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Surprises of the Faraday Cage

By Lloyd N. Trefethen

Nearly everyone has heard of the Faraday cage effect. So when I needed to learn about it, I assumed it would be a matter of looking in some standard physics books, maybe the ones I'd studied as an undergraduate. This was the beginning of a journey of surprises.

The Faraday cage effect involves shielding of electrostatic and electromagnetic fields. A closed metal cavity makes a perfect shield, with zero fields inside, and that *is* in the textbooks. Faraday's discovery of 1836 was that fields are nearly zero inside a wire mesh, too. You see this principle applied in your microwave oven, whose front door contains a metal screen with small holes. The screen keeps the microwaves in, while allowing light, with its much smaller wavelength, to pass through.

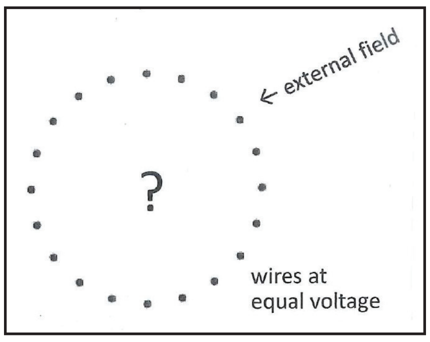
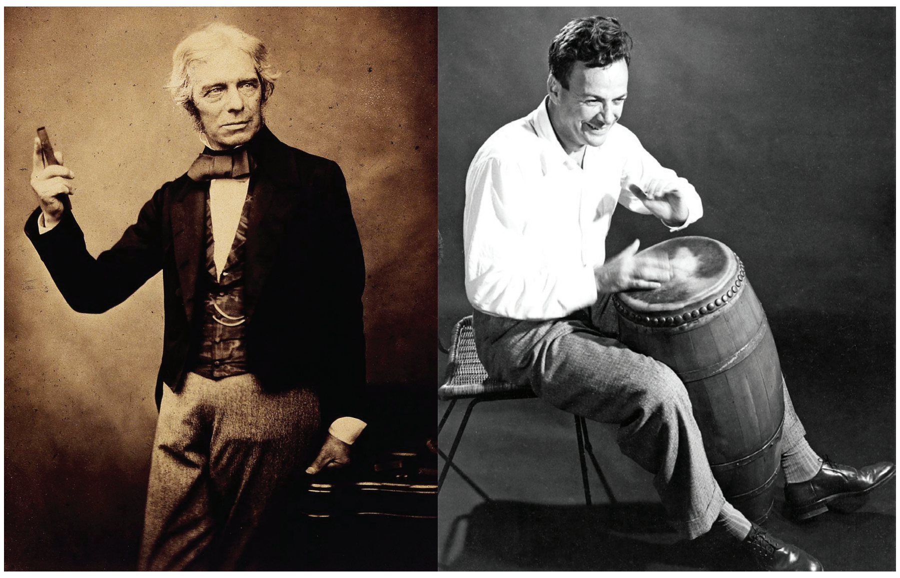


Figure 1. *A Faraday cage can be modelled by a set of dots (cross-sections of wires) spaced around a circle, with equal potential on each. If a potential is applied outside the cage, how close to zero is the field (potential gradient) inside? Figure adapted from [1].*

The essence of the matter can be captured by a two-dimensional model (see Figure 1), where the cage is approximated by a circle or a line of dots representing cross-sections of wires all at the same voltage (connected somewhere in the third dimension). To keep things simple, we focus on electrostatic fields – the Laplace equation.

Let me explain how I got interested in this problem. André Weideman and I were finishing a survey of the trapezoidal rule for periodic analytic functions, which we'd been working on for eight years [5]. We knew the mathematics of that problem: if f is analytic and periodic and you add up sample values at equispaced points, you get an exponentially accurate approximation to its integral. Intuitively, sinusoidal oscillation in one direction corresponds to exponential decay in the direction at right angles in the complex plane. A contour integral estimate of Fourier coefficients exploits this decay to prove exponential accuracy.



English scientist Michael Faraday (left), lends his name to the Faraday cage effect. Photo credit: Wikimedia Commons. Richard Feynman (right), we were surprised to learn, got it wrong. Photo courtesy of the Archives, California Institute of Technology.

To enrich our survey, I thought we should comment on the analogy between this mathematics of the trapezoidal rule and that of the Faraday cage. It seemed obvious the two must be related – it would just be a matter of sorting out the details.

So I started looking in books and talking to people and sending emails. In the books, nothing! Well, a few of them mention the Faraday cage, but rarely with equations. And from experts in mathematics, physics, and electrical engineering, I got oddly assorted explanations. They said the skin depth effect was crucial, or this was an application of the theory of waveguides, or the key point was Babinet's principle, or it was Floquet theory, or "the losses in the wires will drive everything…"

And then at lunch one day, colleague $n+1$ told me, it's in the *Feynman Lectures* [2]!

See **Faraday Cage** *on page 3*

Game Theory and Uncertainty Quantification for Cyber Defense Applications

By Samrat Chatterjee, Mahantesh Halappanavar, Ramakrishna Tipireddy, and Matthew Oster

Cyber system defenders face the challenging task of continually protecting critical assets and information from a variety of malicious attackers. Defenders typically function within resource constraints, while attackers operate at relatively low costs. As a result, design and development of resilient cyber systems that support mission goals under attack, while accounting for the dynamics between attackers and defenders, is an important research problem. The goal of this article is to increase awareness among practitioners and researchers about uncertainty quantification within cybersecurity games, and encourage further advancements in this area.

In order to address cybersecurity challenges, researchers are increasingly adopting game theory-based mathematical modeling approaches that involve strategic decision makers within non-cooperative settings [5-6, 10]. Various taxonomies for classifying game-based modeling approaches exist (see Figure 1). These game formulations contain assumptions about rounds of game plays, past player actions, types of players, number of cyber system states, number of player actions in a given system state, and payoff (reward or penalty) functions associated with player actions.

While game-based attack-defense models consider complex scenarios and effectively represent dynamic interactions, an increased focus on uncertainties in attacker payoff functions could enhance them. In a realistic setting, a defender cannot assume that all necessary information—both about the attackers and their own system—will be available. Since a cyber attacker's payoff generation mechanism is largely unknown, appropriate representation and uncertainty propagation is a critical task. One must also account for the lack or absence of perfect cyber system state information; such uncertainties may arise due to inherent randomness or incomplete knowledge of the behavior of or events affecting the system. For example, partial observability may make a cyber system's state uncertain over time. Moreover, multiple types of attackers could potentially target a system at a given point in time.

Advances in state-space modeling of cyber systems and reinforcement learning approaches for Markov decision processes have inspired the development of partially observable stochastic games (POSGs) and their potential applications for cybersecurity [1, 4, 6-9, 11]. A POSG is comprised of multiple players. Each player independently chooses actions, makes observations, and receives payoffs while the system state transitions based on player-action combinations. A POSG is defined as a tuple (N, A, S, O, P, R, s^0) [4, 7], where:

- N is the set of players
- $A := \Pi_{i \in N} A_i$ is the set of action tuples (pairs when $|N| = 2$), where A_i is the i^{th} player's action set
- S is the set of system states
- $O := \Pi_{i \in N} O_i$ is the set of observations, where O_i consists of the i^{th} player's observations
- P is the probability transition function, where $P(s' | s, a)$ denotes the probability of

