department sizes. Ultimately, the approach will need to be tailored to the individual institution, but having an understanding of how resources are allocated and which constraints are movable will help in one's overall strategy.
- **What is the structure from the student's perspective?** Modeling can be injected into existing courses, new courses can be created, and in some cases minors and/or degree programs are appropriate. In all cases, how will these efforts look to the student? How will they be able to leverage the experience in a meaningful way that opens doors to them when they attempt to enter the workforce?
- **How do these resources and constraints vary when considered at a department level versus across the campus?** The involvement of additional departments and colleges may remove or alter constraints, but also affect objectives and concessions that need to be made to get buy-in. For example, interdisciplinary efforts may open doors to space, external funding, and political capital, but it may also mean that the modeling focus is tied to specific activities or disciplines, which may enhance or detract from the desired goals. Although we recommend expanding as broadly as possible, it may be best to do so slowly if that is necessary to maintain quality and focus.
- **Are there external resources and opportunities that can be used to enhance efforts and programs?** For example, which institutes, centers, conferences, and external funding mechanisms can provide and share resources and expertise that will help? Are there neighboring institutions that can add synergies? Are there professional development and/or sabbatical opportunities that can be used to re-tool?

3.2 Teaching Concepts and Methods

As we embark on developing and improving our modeling efforts in the curriculum, particular consideration on teaching and student activities needs to be considered. There are many successful methods and variations that have been successful. There is a broad array of teaching methods for modeling. While we do not take a position on pedagogy, we do attempt to provide several ideas rooted in successful programs.

- Make use of data: Data should be collected by students; students should be able to identify with the whole process including the data itself.
- Encourage hypothesis formulation: Students take part in this before, during, and after developing models.
- Students need to know what a model is and what modeling is. They should know why it is important and know its limitations.
- Programming and computation can be hard. It is intimidating to many students. Have good processes in place to teach students to make appropriate use of computational resources, including the ability to program well.
- In designing a program, identify the core competencies, standards and desired learning outcomes. Determine how they (and the entire program) should be assessed.
- Recommendations have been made for working in laboratories and actually performing experiments.
- Keep in mind what happens outside the classroom, and bring appropriate outside influences into the classroom.
- Give people the big picture: modeling goes from small to big. Big science starts small.
- Have seminars, particularly aimed toward undergraduates.
- Get the students to give talks so that they learn how to communicate models and how to conduct research in general.
- Research on education: Tie research into educational issues to the development of new materials. Also make sure it is a two way street to help drive research into this area.
- Get students early: Recruiting from high schools and freshman classes is a good idea. There is substantial attrition in STEM. Modeling and applied math research should go a long way to help.
- Broad range of skills: We want students to be able to apply methods to any domain such as biology, engineering, or other context.
- Know how to succeed: Students need to know how to react when they are stuck. They need to "be comfortable with being uncomfortable."
- Teach students to read scientific papers.
- Understand how to quantify uncertainty: Data is an important part of modeling, and to understand the sensitivity that models have to errors in data is important.
- Be inclusive: Modeling is for all students—women, minorities and non-STEM majors. Form a diverse community!
- General education and community colleges: Modeling is an ideal vehicle to motivate quantitative thinking.
- Graduate students: Teaching and research assistants will be teaching classes soon. They need broader training in modeling and knowing how to share that knowledge.

- Interdisciplinary Research Projects:
  Encourage students to take part in a wide variety of projects. For example take part in REU's, seminars, and other opportunities.
- Determine the critical transitions such as:
  High school to college. When is someone first thinking about a STEM major? Determine when the best time is to teach certain principles.
- Re-think "level" and "ability:" Novice/beginner—introduction—run. Apprentice/knowledgeable—introduction with second look—moderate. Accomplished—more advanced—writer.

3.3 Communication and Buy-In

Communication to students, parents, colleagues, potential collaborators, guidance counselors, administrators, and the public is essential. Career options need to be communicated. Courses and programs need to be communicated. Invitations to collaborate need to be communicated. Funding opportunities need to be communicated. Successes need to be communicated. Even failures need to be communicated (to the right audience). In all cases, one should consider how to communicate effectively, and one may need to be clever and concise to do this. We consider the following particulars:

- Many people are unaware of the rich career opportunities in applied mathematics. Indeed, a math degree can prepare students for high-paying and influential careers in finance, insurance, risk management, operations research, computational science and engineering, signal, image, and natural language processing, bioinformatics and computational biology, information science, and machine learning, to name a few. Getting the word out to students, parents, guidance counselors, and administrators, in particular, is important for department growth and for continued support at one's institution.
- Student and faculty success stories need to get out. Unless you have a publicist, you are your own publicist.
- Notation and terminology across disciplines needs to be communicated so that faculty in other departments see the potential that mathematics plays in helping them cross disciplines. This will entice faculty in other departments to want to work with you. By teaching students to be multilingual scientifically, they will have greater opportunities.

3.4 Important Considerations for Success

The success of this effort relies on infrastructure and support. It also requires dedication and preparation of the people involved in the effort. We list some of the challenges and the aspects that must be addressed in order to ensure that modeling becomes a long term fixture in the curriculum:

- Sustainability is a big challenge. Many past efforts and programs have not survived on account of the founders leaving, retiring, or burning out. Therefore, we recommend getting many people involved so that efforts are integrated into the culture and become a community effort, not something being driven by one or two people. The more faculty, departments, infrastructure, etc., the more likely a program or course is to survive in the long run, and the better it likely will be. Along these lines, we recommend reaching out to other departments. By working across campus, faculty can develop strong ties. If done well, modeling will help bind applied mathematics to the rest of STEM. We recommend constant outreach, continually making new connections. An additional advantage of community building is that it allows for better leveraging of existing infrastructure and resources. Intellectual diversification will also make mathematics a central theme and not allow the modeling to become dominated by a specific scientific discipline.
- Excellence breeds excellence; success breeds success. Therefore, we recommend that program and curricular efforts begin with a small group of good students, and let it grow from there. Taking on too much and/or growing too quickly can harm the long-term goals of a program. Achieving quality, even if on a small scale, will serve to attract and retain more students in the long run.
- Programs and curricula need to be student centric to be successful. We recommend that efforts lead to broad experiences, even if they begin with narrow objectives. Students will get more out of a program or course if they are involved in the entire process, even starting with the experiments or data collection process. As the goal is to teach modeling, and not just models, it's important that students fail and learn how to get themselves unstuck. The process is iterative, and students are likely to be more successful if they experience many iterations. Using and integrating data into the modeling curriculum will also help students gain an understanding of the nature of data and potential pitfalls of data.
- It is also recommended that programs and courses involve more senior students and/or graduate students. Having some kind of vertical integration serves three main purposes. First, it takes pressure off of faculty by having help with teaching. Second, it provides senior students with an opportunity to teach. This gives them tremendous opportunities for growth. Third, it gives the students big brothers and sisters, which reduces attrition and gives additional support.

