Newsjournal of the Society for Industrial and Applied Mathematics sinews.siam.org

siam news

Volume 51/ Issue 5
June 2018

Life Sciences Special Issue

Register for the 2018 SIAM Conference on the Life Sciences, to be held August 6-9 in Minneapolis, Minnesota.

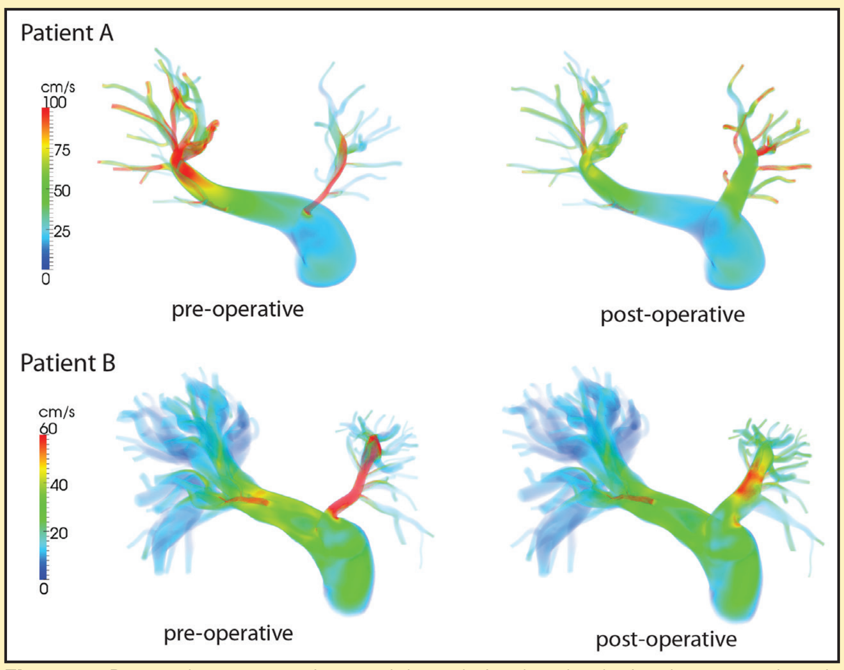


Figure 2. *Pre- and post-operative models and simulated velocity demonstrating virtual surgical planning in the pulmonary arteries of pediatric patients with pulmonary stenosis caused by Alagille syndrome. Image courtesy of [8].*

In an article on page 4, Alison L. Marsden discusses the use of computational modeling to study heart disease progression and treatment planning for personalized medicine.

Mathematical Modelling Disproves Decades-old Hypothesis

A Potential New Way to Treat Heart Disease

By Alona Ben-Tal

Biological systems often display a built-in redundancy—referred to as "plasticity"—that enables more than one mechanism to support the same function. Assessing which mechanism is most important can be difficult, leading to misinterpretations. Here I describe one such misconception and explain mathematics' crucial role in offering clarity and insight.

Our heart rate varies when we breathe, increasing during inspiration (inhalation) and decreasing during expiration (exhalation) — this is called respiratory sinus arrhythmia (RSA). RSA is typically present from birth; it gets stronger in early adulthood and decreases with age. It is also exaggerated during deep and slow breathing and as a result of physical training. RSA is a process that occurs naturally in healthy individuals, and its loss is linked with cardiac mortality. Although researchers have known about the phenomenon for over 150 years, the benefits of RSA have remained a puzzle. A 1996 study by Junichiro Hayano and colleagues [3] suggested that RSA improved oxygen uptake in the lungs of dogs by matching ventilation (the flow of air in and out of the lungs) with perfusion (the flow of blood through the lungs).

Hayano's hypothesis, as the study's conclusions came to be known, seemed very reasonable; increasing one's heart rate as lungs fill up with fresh air could be expected to improve gas exchange in the lungs. The hypothesis became widely accepted by physiologists and clinicians. Consequently, I had no reason to doubt its accuracy when I was able to test it in a mathematical model (a study I conducted with Julian Paton and Sophie Shamailov). Our initial simulations were disappointing; introducing RSA into the model showed only slight, insignificant improvements in gas exchange. We first attributed the discrepancy to our choice of model parameters, and surmised that simplification of the original model would yield an optimal set of parameters. To achieve this, we reduced the model from six nonlinear equations to two linear ordinary differential equations (ODEs) using several assumptions [1]:

See **Decades-old Hypothesis** *on page 4*

Competitive Adaptation Prevents Species from Eradicating Each Other

By Matthew R. Francis

Evolution is frequently rough and unforgiving; individuals within a species compete for food, reproductive partners, or other resources. Species fight each other for survival, especially when preying on one another.

Mathematical biologists often simplify these dynamics to predator versus prey. Real-world populations of predator and prey species within a given ecosystem cycle between booms and busts. In various cases, multiple species—including both predators and prey—coexist with similar diets. For example, a cubic meter of seawater can harbor several species of plankton, consisting of tiny plants and animals (see Figure 1).

One would naively expect reproductive success (more offspring) or competitive performance (eating more than your neighbor) to lead to one species' domination. But that does not occur. While many of these organisms consume the same food, one species does not out-eat the others; the plankton swarm's overall diversity remains fairly constant. Biologists refer to this phenomenon as the "paradox of the plankton" or the "biodiversity paradox," among similar terms.

A possible solution to the paradox is the "kill the winner" (KTW) hypothesis. "We have many species competing with each other, but for each one we also have a predator," Chi Xue, a mathematical physicist at the University of Illinois at Urbana-Champaign, explained. "[If] one species grows to a large population and is poised to win out over the others, the predator will eventually come in and eat the winning species."

In other words, evolution may punish runaway reproductive or competitive success in prey species. Higher population numbers expose organisms to the predators who consume them. KTW is a reasonable hypothesis, but testing it quantitatively to measure whether it matches real population biology is challenging.

Xue and her advisor, Nigel Goldenfeld, modeled multiple strains of bacteria and bacteriophages (or "phages"), which are viruses that prey on bacteria. They found that the standard mathematical version of KTW failed in realistically small populations. Some strains became extinct in the local environment, causing a cascade of other extinctions and the eventual demise of the entire ecosystem.

To fix the problem, Xue and Goldenfeld introduced a way for species to evolve mathematically. "We have an arms race," Xue said. "The prey mutates to escape the predator, but the predator also mutates to catch the prey. New species are constantly introduced, thus avoiding the extinction problem."

The KTW mathematical model is based on a set of equations first proposed for chemical reactions by Alfred J. Lotka in 1920, then adapted to fish populations by Vito Volterra in 1926 (other mathematically-minded biologists discovered similar equations around that time).

When applied to predator-prey systems, the so-called Lotka-Volterra model was simple, elegant, and completely incorrect. KTW adapts the Lotka-Volterra equations to include multiple predator and prey species. Each predator species targets a single prey species, while the prey species simultaneously compete with each other for food. The math produces a result that seems intuitively correct: if a prey species becomes too successful, the predator species targeting it has enough food to increase its own population, therefore restoring balance.

Many KTW formulations assume that all populations are quite large, making it possible to treat the number of organisms in the species as a continuous variable. Xue and Goldenfeld realized that this assumption does not hold in real-world situations. "We took into account the fact that population size is discrete," Xue said. "Because the population size is integer, we found that demographic noise causes species' coexistence to break down."

"Demographic noise" is the ecologists' version of "shot noise": random fluctua-

See **Competitive Adaptation** *on page 3*

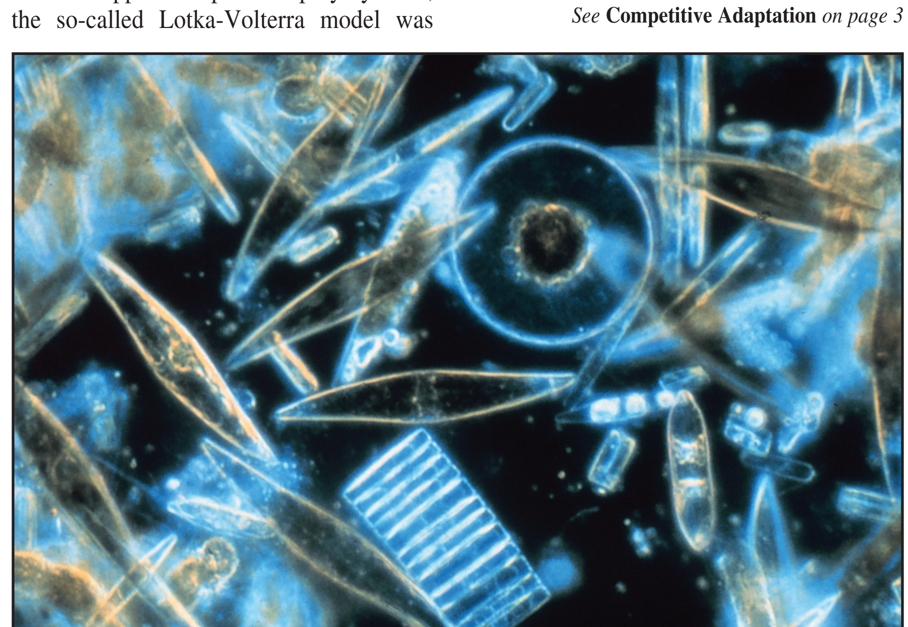


Figure 1. *Many species of plankton, comprised of minuscule plants and animals, successfully coexist in the ocean despite competing for the same food sources. Public domain image.*

Nonprofit Org
U.S. Postage
PAID
Permit No 360
Bellmawr, NJ

siam®
SOCIETY for INDUSTRIAL and APPLIED MATHEMATICS
3600 Market Street, 6th Floor
Philadelphia, PA 19104-2688 USA

siam news
Volume 51/ Issue 5/ June 2018

Celebrating the 50th Volume of the *SIAM Journal on Mathematical Analysis*

On page 9, Felix Otto writes about his top-downloaded paper in the SIAM Journal on Mathematical Analysis and its continued impact on various fields.

Differentiation With(out) a Difference

FROM THE SIAM PRESIDENT

By Nicholas Higham

"The shortest and best way between two truths of the real domain often passes through the imaginary one," mathematician Jacques Hadamard famously said. Here I will discuss an instance of this maxim that deserves to be better known.

A fundamental tension in numerical analysis is the interplay between truncation errors and rounding errors; this is particularly prevalent in the numerical approximation of derivatives. The forward difference approximation

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

has error $O(h)$ for smooth f, so we can make the error as small as we wish by choosing a small enough h. But as numerical analysis textbooks explain, if we take h too tiny then rounding errors incurred in the f-evaluations—which are brought into prominence by subtraction in the numerator and the small denominator—begin to dominate. The h that minimizes the combination of truncation error and rounding error is of order $u^{1/2}$, where u is the unit roundoff ($u \approx 10^{-16}$ for IEEE double-precision arithmetic), and it gives an error in $f'(x)$ of order $u^{1/2}$.

However, most textbooks do not explain that when f is a real function of a real variable, we can do much better by venturing into the complex plane. Provided f is analytic, the approximation

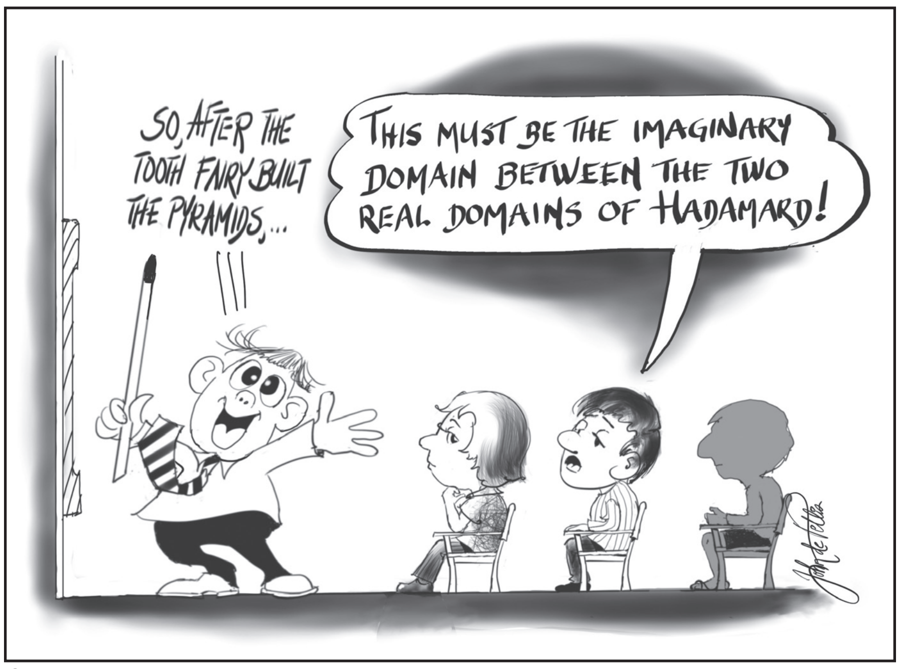
$$f'(x) \approx \operatorname{Im} \frac{f(x+\mathrm{i}h)}{h}, \qquad (1)$$

where i is the imaginary unit, has error $O(h^2)$; h can be taken arbitrarily small without rounding errors vitiating the result. Indeed, the value $h = 10^{-100}$ is used in this approximation at the National Physical Laboratory [2]. The approximation (1) is called the complex-step approximation, and the first occurrence I am aware of is in a 1998 *SIAM Review* paper by William Squire and George Trapp [6]. The authors were inspired by the earlier work of James Lyness and Cleve Moler [5] on the use of complex variables to approximate derivatives.