See **Cyber Defense** *on page 4*

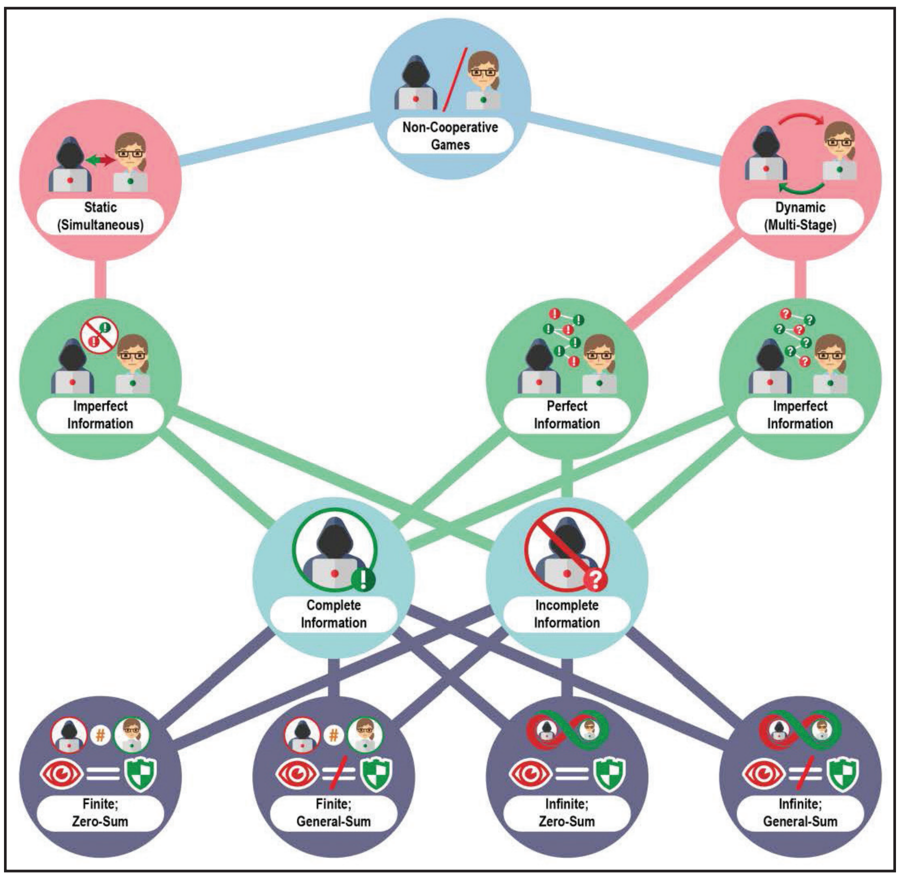


Figure 1. *Types of non-cooperative game models for cybersecurity. Figure created by authors.*

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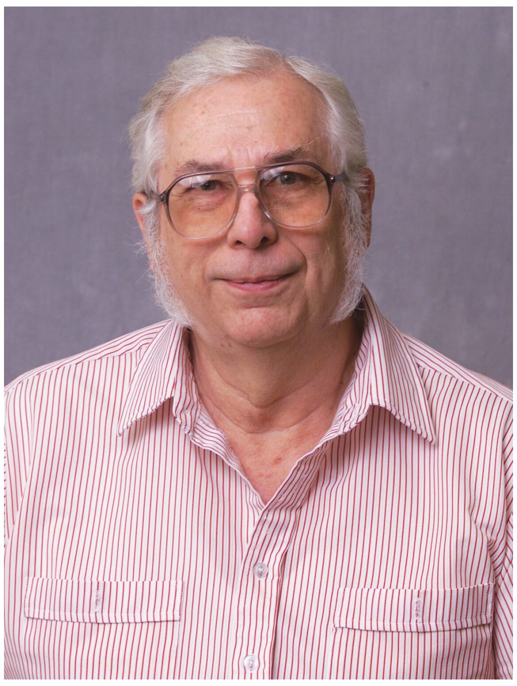
Obituaries

By Douglas N. Arnold, Gregery T. Buzzard, and Bradley J. Lucier

Jim Douglas Jr., the Compere and Marcella Loveless Distinguished Professor Emeritus of Computational Mathematics at Purdue University, passed away on April 27, 2016, after a brief illness. Jim was a highly-regarded applied mathematician whose work influenced the entire spectrum of research in partial differential equations, from purely-theoretical results to very practical applications in the oil industry. He is particularly well-known for his key contributions to the numerical solution of partial differential equations. Jim's early work on alternating direction methods for elliptic and parabolic problems brought many large-scale problems into the realm of practical feasibility, and continues to serve as the foundation for new approaches to optimization and related problems. He was a prolific author and dedicated mentor to dozens of graduate students and postdocs at Rice University, the University of Chicago, and Purdue University. Jim was a SIAM Fellow and an inaugural AMS Fellow, as well as the recipient of numerous awards for his work in the petroleum industry.



Jim Douglas Jr., 1927-2016. Photo credit: Department of Mathematics, Purdue University.

Jim was born in 1927 in Austin, Texas, and obtained an undergraduate and master's degree in civil engineering from the University of Texas in 1946 and 1947, respectively. He continued his studies in mathematics at Rice University, where he earned an M.A. in 1950 and a Ph.D. in 1952, under the supervision of Hugh Brunk.

Jim began his career at Humble Oil, later a part of the ExxonMobil Corporation, where he worked alongside Henry Rachford and Don Peaceman. Their research focused on numerical simulation of the flow of fluids, such as oil or natural gas. Of course, the standards for numerical simulation at that time were different from what they are now. As Peaceman describes in "A Personal Retrospection of Reservoir Simulation," the computing devices used in 1955 had extremely limited storage: they held only 864 words!

The simulations of Jim and his colleagues required the solution of linear systems arising from the approximation of second derivatives in two directions. Gaussian elimination demands about N^3 intermediate storage locations on an $N \times N$ grid, so for N of any reasonable size, their machine did not have enough storage. This led Jim, Peaceman, and Rachford to develop the so-called Alternating Direction Implicit (ADI) method, which could solve finite-difference approximations to parabolic partial differential equations.

As the name implies, ADI is meant to solve auxiliary one-dimensional problems alternately in each direction; each auxiliary problem requires only about N words of intermediate storage, which can be reused. This huge saving in storage brought many problems within computational reach, so the ADI method received much attention. A later realization revealed that this basic technique could be applied to the sum of two nonlinear, nonsmooth operators of the right form (monotone operators). The Douglas-Rachford variant of this method found significance in convex analysis, monotone operators, and most recently in the field of big data. According to the citation database Scopus, in 2015 alone, 230 entries contained the term "Douglas-Rachford," 128 in computer science and 25 in decision sciences.

In 1957, Jim returned to Rice University as an assistant professor of mathematics. He was promoted to full professor in 1961 and named the W.L. Moody Professor in 1964.

In August 1963, Richard Courant chose Jim to attend a meeting on partial differential equations in Novosibirsk, USSR. He was one of 23 American mathematicians to do so, and thus participated in the first large US-USSR mathematics meeting, which was organized with support from the US National Academy of Sciences and the Academy of Sciences of the USSR.

Jim moved to the University of Chicago in 1967, where he turned his attention to the mathematical understanding of the finite element method for partial differential equations. He conducted much of this work with Todd Dupont, who was first Jim's student at Rice and then a colleague at Chicago.

In 1987, Jim became both director of the Center for Applied Mathematics and Purdue's Compere and Marcella Loveless Distinguished Professor of Computational Mathematics, positions he held until his retirement in 2003.

During his distinguished career, Jim wrote more than 200 papers with over 70 co-authors. In addition to his work on ADI, Jim made many other lasting contributions. Among the most important are his pioneering work on interior penalty methods, which grew into the huge field of discontinuous Galerkin methods for elliptic and parabolic problems, the Brezzi-Douglas-Marini mixed finite element methods for second order elliptic problems, and the use of characteristic time-stepping for convection diffusion problems. In recognition of his impact on many fronts, Jim was named both a SIAM and AMS Fellow. He was also the recipient of the Cedric K. Ferguson Medal from the Society of Petroleum Engineers, as well as the Robert Earll McConnell Award from the American Institute of Mining, Metallurgical, and Petroleum Engineers, and a Commemorative Medal from Charles University in Prague.

Jim was a wonderful mentor for young people. He served as an advisor to graduate students and helped shape many professional mathematicians in their early careers. Many of his students and postdoctoral associates are AMS and SIAM Fellows, and leaders in computational science around the world.

Jim is survived by his wife Graça and his sons Jimmy and Craig (himself a professor of mathematics at the University of Wyoming) with his late wife Mary Lou.

Doug Arnold is McKnight Presidential Professor of Mathematics at the University of Minnesota. He was director of the IMA from 2001 to 2008, president of SIAM from 2009 to 2010, and a student of Jim's at the University of Chicago. Greg Buzzard is a professor of mathematics at Purdue University, where he serves as department head. Brad Lucier holds a joint appointment as professor of mathematics and professor of computer science at Purdue University. He is a member of the inaugural class of AMS Fellows.