4 Suggested Initiatives

The suggestions imply a large scale, resource-intensive approach to address an initiative to fill the void in modeling in the high school curriculum. Here we provide three smaller actions that can begin the process. These are initiatives that can be implemented in a timely manner:

- We urge SIAM to consider a new working group in mathematical modeling. This would help provide an ongoing dialog on the topic. This would open doors for collaboration and the sharing of best practices. Modeling is a vast area, and it means different things to different people. There are a wide variety of approaches and aspects to it which should be shared and communicated better. It is a process itself and this is not widely understood. A SIAM working group would help.
- We urge the NSF to fund and/or house a central repository that provides the STEM community a well-maintained information resource of past and existing programs. Along these lines, it may also be worth considering a forum (or blog) that can bring people together, share best practices, and stimulate local efforts across higher education. Such a repository could combine modeling efforts in both higher education and in the K-12 system. This could further open doors between secondary education teachers and university faculty.
- There is a lack of textbooks on mathematical modeling for undergrads. With implementation of the common core, there is a demand for, and there are potential creative authors who can write appropriate textbooks. Start writing!

5 Summary

In higher education, aside from programs driven by accreditation requirements for professional certification, it is relatively rare to encounter coherent discussions focused on what the institutional goals are for particular disciplines and degrees. Although these goals may well be explicitly composed at the college or university level, with carefully constructed guidelines for what range of courses should be included in all degree programs (e.g. the core curricula), it is rare to encounter a similarly explicated program of study within disciplines. The guidelines developed by national programs are most often translated into a set of courses that fit into the silos of disciplines of an institution.

One alternative for curricular development is to focus explicitly on what the objectives are, in terms of concepts and skills, for an undergraduate to leave a particular degree program. A component of the challenge to develop a curriculum appropriate to incorporate these concepts and skills is determining an appropriate procedure to follow. No single course or set of courses can possibly do it; all institutions differ, the potential fields incorporating modeling vary, the student population is diverse and what is needed are multiple routes to success [7], [8]. Over the past two decades a procedure for quantitative curricula development has been used regularly in workshops at a wide array of institutions. Individuals from more than 100 different institutions have been involved in some way with these workshops [9].

The approach is called CPAR: Constraints, Prioritize, Aid, Repeat with components viewed from the perspective of quantitative educators desiring to develop interdisciplinary quantitative education. Though this has been applied mostly to quantitative life science education, the approach is appropriate for modeling:

Understand the **Constraints** under which your colleagues in other disciplines operate—the limitations on time available in their curriculum for inclusion of modeling, the limits on experience their faculty have with modeling, etc.

Work with these colleagues to **Prioritize** the modeling concepts and skills their students really need, and ensure that the quantitative courses these students take (typically in a math department) include these.

Aid these colleagues in developing modeling concepts and skills in their own courses that enhance a student's realization of its importance in their own discipline. This could include team teaching of appropriate courses.

The full curriculum is an opportunity to **Reinforce** modeling concepts and skills, extend the students' appreciation of their applicability, repeat the concepts that are given highest priority by the faculty, and expand on these by introducing them in conjunction with different models and associated quantitative skills. This goes along with cognitive research on the importance of repetition in aiding learning [3].

This approach can

- lead to a coordinated effort across disciplinary silos in higher education,
- reinforce modeling across a variety of formal courses, and
- involve faculty from multiple units in setting their own conceptual and skill objectives that the mathematics and computational science faculty can build upon.

This aligns well with the suggestions outlined in numerous national reports, for example in undergraduate biology education [4] (NRC, 2003), which encourage the interdisciplinary connections that can arise from a modeling perspective across the curriculum.

Evaluation and Assessment of College STEM Readiness and Retention

This group produced many questions and avenues for further investigation. The most basic of these questions are

- What is meant by readiness for college-level study in Science, Technology, Engineering, or Mathematics (STEM)?
- What is the proper role for modeling in the preparation for and implementation of undergraduate STEM education, and what do mean by "modeling" in the context of K-12 education?
- How do we measure success?

In measuring readiness for college-level study in STEM, the focus will be on placement exams. We need to know more about the effectiveness of the MAA's Calculus Concept Readiness project and of the various placement exams. Colleges and universities are employing a wide assortment of such tools. Most have been keeping local data on the effectiveness of their efforts. It would be particularly useful to collect this data and perform a meta-analysis of what has worked under what circumstances.

We need to gather experts in discipline-based undergraduate education to discuss what is needed from mathematics courses. How do they see the role of modeling in the mathematics that their students will take?

Some of the possible measures of success include passing rates, changes in attitudes toward the study of STEM disciplines, persistence into subsequent courses, readiness for and success in subsequent courses, and the identification and retention of key concepts—including the quality of understanding of these concepts—by the time of graduation. We will need to collect and analyze longitudinal data, paying particular attention to study design.

Additional mechanisms for measuring success include analysis of portfolios and interviews with students at key points during as well as at the end of their undergraduate progression. This would involve collecting and analyzing narratives of the experience of STEM education, cataloging the variety of typical experiences, and noting when their mathematical knowledge "clicked." Interviews should also be conducted with those in the sciences, business, or professional masters programs who have been out of college for some time.

The issue of transferability of knowledge is still poorly understood. We need to understand which mechanisms are successful and identify what has been done that is successful. We need better information on whether modeling experiences and a diverse set of applications facilitate this.