Rounding errors still affect the complex step approximation, but they do not inhibit the approximation's ability to achieve the expected accuracy given the uncertainty in the evaluation of f.

As a simple example, for the function $f(x) = \dfrac{\operatorname{atan}(x)}{1+\mathrm{e}^{-x^2}}$, a MATLAB experiment[1] yields the following results:

```
fd =
  0.274623728154858 % Accurate
derivative
fd_cs =
  0.274623728154858 % Complex
step with h = 1e-100.
fd_fd1 =
  0.250000000000000 % Forward
diff. with h = 8u.
fd_fd2 =
  0.274623729288578 % Forward
diff. with h = sqrt(8u).
```



Cartoon created by mathematician John de Pillis.

How does the complex step approximation work? It basically carries out a form of approximate automatic differentiation, with h acting as a symbolic variable. For the process to work properly, it is essential that the computation of f itself not use complex numbers, as rounding errors introduced in the imaginary parts would disturb the process.

Squire and Trapp's paper has garnered over 400 citations on Google Scholar — many from papers that develop more general versions of the approximation. For example, the complex step method is suggested in [4] for the approximation of gradients in the context of deep learning. I have used it to compute Fréchet derivatives of matrix functions [1]. It has also become popular for sensitivity analysis in engineering applications, particularly because its implementation is so simple [3].

The complex step method is an easy-to-appreciate example of the potential gains from "going complex." In programming environments with built-in complex arithmetic, such as MATLAB or Julia, it is trivial to give it a try.

1 The code used can be downloaded from https://gist.github.com/higham.

References

[1] Al-Mohy, A.H., & Higham, N.J. (2010). The Complex Step Approximation to the Fréchet Derivative of a Matrix Function. *Numer. Alg., 53*, 133-148.

[2] Cox, M.G., & Harris, P.M. (2004). *Numerical Analysis for Algorithm Design in Metrology. Software Support for Metrology Best Practice Guide No. 11*. Teddington, U.K.: National Physical Laboratory.

[3] Giles, M.B., Duta, M.C., Mueller, J.-D., & Pierce, N.A. (2003). Algorithm Developments for Discrete Adjoint Methods. *AIAA J., 4*(2), 198-205.

[4] Goodfellow, I., Bengio, Y., & Courville, A. (2016). *Deep Learning*. Cambridge, MA: MIT Press.

[5] Lyness, J.N., & Moler, C.B. (1967). Numerical Differentiation of Analytic Functions. *SIAM J. Numer. Anal., 4*(2), 202-210.

[6] Squire, W., & Trapp, G. (1998). Using Complex Variables to Estimate Derivatives of Real Functions. *SIAM Rev., 40*(1), 110-112.

Nicholas Higham is Royal Society Research Professor and Richardson Professor of Applied Mathematics at the University of Manchester. He is the current president of SIAM.

siam news

ISSN 1557–9573. Copyright 2018, all rights reserved, by the Society for Industrial and Applied Mathematics, SIAM, 3600 Market Street, 6th Floor, Philadelphia, PA 19104-2688; (215) 382-9800; siam@siam.org. To be published 10 times in 2017: January/February, March, April, May, June, July/August, September, October, November, and December. The material published herein is not endorsed by SIAM, nor is it intended to reflect SIAM's opinion. The editors reserve the right to select and edit all material submitted for publication.

Advertisers: For display advertising rates and information, contact Kristin O'Neill at marketing@siam.org.

One-year subscription (nonmembers): Electronic-only subscription is free. $73.00 subscription rate worldwide for print copies. SIAM members and subscribers should allow eight weeks for an address change to be effected. Change of address notice should include old and new addresses with zip codes. Please request address change only if it will last six months or more.

Printed in the USA.
siam *is a registered trademark.*

Remembering Cathleen Morawetz: A Champion of Mathematics at a Tumultuous Time

By John Ewing and Margaret Wright

This article is a tribute to renowned mathematician Cathleen Morawetz, who passed away last summer on August 8. We are looking back at her tireless advocacy for mathematics during a particularly trying period. Cathleen spent much of her respected career at New York University's Courant Institute of Mathematical Sciences, and is remembered for her contributions to the study of partial differential equations governing fluid flow. Notably, she was also only the second woman to be elected president of the American Mathematical Society (AMS).

Society presidents are busy people. They serve two-year terms, but also function as president-elects the year before and past-presidents the year after. They travel to meetings around the country, learn how new federal policy affects the mathematical sciences, communicate with other societies around the world, and respond to changes in the academy. It's a big job.

Cathleen was elected president of the AMS in 1993. Her presidency (1995-1996) was especially challenging given the turbulent time period, which was politically fraught and replete with controversial issues for the mathematical sciences.

The U.S. government shut down twice in quick succession because of standoffs between President Clinton and Congress. The shutdowns led to general reductions in federal spending — particularly in science funding, which could no longer be justified by the Cold War. Rather, research in basic science was under pressure to demonstrate its contributions to "economic competitiveness."

Members of the mathematical sciences community feared that these trends would lead to minimal or no support for "core mathematics" and an overall decline in the long-term health of the mathematical sciences. It was a perilous time — a time that might have led to disastrous infighting between pure and applied mathematics. By good fortune, it did not.

During Cathleen's AMS presidency, coauthor Margaret Wright was president of SIAM. The two worked together to implement agendas for both the AMS and SIAM that emphasized unity among the mathematical sciences, thus strengthening the community and making a case for the importance of mathematics in the larger scientific endeavor. Because the two societies did not have a history of cooperation or even goodwill, they first had to overcome nearly half a century of apprehension.

Margaret was a Distinguished Member of Technical Staff at Bell Laboratories, only a few miles away from Cathleen at Courant. They began to meet for lunch in Greenwich Village (on their own dime!), and talked about issues and hazards, politics and agencies, and goals and strategies. They formulated carefully-worded statements for Congress and agency leaders, always stressing the remarkable track record of useful mathematics and the unexpected benefits of undirected basic research. Cathleen and Margaret conveyed that same message to the director of the National Science Foundation, leaders of other agencies that support mathematics research, congressional staff, and members of Congress whenever they spoke on behalf of their societies and the mathematical sciences. As they campaigned, they became good friends. That interaction and camaraderie brought together not only the two leaders, but their societies as well. Consequently, mathematical factions—which might have grown farther apart—instead grew closer and worked together, benefiting every element of the mathematical sciences.

Cathleen faced many other problems throughout her presidency. Employment prospects for young mathematicians were poor. She brought attention to the dilemma and set people to work on enhancing career opportunities for trained mathematicians. The public (and Congress) increasingly believed that universities were not taking teaching seriously, and Cathleen engaged the AMS to counter that perception. The internet was in its infancy, and few people were concerned with making older literature available online. As a board member of JSTOR, Cathleen was determined to put the mathematical sciences at the forefront of this new effort.

One problem in particular illustrated the tenor of the times. In November of 1995, the University of Rochester announced that it was ending its graduate program in mathematics. The plan was to eventually cut the size of the faculty in half and move some undergraduate professors to other departments. Rochester higher-ups did not see mathematics as essential to a research program. Cathleen jumped in, creating a task force that ultimately convinced the school to reverse the decision. Fittingly, one of the outcomes was a clear statement of support from all parts of the mathematical sciences community.

Soon after her term as president, Cathleen was awarded the National Medal of Science; she was the first woman mathematician to receive this honor. The award primarily recognized her many research contributions, which extended over a remarkable five decades. But the honor also confirmed her position as an equally remarkable leader in the mathematical sciences. The legacy of that leadership remains.

From left to right: Coauthor Margaret Wright, Donald J. Lewis, and Cathleen Morawetz at the "Celebration for Donald J. Lewis" in 1999. Image courtesy of Marty LaVor.

John Ewing served as executive director of the American Mathematical Society from 1995-2009, and is currently president of Math for America. Margaret Wright is a Silver Professor of Computer Science at New York University's Courant Institute of Mathematical Sciences. Prior to that, she was a researcher at Stanford University and a Distinguished Member of Technical Staff at Bell Laboratories.

Competitive Adaptation

Continued from page 1

tions that arise simply because the number of objects in a model are integer (see inset on the right). It is important even for abundant species like bacteria because a real-world ecosystem does not include every member of a bacterial species, and predator-prey interactions occur in relatively small local populations.

When Xue and Goldenfeld modified KTW to treat discrete populations, species became sequentially extinct and the model failed. Although this sort of thing *does* happen in nature sometimes, it is not a feature of normal ecosystems. Diversity exists among species, and our mathematical models should reflect that.

To solve the problem, they looked to evolution. "If we introduce coevolution into the system, we can avoid the extinction situation," Xue said. "Coevolution here means that prey and predator are both mutating."

In strict Lotka-Volterra or KTW models, all organisms within a specific group (prey species, for example) are identical. But according to Darwinian evolution, mutation ensures that descendants are not always identical to their parents, even in species like bacteria that reproduce asexually (bacteria can also transfer genes between individuals, which helps spread new traits). These small changes between generations are often enough to give the offspring an edge over other species.

Xue and Goldenfeld modeled descent with modification by allowing their various bacterial prey species to mutate into new strains at a certain rate. The predatory phage that targets the original strain cannot prey on the new strain, which then "wins." However, phages also evolve at a similar rate, allowing them to consume the new strain (viruses are not "alive" in the normal biological sense, but they still evolve).

The simplified KTW model is a set of coupled differential equations

$$\dot{B}_i = B_i\left(b_i - e\sum_{j=1}^{m} B_j - pV_i\right)$$

$$\dot{V}_i = V_i(\beta p B_i - d),$$

where B_i and V_i are the populations of m species of bacteria and viruses respectively. Each virus strain preys on only one bacterial strain with rate p, while the bacteria reproduce at rate b_i and compete with other strains at rate e. The viruses can also "burst" with rate β and perish at rate d. One can obtain the original Lotka-Volterra model by setting e to zero and restricting the equations to one predator and prey species.

The previous equations model populations as continuous. Xue and Goldenfeld modified them to handle discrete numbers for each population. They also included *potential* species in the overall species set and added a mutation rate, so that offspring from population i could be part of population $i+1$.

The researchers discovered several distinct outcomes upon adjusting mutation rates:

1. When the mutation rate was made too small, the demographic noise problem returned. The prey species did not evolve quickly enough to avoid extinction, and the whole ecosystem collapsed.
2. Species maintained diversity at moderate mutation rates, but average populations remained low. Individual prey species sometimes spiked when mutation allowed them to reproduce without predatory interference, only to crash again when predators evolved to catch up.
3. A rapid mutation rate kept overall diversity high; no species had a chance to dominate another. Prey mutated quickly enough to avoid being eaten into extinction, while predators adapted to new strains before prey populations grew out of hand.

Xue and Goldenfeld focused on bacteria/phage competition, since they could study these diverse and rapidly-mutating species in the laboratory. But Xue argues that adjustment of mutation and prediction rates can extend the model to other systems, such as mammals. "If you look at an ordinary predator and prey—the fox and hare, for example—their coevolution rate is very slow compared to their predation rate," she said.

Demographic Noise

Plotting the number of adult women of a given height on a graph yields very different plots for small and large sample sizes. For large numbers, the distribution is Gaussian: symmetrical around the population's mean value. Smaller populations, however, do not exhibit a clear pattern. Because an individual's height does not correlate with the height of anyone else, fluctuations dominate the data when the population is small; this is known as "demographic noise." Demographic noise in species represents randomness in reproduction, deaths from disease or accident, and other factors arising from classification of populations as discrete rather than continuous.

Whatever those rates, coevolution is key to understanding how ecosystems maintain species diversity, even among competitors. "Evolution is the universal law behind everything in biology," Xue said. "Diversity is universal for basically every ecosystem on Earth."

Further Reading

[1] Murray, J.D. (1989). *Mathematical Biology*. Berlin, Germany: Springer-Verlag. *The Lotka-Volterra model, its problems, and its generalizations are available in chapter 3.

[2] Xue, C., & Goldenfeld, N. (2017). Coevolution Maintains Diversity in the Stochastic 'Kill the Winner' Model. *Phys. Rev. Lett., 119*, 268101.

Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Clinically Useful Computational Models for Personalized Treatment Planning in Cardiovascular Disease

By Alison L. Marsden

Cardiovascular disease is the leading cause of death worldwide, and heart disease alone is responsible for one in four deaths in the U.S. Congenital heart disease in children is the primary reason for infant mortality and affects roughly one percent of U.S. births, with more than half of patients requiring at least one invasive surgery during their lifetime. The complex interplay of mechanical stimuli (blood flow, mechanical forces) acting on tissues, and the subsequent biological responses, influence cardiovascular disease. Altered pressure and flow lead to extensive remodeling of vascular and cardiac morphology and wall properties. These interactions are crucial to progression of coronary artery disease, bypass graft failure, cardiomyopathy, pulmonary hypertension, cardiac development, and congenital heart disease. Computational modeling can noninvasively quantify blood flow and pressure in image-based models for individual patients, and is increasingly able to capture biological responses to changing mechanical forces. One can use these models to determine the impact of mechanical forces on disease progression and in virtual treatment planning for personalized medicine.