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SIAM Fellows Honored in White House Ceremony

SIAM Fellows Simon Levin and Michael Artin were among nine renowned researchers and scientists awarded the 2015 National Medal of Science by President Barack Obama during a White House ceremony on May 19th. Levin, the George M. Moffett Professor of Biology at Princeton University, was honored for his work in environmental science and ecological complexity, while Artin, a professor emeritus at the Massachusetts Institute of Technology, received the medal for his accomplishments in modern algebraic geometry. The National Medal of Science is presented annually to those exhibiting both leadership and remarkable contributions to the fields of science and engineering. It is the highest award for scientific achievement in the United States. For a more detailed article, see the May issue of *SIAM News*.

President Barack Obama awards Simon Levin with the 2016 National Medal of Science for his work in ecological complexity. Photo credit: NICHOLAS KAMM/AFP/Getty Images.

Simon Levin at the 2016 National Medals Gala. Photo courtesy of Ryan K. Morris and the National Science and Technology Medals Foundation.

Michael Artin receives the 2016 National Medal of Science for his work in algebraic geometry. Photo courtesy of Ryan K. Morris and the National Science and Technology Medals Foundation.

Faraday Cage

Continued from page 1

And sure enough, Feynman gives an argument that appears to confirm the exponential intuition exactly. He sets up a model of an array of charged wires (see Figure 2) and shows with simple formulas that electrostatic shielding is exponentially effective for just the reason I had imagined: because periodic in one direction means exponential decay at right angles. He writes:

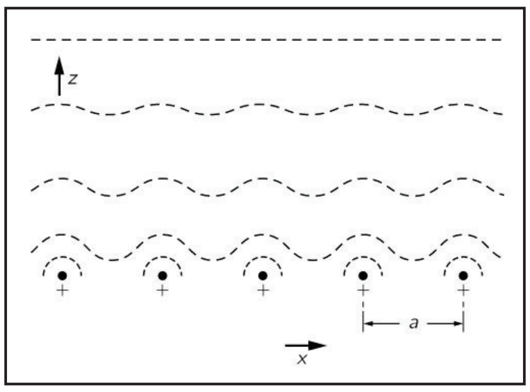

Figure 2. *Equipotential surfaces above a uniform grid of charged wires. Excerpted from The Feynman Lectures on Physics, Volume II by Richard P. Feynman. Copyright © 1964. Available from Basic Books, an imprint of Perseus Books, LLC, a subsidiary of Hachette Book Group, Inc.*

> The method we have just developed can be used to explain why electrostatic shielding by means of a screen is often just as good as with a solid metal sheet. Except within a distance from the screen a few times the spacing of the screen wires, the fields inside a closed screen are zero.

Now Feynman is a god, the ultimate cool genius. It took me months, a year really, to be confident that the great man's analysis of the Faraday cage, and his conclusion of exponential shielding, are completely wrong.

The error is that Feynman's wires have constant *charge*, not constant *voltage*. It's the wrong boundary condition! I think that Feynman, like me and most others beginning to think about this problem, must have assumed that the wires may be taken to have zero radius. The trouble is, a point charge makes sense, but a point voltage does not. (Dirichlet boundary conditions for the Laplace equation can only be applied on sets of positive capacity.) Since the correct boundary condition cannot be applied at points, I'm guessing Feynman reached for one that could, intuiting that it would still catch the essence of the matter. This is a plausible intuition, but it's wrong.

Feynman's calculation is arithmetically correct: an infinite array of equal point charges generates a far field that is exponentially close to uniform. However, this isn't the configuration of Faraday shielding. In fact, the point charge model doesn't include a source to be shielded, or a wavelength. As soon as you realize these things, if you are a numerical analyst like me, you want to compute some solutions of the true PDE problem, like those shown in Figure 3.

The computations reveal two big facts. First of all, *the radius of the wires matters*. As $r \to 0$, the shielding goes away. This, we now realize, must be why your microwave oven door has so much metal in it, and is not just a sheet of glass with a thin wire grid.

Secondly, *the shielding is linear in the gap size, not exponential*. If it were exponential, the field strength inside the cavity would square when you halve ε, the gap between the wires. In fact, it just cuts in half. This may be why your cell phone often works in surprising places, like inside an elevator. The analysis shows that in the limit $r << \varepsilon << 1$, the field scales as $\varepsilon \log r$.

I have started to speak of "we." As the study progressed, I knew I had to get more serious mathematically. This was the beginning of a happy collaboration with Jon Chapman and Dave Hewett, who share a hallway with me at Oxford. As a threesome with varied backgrounds, we talked to more people and learned more. For example, we learned that Maxwell in his treatise from the 1870s considered an infinite array of wires and got the physics right, including the logarithmic dependence on radius [4]. Why has Maxwell's work been forgotten?

Most importantly, Chapman and Hewett developed an analysis in which a wire cage is modeled by a continuum boundary condition. Intuitively, it cannot be necessary to describe your microwave oven door hole-by-hole; there must be a homogenized boundary condition that has the same effect. Using multiple-scales analysis, Chapman and Hewett found this boundary condition, involving the jump in the normal derivative of the potential, which makes precise the idea that a metal screen behaves like a continuous substance that is not quite a metal. A physical interpretation involves energy minimization in surfaces of restricted electric capacity. The figures in [1] show strikingly that the homogenized model and an energy minimization calculation match the true behavior as found in the numerical simulations, and Hewett and Ian Hewitt have gone on to extend the analysis to electromagnetic waves [3].

In closing, I want to reflect on some of the curious twists of this story, first, by mentioning three lessons:

L1. There are gaps out there. If you find something fundamental that nobody seems to have figured out, there's a chance that, in fact, nobody has.

L2. Analogies are powerful. I would never have pursued this problem had I not been determined to understand the mathematical relationship between the Faraday cage and the trapezoidal rule.

L3. Referees can be useful. Thank you, anonymous man or woman who told us the Faraday cage section in our trapezoidal rule manuscript wasn't convincing! We removed those embarrassing pages, and proper understanding came months later.

And then three questions:

Q1. How can arguably the most famous effect in electrical engineering have remained unanalyzed for 180 years?

Q2. How can a big error in the most famous physics textbook ever published have gone unreported since 1964?

Q3. Somebody must design microwave oven doors based on laboratory measurements. Where are these people?

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Nick Trefethen is Professor of Numerical Analysis at the University of Oxford. During 2011-2012 he served as President of SIAM.

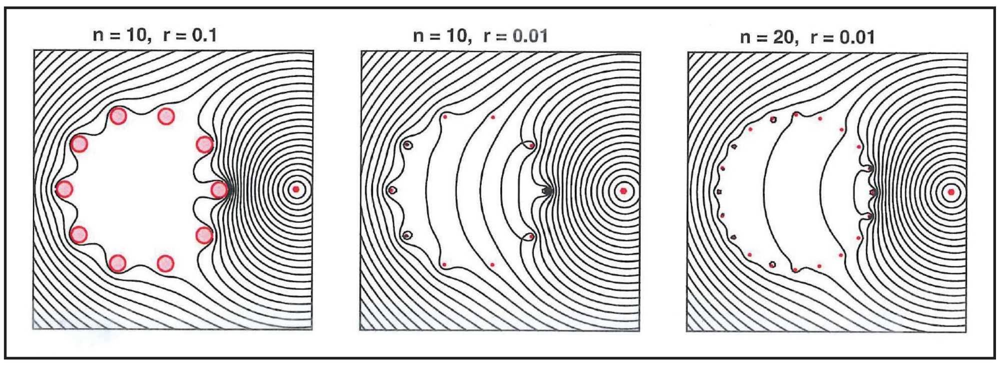

Figure 3. *Computed equipotential lines in a Faraday cage. With thick wires (left), the shielding is good. With thin wires (center), the shielding is weak. As the gap between the wires is reduced (right), the shielding improves only linearly, not exponentially. Figure adapted from [1].*

Fighting Terrorism with Mathematics

By Stephen Tench

In terms of security threats, terrorism is a prominent issue in today's world. Reports of terrorist attacks in varied locations pervade news coverage on an almost daily basis, and the threat doesn't appear to be going away anytime soon. Thus, new insights about terrorist activity and its evolution are greatly needed.