We need to pay attention to approaches that are particularly successful with critical subpopulations, especially women, minorities, students transferring from 2-year institutions, and other at-risk populations. We need to determine the effect of the incorporation of modeling experiences on retention and diversity.

There are a variety of programs that are designed to improve preparation for STEM study as well as retention and success. These include high school teamwork experiences (e.g. Moody's, Shodor, Science Olympiad, Toshiba, Disney, robotics competitions), GEAR-UP, Emerging Scholars Programs, Supplemental Instruction, Upward Bound, REMACS, the McNair Scholars Program, ATE (Advanced Technological Education), internships, interdisciplinary teamwork experiences (e.g. UBM, Krell, MCM programs), modeling competitions, the innovative West Point first-year mathematics curriculum, and programs in computational science.

We also need a better understanding of how to incorporate modeling most effectively. Should it be discipline specific? Is modeling most effective when presented in a stand-alone course or incorporated into several required courses? What should be the role of modeling in preparing future teachers, and what if any effect does it have on their experience as teachers?

Conclusions and Recommendations

The most obvious conclusion to draw from the foregoing is perhaps that this short workshop could do little more than scratch the surface of a potentially important development in the recruitment, retention and education of a strong pool of STEM undergraduate majors. However that does little justice to those scratches which suggest some potentially exciting developments for curriculum and workforce development.

Several specific suggestions are included in the Executive Summary at the beginning of the report.

Each of the topical discussions was very fruitful. Important questions for future investigation have been presented, and even some tentative possible responses to those questions. One of the difficulties resulting from the day-and-a-half format is that there was insufficient opportunity for discussions on the interrelationships among the themes.

Arguably the strongest single recommendation is therefore:

Recommendation 1

There should be a follow-up workshop of longer duration that can explore questions raised in this report, the linkages between the different themes, and reach greater specificity on research questions.

- a minimum of 2.5 days seems appropriate, with
- expanded participation including many of the attendees from the first workshop, and including
- both pairwise and three-way interactions among themes to explore connections.

An important outcome of the second workshop will be to identify small leadership teams for each theme. The workshop steering committee would begin that process in the planning stage.

Unsurprisingly, this issue of linkage among the themes emerges in each of the sections of the report. This interplay can be reflected in the membership of the leadership teams referred to above. Clearly changes in K-12 and undergraduate college curricular cannot be totally disjoint, and a key element of assessment and evaluation will be the vital transition between those.

At the same time there are topics that belong appropriately in each of the individual theme areas and so some sessions devoted to those are also appropriate. These are reflected in the recommendations from the different groups.

Realization, and subsequent exploitation, of the interplay among the thematic areas will facilitate meaningful changes that would not result from disjoint investigations.

Recommendation 2

In the context of expanding modeling and applied mathematics in the K-12 educational sphere, some major topics to research further are:

- Close examination of approaches to expanding *Modeling across the Curriculum* content, and selecting a few particularly good candidates for experimental implementation.
- Developing appropriate materials and opportunities for training guidance counselors and teachers on the importance of applied mathematics and modeling, and indeed the meaning of those terms.
- Exploring opportunities for outreach and interaction with informal education.
- Developing a mentoring network to help with development and implementation of modeling throughout the K-12 system.

Again, it is apparent that such developments are not independent of the undergraduate STEM educational opportunities that follow.

Recommendation 3

Some major topics to be explored further at the undergraduate STEM education level are:

- Development of a "modeling education" working group within SIAM and the broader mathematics and applications community. One goal of the second workshop should be to develop a more detailed charge for this working group.
- Development of a well-maintained peer-reviewed repository of good curriculum models, modules, projects in all areas of the undergraduate STEM spectrum. One relevant example that might be emulated is the National Computational Science Institute's curricular content at *http://computationalscience.org/.*
- Investigation of the feasibility of STEM programs that are not necessarily housed in individual disciplines. This would include researching issues such as obstacles facing multi-disciplinary efforts, characteristics of successful programs, and how to leverage such successes elsewhere.
- Encourage publishers and authors to develop appropriate textbooks and related materials for modeling, computational and applied mathematics.

- The assessment and evaluation theme is perhaps the least independent of the three. That does not of course imply an absence of recommendations or future research questions.

Recommendation 4

Clearl,y counts of students entering, continuing, and graduating from STEM majors will provide some of the base data for assessment. Among the major questions to be investigated are:

- What is meant by readiness for college-level study in Science, Technology, Engineering, or Mathematics (STEM)? And how do we measure it?
- How do we gauge success?
- How do we merge learning from local, largely independent, studies and broader national ones?

The second workshop will start to frame the answers and point to specific research questions and suggested approaches to all of these. The Executive Summary points to some of those specific measures that are achievable within a relatively short time and which can lead to an expanded pipeline of well-prepared STEM college majors, and to improved retention of those students helped by a relevant applied mathematics experience in the vital transition between high school and college. The workshop plan predated the *Engage to Excel* report [6] by several months but clearly begins to address some of the issues raised in a very constructive manner.

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Appendix A
Participants and Contributors

Principal Investigators

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Peter R. Turner, *Clarkson University*

Steering Committee Members

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Jeff Humpherys, *Brigham Young University*
Robert M. Panoff, *Shodor Foundation*
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Amina Eladdady, *The College of Saint Rose*

Kathleen Fowler, *Clarkson University*

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Darryl Yong, *Harvey Mudd College*

Lizette Zietsmann, *Virginia Tech*

Appendix B Workshop Agenda

SIAM-NSF Workshop on Modeling across the Curriculum

August 30-31, Arlington, Virginia USA

Themes

- Undergraduate Curricula
- Development of high school, and potential AP, STEM courses based on modeling and computation
- Assessment of college STEM readiness

Thursday, August 30

1:00pm	Arrival and Registration	
1:30pm	Welcome and Overview of the Meeting Joan Ferrini-Mundy, Assistant Director for Education and Human Resources, NSF Introduction to the Meeting and Issues Peter Turner, VP for Education, SIAM; Clarkson University	
2:30pm	Introductions and single slide presentations (all participants)	
4:00–6:00pm	Introductory Presentations on the Themes	
	Katherine Socha	High school STEM course and curriculum development
	David Bressoud	College STEM readiness
	Jeff Humpherys	Undergraduate curriculum: applied and computational math
	Bob Panoff	Undergraduate curriculum: Modeling and computation in application fields
6:30–8:30pm	Reception and Networking Discussions — Hilton Hotel — provided by SIAM	