Partnerships between engineers and clinicians have yielded numerous advances in the treatment of adult and pediatric cardiovascular disease. One prominent example is the development of the first battery-operated wearable pacemaker, pioneered by surgeon Clarence Walton Lillehei and electrical engineer/TV repairman Earl E. Bakken. Engineering-clinical collaborations have also resulted in other high-impact technologies, such as cardiopulmonary bypass, coronary stents and balloon angioplasty, advanced medical imaging techniques, and transcatheter devices to treat structural diseases. Over the past decade, cardiovascular blood flow simulation has emerged as one of the most dynamic examples of this critical partnership, and is now poised to significantly impact cardiovascular patient care.

Computational models of blood flow were first developed for pediatric cardiology in the late 1980s, and initially combined idealized anatomic models with computational fluid dynamics (CFD) simulations. CFD quickly influenced surgical practice for those born with a severe form of single-ventricle congenital heart disease, in which patients are missing half of their heart at birth [1]. The introduction of patient-specific modeling—where anatomic models are reconstructed from medical image data—subsequently allowed blood flow simulations in customized anatomic models for individual patients [7]. High-performance computing and numerical analysis tools developed in the fluid dynamics and aerospace communities enabled fast parallel solution of the Navier-Stokes equations — the partial differential equations (PDEs) governing blood flow. Recent clinical trials demonstrate that CFD simulations of coronary flow—coupled with physiologic models—can noninvasively predict fractional flow reserve, a measure of the severity of coronary blockages caused by atherosclerotic plaques [3].

Cardiovascular simulations fill several crucial gaps in current clinical capabilities. First, while predictive simulations are now used routinely in aerospace and other engineering industries, the medical field still relies primarily on statistical outcomes and trial-and-error approaches to advance surgical methods. Simulations now offer a powerful way to predict surgical outcomes, systematically test and optimize new surgical approaches and devices, perform risk stratification, and personalize treatments for individual patients. Second, one can use simulations to characterize the in vivo mechanical environment, providing crucial hemodynamics and mechanical stimuli data that is not readily attainable from medical imaging. This data is a key component of the mechanobiological puzzle relating the mechanical environment to subsequent disease progression.

One can model blood flow in the cardiovascular system with varying levels of fidelity, typically trading computational cost for spatial and temporal resolution (see Figure 1, on page 5). The simplest of these are zero-dimensional lumped parameter circuit models, which are governed by ordinary differential equations. They are surprisingly effective at capturing vital features of cardiovascular physiology and heart function, producing realistic pressure-volume loops and flow and pressure waveforms. But they lack spatial information. One can include limited spatial information in intermediate-complexity one-dimensional models, which integrate the full PDEs over the blood vessel cross-section and produce a reduced one-dimensional system with unknowns of flow rate and cross-sectional area, together with constitutive material models. The resulting system captures vessel wall deformation and wave propagation with more realistic geometry and one spatial dimension, and is usable in extensive "full-body" models of the cardiovascular system. Finally, one can solve the full three-dimensional equations by discretizing the Navier-Stokes equations with finite-element or finite-volume methods and fully resolving pressure and velocity at every spatial point in a three-dimensional, patient-specific anatomic model. These models produce high-fidelity spatial and temporal data, but typically require large computations and are thus limited to local anatomical regions.

Despite the aforementioned advances, there is an increasing need to incorpo-

See **Cardiovascular Disease** *on page 5*

Decades-old Hypothesis

Continued from page 1

$$\frac{dh_1}{dt} = \alpha q_{in}(t) - D(h_1 - h_2)$$

$$\frac{dh_2}{dt} = D(h_1 - h_2) - \beta u(t).$$

As is often the case in mathematical studies, we realised that this simplified model can describe a completely different system (see Figure 1). h_1 and h_2 can represent either the heights of water in two interconnected containers or the partial pressures of oxygen (or alternatively, carbon dioxide) in lung air sacs and blood capillaries. $q_{in}(t)$ can portray water flow into the left chamber or inspired air flow into the lungs, and $u(t)$ can depict the action of a water pump or heart rate over time; D, α, and β are appropriate parameters for each of these systems.

We could now test Hayano's hypothesis in the hydrodynamic model: if we let water flow into the left container for the first half of the period and stop the flow during the second half, could we operate the pump in a way that would maximize the amount of water flowing out of the right container? Similarly, if the lungs are filled with oxygen during inspiration, could we operate the heart in a way that would transport more oxygen into the bloodstream? The answer is clearly no; if we wish to maintain steady state, the same amount of water that comes in over one period must also flow out during that same time period, regardless of the manner in which the pump functions. This understanding ruled out Hayano's hypothesis in the simplified model and revived the initial question: what is the benefit of RSA?

We searched for another solvable optimization problem. First we estimated the amount of work carried out by the heart during one breathing period. Suppose the heart pumps a volume of blood V_c over each heartbeat against a resistance to flow R_b. We can then show [1] that the work expended by the heart over one breathing period T is

$$W_T = \int_0^T V_c^2 R_b HR^2 dt.$$

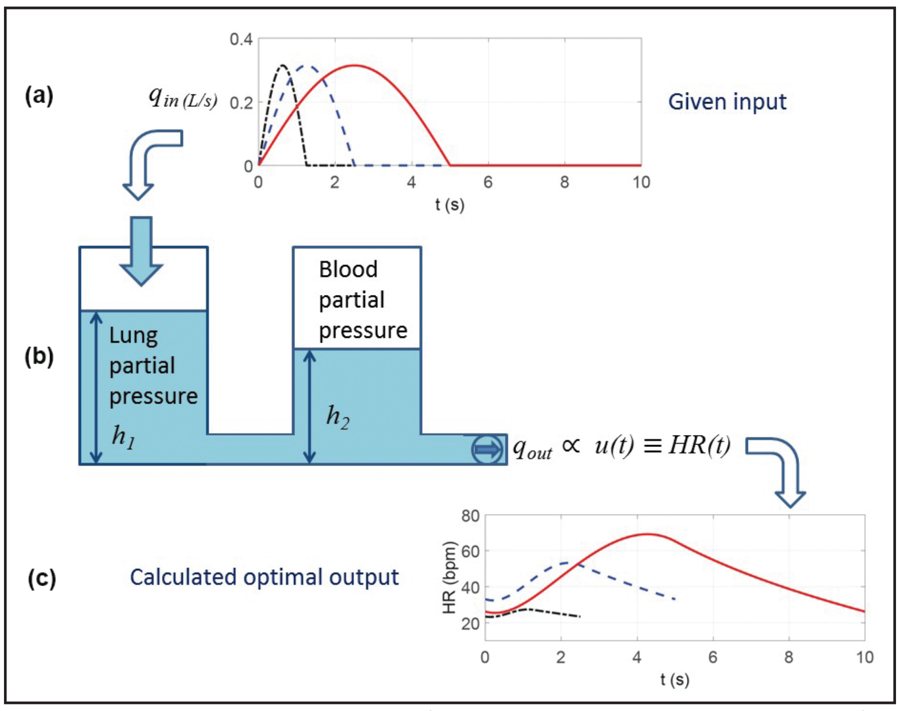


Figure 1. *A hydrodynamic model analogy of gas transport in the lungs.* **1a.** *Three given flow patterns; only one breathing period is shown for each pattern.* **1b.** *The container on the left represents air sacs and the container on the right depicts blood capillaries in the lungs.* **1c.** *The resulting optimal calculation of heart rate. The solid red line portrays slow, deep breathing and the black dotted line represents fast, shallow breathing. Image adapted from [1].*

If we assume that V_c and R_b are constant, minimizing W_T is the same as minimizing $E = \int_0^T HR^2 dt$. This allowed us to state the following optimal control problem:

Find $HR(t)$, such that E is minimized subject to the following constraints:

1. The differential equations of the mathematical model are satisfied
2. The system is in steady state
3. The partial pressure of either oxygen or carbon dioxide in the blood has a given average value over one breathing period.

We transformed this optimal problem into a larger system of ODEs with boundary conditions, which we solved numerically [1]. To our delight, the optimal solution (shown in Figure 1c) had the form of RSA. Moreover, the calculations showed that the amplitude of RSA increases under slow and deep breathing (indicated by the solid red line in Figures 1a and 1c), consistent with experimental observations. We also found that maintaining blood carbon dioxide levels had a significantly stronger effect on RSA amplitude than maintaining oxygen levels.

These realizations led to a new hypothesis: RSA minimizes the work executed by the heart while maintaining physiological levels of blood partial pressures of carbon dioxide. We tested and validated the hypothesis on models with increasing degrees of complexity [1, 2]. Our calculations (over one breath) predicted RSA-related energy savings of around three percent. This prompted us to wonder whether the loss of RSA results in more work for the heart and hence contributes to heart damage; if so, would reinstating RSA help the heart recover? Our group is currently contributing to a new study that will test the effect of RSA on heart function and could potentially change the way cardiac pacemakers operate in people with heart disease.

Why did Hayano's hypothesis endure for so long? First, Hayano and his colleagues' experiments in dogs were compelling, revealing a clear difference between mean oxygen consumption with and without RSA. Upon re-examination, it appears that the results might have been statistically insignificant; the margins of error were much larger than the difference in the means; calculation of the mean of the differences may have yielded more conclusive answers. Additionally, some of the experiments that confirmed Hayano's hypothesis used deep and slow breathing to generate stronger RSA. However, our theoretical study illustrates that such breathing improves gas exchange efficiency regardless of RSA, which explains these previous experimental findings. Lastly, our intuition tells us that RSA should make gas exchange more efficient; it does, but the more complicated mathematical model reveals that the improvement is insignificant.

Ultimately, theoretical studies and mathematical modelling are essential for the quantification and comparison of the different effects that co-exist in biological systems.

Acknowledgments: This work was partly supported by NIH grant R01 NS069220. The research of my collaborator, Julian Paton, was supported by the University of Bristol and the Royal Society.

References

[1] Ben-Tal, A., Shamailov, S.S., & Paton, J.F.R. (2012). Evaluating the physiological significance of respiratory sinus arrhythmia: looking beyond ventilation-perfusion efficiency. *J. Physiol., 590*(8), 1989-2008.

[2] Ben-Tal, A., Shamailov, S.S., & Paton, J.F.R. (2014). Central regulation of heart rate and the appearance of respiratory sinus arrhythmia: New insights from mathematical modeling. *Math. Biosci., 255*, 71-82.

[3] Hayano, J., Yasuma, F., Okada, A., Mukai, S., & Fujinami, T. (1996). Respiratory sinus arrhythmia. A phenomenon improving pulmonary gas exchange and circulatory efficiency. *Circ., 94*(4), 842-847.

Alona Ben-Tal was initially trained as a mechanical engineer and worked for a few years in industry before pursuing a Ph.D. in mathematics, followed by a New Zealand Science and Technology Postdoctoral Fellowship at the Auckland Bioengineering Institute. She is now a senior lecturer in mathematics and Deputy Head of Institute at Massey University's Institute of Natural and Mathematical Sciences in Auckland, New Zealand.

California Students Clinch Top Prize for Mathematical Approach to Food Insecurity

2018 MathWorks Math Modeling Challenge Addresses Food Waste

By Lina Sorg

Many Americans take for granted consistent availability of a nutritious food supply. Yet according to the United States Department of Agriculture (USDA), nearly 13 percent of U.S. households[1]—equivalent to over 41 million Americans[2]—experienced food insecurity in 2016. Food insecurity is defined as a state of living in which one lacks reliable access to a balanced source of food that permits a healthy, active lifestyle.

Furthermore, the Food and Agriculture Organization of the United Nations (FAO) estimates that approximately one third of all food produced in the world is wasted each year[3] due to disfigurement, overstocking, impulse buying, excessive portioning, and unreasonable quality standards. This uneaten food comprises the bulk of landfill and incinerator content in the U.S., thus squandering the valuable labor, land, and water resources necessary for production. Can wasted food be used to feed food-insecure populations? If so, how can agencies effectively and economically repurpose the waste?

These are the kinds of complicated, multifaceted questions explored in this year's MathWorks Math Modeling (M3) Challenge,[4] which tackled food insecurity and the reduction of food waste in America. Sponsored by MathWorks and organized by SIAM, the contest invites participating teams of U.S. high school juniors and seniors to address a complex, real-world problem—akin to those faced by professional mathematicians and computational scientists—in 14 hours using mathematical modeling. After two intense rounds of judging—one via an online platform, followed by a second at SIAM offices—collectively involving roughly 130 SIAM member judges, the top six teams were invited to travel to New York to present their solution papers to a final panel of judges and compete for scholarship money. The concluding round was hosted by Jane Street, a quantitative trading firm.

This year's Challenge problem asked teams to create a mathematical model to determine whether the state of Texas could feed its food-insecure population using food waste from the state. Participants then had to develop a second model to calculate the amount of food waste produced yearly by different types of households with varying income levels. Finally, they were to suggest a strategy for repurposing the maximum amount of food for the least cost.

"These are the very types of questions that we look at in our research," guest judge and USDA agricultural economist Pat Canning said. "We look not at short-term, but medium- and long-term policy-relevant issues. Food security is obviously one of our most important issues, and many have identified food waste as a promising area of research for achieving technological advancements."