One way to gain such knowledge is via mathematical modelling. This involves taking a quantitative approach by studying datasets of terrorist events with different models and interpreting the results qualitatively for new information. Modelling data is an economically cheap option that lets the data speak for itself, extricating the researchers' emotions from the equation.

Of particular interest in the field of crime and terrorism studies is a specific model known as the Hawkes self-exciting point process. The model's basic principle involves looking at how events in the past can influence those in the future; it was initially developed in the field of seismology to study the relation between earthquakes and their aftershocks. But recent studies have identified similar patterns of main and follow-up events in datasets describing areas such as gang violence [1] and violent civilian deaths in conflicts [2]. For a given set of event times $\{t_i\}$, the equation form of the Hawkes process can be expressed as

$$\lambda(t) = \mu + k_0 \sum_{t>t_i} g(t-t_i),$$

where

$$g(t) = \omega e^{-\omega t}.$$

The three parameters (μ, k_0, ω) must be computed to specify this model. One can obtain this parameter set via maximum likelihood estimation (MLE), which finds the parameters maximising the equation

$$\log L = \sum_i \log(\lambda(t_i)) - \int_0^T \lambda(t)dt.$$

This maximisation can be calculated computationally by several different programs or found via a maximising routine coded in a programming language.

Finding the parameter values of the Hawkes process is the first step of the modelling procedure, but turning those values into something actionable requires a qualitative interpretation. Consider, for example,

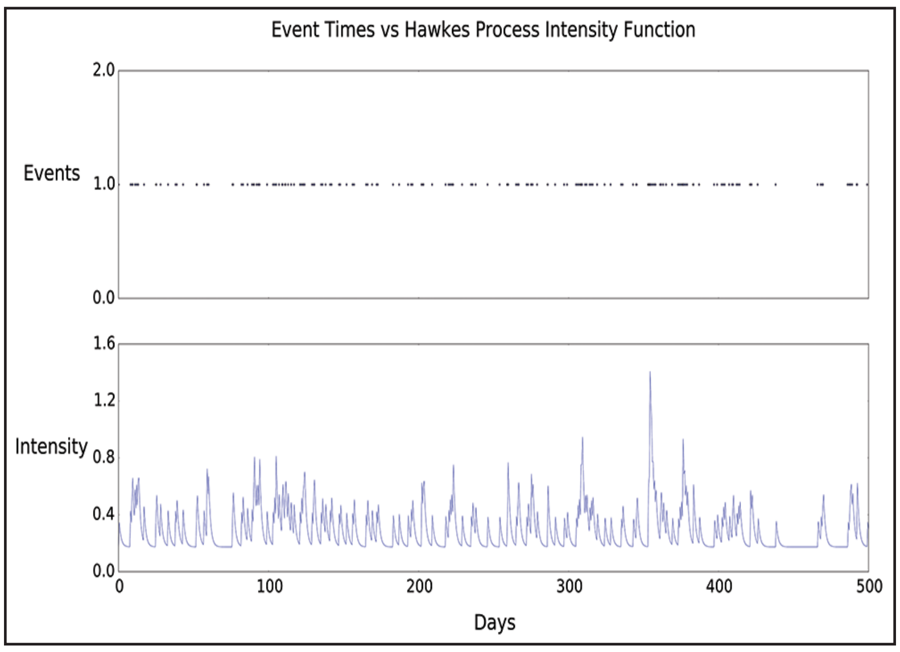

Figure 1. *A graph comparing the times of IED events during "The Troubles" in Northern Ireland (top) with the intensity function of a fitted Hawkes process (bottom). Image credit: Stephen Tench.*

improvised explosive device (IED) attacks carried out by a terrorist group over some period of time, and assume that the Hawkes process has been fitted to the timestamps of the events. The first parameter μ describes a background rate of events. This term is time independent and explains the rate of new IED attacks not related to past occurrences.

See **Terrorism** *on page 5*

Cyber Defense

Continued from page 1

reaching state s' given a starting state of s and an action tuple a chosen by the players

- R is the reward function, where R_i denotes the individual reward function of the i^{th} player
- s^0 is the initial system state.

POSGs are very general formulations, and thus become intractable. Identifying joint policies (that map from observation history and system states to actions) of players forming a *Nash* equilibrium is the decision-making goal. Under equilibrium conditions, no player gains by unilaterally changing his/her policy. Typically, these problems may be categorized into the following two categories: (1) *Planning* – where complete specification of the cyber-system environment is known and optimal joint policies are desired; and (2) *Learning* – where players need to interact with the cyber-system environment to learn about the system and each other, while updating their policies based on these interactions. Solving such problems involves iteratively finding policies that achieve high rewards, on average, over the long run. A POSG's typical objective is to maximize the expected cumulative value (i.e. a function of payoffs) for each player [8]:

$$V_{p_1}(\boldsymbol{\pi}) = \mathbf{E}\left[\sum_t R_{p_1}(s,\boldsymbol{a}) \mid \boldsymbol{\pi}, b^0\right],$$

where:

- $V_{p_1}(\boldsymbol{\pi})$ is the value function for the first player, i.e p_1, associated with a tuple of policies $\boldsymbol{\pi}$
- $R_{p_1}(s,\boldsymbol{a})$ is the reward over time t for the first player in state s for a joint action $\boldsymbol{a}$
- b^0 is the initial system state distribution.

Researchers have proposed various approaches for solving POSGs, including dynamic programming with iterative elimination of weakly dominated strategies [1] and transformations of POSGs to a series of Bayesian games (with incomplete information about other player payoffs) that have properties similar to the original POSG [7].

In realistic cybersecurity settings, insufficient and uncertain information about system properties and attacker goals may be available to a defender. A recent approach proposed a probabilistic framework for quantifying attacker payoff uncertainty within a stochastic game setup that accounts for dependencies among a cyber system's state, attacker type, player actions, and state transitions [2-4]. This approach adopts conditional probabilistic reasoning to characterize dependencies among these modeling elements. The application of probabilistic theories (such as total probability theorem) and functions (such as marginal and conditional) may then lead to simulation of attacker payoff probability distributions under various system states and operational actions. The framework is flexible and accounts for multiple types of uncertainties—such as *aleatory* (statistical variability) and *epistemic* (insufficient information)—in attacker payoffs within an integrated probabilistic framework (see Figure 2).

Mathematically, as presented in [2-4], the discrete version of the marginal probability of attacker payoff utility (involving notions of time and cost), $Pr(u_{p_1})$, is:

$$Pr(u_{p_1}) = \sum_i \sum_j \sum_k \sum_l Pr$$

$$(u_{p_1} \mid s'_l, a_k, \alpha_j, s_i) \cdot Pr(s'_l \mid a_k, \alpha_j, s_i) \cdot$$

$$Pr(a_k \mid \alpha_j, s_i) \cdot Pr(\alpha_j \mid s_i) \cdot Pr(s_i)$$

where:

- $Pr(s_i)$ is the initial (prior) probability of system states s_i
- $Pr(\alpha_j \mid s_i)$ is the conditional probability of attacker type α_j for a given system state
- $Pr(a_k \mid \alpha_j, s_i)$ is the conditional probability of attacker and defender action combinations a_k for a given attacker type and initial system state
- $Pr(s'_l \mid a_k, \alpha_j, s_i)$ is the conditional probability of system state transition from s_i to s'_l for given action combinations, attacker type, and initial system state
- $Pr(u_{p_1} \mid s'_l, a_k, \alpha_j, s_i)$ is the conditional probability of attacker payoff utility.