Friday, August 31

8:00am Briefing on working groups

8:30am–12:00pm

Working Group Session	***Moderator***	***Recorder***
Undergraduate Curricula	Jeff Humpherys and Bob Panoff	Kelly Black
HS STEM Curriculum Development	Katherine Socha	Michelle Montgomery
College STEM Readiness	David Bressoud	Bill Kolata

12:00–1:00pm Lunch and Informal Discussions

1:00–2:30pm Working Groups Prepare to Report Out

2:30–3:30 pm Report Out from Working Groups

3:30–4:00pm Next Steps and Closing

Appendix C The Working Groups

The principal investigators and NSF participants were not assigned to particular working groups but were observers and occasional participants in all. The overriding theme of all groups was modeling across the curriculum considered in the three different thematic areas.

K—12 STEM Curriculum Development

Moderator:	Katherine Socha
Recorder:	Michelle Montgomery
Participants:	Sol Garfunkel
	Katie Fowler
	Ben Galluzzo
	Sharon Hessney
	Holly Hirst
	Patrick Honner
	Oana Pascu
	Susan Sclafani
	Richard Sisley
	Dan Teague
	Zalman Usiskin
	Ted Wendt
	Susan Wildstrom

College STEM Readiness and Evaluation

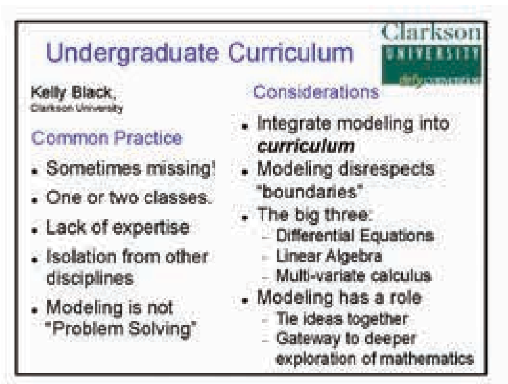
Moderator:	David Bressoud
Recorder:	Bill Kolata
Participants:	Alan Knoerr
	Mike Long
	Michael Pearson
	Angela Shiflet
	Troy Siemers
	Ted Stanford
	Dave Wick
	Darryl Yong

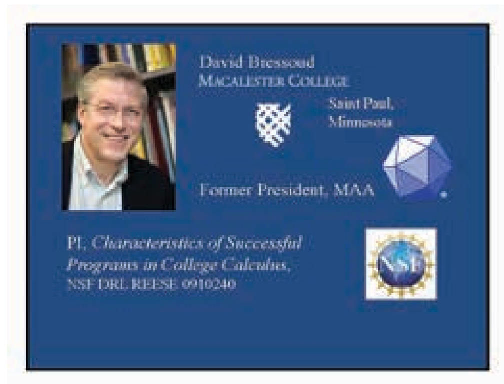
Undergraduate Curricula

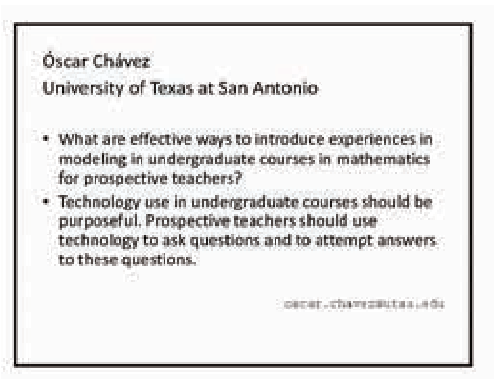
Moderators:	Robert Panoff Jeff Humpherys
Recorder:	Kelly Black
Participants:	Oscar Chavez
	John David
	Amina Eladdady
	Lou Gross
	Erich Kostelich
	Scott Lathrop
	Rachel Levy
	Reza Malek-Madani
	Luis Melara
	Rebecca Nichols
	George Shiflet
	Lizette Zietsmann

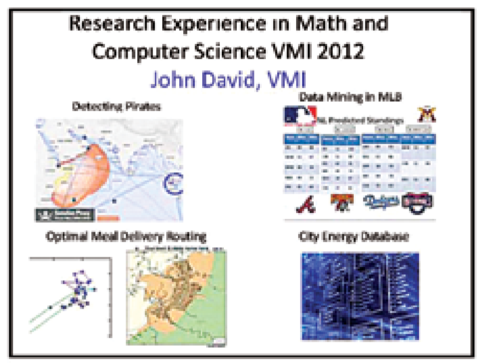
Appendix D Participants' Introductory Slides

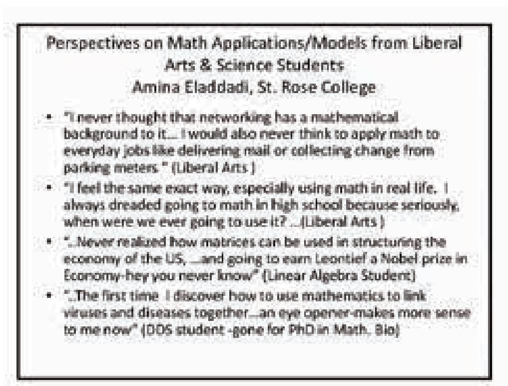
The following are reproductions of the single slide presentations prepared by the participants to introduce themselves and their relevant experience and expertise. There were no particular rules governing these, with the intention that individuals would speak to what they felt was important.