In his opening remarks, Canning emphasized mathematics' relevance to the social sciences—specifically agricultural, natural resource, ecological, and environmental economics—by describing his recent projects. One task involved the use of an environmental input-output multiplier model to gauge whether healthier diets generate lower greenhouse gas emissions and use less fossil fuels and fresh water. For another, he utilized computable general equilibrium models to examine the Supplemental Nutritional Assistance Program's effect on farm output and farm employment.

This year's first-place team, from Los Altos High School in Los Altos, Calif., began by consulting government databases and FAO resources to become familiar with food waste and food insecurity. Using this data, the students identified a production waste vector, calculated the percentage of different types of food waste in Texas, and converted the resulting numbers to kilocalories per dollar. They summed up these values and divided them by one's average daily caloric needs to determine the number of people that the state's food waste could nourish. Based on a 2,000-calorie diet, the team concluded that Texas' waste could feed 1.95 million of the 4.32 million food-insecure residents. "That's on the assumption that we're feeding food-insecure individuals their entire diet," team member Michael Vronsky said. "If we wanted to actually distribute whatever food we had across the population, we could have covered about 40 to 50 percent of their diet."

For the second question, the California team calculated the average number of

See **Food Insecurity** *on page 6*

[1] https://www.ers.usda.gov/topics/food-nutrition-assistance/food-security-in-the-us/key-statistics-graphics.aspx
[2] http://map.feedingamerica.org/
[3] http://www.fao.org/save-food/resources/keyfindings/en
[4] https://m3challenge.siam.org/

Cardiovascular Disease

Continued from page 4

rate multiphysics models and biological responses into computational predictions of cardiovascular disease. The field is thus undergoing a paradigm shift to move beyond "pure" CFD towards validated models that integrate mechanobiological mechanisms and whole-heart physiology. This capability will be key to understanding, predicting, *and altering* cardiovascular disease progression. Researchers have recently paid much attention to the advancement of fluid structure interaction methods to capture large deformations in the heart and valves, and more realistic material models for biological tissues with good computational efficiency. One can expand these efforts to whole-heart modeling, including heart contraction and electrophysiology.

Compared to traditional CFD models, these methods pose additional challenges of multiple time and spatial scales, multiple parameters, and the need for fast cardiac image segmentation. Further efforts have attempted to couple CFD with vascular growth and remodeling to capture the interplay of mechanical forces, such as wall shear stress and pressure with the vessel wall's mechanobiologic response. These forces lead to changes in vessel morphology and composition. Disparate time scales; complex geometries; and inhomogenous, viscoelastic, incompressible biological tissues give rise to further computational and methodological challenges. Finally, scientists are developing thrombosis models to measure the complex series of chemical reactions that comprise the blood coagulation cascade.

The use of uncertainty quantification as a systematic framework for handling uncertain clinical data in model inputs has also garnered recent attention. Researchers accomplish this via parameter estimation to determine model parameter distribution from clinical data, and uncertainty propagation to produce output statistics on quantities of interest. Model parameters are tuned to match the clinical data for individual patients, enabling the computation of personalized output statistics. Additionally, multi-fidelity models—leveraging the aforementioned zero-dimensional and one-dimensional models—have shown recent promise in the convergence acceleration of output statistics and computational cost reduction [6].

The above directions can potentially enable application of predictive simulations to increasingly complex and high-impact clinical problems. Using patient-specific simulations and vascular growth and remodeling, our group has recently hypothesized new mechanisms for the prevention of vein graft failure after coronary artery bypass graft surgery [4, 5]. Recent work has also yielded predictive growth and remodeling models for tissue-engineered bypass grafts, which surgeons are currently implanting in pediatric patients with single-ventricle physiology [2]. Practitioners have used simple models of right ventricular stroke work to risk-stratify pediatric patients with pulmonary hypertension and predict their need for heart transplants. Virtual surgery has likewise been effective in predicting the need to relieve pulmonary stenosis in pediatric patients with Alagille syndrome (see Figure 2, on page 1) [9]. Future clinical applications that will leverage the aforesaid efforts in integrative modeling will include hypertrophic cardiomyopathy, surgical planning in pulmonary reconstructive surgery for congenital heart patients, thrombotic risk in devices and grafts, and aortic dissection. It is crucial that such applications continue to drive the development of new computational technologies, including multiscale modeling, fast image segmentation, linear solver technology, and uncertainty quantification.

References

[1] de Leval, M.R., Kilner, P., Gewillig, M., & Bull, C. (1988). Total cavopulmonary connection: a logical alternative to atriopulmonary connection for complex Fontan operations. Experimental studies and early clinical experience. *J. Thorac. Cardiovasc. Surg., 96*, 682-695.

[2] Miller, K.S., Lee, Y.U., Naito, Y., Breuer, C.K., & Humphrey, J.D. (2014). Computational model of the in vivo development of a tissue engineered vein from an implanted polymeric construct. *J. Biomech., 47*(9), 2080-2087.

[3] Nakazato, R., Park, H.-B., Berman, D.S., Gransar, H., Koo, B.-K., Erglis, A.,… Min, J.K. (2012). Fractional flow reserved derived from computed tomographic angiography (FFRCT) for intermediate severity coronary lesions: Results from the DeFACTO trial (determination of fractional flow reserve by anatomic computed tomographic angiography). *J. Am. Col. Cardio., 60*(17), B6.

[4] Ramachandra, A.B., Humphrey, J.D., & Marsden, A.L. (2017). Gradual loading ameliorates maladaptation in computational simulations of vein graft growth and remodeling. *J. R. Soc. Int., 14*(130).

[5] Ramachandra, A.B., Kahn, A.M., & Marsden, A.L. (2016). Patient specific simulations reveal significant differences in mechanical stimuli in venous and arterial coronary grafts. *J. Cardio. Trans. Res., 9*(4), 279-290.

[6] Schiavazzi, D.E., Doostan, A., Iaccarino, G., & Marsden, A.L. (2017). A generalized multi-resolution expansion for uncertainty propagation with application to cardiovascular modeling. *Comp. Meth. Appl. Mech. Eng., 314*(1), 196-221.

[7] Taylor, C.A., Draney, M.T., Ku, J.P., Parker, D., Steele, B.N., Wang, K., & Zarins, C.K. (1999). Predictive medicine: Computational techniques in therapeutic decision-making. *Comput. Aid. Surg., 4*(5), 231-247.

[8] Yang, W., Feinstein, J.A., & Vignon-Clementel, I.E. (2016). Adaptive outflow boundary conditions improve post-operative predictions after repair of peripheral pulmonary artery stenosis. *Biomech. Model Mechanobiol., 15*(5), 1345-1353.

[9] Yang, W., Hanley, F.L., Chan, F.C.P., Marsden, A.L., Vignon-Clementel, I.E., & Feinstein, J.A. (2018). Computational simulation of post-operative pulmonary flow distribution in alagille patients with peripheral pulmonary artery stenosis. *Cong. Heart Dis., 13*(2), 241-250.

Alison Marsden is an associate professor in the Department of Pediatrics, the Department of Bioengineering, and the Institute for Computational and Mathematical Engineering at Stanford University.

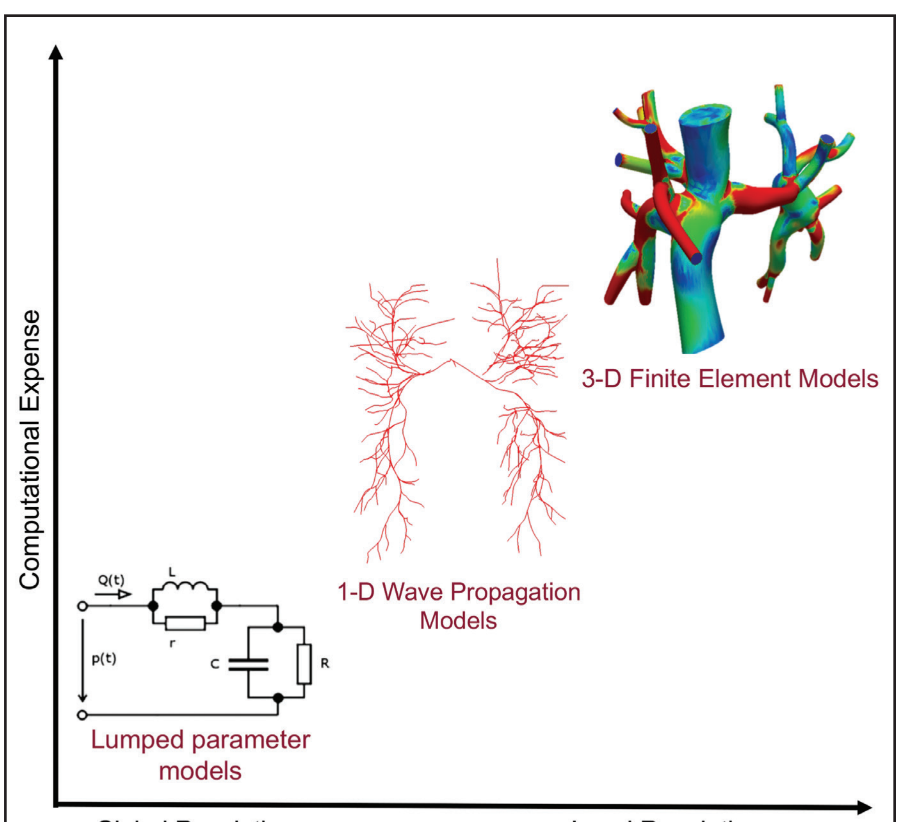


Figure 1. *Multi-fidelity models of the cardiovascular system include zero-dimensional ordinary differential equation models using lumped parameter circuit elements, one-dimensional partial differential equation (PDE) models that capture wall deformation and wave propagation at reduced computational cost, and full three-dimensional PDE models that provide high-fidelity resolved simulations of velocity and pressure in anatomic models. Figure courtesy of the Cardiovascular Biomechanics Computation Lab at Stanford University.*

Reflections from a SIAM Science Policy Fellowship Recipient

By Natalie Sheils

In high school and college, I was heavily involved in student government. In fact, before pursing math I initially majored in public affairs with a minor in nonprofit leadership. I planned to one day run for public office. A month into college free of math classes, I realized I missed mathematics. Without knowing what a future career could look like, I changed my major to applied mathematics. While I retained my love for politics and desire to affect change, these motivations slowly faded into the background as my time and energy shifted to mathematics research, graduate school, and the search for a postdoctoral fellowship.

When I received an email from SIAM advertising the Science Policy Fellowship[1] last summer, I immediately knew I wanted to apply. After years of confining my political inclinations to marches and social media posts, I was excited at the opportunity to do something impactful. I sought advice from Tom Grandine, a member of SIAM's Committee on Science Policy (CSP),[2] whom I knew from a past summer internship at the Boeing Company. "U.S. policy decisions will likely have a big impact on you and your career," Tom advised. "Having an insider's perspective on how the federal funding agencies work and how to navigate those processes can only benefit you going forward." After attending the CSP's spring session and meeting with congressional representatives, I can already attest to Tom's guidance.

I had no idea what to expect when we—the first class of SIAM Science Policy Fellowship recipients—attended the biannual CSP meeting. Rosalie Bélanger-Rioux (Harvard University), Robert Edman (Adventium Labs), Emily Evans (Brigham Young University), Sheri Martinelli (Pennsylvania State University), Jason Pries (Oak Ridge National Laboratory), Sucheta Soundarajan (Syracuse University), and I (University of Minnesota) gathered in Washington, D.C., in April for a half-day orientation before the regular members of the CSP arrived and the real work began. Staff members at Lewis-Burke Associates, the firm SIAM employs to represent us in Washington, led the orientation. We learned about SIAM's history of advocacy and current congressional priorities. We also covered fundamentals, such as the outline of the federal budget, executive and congressional roles in budget creation, and investments in mathematics research. Finally, we talked about CSP goals and the general progression of the meeting, including a visit to legislators' offices.

The next two days were some of the best in my career. I made no breakthroughs in my research, I fell behind on email, and I skipped teaching and advising duties. However, I learned that many senior mathematicians—whom I admire and respect—share my interest in policy and possess a similar desire to impact change on a national scale through advocacy. When I decided to pursue an academic career, I had presumed that with much of my time devoted to research, advocacy would be limited to student mentoring. I had no idea that opportunities like the CSP existed, and certainly doubted that such contributions would be valued in academia. My conversations with CSP members over the course of the meeting drastically changed my perception and gave me hope that I could continue working as a research academic while advocating policy for many years to come.

During the first day of the meeting, we heard presentations by representatives from the White House Office of Science and Technology Policy, the National Science Foundation (NSF), the Department of Energy, and the Office of Naval Research. On the second day, we split into smaller groups and visited our legislators' offices. Our group met with staffers for two House representatives and four senators. Many of them were American Association for the Advancement of Science Congressional Science and Engineering Fellows, well-versed in technical fields and similarly committed to securing funding for basic research. Others were unfamiliar with SIAM, which gave us the chance to share exciting research and note the critical federal support that enabled these developments.

I applied for the Science Policy Fellowship to increase my knowledge of the policy and decision-making processes in Washington, D.C., where I hoped that networking opportunities would further my academic career. Additionally, I wanted to use my voice to represent applied mathematics and SIAM to lawmakers and staff while championing policy positions that would further diversity and increase federal investment in mathematical research, outreach, and education. As noted in the recent American Mathematical Society Report on 2015-2016 Academic Recruitment, Hiring, and Attrition,[3] just 32 percent of all hires in the mathematical sciences during this time were female, and only 31 percent of mathematical sciences Ph.D.s were granted to women. Furthermore, a mere 22 percent of tenured doctoral full-time faculty are female.[4] We need policies and programs that emphasize mentorship and allow flexibility for maternity leave and child care to help women advance to tenured professorship. For example, the NSF currently does not allow for travel costs or associated dependent care expenses for individuals traveling on NSF award funds.[5] This places a considerable financial burden on nursing mothers, among others.