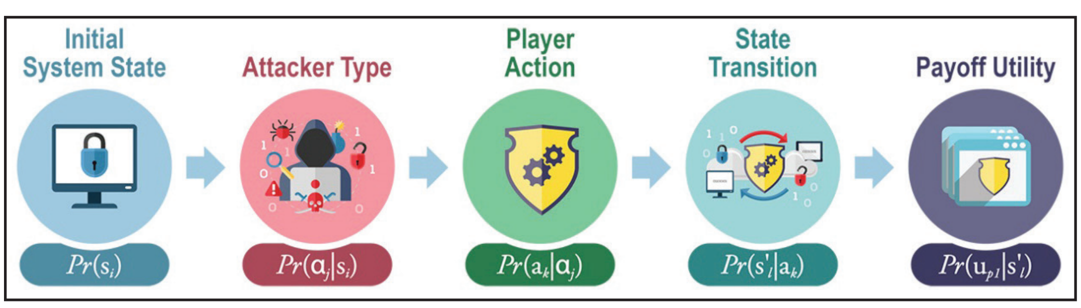

Figure 2. *Probabilistic attacker payoff framework. Figure created by authors.*

Statistical probability distributions typically address aleatory uncertainty, while mathematical intervals address epistemic uncertainty. Depending on these representations, uncertainty propagation methods may include Monte Carlo sampling analysis, interval analysis, and/or probability bounds analysis. Application of uncertainty propagation techniques generates probability distributions, intervals, or intervals of distributions associated with attacker payoffs that serve as critical inputs within stochastic cybersecurity games. These probabilities may be informed and updated based on empirical event and system data, simulation experiments, and/or informed judgments of subject matter experts.

The game-theoretic and uncertainty quantification methods outlined above model the dynamics between cyber attackers and defenders, and have real-world potential to address proactive resource allocation challenges within resilient cyber systems. However, challenges to their implementation exist, including real-time, data-driven system state determination, "realistic" payoff uncertainty representations, and scalability of uncertainty propagation and stochastic game algorithms. Nevertheless, these approaches represent steps toward practical uses of game theory as an effective tool for rigorous cyber defense analysis.

Acknowledgments

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The FEniCS Project

Towards Automated Scientific Computing

SOFTWARE AND PROGRAMMING

By Robert C. Kirby

The numerical approximation of differential equations to solve problems in science and engineering was a driving force in the development of early computers. Classical and modern techniques alike work to produce solutions by the repetition of vast amounts of arithmetic, but performing this arithmetic by human effort is a tedious and error-prone task. The world now takes for granted the revolution begun by Alan Turing, John von Neumann, and other pioneers of computer science; machines can carry out simple computations with speed and accuracy far surpassing even a team of humans, the original parallel computers [2].

However, programming these early computers in machine code proved to be tedious and error-prone as well. This led to automated programming by assemblers, and later by FORTRAN compilers. In the 1950s, a machine-independent language with high-level data structures, such as arrays, offered scientists new freedom. Ongoing improvements in programming languages and compilers, machine architecture, and algorithms led scientists to tackle bigger challenges that further stretched boundaries in all these areas.

```
from dolfin import *
mesh = UnitSquare(32, 32)
V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")
a = dot(grad(u), grad(v))*dx
L = f*v*dx
bc = DirichletBC(V, 0.0, DomainBoundary())
u = Function(V)
solve(a == L, u, bc)
plot(u)
```

Figure 1. *Complete working Python script for solving Poisson's equation with FEniCS.*

Moving through several generations of progress to the present day, one may rightly say that implementing algorithms for inverse problems dealing with finite element discretizations of coupled systems of nonlinear partial differential equations (PDEs) in unstructured three-dimensional geometry is a tedious and error-prone task. Can this be helped by computers?

The resolution of our (recursive) conundrum begs the development of even higher-level descriptions of our tasks, along with tools for reducing these descriptions into lower-level code. Much like how FORTRAN compilers turn mathematical formulas and array syntax into assembly code, higher-level, domain-specific languages can describe higher-level mathematical structures. MATLAB is a quintessential domain-specific language, concealing implementation details of linear algebra behind a custom, operator-overloaded syntax. Similarly, one could "embed" a domain-specific language in existing high-level languages such as C++ or Python. For example, looking at expression template libraries such as Blitz++[1] for use of C++ facilities can help implement linear algebra syntax.

Several modern software projects seek to bridge the even greater distance between machines and PDEs via domain-specific languages. A leading example of this approach is the FEniCS project.[2] For example, the "Hello World" program in FEniCS offers a complete numerical solution of the weak form of Poisson's equation (see Figure 1).

Many things happen when this deceptively-simple Python program is run. The expression **inner(grad(u), grad(v))*dx** is a symbolic representation of the weak form of the Laplace operator. When the solve function is called, a special-purpose just-in-time compiler generates low-level code based on this expression, links it into the environment, builds the sparse stiffness matrix, and invokes a linear solver. The matrix format and solver library support many options, including PETSc and Trilinos. With a bit more code, it is possible for users to configure exactly how these libraries are utilized.

FEniCS was created in 2003 to achieve automated computational mathematical modeling [1]. The two original FEniCS components were DOLFIN, a C++ library originally written by Anders Logg and Johan Hoffman for describing meshes, solvers, and bilinear forms, and FIAT, a novel if somewhat esoteric Python package for automatically constructing high-order bases for Raviart-Thomas and other complicated finite element spaces. Ridgway Scott brought prior experience to the project with a Scheme-based system called Analysa [2].

FEniCS progressed rapidly over the next several years. The FEniCS Form Compiler (FFC) [3] enabled the generation of low-level code for element matrix and vector construction linkable against DOLFIN. Later, its input language was lifted out and extended to become the Unified Form Language (UFL). Garth Wells joined the project early on, expanding FEniCS' boundaries by tackling new problems and building new capabilities as needed. When Simula Research Laboratory received Norwegian Center of Excellence status and built the Center for Biomedical Computing, FEniCS advanced to the forefront of the Center's modeling efforts in problems related to blood flow, aneurisms, and cardiac mechanics and electrophysiology.

While FEniCS is now a global, visible project, we cannot overlook the work of similar projects. Even older than FEniCS is Sundance [4], a C++ library developed by Kevin Long that provides a high-level description of variational forms and their symbolic derivatives. Rather than generating customized code, it relies on a highly-optimized interpreter for bilinear forms. In contrast to these embedded languages, the FreeFEM++ project[3] of comparable vintage provides an entire domain-specific language and user environment. This offers a fairly complete user environment (more like MATLAB), but requires the recreation of many language features, making interaction with external codes more challenging. Deal.II,[4] a more traditional C++ library from the 1990s that won the 2007 Wilkinson Prize, delivers a rich set of finite element classes with special attention to adaptive methods but lacks an automation layer present in these other projects. The Deal.II community has produced an incredible set of documentation and tutorials for learning finite element methods through the software's many features.

Also worth noting is a new generation of projects inspired by the vision and success of those before them. Employing FEniCS as a point of departure, Firedrake[5] attempts to provide compatible and enhanced functionality, engineered accordingly to different design principles. A notable result of the Firedrake/FEniCS collaboration is the dolfin-adjoint project,[6] featuring lead developers of both software. Using UFL, the team, which won the 2015 Wilkinson prize,[7] is able to automatically derive and discretize adjoints and other operations required for inverse problems.

It is an exciting time for numerical software. Much as early computers and FORTRAN lifted us above the slide rule and assembly code respectively, projects like FEniCS allow us to think of variational forms rather than arrays. Enduring challenges such as uncertainty quantification, accelerator-based architectures, and even more challenging applications will continue to spur future developments. The FEniCS project welcomes new users and developers and invites users, developers, colleagues, competitors, and other interested parties to its annual meeting.

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[1] https://sourceforge.net/projects/blitz/

[2] http://fenicsproject.org/

[3] http://www.freefem.org

[4] https://www.dealii.org

[5] http://www.firedrakeproject.org

[6] http://www.dolfin-adjoint.org/en/latest/

[7] http://fenicsproject.org/featured/2015/wilkinson_prize.html

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Terrorism

Continued from page 4

A temporary jump in the rate of events might follow an IED attack. For example, a successful attack may encourage follow-up attacks or prompt some kind of counterterrorism response from security forces, leading to a tit for tat escalation in violence. In these or similar cases, the parameter k_0 captures the jump in the rate of events. Finally, since it is unrealistic for events to remain high indefinitely—the resources to make IEDs will start running out, for example—there must be a term that reduces the influence of past events in the very-distant future. This idea is contained in the response function g, which is an exponential decay; the third parameter ω controls the speed of decrease.

The following transformation can be used to test whether the Hawkes process is a good model for the event times:

$$\tau_i = \int_0^{t_i} \lambda(t)dt.$$

Theory states that if the Hawkes process is a good-fitting model, the residuals $\{\tau_i\}$ should be a Poisson process with unit rate. The Kolmogorov-Smirnov test can assess this assumption.