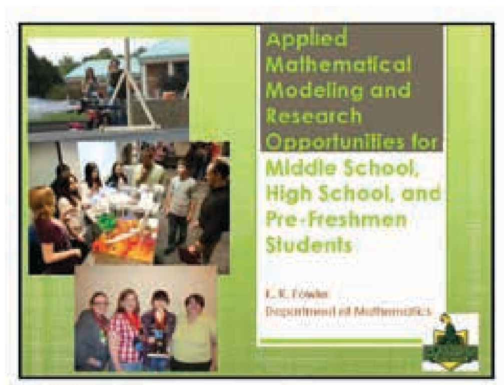


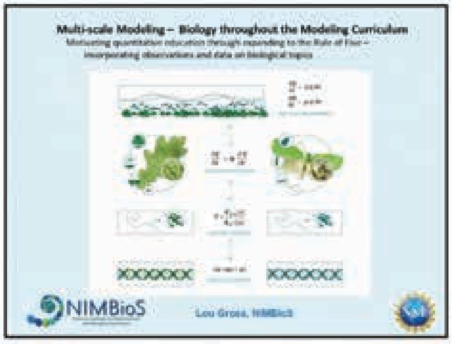


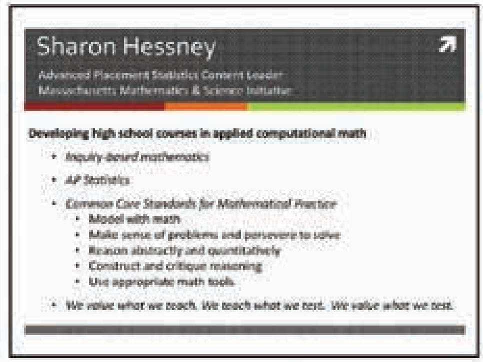


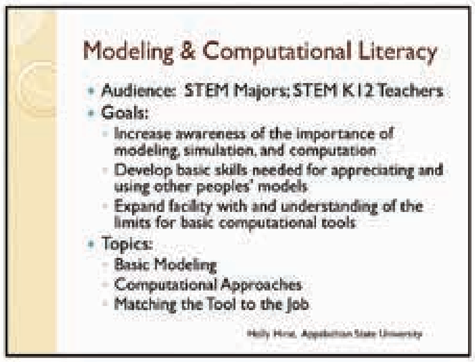




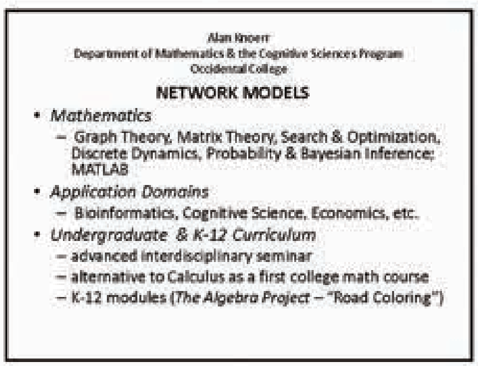


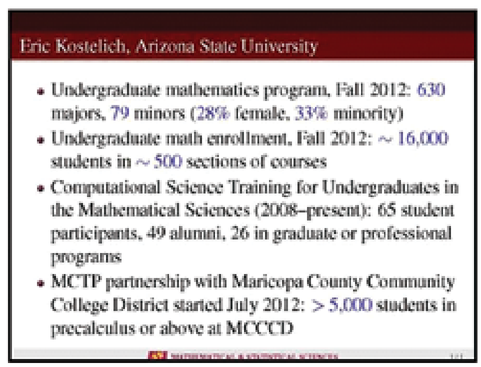

Multi-scale Modeling – Biology throughout the Modeling Curriculum
NIMBioS
Lou Gross, NIMBioS


Sharon Hessney
Advanced Placement Statistics Content Leader
Massachusetts Mathematics & Science Initiative
Developing high school courses in applied computational math
Inquiry-based mathematics
AP Statistics
Common Core Standards for Mathematical Practice
Model with math
Make sense of problems and persevere to solve
Reason abstractly and quantitatively
Construct and critique reasoning
Use appropriate math tools
We value what we teach. We teach what we test. We value what we test.

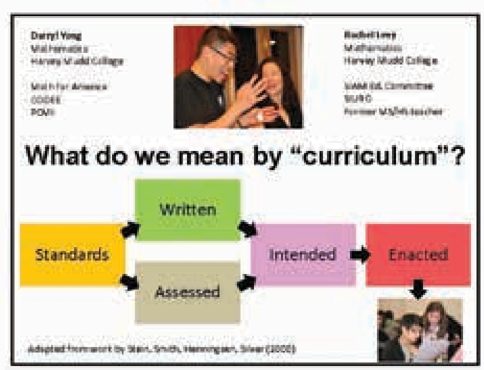

Modeling & Computational Literacy
Audience: STEM Majors; STEM K12 Teachers
Goals:
Increase awareness of the importance of modeling, simulation, and computation
Develop basic skills needed for appreciating and using other peoples' models
Expand facility with and understanding of the limits for basic computational tools
Topics:
Basic Modeling
Computational Approaches
Matching the Tool to the Job
Holly Hirst, Appalachian State University

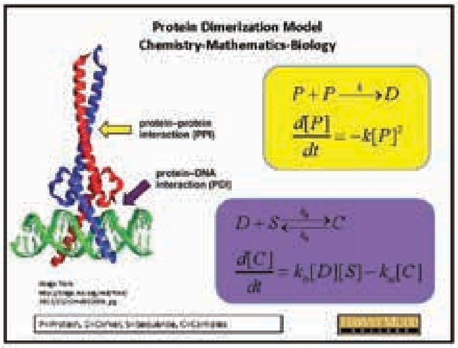

Patrick Honner
Brooklyn Technical HS
www.MrHonner.com
Courses:
Calculus (Single / Multi / Vector)
Math Research
Topics ("Advanced" Geometry, Combinatorics, etc)
Interests:
Student Research / Writing
Teaching / Learning Technologies
Art


Alan Knoerr
Department of Mathematics & the Cognitive Sciences Program
Occidental College
NETWORK MODELS
Mathematics
– Graph Theory, Matrix Theory, Search & Optimization, Discrete Dynamics, Probability & Bayesian Inference; MATLAB
Application Domains
– Bioinformatics, Cognitive Science, Economics, etc.
Undergraduate & K-12 Curriculum
– advanced interdisciplinary seminar
– alternative to Calculus as a first college math course
– K-12 modules (The Algebra Project – "Road Coloring")