As a SIAM Science Policy Fellowship recipient, I am learning how federal funding agencies—upon which I rely to further my academic career—operate. I am also observing firsthand the effect of federal politics on these agencies' budgets. This information will be vital to me as I continue to work in applied mathematics and campaign for diversity in the field. Although I am only a few months into my term, I am already reaping the benefits of this unique opportunity. I am reenergized in my work knowing that I am connected to a community of dedicated mathematicians and policy advocates, with whom I cannot wait to reunite in D.C. this fall at our next committee meeting.

Natalie Sheils is a visiting assistant professor in the School of Mathematics at the University of Minnesota. She earned her Ph.D. from the Department of Applied Mathematics at the University of Washington in 2015.

1 https://www.siam.org/about/science/sci-pol-fellowship.php
2 https://www.siam.org/about/science/sci-pol.php
3 https://www.ams.org/publications/journals/notices/201706/rnoti-p584.pdf
4 http://www.ams.org/profession/data/annual-survey/annual-survey
5 https://www.nsf.gov/pubs/policydocs/pappguide/nsf11001/aag_5.jsp#VB4

SIAM Science Policy Fellowship recipients attended the biannual meeting of the Committee on Science Policy in Washington, D.C., this spring. SIAM photo.

Food Insecurity

Continued from page 5

calories required per day for people of different ages, genders, and activity levels; this established a baseline for four different household types with a range of incomes. The students then used a nonlinear regression on the consumer behavior data set to predict the proportion of income each household spent on different food types, such as vegetables or grains. From this information, they estimated the amount of food (in pounds) wasted by a single parent with a toddler, a family of four, an elderly couple, and a single 23-year old: 256.4, 839.9, 366.8, and 217.2 pounds respectively.

Identifying the delivery of repurposed food waste to the food-insecure as a primary setback, the Los Altos students experimented with three delivery strategies for their county: one central distribution center, multiple distribution points, and a centralized distribution facility with mobile freezer trucks. Employing computer simulations to measure the strategies' effects on both recipients and the state, the team examined the availability of distribution methods, the distance and travel necessary to obtain repurposed food, and the associated economic costs and benefits.

"We used results from the simulation, along with some of the cost data associated with created distribution centers, like the cost of actually building one and wages for truck drivers who work in the centers," team member Joanne Yuan said. "With this information, we calculated a net gain function to see which strategy would be most effective." Assuming that food-insecure individuals would only travel to distribution centers if the value of the food was greater than the cost of transportation and the potential loss of work due to travel, the team concluded that a multi-hub model—of which 90 percent of food-insecure people would take advantage—would be most effective in the long-term.

In addition to Vronsky and Yuan, the Los Altos champion team—which will split $20,000 in scholarship funds—included seniors Ryan Huang, Daniel Wang, and Justin Yu. The group also nabbed the top M3 Technical Computing Scholarship Award, new in 2018 and presented for "outstanding use of programming to analyze, design, and conceive a solution for the problem," which yielded an additional $3,000 in prize money. Members agreed that seeing a concrete result of their hard work was gratifying, and praised the competition's practical nature.

"The problem is good prep for a real-world experience," Huang observed. "What we did in the Challenge is similar to what a lot of consulting firms do—analyzing problems with a team, writing a paper that explains our ideas, doing analysis, and creating assumptions—which is also really important for modeling in the real world. It's definitely good practice for the future."

Los Altos coach Carol Evans echoed her students' approval, acknowledging that high school curricula typically do not expose students to much mathematical modeling. "With the exception of statistics, other high school math classes tend to be more formulaic and very procedural," she said. "So this is a great opportunity for the kids to put it all together. I see skills from their algebra II classes, their stats classes, and a variety of other classes; that doesn't happen a lot in high school math curricula."

Champion team members expressed interest in pursuing careers in science, technology, engineering, and mathematics—from bioengineering to computer science—and their success inspired a hearty dialogue about potential employment opportunities and future areas of study as they move on to college in the fall. This is exactly what the M3 Challenge hopes to achieve. "Students have tremendous tensions pulling on them, and we want to keep them looking at math as something they can do with their lives," MathWorks cofounder and Challenge judge Cleve Moler said. "Throughout their elementary and junior high school education, they haven't seen much that is exciting about math. This is one of the first examples they encounter of how math can be exciting and important."

Lina Sorg is the associate editor of SIAM News.

The 2018 MathWorks Mega Math (M3) Challenge championship team from Los Altos High School in California will split $20,000 in scholarship money for their first-place finish. Here they accept the top M3 Technical Computing Scholarship Award from Cleve Moler (far right). From left: team coach Carol Evans, Michael Vronsky, Justin Yu, Joanne Yuan, Daniel Wang, and Ryan Huang. SIAM photo.

Further Upon Reflection: Caustics in a Ball

What can one say about the location of the sun's reflection in a shiny ball? Before discussing the answer, let us first consider a circular mirror with a reflecting inner (rather than outer) surface (see Figure 1), illuminated by a pencil of incoming parallel rays — like the top view of a cup in the sun. The bright line on the bottom of the cup is the envelope of the family of reflected rays; such envelopes are called *caustics*.

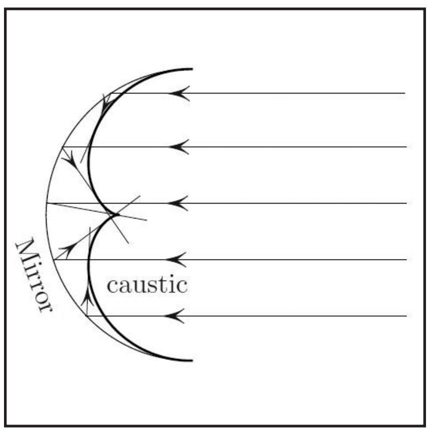


Figure 1. *A circular mirror "focuses" the incoming beam on a caustic.*

MATHEMATICAL CURIOSITIES
By Mark Levi

Interestingly, the exact same caustic is the locus of the sun's reflections from the *outside* surface of the same circular mirror (see Figure 2). The caustic in Figure 2 is the envelope of (the lines defined by) reflected rays. Since the same words describe the caustic in Figure 1, the two caustics are indeed identical; Figure 3 depicts the outer and inner reflections side-by-side, illustrating the identity of the two caustics in one picture.

Now, why do the sun's reflections lie on the caustic? Figure 2 provides the answer: infinitesimally speaking, each point (such as a or b) on the caustic/envelope acts as the point source of light. To the observer A, it will thus appear as if a point source of light exists at a. To summarize, *every reflection in any shiny mirror lies on a caustic, i.e., on the envelope of reflected rays.* This holds true for a mirror of any shape, not only a circle.

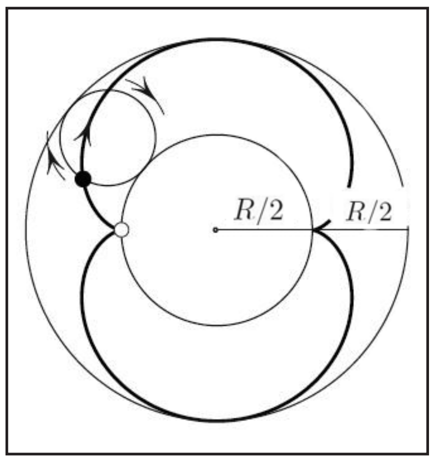


Figure 4. *The nephroid is a hypercycloid traced by a point of a wheel rolling on a circle, with the* 1:2 *ratio of the radii.*

For the circular mirrors in Figures 1 and 2, the caustic happens to be a nephroid, i.e., a hypercycloid traced by a point on the rolling wheel, as shown in Figure 4. I provided a short proof of this fact in my April[1] column.

The figures in this article were provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

[1] https://sinews.siam.org/Details-Page/focusing-on-nephroids

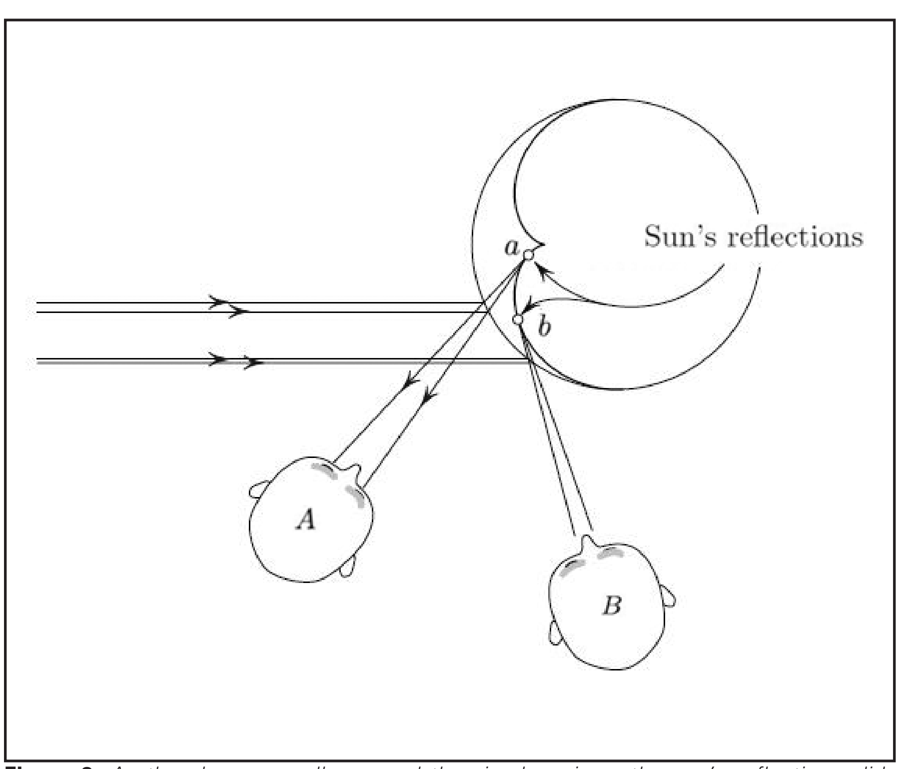


Figure 2. *As the observer walks around the circular mirror, the sun's reflections slide along the caustic.*

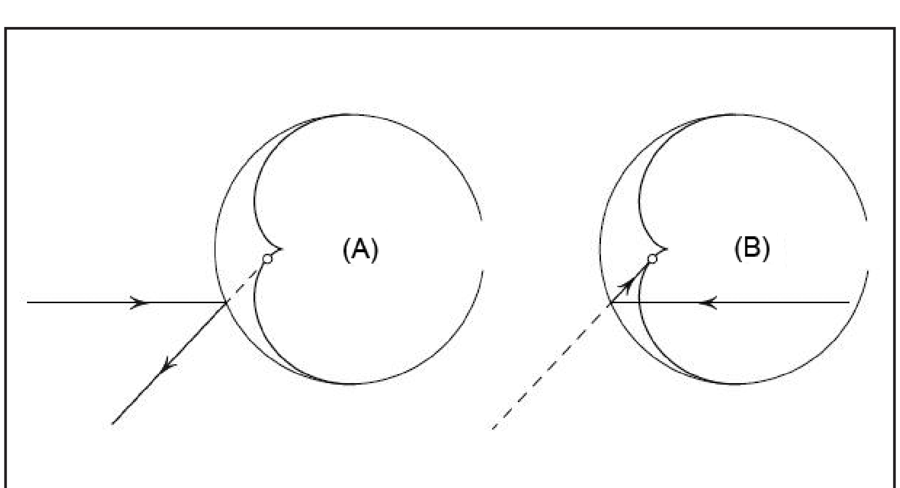


Figure 3. *The locus of the sun's images in a circular mirror (A) coincides with the caustic on the bottom of a coffee cup (B).*

Attending AN18? Want to Write for SIAM News Online?

Interested in reporting on technical sessions from the 2018 SIAM Annual Meeting, to be held July 9-13 in Portland, Ore., for SIAM News Online? Email **sinews.siam.org** with the subject line "AN18 Reporting."

New Foreign Associate for U.S. National Academy of Sciences

Simon Tavaré, former director of the Cancer Research UK Cambridge Institute, has been elected as a foreign associate of the U.S. National Academy of Sciences. Tavaré is a professor in the Department of Applied Mathematics and Theoretical Physics, and Professor of Cancer Research in the Department of Oncology, at the University of Cambridge.

Simon Tavaré of the University of Cambridge was elected as a foreign associate of the U.S. National Academy of Sciences.

"I have worked for many years at the interface between the mathematical sciences and the biological and medical sciences," Tavaré said of the research that won him the distinction. "This work has led to the development of methods for understanding modern molecular data in biomedicine."

His research has included probabilistic and statistical aspects of coalescent theory in population genetics, likelihood-based methods for sequence variation data, evolutionary approaches to cancer, stem cell biology, approximate Bayesian computation for inference in complex stochastic processes, and statistical bioinformatics (particularly in the cancer field).