The Hawkes process was used to study terrorism in Northern Ireland, particularly the period known as "The Troubles" occurring from 1969-1998 [3]. During this time, a group called the Provisional Irish Republican Army (PIRA) fought against British rule in Northern Ireland. One of the group's main weapons was IED attacks, and members went to great lengths to develop and enhance their IED arsenal over the course of their campaign. The PIRA evolved both on the tactical weapon side and as an organization, and sociological research describes the group's change in five phases:

- *Phase 1* (1969-1976): The PIRA maintained a military structure organised in terms of brigades, battalions, etc.
- *Phase 2* (1977-1980): The Phase 1 structure allowed heavy infiltration by security forces, so the PIRA moved to a cell-based structure consisting of many small units.
- *Phase 3* (1981-1989): The PIRA launched a big wave of IED attacks with the hope of finally securing a victory against the British Armed Forces.
- *Phase 4* (1990-1994): PIRA leadership began secret negotiations with the British Government to end hostilities.
- *Phase 5* (1995-1998): Negotiations for Phase 4 were made public and the conflict ended for many with the ratification of the "Good Friday" agreement.

This description of the phases of the PIRA comes from sociological and historical accounts. The start and end times of each phase were defined qualitatively for convenience. Having a somewhat arbitrary cut-off between phases may be unrealistic, however, as factors influencing IED attacks could extend further back in time. Such a problem lends itself naturally to the Hawkes process, which captures historical dependencies. This is precisely the outcome of [3], which used the aforementioned techniques to find mathematical boundaries for the phases of the PIRA. Figure 1 (on page 4) demonstrates graphically the relationship between IED event times and a fitted Hawkes process intensity function.

The mathematically-determined change points for the organisation were found to be fairly consistent with the sociological boundaries for phases 2 and 3. However, the mathematical boundaries differed considerably from the sociological ones in phases 4 and 5. The new boundaries found by the Hawkes process raise interesting questions about the potential for novel sociological research to reconcile the qualitative theory of the PIRA with these quantitative results. This last point illustrates the wide applicability of the Hawkes process, which can act as a bridge between the tools and techniques of mathematics and other applied research areas. Please see the online version of this article[1] for a video that further discusses the results obtained in [3].

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[1] sinews.siam.org

Nautical Numbers

The Influence of Nathaniel Bowditch

BOOK REVIEW

By Ernest Davis

Nathaniel Bowditch and the Power of Numbers: How a Nineteenth Century Man of Business, Science, and the Sea Changed American Life. *By Tamara Plakins Thornton, University of North Carolina Press, Chapel Hill, NC, 2016, 416 pages, $35.00.*

In chapter 35 of *Moby Dick*, Herman Melville admonishes the ship owners of Nantucket, "Beware of enlisting in your vigilant fisheries any lad . . . who offers to ship with the Phaedon instead of Bowditch in his head."

The *Phaedon*, now usually called the *Phaedo* in English, is the most moving of Plato's dialogues; it is set on the last day of Socrates' life and ends with his execution. "Bowditch" refers to *The New American Practical Navigator* by Nathaniel Bowditch. And Nathaniel Bowditch (1773-1838), the subject of a new, fascinating, and scholarly biography by social historian Tamara Thornton, was a remarkable man.

Bowditch was born in 1773 to Mary Ingersoll and Habakkuk Bowditch in Salem, Massachusetts, the second largest port in New England and one of the largest cities in America. Because of the sea trade, it was rather cosmopolitan by the standards of time and place.

Bowditch's mathematical gifts were apparent from a young age, and he was soon widely known as a prodigy, able to quickly solve problems and puzzles that his elders found difficult. By extraordinary chance, a library of over 200 scientific and mathematical volumes came to Salem (as loot captured by a Patriot privateer). The young Bowditch worked through this library intensely, and taught himself Latin in order to read Newton's *Principia*.

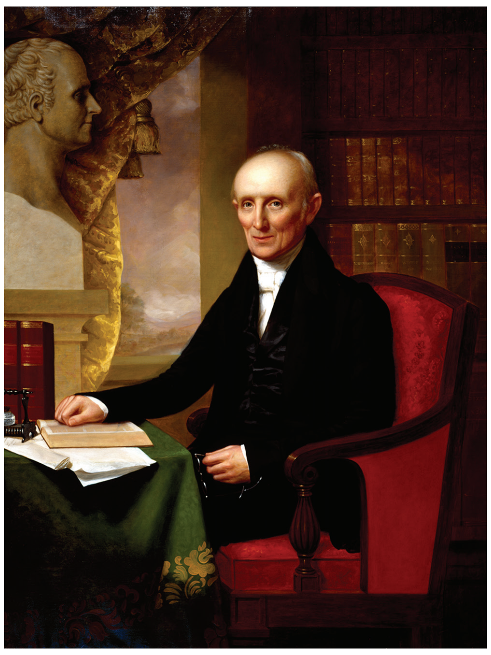
Between 1795 and 1803, Bowditch made five sea voyages to destinations as remote as the Philippines and Réunion Island, first as the officer in charge of the cargo, and on his final voyage as master and part owner. Then he got married and began a career in business.

Bowditch had two great scientific accomplishments. The first was the one that got him into *Moby Dick*, his *New American Practical Navigator*, first published in 1802. The publication's key element was the presence of tables for the lunar method of computing longitude. At sea, latitude is easy to compute; it's the angle of the North Star above the horizon. Longitude, on the other hand, is very difficult; as Earth rotates, the position of the stars at a given local time and latitude remains the same at all longitudes. However, if you know the time at some fixed location—Greenwich, for example—you can compute the local time by astronomical observation. The difference between your time and Greenwich time in minutes is four times your longitude in degrees. Thus, if you know Greenwich time within 4 minutes, you can determine your longitude to within 1 degree.

So how can you determine the time in Greenwich? The obvious solution is to carry a clock that reliably keeps Greenwich time. John Harrison's marine chronometer accomplished this in the mid-1700s. That would have been the end of the question, except that chronometers were extremely expensive; most ships could not afford one until the 1830s.

In the lunar method, the moon is the "clock." In a very crude first approximation, the position of the moon against the fixed stars is the same everywhere on Earth; however, you have to correct for the parallax and atmospheric refraction. Bowditch's tables precalculated all this, allowing a mariner to determine his longitude by measuring the angle between the moon and specified stars.

Contrary to popular belief, Bowditch did not invent the lunar method, and was not even the first to publish this kind of table. The standard of his day was John Hamilton Moore's *New Practical Navigator*, first published in 1772 and now in its twelfth edition. However, Bowditch corrected thousands of Moore's errors and incorporated all kinds of additional useful information, including instruction in the basic mathematics needed to carry out the lunar method. Bowditch's book soon became *the* standard reference for navigation. The book is supposedly still carried aboard every commissioned U.S. Naval vessel, though presumably at this point for tradition rather than use.[1]



A portrait of Nathaniel Bowditch by Charles Osgood. This photo is featured on the book's jacket. In the top left, the marble bust of Pierre Laplace that his widow gifted to Bowditch is visible. © 2006 Peabody Essex Museum. Photo by Mark Sexton.

Bowditch's second major mathematical contribution was a translation and annotation of the first four volumes of Pierre Laplace's five-volume *magnum opus*, *Mécanique Céleste*, the definitive analysis of the solar system in terms of Newtonian gravity. Bowditch began his study of Laplace in 1803 and ultimately published his 3,000 page translation between 1828 and 1839, in four volumes at his own enormous expense. The work was a careful exposition of the theory for the beginning mathematical student, and incorporated an extensive discussion of the relation between Laplace's analysis and the later, more mathematically sophisticated studies of problems by mathematicians such as Carl Friedrich Gauss and Heinrich Wilhelm Olbers.

Bowditch's translation earned him an international reputation. The likes of Sylvestre Lacroix, Adrien-Marie Legendre, and John Herschel showered him with plaudits, and Charles Babbage engaged him in an extended correspondence to discuss his ideas for the Analytical Engine. Most cherished of all, Laplace's widow sent Bowditch a large marble bust of her former husband (see photo).

Bowditch was also a very successful man of affairs, and Thornton argues vigorously that his contributions in this sphere were ultimately more significant than his mathematical work. He imposed a strict mathematical order—then a radical innovation—on the doings of the life insurance company that he headed: records were carefully kept and filed, business was carried out on preprinted forms, and bills were collected on time with penalties for late payment.