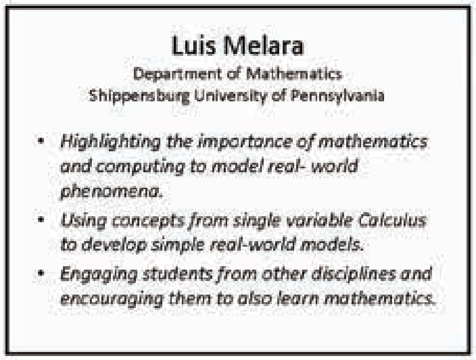

Eric Kostelich, Arizona State University
Undergraduate mathematics program, Fall 2012: 630 majors, 79 minors (28% female, 33% minority)
Undergraduate math enrollment, Fall 2012: ~ 16,000 students in ~ 500 sections of courses
Computational Science Training for Undergraduates in the Mathematical Sciences (2008–present): 65 student participants, 49 alumni, 26 in graduate or professional programs
MCTP partnership with Maricopa County Community College District started July 2012: > 5,000 students in precalculus or above at MCCCD

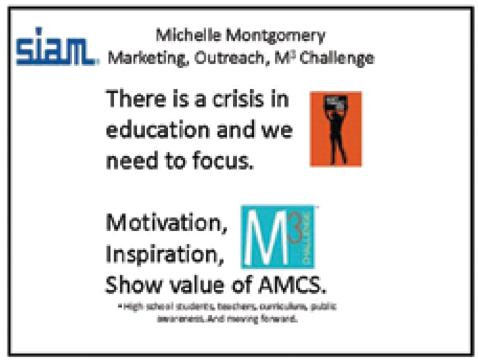

Scott Lathrop
XSEDE/Blue Waters/Shodor/NCSA
Goal is to incorporate computational methods, quantitative analysis, and parallel reasoning into all fields of study.
We have a rich history of working with individual faculty and teachers to assist them with developing curricular materials, sharing those materials with others, and anecdotal evidence of enhanced student motivation and learning.
On-going series of live, web-based, and self-paced formal and informal learning opportunities. We welcome collaborations to expand the breadth and depth of offerings.
Core set of CS&E and HPC competencies have been developed. We welcome/need on-going community input to evolve these and to address domain specific aspects.
Sustained systemic adoption requires institutional buy-in and support. We are working with institutions to develop under/grad certificate and degree programs.
Updates to the GRE process can further stimulate national-scale adoption.

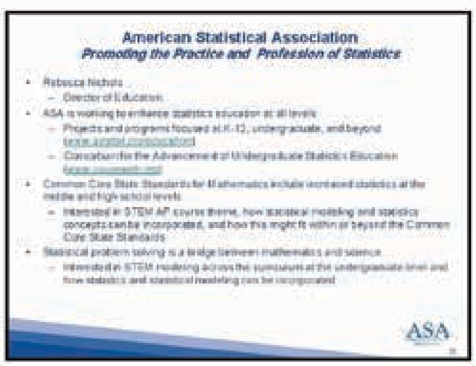

Darryl Yong
Mathematics
Harvey Mudd College
Math for America
POMI
Rachel Levy
Mathematics
Harvey Mudd College
SIAM Ed. Committee
Former MS/HS teacher
What do we mean by "curriculum"?
Standards
Written
Assessed
Intended
Enacted
Adapted framework by Stein, Smith, Henningsen, Silver (2000)


Protein Dimerization Model
Chemistry-Mathematics-Biology
protein-protein interaction (PPI)
protein-DNA interaction (PDI)
$P+P \xrightarrow{k} D$
$\frac{d[P]}{dt} = -k[P]^2$
$D+S \rightleftharpoons C$
$\frac{d[C]}{dt} = k_b[D][S] - k_u[C]$

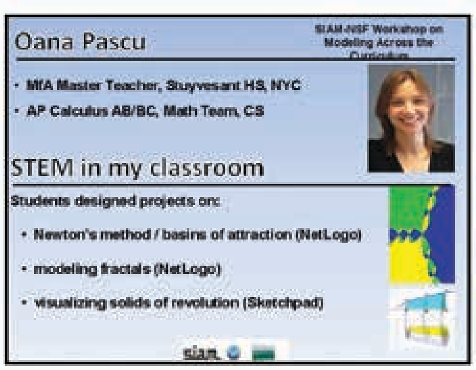

Reza Malek-Madani
US Naval Academy
Goal: Develop interdisciplinary courses to introduce the fundamental concepts of applied mathematics in the context of applications from oceanography, mechanics, optics, and systems engineering
Two examples:
Mathematical Modeling of Oceanic and Atmospheric Processes
Mathematics of Light

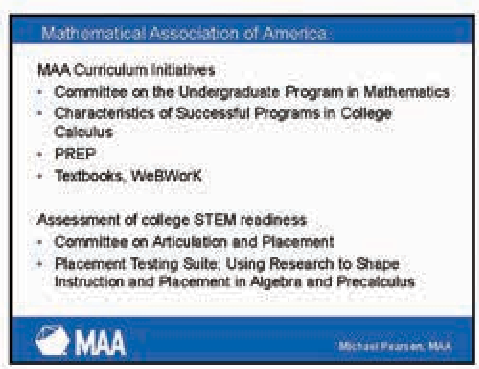

Luis Melara
Department of Mathematics
Shippensburg University of Pennsylvania
Highlighting the importance of mathematics and computing to model real- world phenomena.
Using concepts from single variable Calculus to develop simple real-world models.
Engaging students from other disciplines and encouraging them to also learn mathematics.


siam
Michelle Montgomery
Marketing, Outreach, M³ Challenge
There is a crisis in education and we need to focus.
Motivation,
Inspiration,
Show value of AMCS.
•High school students, teachers, curriculum, public awareness. And moving forward.