"I am delighted to have been elected as a foreign associate of the National Academy of Sciences," Tavaré said. "It is a singular honour to be a member of so august a scientific society. But much more importantly, I hope this gives further recognition to the important role the mathematical disciplines can and do play in science more generally."

Tavaré is a fellow of the Royal Statistical Society, the American Mathematical Society, the Institute of Mathematics and its Applications, and the Society of Biology. He also served as president of the London Mathematical Society from 2015 to 2017.

Tavaré was among 21 foreign associates elected by the U.S. National Academy of Sciences in May. The Academy elected 84 new members as well.

Race and Gender in the Scientific Community

Björn Sandstede offers a class titled "Race and Gender in the Scientific Community" at Brown University, where he is a professor in the Division of Applied Mathematics. Mary Silber of the University of Chicago recently chatted with Sandstede about his inspiration behind and vision for the class.

Silber: *I was surprised to learn that you—a white guy from Germany—were teaching a course called "Race and Gender in the Scientific Community." How did that come about?*

Sandstede: A group of Brown University undergraduate students from a range of science, technology, engineering, and mathematics (STEM) departments started it all in the fall of 2014. They wanted to learn more about representation in the scientific community and use this knowledge to advocate for change on campus. To accomplish this, they developed the course syllabus and ran it as a Group Independent Study Project (GISP), advised by many faculty members — including then-GISP faculty advisor Cornelia Dean. I became aware of the GISP much later while chairing a departmental committee that developed a diversity and inclusion action plan (as part of Brown's initiative to create such plans in every unit across the university). Amy Butcher, Jasmine McAdams, and Jamelle Watson-Daniels (who were all part of the team that created the GISP) advocated for its upgrade to a regular course. I agreed that this was an important step to take, and also felt that pursuing this would give me an invaluable opportunity to engage with students on these topics. The initial plan was to co-facilitate the course with a faculty member or graduate student from Africana Studies, but when this did not pan out I offered it alone. 30 students enrolled last spring and 15 enrolled this spring, with representation across all STEM fields. All students felt strongly about the need to create a more inclusive and diverse environment in STEM disciplines.

Silber: *I have taught some classes where I thought "never again" because they did not play to my interests and strengths, and others where I could not wait to teach a second and third time — to make the most of my initial investment, and also to improve on the first iteration. Would you teach this class again?*

Sandstede: This spring was the second time I offered it — I was very nervous (scared is probably a better word!) when offering it the first time as I was all too aware of my nonexistent background in the areas we covered. I learned a tremendous amount and thoroughly enjoyed facilitating this class ("teaching" implies that I knew what I was doing…). Ideally, I would like to see the class rotate through STEM departments at Brown, perhaps co-taught with faculty from our Science, Technology, and Society Program. There are aspects I would like to change: allocating more time for guest speakers and more readings on intersectionality, for instance.

Silber: *What did you find most challenging about teaching this class? What did you find most interesting?*

Sandstede: My main fear was that I would say the wrong things in class. I worried about facilitating a course that consists exclusively of assigned reading and classroom discussions of potentially sensitive topics — I felt very inadequate. I tried to address this by being open about these misgivings with my students (I included a discussion of our hopes and fears for the class, which helped me articulate these concerns). The most interesting aspect was our ability to engage in these topics from an academic viewpoint whilst also discussing personal experiences.

Silber: *The reading list for your course looks very challenging, especially for a mathematician accustomed to thorough reading. How did you generate it? One book I recognized was Claude Steele's Whistling Vivaldi about stereotype threat. That book had a big impact on me; I was left reeling from it. Did a particular book or article have that kind of impact on you? How about the students? Any recommended reading for SIAM members?*

Sandstede: Fortunately, I did not have to create the reading list! A group of undergraduate students designed the list in 2014 with help from many faculty members, including (notably) Cornelia Dean. I had read *Whistling Vivaldi* as part of Brown's Team Enhanced Advising and Mentoring program — the book had a big influence on me too, and I would definitely recommend it to everyone. I think what impacted me most were the experiences that my students shared; many of these were very shocking, and they reinforced how far we are from an environment where everybody feels welcomed, respected, and included.

Silber: *How did teaching this course change the way you think about diversity in applied mathematics? Why do you think it is important to our field and to STEM?*

Sandstede: If anything, offering the course probably made me more pessimistic and disheartened. Most of my students came into this class hoping/thinking that we could identify a few major issues that—once resolved—would change everything; seeing their frustration at the systemic nature of the lack of diversity was depressing. I also felt very inadequate when they asked me about the stagnation of applied math and other STEM departments at Brown, and I could not give any satisfactory answers. There are many reasons to create a more diverse and inclusive environment in the sciences. A consistent theme in our class discussions is the challenge students encounter in persevering in a field where it is rare to see STEM faculty that look like them. For me, the lack of equal opportunity this implies is reason enough to work towards more diversity.

Silber: *With so few women and underrepresented minorities in faculty positions, there is sometimes an expectation that they will advocate on behalf of diversity within departments—perhaps unfairly—freeing up others to focus on a different set of priorities. It seems that you might now be in a position to champion diversity on committees. Do you agree?*

Sandstede: University of Michigan's theatre group, which creates plays that cover a variety of situations in academia, performed one about a tenure case of a female faculty member — the committee consists of one woman, and she argues against tenure in the end. When I saw that play as part of a "Women in Neuroscience" workshop, several people asked why the female member did not support the faculty member more strongly. We were then told that another version of the play has the female committee member advocating strongly for tenure. In this case, the group is usually asked why she endorses the candidate; nobody questioned the male committee members in this way. So, I agree that we often put the burden on historically underrepresented faculty to advocate on behalf of their groups — which is wrong, as nobody should be expected to be a spokesperson for "their" group. I have started to

See **Race and Gender** *on page 10*

Networks through History: From Ancient Hierarchies to Modern Online Communities

The Square and the Tower: Networks and Power, from the Freemasons to Facebook. *By Niall Ferguson. Penguin Press, New York, NY, January 2018. 592 pages, $30.00.*

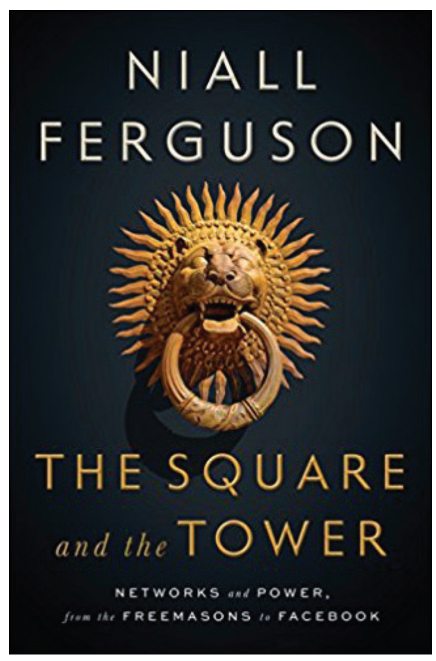
BOOK REVIEW
By James Case

The modern world is dependent on power grids, transportation and communication networks, and water and sewer systems, while facing credible threats from the National Rifle Association, the Irish Republican Army, the Islamic State of Iraq and Syria (ISIS), Al-Qaeda, and various terrorist networks. In graph-theoretic terms, hierarchies are themselves networks—specifically trees—with a single "root node" at the top, connected by sequences of edges to a multitude of "leaf nodes" at the bottom.

Financial historian Niall Ferguson's fifteenth book, *The Square and the Tower*, explores the tensions between social networks and political hierarchies in human history. Ferguson asserts that his fellow historians have somewhat overlooked networks, despite extensive written records left by hierarchies like the Roman Empire, the Third Reich, and General Motors; other networks, such as the early Christian church, the Protestant Reformation, and the Populist Movement, began as loosely-connected grassroots organizations with minimal paper trails.

The book's title is inspired by the majestic Torre del Mangia, which looms allegorically over the Piazza del Campo in Siena, Italy. The piazza was designed to host voluntary interactions among relative equals, while the tower symbolizes the imposition of hierarchic secular power. Together they remind us that social networks and political hierarchies have coexisted in varying degrees of harmony since as early as the 14th century.

The early chapters of *The Square and the Tower* present the rudiments of network theory, which is at once an empirical science and a branch of mathematics. Ferguson is careful to introduce only mathematical principles that are easily absorbed and obviously relevant to the analysis of historical trends. Perhaps the simplest concept is the "density" (edge density) of a network, which is merely the number of edges in a particular graph on n vertices divided by the number $n(n-1)/2$ of potential edges. Duncan Luce and Albert Perry's "clustering coefficient"—the number of complete triangles divided by the number $n(n-1)(n-2)/6$ of potentially-complete triangles—is a related notion, as both measure global properties of a graph.



The Square and the Tower: Networks and Power, from the Freemasons to Facebook. By Niall Ferguson. Courtesy of Penguin Press.

Ferguson also presents three measures of "centrality," a local property pertaining to a single node. The simplest is "degree centrality," which counts the number of edges on a given node or vertex. It differs from both "betweenness centrality," which counts the number of "complete paths" in the network that pass through a given node, and "closeness centrality," which computes the number of edges in the shortest path connecting the node of interest to another — averaged over all other nodes. He makes no mention of more sophisticated centrality measures, such as Katz, eigenvector, or PageRank centrality.

Though Ferguson avoids the term "graph theory," he begins his exposition with Euler's analysis of the Königsberg Bridge problem, followed by a description of social psychologist Stanley Milgram's "small world experiment." Milgram asked randomly-chosen residents of Wichita, Kan., and Omaha, Neb., to forward letters to designated recipients in or near Boston, Mass. Residents were to send the letters to their planned recipients if they knew them on a first-name basis; if not, they were to direct them to people who potentially knew the recipients, aiming to bring the letters closer to their intended targets. 44 of the original 160 letters reportedly reached their destinations, having been passed along by an average of five intermediaries. Since a path with two terminal and five intermediate nodes contains six edges, the situation came to be known as "six degrees of separation." Since that time, six degrees of Marlon Brando, Monica Lewinsky, Kevin Bacon (which later became a board game) and mathematician Paul Erdös have been catalogued. Subsequent research suggests that the national degree of separation is closer to five than six. The number is 4.6 for directors of Fortune 1000 companies and 3.52 for Facebook users. Of course, this is only a restricted version of closeness centrality.

Ferguson extracts seven network insights relevant to the analysis of social networks, two of which seem particularly worthy of mention here. One is his observation that "structure determines virality." Ferguson argues that some ideas "go viral" due to structural features of the networks through which they propagate. To illustrate the point, he invokes an example involving a graph on n nodes arranged in a circle, in which each node is connected by an edge to only its two nearest and two next-nearest neighbors. Without appreciably increasing the edge density of such a graph, the addition of a few edges connecting widely separated nodes could substantially increase the likelihood of virality.

The other noteworthy insight is that the majority of social networks are profoundly *inegalitarian*, in the sense that new edges are most often added to nodes that are already abundantly connected. This leads to the emergence of the type of "hubs" that populate airline route maps, and immediately distinguishes social networks from those studied by Erdös and Alfréd Rényi, in which edges are added at random to a preexisting set of vertices. The degrees of the several vertices in Erdös-Rényi networks tend toward equality as additional edges are added.

The latter chapters of *The Square and the Tower* consist of descriptions and analyses of historically significant networks. Of these, few have rivaled the impact of the Freemasons' network. Freemasonry began in the late middle ages as a fraternal

See **Networks through History** *on page 11*

The Impact of the *SIAM Journal on Mathematical Analysis'* Most Popular Paper

By Felix Otto

On the occasion of the SIAM Journal on Mathematical Analysis' 50th volume, a coauthor of the journal's most popular article offers insight as to why the paper continues to spark so much interest.

My article[1] with Richard Jordan and David Kinderlehrer highlighted the gradient flow structure of the Fokker-Planck equation describing overdamped Langevin dynamics of a particle in a potential. It did so via an (implicit) time discretization in the form of a sequence of variational problems. The insight that such time discretizations—called "minimizing movements" by Ennio De Giorgi's school—are helpful in understanding dissipative dynamics became popular in geometric analysis through Fred Almgren, Jean Taylor, and Lihe Wang's work on the flow of a hypersurface by its mean curvature. My Ph.D. advisor Stephan Luckhaus' convergence result with fellow Ph.D. student Thomas Sturzenhecker provided me with an understanding of this research.

Surprisingly, our minimizing movement scheme made the Wasserstein metric (as recognized by Richard Jordan)—the transportation cost between two particle densities—appear. Our analysis relied on Yann Brenier's work with optimal transportation, which was motivated by Vladimir Arnold's geometric view of Euler's equations for an inviscid incompressible fluid. Incidentally, I learned of Brenier's study in Luis Caffarelli's inspiring class on the regularity of the Monge-Ampère equation, observable as the Euler-Lagrange equation of optimal transportation.

Our article fed into two recent developments, the first classical theme being the thermodynamically-consistent modelling of quasistatic dissipative evolutions. This process properly separates driving energetics from limiting dissipation — an area somewhere between applied partial differential equations and rational mechanics. In particular, Alexander Mielke and his collaborators developed a general theory partly inspired by rate-independent evolutions like crack propagation. More recently, Mark Peletier and his colleagues showed that large deviation principles of the underlying stochastic process should guide the proper separation between the energy functional and the dissipation metric. A beautiful extension of optimal transportation suitable for *reaction*-diffusion equations has recently come to light. The proper extension of the minimizing movement structure to discrete Markov chains—and somewhat related dissipative quantum evolutions—is an active area of research.