The finances of Harvard College at the time were scandalously badly managed. Bowditch was among the leading figures involved in placing the finances on sound footing. His reforms were proper and necessary, but his methods were somewhat ruthless; the well-respected incompetents Bowditch replaced were not allowed to back out gracefully and save face.

Thornton's account of Bowditch's role in the aforementioned business and financial activities is, I gather, the major original historical contribution of her biography. She has researched these deeply, and makes them as interesting as the details of business transactions from two centuries ago can be.

One of the most interesting aspects of Thornton's book is the account of Bowditch's slightly uneasy social position vis-á-vis the Boston Brahmins of his time. (Oliver Wendell Holmes Sr. didn't coin the phrase until three decades later, but the caste was very much in place at the time.) Bowditch was universally respected for his mathematical gifts, business ability, and absolute integrity. But he was not truly of the elite: his family, while related to the top families, was not quite one of them. He had not gone to college, his Latin was just adequate for reading science, and he knew no Greek. Moreover, Bostonians regarded the people of Salem as crass philistines, interested only in making money, with no regard for the Higher Things. Harvard offered Bowditch a professorship, but he declined it, partly because it would have involved a substantial loss of income, but partly, Thornton speculates, because he may have felt inferior to his colleagues with classical educations.

The strangest part of Bowditch's life is the trajectory of his fame. I had never heard of him until I read Thornton's biography—I presume that the same is true of most readers of *SIAM News*—and I am entirely confident that it is true of most readers of *Moby Dick*. But in his own time, and in Melville's time, and for a century after Melville, Bowditch was extremely famous. Thornton writes, "Thomas Jefferson praised him as 'a meteor of the hemisphere' in which he lived. James Madison paid homage to his 'distinguished genius.' His countrymen compared him to Benjamin Franklin and even Isaac Newton, and his name became synonymous with genius, not unlike Einstein's is today."

Numerous biographical accounts of Bowditch were written over the years, and according to Thornton, the level of hero worship steadily increased even while accuracy diminished. Most remarkably, a children's biography, *Carry On, Mr. Bowditch,* by Jean Lee Latham and with charming woodcut illustrations, won the prestigious Newbery Medal in 1955.

In his own time and for many years afterward, the United States clearly and urgently needed a native-born mathematician hero, so Bowditch was venerated. Now that need is apparently no longer urgent, so he is largely forgotten. But Thornton's account of Bowditch's life still sheds a striking light on the science and activities of the early United States, and therefore is an important contribution to our historical understanding.

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[1] This statement is not in Thornton's book; however it is repeated on many websites, including here: http://msi.nga.mil/MSISiteContent/StaticFiles/NAV_PUBS/UNTM/201541/Important_Info.pdf

MATLAB Creator Cleve Moler Visits Texas A&M University

By Sourav Dutta

The Texas A&M University Chapter of SIAM had an impressive inaugural year. This April, with the enthusiastic support of the Department of Mathematics, the chapter took on the ambitious task of inviting Dr. Cleve Moler, the creator of MATLAB, to Texas A&M for the first time. Moler is the original author of MATLAB, and one of the founders of MathWorks. He is currently chairman and chief scientist of the company, as well as a member of the National Academy of Engineering and recipient of the 2012 IEEE Computer Pioneer Award and the 2014 IEEE John von Neumann Medal. Moler is also a past president of SIAM. In honor of this visit, the SIAM student chapter worked hard to organize a workshop showcasing the widespread application of MATLAB in research at Texas A&M, followed by a public lecture by Moler at the Hawking Auditorium of the Mitchell Physics Institute. Naturally, it was the most anticipated and biggest chapter event of the year.

Moler was hosted by his longtime friend and colleague, Dr. Tim Davis (Department of Computer Science & Engineering). The day began with a 1.5 hour campus tour in a golf cart, during which three excited students accompanied Moler. Kevin Andrews, Texas A&M's very own campus connoisseur and Ph.D. student in Agricultural Leadership, Education, and Communications, led the tour. Moler greatly enjoyed the tour and the brief anecdotes and campus trivia offered by Andrews, repeatedly expressing his admiration for the rich tradition and spirit of the University.

Dr. Peter Howard (Graduate Director of Mathematics) kicked off the "Workshop on Scientific Computing with MATLAB at Texas A&M" with a warm welcome to Moler. The workshop featured a diverse set of presentations on topics such as biomedical imaging research, rehabilitation robotics, and cancer therapy design. Talks were divided into two sessions, both of which filled rooms to capacity. The highlight of the workshop was during a talk by Dr. Simon Foucart (Department of Mathematics) on compressed sensing research. An impressed Moler stated, "That was one good reason to come to Texas today. It was well worth the effort."

Cleve Moler and students at Texas A&M University tour the campus during his visit. Photo credit: Sourav Dutta.

Finally, it was time for the main event: Moler's talk on the "Evolution of MATLAB." The Department of Physics and Astronomy had graciously organized a reception before the talk. As was expected, Moler spoke to a packed auditorium of professors, postdoctoral fellows, and graduate and undergraduate students. He began by going back to his college days at the California Institute of Technology, when Caltech's computer was one of only a couple dozen computers in Southern California. Moler later attended graduate school at Stanford University and studied under George Forsythe, the founder of Stanford's computer science program, which went on to pioneer the industry. It all began with the enterprising and hardworking students who tinkered with *Space War*, the world's first video game, to blow off steam. Moler recounted an attempt by Stanford and the Massachusetts Institute of Technology to hold a *Space War* competition over the phone, which was unsuccessful because of the slow connection. During his time at Los Alamos, Moler created a film to demonstrate the efficient computation of singular value decomposition of matrices. The audience gasped in wonder at a clip showing a part of this film being used on one of the many screens of the SS Enterprise in the original 1979 *Star Trek* movie.

Moler originally wrote MATLAB as a side project. His numerical analysis students at Stanford in 1979 were not particularly impressed with it; however, the program somehow made its way to Jack Little, the current president and co-founder of MathWorks. "Three years later he came up to me and said he wanted to commercialize MATLAB," Moler reminisced. Little quit his job, moved up to the hills behind Stanford, and co-founded MathWorks in 1984. Moler explained that for the first few years, MATLAB grew steadily at a rate of 2^n employees for the n^{th} year; currently it has 3,500 employees worldwide. The software's diverse areas of application include biology in RNA sequencing, Wall Street finance calculations, medical devices like hearing aids, electronics and circuit design, control systems for things like quadcopters, and computing systems in cars like the Chevrolet Volt. Moler signed off to thunderous applause and a standing ovation from the 200-strong audience. He tirelessly posed for hundreds of photographs throughout the day, greeting everyone with a big smile and offering warm words of encouragement to the students. Overall, it was an unforgettable experience for the students to interact with an iconic figure in the field of mathematics and computer science; afterwards, many expressed sincere interest in getting involved with the student chapter. Undoubtedly, it was the best possible way the newly-formed Texas A&M University Chapter of SIAM could have hoped to end the academic year.

Sourav Dutta is a Ph.D. student in the Department of Mathematics at Texas A&M University. He was the president of the SIAM student chapter during the 2015-16 school year, and is currently an active member.

RWTH Aachen University's SIAM Student Chapter Hosts Jeopardy Night

The RWTH Aachen University Chapter of SIAM (Aachen, Germany) held its first Science Jeopardy Night in April. In the first round, teams of four candidates were quizzed on questions from more or less scientific categories like "Algorithms," "Plots," and "May the force be with you." When no candidate could provide the correct answer, somebody from the audience could partake and win a special prize.

After completion of the initial rounds, the remaining competitors participated in the final round. In the end, SIAM student chapter officer Alexander Jaust emerged the clear victor. All participants won prizes, ranging from SIAM mugs and pens to MathWorks shirts and backpacks. Local company d-fine graciously sponsored the evening, which included snacks and beverages for everybody. The event was a great success and will surely be repeated soon! For those interested in learning more, the LaTeX template for the event can be found at GitHub under the project name "SIAMSC_jeopardy." To read more about activities at the RWTH Aachen University Chapter of SIAM, visit the chapter blog.[1]

– Julian Köllermeier, for RWTH Aachen University Chapter of SIAM

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[1] http://blog.rwth-aachen.de/siamsc/

Contestants hard at work at RWTH Aachen University Chapter of SIAM's Science Jeopardy Night. Photo credit: Alexander Jaust.