American Statistical Association
Promoting the Practice and Profession of Statistics
Rebecca Nichols
Director of Education
ASA is working to enhance statistics education at all levels
ASA


SHODOR
Robert M. Panoff, Shodor Foundation

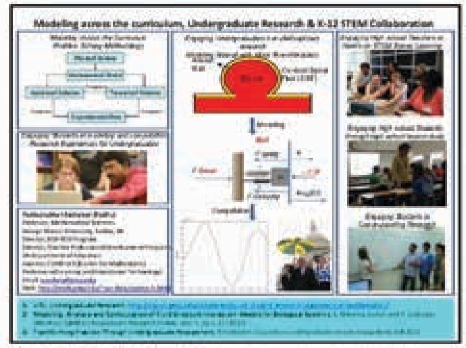

Oana Pascu
SIAM-NSF Workshop on Modeling Across the Curriculum
MfA Master Teacher, Stuyvesant HS, NYC
AP Calculus AB/BC, Math Team, CS
STEM in my classroom
Students designed projects on:
Newton's method / basins of attraction (NetLogo)
modeling fractals (NetLogo)
visualizing solids of revolution (Sketchpad)
siam


Mathematical Association of America
MAA Curriculum Initiatives
Committee on the Undergraduate Program in Mathematics
Characteristics of Successful Programs in College Calculus
PREP
Textbooks, WeBWorK
Assessment of college STEM readiness
Committee on Articulation and Placement
Placement Testing Suite: Using Research to Shape Instruction and Placement in Algebra and Precalculus
MAA
Michael Pearson, MAA

Susan Sclafani
Pearson Education

Development of HS, and potential AP, STEM courses based on modeling and computation

- HS for Engineering Professions '75-83
- Technology and Curriculum Development
- Urban Systemic Initiative: HUUNC
- Math-Science Initiative US ED 2001-03
- High School Redesign Initiative US ED 2003-05
- Current Project: Common Core System of Courses aligned to CCSS for ELA and Math
  - Digital curriculum
  - One-to-one tablets
  - Mathematical Modeling



Modeling across the Curriculum
Angela Shiflet, Wofford College

- Emphasis in Computational Science (ECS)
  - One of 1st programs in U.S. – 1st graduate in 2001
  - 42 graduates; almost 70 currently pursuing ECS
  - Internships: DoE, NASA, REU, national & international
- *Introduction to Computational Science: Modeling and Simulation for the Sciences*, Angela & George Shiflet, Princeton U. Press, 2006
  - 1st textbook designed specifically for intro course
  - System dynamics modeling, cellular automaton simulations
  - Working on 2nd edition
    - To include agent-based modeling and modeling with matrices

Modeling Across the Curriculum
George Shiflet, Wofford College

- Integration of Modeling into Biology Courses
  - Cellular & Molecular Biology – Enzyme Kinetics
  - Microbiology – Spread-of-Disease Modeling
- Development of Undergraduate Educational Modules (with Angela Shiflet)
  - Keck
  - NASA
  - Blue Waters

Support for the theme of

Development of a STEM AP course based on modeling and computation
Troy Siemers, VMI

At VMI, we focus our Applied Mathematics degree around three tracks:
- Operations Research
- Statistics
- Numerical Methods

which fit very well with the CCSSM emphases on statistics and modeling.

Recommendations for material for a STEM AP course for high schools can come from modified projects in these areas to generate excitement, interest and relevancy to society.

A brief list of possibilities might include traveling salesman problems, scheduling and allocation problems, and techniques of experimental design, and could/should involve supporting computing tools or software.

Richard Sisley
Loudon Academy of Science

THE W. M. KECK CURRICULUM PROJECT COURSE SEQUENCE
with
MODELING AS A CENTRAL THEME

COURSE I: ***GEOMETRY AND ALGEBRA WITH TRANSFORMATIONS***

COURSE II: ***DATA, MODELS, AND PREDICTIONS***

COURSE III: ***THE MATHEMATICAL ANALYSIS OF CHANGE***

COURSE IV: ***CALCULUS AS A LANGUAGE OF CHANGE***

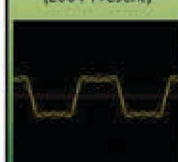
MfA Math for America

Katherine Socha
Director of Education Policy, MfA
Member, SIAM Education Committee
Formerly:
AAAS Science & Technology Policy Fellow, NSF-DMS
Associate Professor of Mathematics, St. Mary's College of Maryland

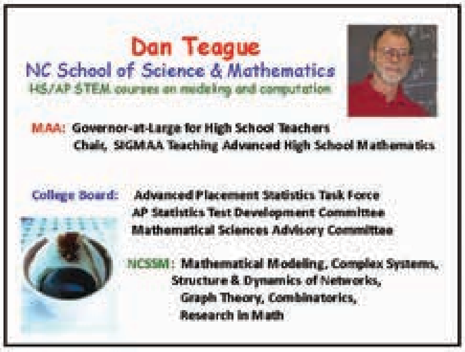
At this workshop, I will facilitate the working group on high school modeling. More from me later.

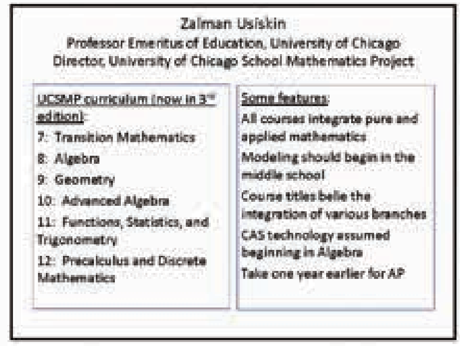
Ted Stanford

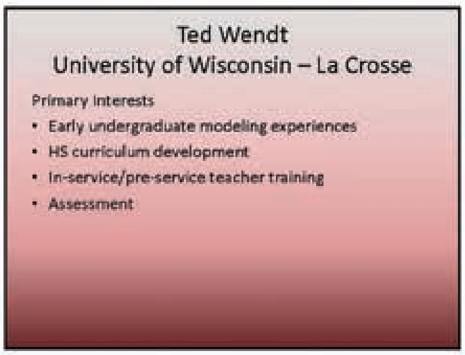
- Math Department at New Mexico State University
- Research background in Topology (Ph.D in 1993)
- Work with pre-service and in-service K-12 teachers (2004-Present)