Perhaps our article had its broadest impact in geometric analysis: companion articles put Robert McCann's displacement convexity of entropy-functionals and ensuing contractivity properties of (nonlinear) diffusions into an infinite-dimensional geometric perspective. The intimate connection between these and the Ricci curvature of the underlying manifold (which on the infinitesimal level was already present in the Bakry-Emery calculus) eventually allowed for a "synthetic" definition of Ricci curvature on metric measure by John Lott, Cédric Villani, and Karl-Theodor Sturm. The work of Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré again made the connection to De Giorgi's program.

Table 1 displays the 10 most-downloaded articles from the *SIAM Journal on Mathematical Analysis*, all of which are freely accessible through the end of the year. This information is also available online.[2]

Rank	Article	Author(s)	Year
1	The Variational Formulation of the Fokker–Planck Equation	Richard Jordan, David Kinderlehrer, and Felix Otto	1998
2	The Lifting Scheme: A Construction of Second Generation Wavelets	Wim Sweldens	1998
3	Homogenization and Two-Scale Convergence	Grégoire Allaire	1992
4	Boundary Integral Operators on Lipschitz Domains: Elementary Results	Martin Costabel	1988
5	Global Bifurcation of Solutions for Crime Modeling Equations	Robert Stephen Cantrell, Chris Cosner, and Raúl Manásevich	2012
6	Modulational Stability of Ground States of Nonlinear Schrödinger Equations	Michael I. Weinstein	1985
7	On the Cahn-Hilliard Equation with Degenerate Mobility	Charles M. Elliott and Harald Garcke	1996
8	Decomposition of Hardy Functions into Square Integrable Wavelets of Constant Shape	Alexander Grossmann and Jean Morlet	1984
9	Discretized Fractional Calculus	Christian Lubich	1986
10	New and Improved Johnson-Lindenstrauss Embeddings via the Restricted Isometry Property	Felix Krahmer and Rachel Ward	2011

Table 1. *The 10 most downloaded articles from the SIAM Journal on Mathematical Analysis.*

Felix Otto is a director at the Max Planck Institute for Mathematics in the Sciences in Leipzig, Germany.

[1] https://epubs.siam.org/doi/abs/10.1137/S0036141096303359

[2] https://epubs.siam.org/page/sima_celebrates_50_volumes

Stand Out in the Industry Job-Hunting Process and Optimize Your Career Potential

Attend the Professional Development Evening on Thursday, July 12th at AN18

By Di Ye

On average, any given job receives 118 applications. However, only 20 percent of applicants are granted an interview.[1] With the number of graduating Ph.D. students exceeding the number of available academic employment opportunities fivefold, the job market is certainly competitive.

Landing a rewarding job requires both technical competency and a spectrum of soft skills, including self-confidence and awareness, career adaptability, resilience, communication, empathy and relationship-building, professionalism, time management, and emotional regulation. These skills account for up to 80 percent of career success, but are often underdeveloped in today's college graduates.[2]

Recognizing the need to better equip graduate students to secure industry jobs, SIAM has partnered with Zhennovate,[3] a talent development company that specializes in helping science, technology, engineering, and mathematics students and working professionals achieve their career and leadership goals. Together, they will provide a hands-on learning opportunity for the industry job-hunting process at the Professional Development Evening, to be held on Thursday, July 12th during the 2018 SIAM Annual Meeting (AN18).

In anticipation of the event, let us examine the reality of the job market and review some tips to maximize career potential.

#1: Adapt with Self-awareness and a Growth Mindset

Take command of the job-hunting process by identifying jobs that would be fulfilling to you. Many exciting opportunities exist beyond academia in private industries, nonprofit settings, and public service sectors. The ability to adapt to a new career is fundamental to navigating today's rapidly-changing economy. Only 25 percent of college graduates work in an occupation related to their major.[4] The other 75 percent must learn to quickly adjust to a new field, a new set of skills, and a new network of people. To keep up with these changes, learn to thrive on challenges and embrace setbacks as a springboard for personal growth.

#2: Build Your Support Network Beyond Academia

In reality, at least 70 percent of available job openings are not published,[5] and about 50 percent of employment at top companies comes through referrals. Referred candidates are twice as likely to land an interview and have a better chance of being hired. If you want an industry job, connecting with industry insiders is imperative. Effective networking is never transactional, but rather an authentic relationship-building process.

In addition to a professional network of peers, a support network—a group of people who believe in you, challenge you to grow, and introduce new possibilities—is essential to success. Coaching can be a complementary and equally-important part of your support network. Professional coaches are human behavior experts who help deepen self-awareness, overcome limiting beliefs, and cultivate constructive mental and behavioral habits. Once reserved only for senior executives, coaching is now leveraged by high-potential professionals as a proven strategy for soft skills development.

#3: Identify and Effectively Communicate Your Value

Effectively communicating your value is essential to securing new opportunities, including funding your research proposal, earning a promotion, establishing a collaboration, or landing a great job. The mindset and priorities of industry-based careers vary from those of academia. In industry, relevant skills and experiences are more important than academic credentials. Many applicants set themselves up for failure by applying to an industry job with a CV, which lists academic accreditations instead of background and competencies.

The application materials and workflow necessary for success in industry differ from academic requirements. An elevator pitch to articulate your skills and career objectives for networking occasions, a resume tailored to the job rather than a general CV, and a set of compelling stories to illustrate how your experiences fit the needs of the employer are good starting points.

#4: Begin Your Search Early

On average, applicants spend anywhere from two to eight months seeking an industry position. The earlier you start preparing, the better positioned you are. Those who are desperate to find a job quickly have a harder time managing their emotional stress, and desperation drives employers away. Give yourself some breathing room rather than waiting until the last minute.

About the Professional Development Evening

Drawing on the domain knowledge and experiences of hiring managers, industry and academia insiders, and career and leadership coaches, Zhennovate has developed two workshops for the Professional Development Evening at AN18. These workshops are best suited for graduate students, postdocs, and other early-career professionals interested in introductory preparation for entering the industry job market.

Workshop A: How to Build a Strong Support Network

Networking is essential to finding job opportunities, establishing collaborations, and building support. Attendees will learn how to do the following:

- Strategically plan and build their support network
- Conduct informational interviews
- Network for meaningful relationships
- Network for job opportunities at conferences, career fairs, and beyond.

Workshop B: How to Develop a Winning Application for Industry Jobs

Via a hands-on walk-through of the industry job application process, attendees will practice the subsequent techniques:

- Interpreting job descriptions to guide their application process
- Framing their experiences and skills from graduate school as valuable assets to potential employers
- Preparing documents for career fairs and job applications
- Preparing for job interviews in industry.

The workshops, which I will lead, have 50 available spots. A separate application is required for workshop participation, and graduate students and postdocs are encouraged to apply. To apply, please indicate your interest in the Professional Development Evening when you register for AN18. All interested candidates will then receive application instructions via email in mid-June and be notified of admission status on July 1st. If you neglected to specify your interest in the Professional Development Evening when registering for AN18, you may contact Kimberly Haines at khaines@siam.org before application closes on June 25th.

Di Ye is a professional trainer and certified coach at Zhennovate, where she provides high-performance training and 1:1 coaching to clients from various institutions, such as Oracle, Boeing, Microsoft, the University of Washington, and the Massachusetts Institute of Technology (MIT). She graduated from MIT with a degree in electrical engineering and computer science, and has industry experience at Microsoft, Qualcomm, and Seagate and research experience with the Athinoula A. Martinos Center for Biomedical Imaging at Harvard Medical School.

[1] https://bit.ly/2rBHKki
[2] https://www.luminafoundation.org/files/publications/Closing_the_skills_gap.pdf
[3] https://zhennovate.com/
[4] https://wapo.st/2ryW4eo
[5] https://www.npr.org/2011/02/08/133474431/a-successful-job-search-its-all-about-networking

Race and Gender

Continued from page 8

champion for diversity more strongly; I do find it very challenging, but I also care less about pushback than I used to.

Silber: *Thanks for your candid responses. I learned a number of things from this exchange, and thought of a few more questions. Did this class change the way you teach mathematics or mentor and advise students? How can people find out more about this class and acquire the reading list? Is there a broader community of STEM educators and students developing these materials?*

Sandstede: Offering this class has changed the way in which I structure my classes and mentor students. Some of the changes involve simply talking more about challenges and expressing support more explicitly. The syllabus and reading list are available on my webpage.[1] This summer, I plan to work with three former students to develop a guide for this course. There is a broader STEM community interested in these kinds of courses; for instance, Amherst College and Yale University offer courses on "Being Human in STEM" (HSTEM) that have similar goals to the Brown course. We had an HSTEM summit at Yale in April, and one of the discussion items was the creation of a central repository for these courses to share instructor guides and class materials.

[1] http://www.dam.brown.edu/people/sandsted/

Björn Sandstede is a professor in the Division of Applied Mathematics at Brown University. He is a former Sloan Research Fellow, a SIAM Fellow, and the 2001 recipient of the SIAM Activity Group on Dynamical Systems' J.D. Crawford Prize. Mary Silber is a professor in the Department of Statistics and director of the Committee on Computational and Applied Mathematics at the University of Chicago. She is a SIAM Fellow, an American Physical Society Fellow, and recipient of a National Science Foundation Division of Mathematical Sciences CAREER Award. Silber and Sandstede are acquainted through their shared research interests in pattern formation.

SIAM Participates in Annual CNSF Exhibition

Emily Evans of Brigham Young University (BYU)—one of SIAM's new Science Policy Fellowship recipients—represented SIAM at the 24th Annual Coalition for National Science Funding (CNSF) Capitol Hill Exhibition, held this May in Washington, D.C.

Evans discussed mathematical science research and pitched ideas to Representatives Jerry McNerney (D-CA) and James Comer (R-KY). She met with Jim Lewis, Deputy Assistant Director for the Education and Human Resources Directorate at the National Science Foundation (NSF); Deborah Lockhart, Deputy Assistant Director for the Directorate for Mathematical and Physical Sciences at the NSF; and Ron Buckmire, Lead Program Director of the NSF's Scholarship for Science, Technology, Engineering and Mathematics program.

Prior to the exhibition, Evans met with Utah delegation officials and Congressional Representative John Curtis, an alumnus of BYU who represents the district in which the university is located. Evans also had the opportunity to meet with Senator Mike Lee (R-UT) at a constituent meet-and-greet.

Evans fielded a constant stream of attendees interested in learning about the Applied and Computational Mathematics Emphasis program at BYU, including members of Congress, congressional staff, NSF officials, mathematics colleagues, and others.

Emily Evans (left) poses with Senator Mike Lee (R-UT) and Miriam Quintal (right) at a constituent meet-and-greet during the 24th Annual Coalition for National Science Funding (CNSF) Capitol Hill Exhibition, which took place in Washington, D.C., this May. Photo credit: Senator Lee staff.

Emily Evans (left) engages Representative James Comer (R-KY) in discussion at the 24th Annual Coalition for National Science Funding (CNSF) Capitol Hill Exhibition, which took place in Washington, D.C., this May. Photo credit: Eliana Perlmutter.

SIAM Has a New Class of Fellows!

If you missed our announcement of the 2018 Class of SIAM Fellows in the May issue, head over to **http://www.siam.org/prizes/fellows/** for the full list.

The SIAM Fellows Program:

– Honors SIAM members who are recognized by their peers as distinguished for their contributions to the discipline

– Helps make outstanding SIAM members more competitive for awards and honors when they are compared with colleagues from other disciplines

– Supports the advancement of SIAM members to leadership positions in their own institutions and in the broader society.

Information about all Fellows is available at **http://fellows.siam.org/**.

Nominations for the 2019 Class of SIAM Fellows are open until November 5, 2018. Visit **nominatefellows.siam.org** to nominate a qualified individual; you may nominate up to two people per year.

Networks Through History

Continued from page 9

organization of stone masons and other skilled craftsmen, ultimately joined by members of the nobility. Local Masonic lodges were among the few places where noblemen and affiliates of the bourgeoisie could freely mingle. Following the appearance of *The Constitutions of the Free-Masons* in 1723, the organization entered a period of rapid growth.

Leading up to the American Revolution, five associations in Boston were more or less sympathetic to the revolutionary cause. Aside from the local committee of correspondence, the Masonic Lodge that met at the Green Dragon Tavern was probably the most active. A total of 137 men belonged to one or more of the five associations, though the majority belonged to only one. Joseph Warren belonged to four, while Paul Revere, Samuel Adams, and Benjamin Church each belonged to three. Considering two members of the set $S = \{1, \ldots, 137\}$ of association members to be "equivalent" if they belong to exactly the same associations, Ferguson partitions S into a mere 14 equivalence classes: $S_1 \cup \cdots \cup S_{14}$. He then constructs a graph on 14 points in which each node i represents an equivalence class S_i, and an edge connects node i with node j only if S_i and S_j share a member. Because the subsets containing Warren and Revere exhibit the most betweenness centrality, Ferguson concludes that (i) they were the two most important revolutionaries in Boston and (ii) the Freemasons were the primary instigators of the American Revolution. Though historians have disputed the pros and cons of the latter proposition many times, Ferguson shines new light on the long-running debate.