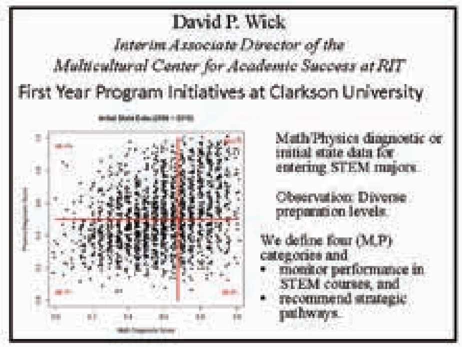


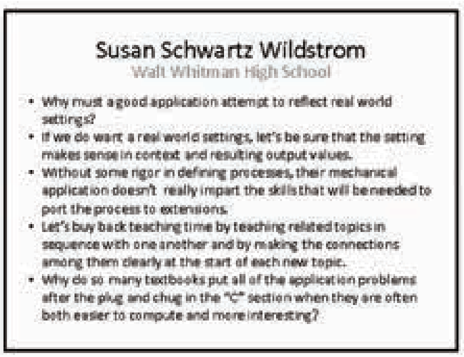
Dreams
- Research mathematical proficiency in the real world
- Computer programming in high school
- Math lessons with light, sound, and Fourier Series


Dan Teague
NC School of Science & Mathematics
HS/AP STEM courses on modeling and computation
MAA: Governor-at-Large for High School Teachers
Chair, SIGMAA Teaching Advanced High School Mathematics
College Board: Advanced Placement Statistics Task Force
AP Statistics Test Development Committee
Mathematical Sciences Advisory Committee
NCSSM: Mathematical Modeling, Complex Systems, Structure & Dynamics of Networks, Graph Theory, Combinatorics, Research in Math

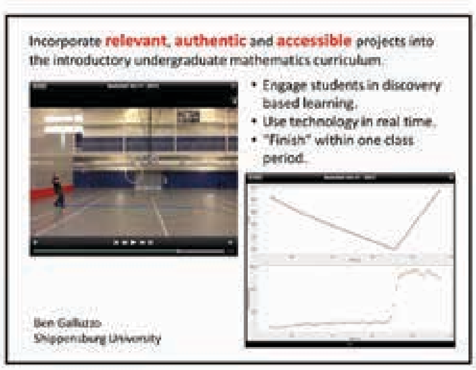

Zalman Usiskin
Professor Emeritus of Education, University of Chicago
Director, University of Chicago School Mathematics Project
UCSMP curriculum (now in 3rd edition):
7: Transition Mathematics
8: Algebra
9: Geometry
10: Advanced Algebra
11: Functions, Statistics, and Trigonometry
12: Precalculus and Discrete Mathematics
Some features:
All courses integrate pure and applied mathematics
Modeling should begin in the middle school
Course titles belie the integration of various branches
CAS technology assumed beginning in Algebra
Take one year earlier for AP

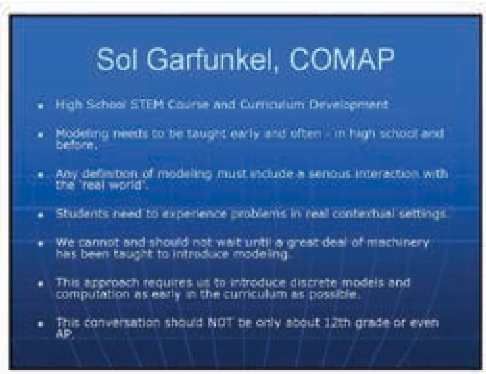

Ted Wendt
University of Wisconsin – La Crosse
Primary Interests
• Early undergraduate modeling experiences
• HS curriculum development
• In-service/pre-service teacher training
• Assessment


David P. Wick
Interim Associate Director of the Multicultural Center for Academic Success at RIT
First Year Program Initiatives at Clarkson University
Math/Physics diagnostic or initial state data for entering STEM majors.
Observation: Diverse preparation levels.
We define four (M,P) categories and
• monitor performance in STEM courses, and
• recommend strategic pathways.

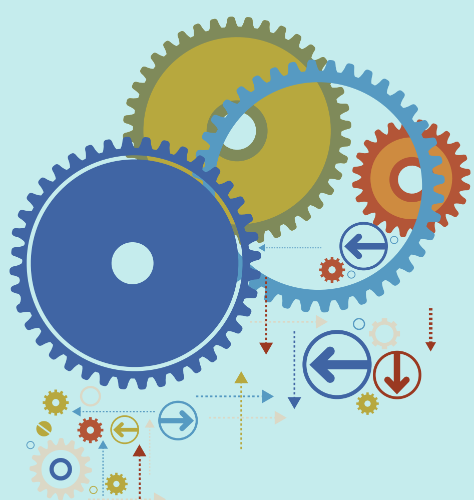

Susan Schwartz Wildstrom
Walt Whitman High School
• Why must a good application attempt to reflect real world settings?
• If we do want a real world settings, let's be sure that the setting makes sense in context and resulting output values.
• Without some rigor in defining processes, their mechanical application doesn't really impart the skills that will be needed to port the process to extensions.
• Let's buy back teaching time by teaching related topics in sequence with one another and by making the connections among them clearly at the start of each new topic.
• Why do so many textbooks put all of the application problems after the plug and chug in the "C" section when they are often both easier to compute and more interesting?


Lizette Zietsman
Department of Mathematics
Interdisciplinary Center for Applied Mathematics
Virginia Tech
Recent experience with undergraduate curriculum devlopment:
Applied Mathematical Modeling. (Junior/Senior level.)
Math in a Computational Context. (Sophomore level.)
Part of group preparing a proposal for a new Computational Science degree.


Incorporate relevant, authentic and accessible projects into the introductory undergraduate mathematics curriculum.
• Engage students in discovery based learning.
• Use technology in real time.
• "Finish" within one class period.
Ben Galluzzo
Shippensburg University


Sol Garfunkel, COMAP
• High School STEM Course and Curriculum Development
• Modeling needs to be taught early and often - in high school and before.
• Any definition of modeling must include a serious interaction with the 'real world'.
• Students need to experience problems in real contextual settings.
• We cannot and should not wait until a great deal of machinery has been taught to introduce modeling.
• This approach requires us to introduce discrete models and computation as early in the curriculum as possible.
• This conversation should NOT be only about 12th grade or even AP.

Society for Industrial and Applied Mathematics
3600 Market Street, 6th Floor
Philadelphia, PA 19104-2688 USA
www.siam.org • siam@siam.org



Modeling Across the Curriculum

07/2013