Freemasons, along with their kindred spirits, the *Illuminati*, are frequently blamed for both the French and American revolutions. Ferguson offers a concise history of the Illuminati, which began in 1748 and was dissolved by Pope Clement XIV in 1773, well before the start of the uprising in France. Still, conspiracy theorists continue to blame them for the misdeeds of the Paris mob.

Ferguson's closest citation to a practical application of network theory is—unsurprisingly—to warfare. Its post-World War II experience in the jungles of Malaya taught the British military that insurgencies are difficult to combat, in part because they are led by loosely-affiliated tribal chieftains and village elders — no one of whom is indispensable to the cause or has the authority to conclude a negotiated settlement. They tried with little success to explain this to the American generals in Vietnam, and then Afghanistan. When the U.S. Army did at last catch on, it issued a field manual entitled *Counterinsurgency: FM 3-24*. "To a striking extent, *FM 3-24* set out to educate the U.S. military about network theory, explaining concepts such as network density, degree centrality, and betweenness," Ferguson writes. The first edition even contained an appendix entitled "Social Network Analysis."

Ultimately, it seems likely that Ferguson's revelations will prove most useful to his fellow historians, as a great deal of information concerning historically influential networks already is—or will soon be—readily available. Ferguson considers a range of networks, including the Early Christian Church, the Knights of the Round Table, the House of Saxe-Coburg-Gotha, the Protestant Reformation, the Rothschild family, the internet, the warlords of Afghanistan, ISIS, and Al-Qaeda. In almost every case, he brings fresh perspective to familiar historical episodes and sometimes challenges conventional wisdom. Though networks have been around as long as kings, princes, and military leaders, historians might have underestimated their overall significance.

James Case writes from Baltimore, Maryland.

Forecasting and Modeling Techniques to Study Climate's Impact on Public Health

By Cory Morin and Kristie L. Ebi

Weather conditions impact human health in a variety of ways [1]. The risk and severity of health outcomes depend on exposure (atmospheric conditions), the individual or community's sensitivity to the hazard in question, and the capacity to prepare for and manage the hazard. The major pathways through which weather and climate directly affect human health include extreme temperatures, severe weather events, and poor air quality. Climate also indirectly influences nutrition, infectious disease, and reproductive health, all of which are major areas of study. Researchers seek to identify relationships between meteorological conditions and human health, develop forecasting methods and projections to help governments and citizens better prepare for potential hazards, and understand the physical benefits (co-benefits) of mitigating climate change with renewable energy and other technologies [7].

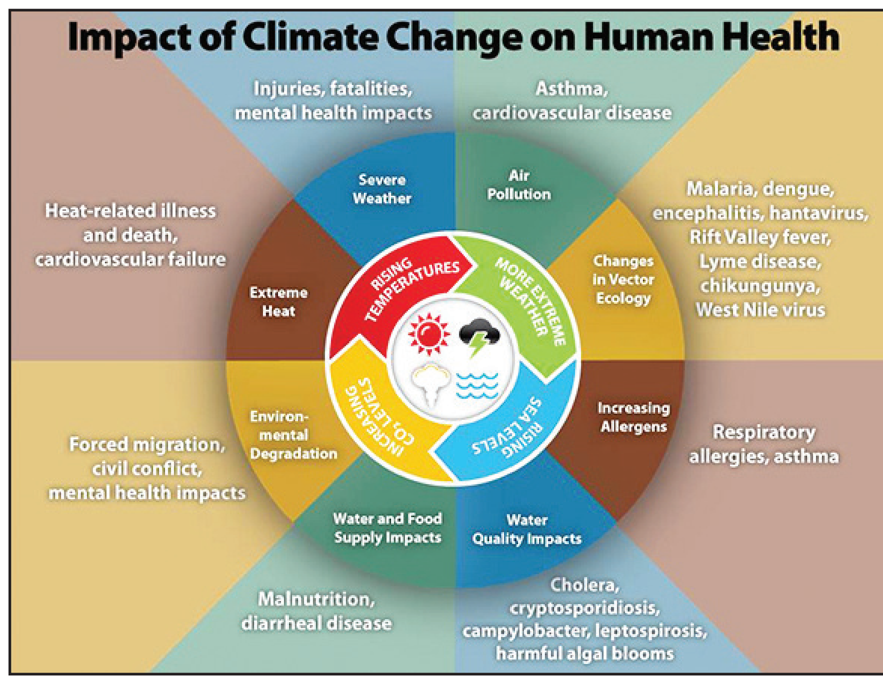
The earliest studies of climate change and health focused almost exclusively on recognition of relationships between meteorological conditions and select health outcomes, and prediction of future consequences. For example, connections between extreme heat and maladies such as heat exhaustion, heat stroke, and cardiovascular disease are now well established. Recent research centers on identification of individual heat-related risk due to increased exposure (e.g., in outdoor workers), limited adaptive capacity (e.g., lack of air conditioning or other cooling), and exacerbating health problems (e.g., existing cardiovascular conditions). This illustrates an important transition from establishing a relationship to implementing related policy to reduce morbidity/mortality in high-risk groups.

Developing a deeper understanding of the mechanisms through which weather and climate change impact human health is often challenging because a single meteorological variable (such as temperature, precipitation, or humidity) can affect numerous aspects of disease ecology, requiring the development of increasingly complex models [5]. Temperature can regulate (i) pathogen development and replication within a vector (e.g., dengue virus replication within a mosquito) or the environment (e.g., salmonella bacterium replication on food), (ii) vector population dynamics in species such as flies, and (iii) reservoir species' behaviors and populations, like bird migration. Precipitation influences disease transmission dynamics by (i) providing water sources for vector breeding (e.g., for mosquito larvae and pupae), (ii) causing flooding that contaminates water and leads to diseases such as cholera, and (iii) triggering changes in population and behaviors of reservoir species (e.g., rodents) by flooding them out of their homes or altering their food supply. Humidity also significantly influences vector survival (e.g., in ticks) and pathogen viability in the environment (e.g., influenza).

Additionally, climate change poses long-term risks to overall wellbeing. Climate variability can adversely affect food security, especially in low- and middle-income countries. Malnutrition suppresses an individual's ability to combat disease, thus increasing susceptibility to other adverse health outcomes. New research shows that changes in carbon dioxide concentrations driving anthropogenic climate change are reducing the quantities of important micronutrients in cereal crops, thus lessening the quality of widely-consumed staples such as wheat and rice.

Furthermore, climate change may interact with other global trends, such as increased human mobility/migration and economic insecurity, to influence demographics and health [6]. For example, as household incomes and living conditions change, women may make different choices about their fertility. Recent research from Tanzania suggests that women who experienced crop failures were less likely to get pregnant or give birth. Weather conditions can also affect pregnancy outcomes. Extreme heat exposure during the third trimester is associated with higher rates of stillbirth, increased risk of preterm delivery, and lower birth weights.

Greater comprehension of complex human-environment relations has yielded more advanced modeling techniques to create early-warning health systems [4]. For instance, climate change is exacerbated by anthropogenic and naturally-occurring emissions—such as particulate matter and surface-level ozone—that contribute to poor air quality. These emissions substantially and adversely impact respiratory conditions like asthma. A large portion of the population also suffers from seasonal allergies, possibly due to increased pollen concentrations from earlier seasonal flowering in a warmer world. Indicators such as pollen counts and air quality indices are grounded in the latest science and warn the general public of unhealthy conditions. A variety of metrics, based mostly on temperature and humidity, communicate heat risk and provide recommendations to reduce exposure and negative health outcomes (e.g., avoid outdoor work or drink sufficient amounts of water). Researchers are currently focused on generating finer-scale monitoring and modeling, which can inform policymakers of new or increasing health concerns and allow them to deploy proactive interventions. Remotely-sensed data, such as the normalized difference vegetation index, provides a signifier and facilitates better monitoring of phenology changes, which indicate pollen level variations that can affect allergy sufferers.



Climate change, when combined with natural and human-induced stressors, influences health and disease in numerous ways. Image courtesy of the Centers for Disease Control and Prevention.

The development of forecasting methods to enable earlier and more targeted surveillance and intervention strategies requires overcoming several challenges. Differentiating the climate's contribution from other influences is one such challenge. Trade and travel, socioeconomic and demographic conditions, human mobility, and pathogen evolution are some of the many non-environmental factors impacting human health. Short and long-term forecasting systems must anticipate how these other components could change and interact with meteorological conditions. Therefore, models should account for these supplementary factors, in addition to uncertainty in weather and climate forecasts.

Numerical simulations can both enable the forecast of important health outcomes through climate data and provide insight into the mechanisms through which climate impacts health. However, simulation modeling is often highly dependent on parameter values, which are usually difficult to identify [5]. Methods to estimate and evaluate unknown parameter values are needed to validate models and better understand their relative importance. The significance of seasonality in climate and health research also necessitates spatiotemporal simulations, especially for diseases. Static maps of risk are frequently of limited value because risk can vary greatly throughout a season. But climate can alter substantially in space as well; therefore, the best dynamic simulations include location-specific data. Information on neighboring areas is likewise important for modeling pathogen risk and diffusion. Unfortunately, models that include location-specific parameters and interactions between nearby locations can be complex and require considerable computing resources.

Simulations help researchers develop insights into unexpected dynamics with consequences for human health in other areas, such as toxicology. For instance, temperature can interact with toxicants to amplify disruptions to the health of organism populations in certain ecosystems, ultimately impacting the ecosystem services these populations provide [3]. Mechanistic effect models—dynamic models based on mechanistic understanding of chemical effects on individuals—can improve chemical risk assessment and increase stakeholder and regulator utility [2].

Despite significant challenges, advances in weather and climate simulations, forecasting and modeling techniques, and observational tools—such as remotely-sensed data—are improving meteorologically-driven forecasts and projections of human health risks. Producing quantitative and location-specific research will be essential to helping public health professionals and policymakers make informed decisions with the greatest health benefits for their communities. This requires a robust interdisciplinary effort with contributions from a number of fields, including public health, environmental science, the social sciences, and applied mathematics. Though ambitious, such a goal is critical for assuring a healthier future for all.

References

[1] Ebi, K.L., Frumkin, H., & Hess, J.J. (2017). Protecting and promoting population health in the context of climate and other global environmental changes. *Anthropoc., 19*, 1-12.

[2] Forbes, V.E., & Galic, N. (2016). Next-generation ecological risk assessment: Predicting risk from molecular initiation to ecosystem service delivery. *Enviro. Int., 91*, 215-219.

[3] Galic, N., Grimm, V., & Forbes, V.E. (2017). Impaired ecosystem process despite little effects on populations: modeling combined effects of warming and toxicants. *Glob. Chang. Biol., 23*, 2973-2989.

[4] Hess, J.J., & Ebi, K.L. (2016). Iterative management of heat early warning systems in a changing climate. *Ann. N.Y. Acad. Sci., 1382*, 21-30.

[5] Morin, C.W., Monaghan, A.J., Hayden, M.H., Barrera, R., & Ernst, K.C. (2015). Meteorologically driven simulations of Dengue epidemics in San Juan, PR. *PLoS Negl. Trop. Dis., 9*, e0004002.

[6] Sellers, S., & Ebi, K.L. (2018). Climate Change and Health under the Shared Socioeconomic Pathway Framework. *Int. J. Environ. Res. Public Health, 15*, 3.

[7] Watts, N., Adger, W.N., Agnolucci, P., Blackstock, J., Byass, P., Cai, W.,…, Hunter, A. (2015). Health and climate change: policy responses to protect public health. *Lanc. Commiss., 386*, 1861-1914.

Cory Morin is an acting assistant professor in the Center for Health and the Global Environment (CHanGE) at the University of Washington (UW). His research focuses on the impact of climate and environmental factors on infectious disease ecology. Kristie L. Ebi is director of CHanGE at UW. Her research focuses on the health risks of a changing climate, and adaptation and mitigation options to prepare for and effectively manage risks in coming decades.

SIAM Fellow Elected to U.S. National Academy of Sciences

SIAM Fellow Andrea Bertozzi has been elected as a member of the U.S. National Academy of Sciences in recognition of her "distinguishing and continuing achievements in original research."

"Becoming a member of the National Academy of Sciences is a lifetime goal that signifies the importance of my work's impact on applied mathematics," Bertozzi said. "Much of this research was collaborative and thus imparts some recognition to my colleagues, students, and postdoctoral fellows in addition to myself. It is an honor and I hope to utilize this new connection to further promote the use and development of mathematics in a variety of disciplinary areas of science."

Andrea Bertozzi of the University of California, Los Angeles was elected as a member of the U.S. National Academy of Sciences.

Bertozzi holds the Betsy Wood Knapp Chair for Innovation and Creativity at the University of California, Los Angeles, with a primary faculty appointment in mathematics and a secondary appointment in mechanical and aerospace engineering. Her nomination highlights her multifaceted achievements in fluid dynamics, swarming, crime modeling, and graphical models in machine learning.

"What I love about being an applied mathematician is that we can use mathematics as a unifying language to study many different phenomena," Bertozzi remarked. "I have used ideas from fluids to study swarms and image processing. I work on models from materials science to study clustering of high-dimensional data. Mathematics is the common language that brings these fields together and provides a framework to use models developed for one discipline to make new contributions in a different area of science. These advances often push forward new and interesting mathematics."

The U.S. National Academy of Sciences elected 84 new members and 21 foreign associates in May.