Students enjoy Science Jeopardy Night organized by the RWTH Aachen University Chapter of SIAM. Photo credit: Alexander Jaust.

The First Central Valley Regional SIAM Student Chapter Conference

The SIAM student chapter at the University of California, Merced hosted "The First Central Valley Regional SIAM Student Chapter Conference" at the end of April. The conference attracted approximately 50 participants from UC Merced and other local universities. The one-day event featured a keynote talk on "Large-scale Bayesian Inverse Problems and Applications to Flow of the Antarctic Ice Sheet" by professor Omar Ghattas (The University of Texas at Austin), as well as a poster session showcasing graduate and undergraduate research from UC Merced and California State University, Fresno.

UC Merced SIAM student chapter president Julia Clark, secretary Jessica Taylor, treasurer Eric Roberts, undergraduate representative Derick Garcia, and faculty advisor Noemi Petra collaborated extremely well and made this event a great success.

Another chapter event of note is the weekly "Some Applied Math for People to LEarn" (SAMPLE) seminar series, where students from UC Merced and UC Davis presented their research.

– University of California, Merced SIAM Student Chapter

From left to right: Keynote speaker Omar Ghattas (The University of Texas at Austin), the University of California, Merced SIAM Student Chapter Certificate of Recognition recipient Julia Clark, and SIAM student chapter faculty advisor Noemi Petra. Photo credit: Blake Williams, UC Merced University Communications.

UC Merced graduate student Johannes Brust presents his poster to fellow grad student Madushani Rajapaksha. Photo credit: Blake Williams, UC Merced University Communications.

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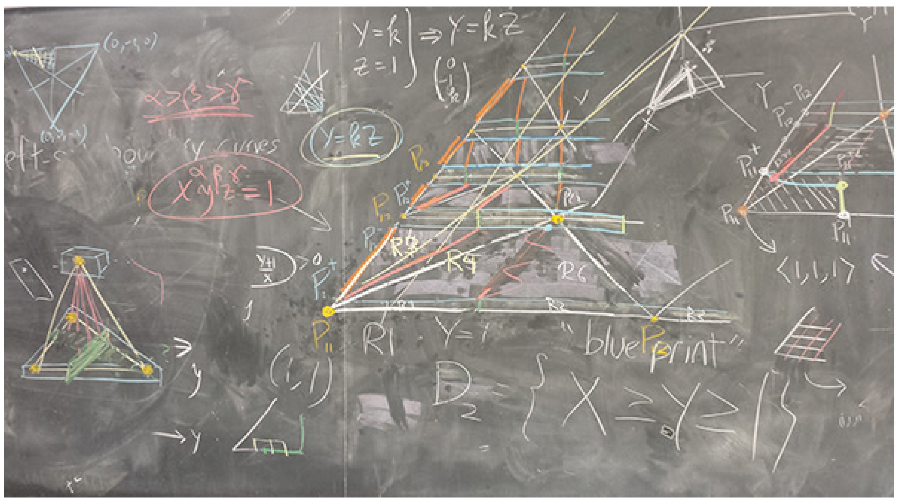
Discussing the Proof of the Global Attractor Conjecture

By Elizabeth Gross, Matthew Johnston, and Nicolette Meshkat

The Global Attractor Conjecture (GAC) concerns an important class of ordinary differential equation models known as toric dynamical systems—originally called complex-balanced systems—that arise from chemical kinetics. Ludwig Boltzmann first introduced the complex-balanced condition [3] for modeling collisions in kinetic gas theory; Fritz Horn and Roy Jackson later applied the condition to chemical kinetics in their 1972 seminal paper [9], which is frequently credited with founding the research area popularly known as Chemical Reaction Network Theory. This area attempts to establish connections between the network properties of the reaction graph and the permissible behaviors of the resulting dynamical system. Related research has seen a swell of activity since the rise of systems biology over the last two decades, and has helped establish structural motifs underlying cellular regulation [1]. The adjective "toric" was proposed in [5] to underline the tight connection to the algebraic study of toric varieties.

San José State University (SJSU), with cooperation from Santa Clara University and the University of California, Berkeley, hosted a weekend workshop in March about Gheorghe Craciun's recent proof of the GAC [4]. Intensive reading seminars at UC Berkeley and SJSU, as well as a series of YouTube videos by Craciun, preceded the workshop, which included participants from nearly a dozen universities around the world.



A blueprint for a zero-separating surface, an important construction in Craciun's proof of the Global Attractor Conjecture. Photo credit: Nida Kazi Obatake.

A polynomial dynamical system arising from a reaction network (i.e. a weighted, directed graph) is said to be a *toric dynamical system* if the net inflow and net outflow across each vertex are equal at steady state. For example, consider the following reaction network:

$$2X_1 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} X_1 + X_2 \underset{k_4}{\overset{k_3}{\rightleftharpoons}} 2X_2.$$

Under mass-action kinetics, we can associate this network to the following system of polynomial differential equations:

$$\begin{aligned}\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} &= k_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} x_1^2 + k_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} x_1 x_2 \\ &+ k_3 \begin{bmatrix} -1 \\ 1 \end{bmatrix} x_1 x_2 + k_4 \begin{bmatrix} 1 \\ -1 \end{bmatrix} x_2^2. \end{aligned} \tag{1}$$

In order for this system to be a toric dynamical system, it must satisfy the steady state conditions implied by (1) as well as $k_2 x_1 x_2 = k_1 x_1^2$ (vertex $2X_1$), $k_1 x_1^2 + k_4 x_2^2 = (k_2 + k_3) x_1 x_2$ (vertex $X_1 + X_2$), and $k_3 x_1 x_2 = k_4 x_2^2$ (vertex $2X_2$). Given this network-balancing property, the authors of [9] were able to prove the following powerful dynamical result.

According to the theorem, *"Within every strictly positive stoichiometric compatibility class (linear invariant space) of a toric dynamical system, there exists exactly one strictly positive steady state; this steady state is locally asymptotically stable with respect to its compatibility class"* [9].

However, the theorem only guarantees local stability of the steady state, so one might naturally wonder whether this stability may be extended globally throughout the compatibility class. The GAC offers the resolution to this question:

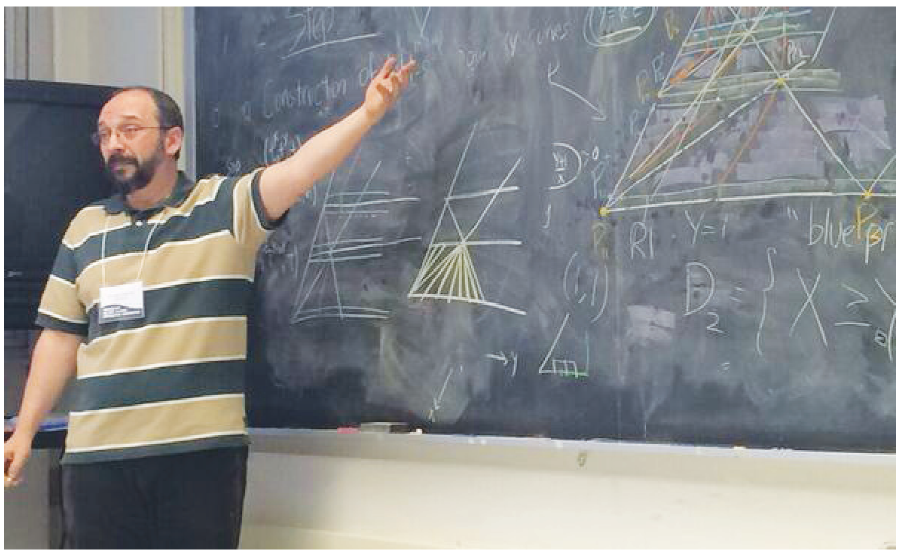
The unique positive steady state $\mathbf{x}^* \in \mathbb{R}^m_{>0}$ *in every stoichiometric compatibility class of a toric dynamical system is the global attractor for its stoichiometric compatibility class.*

The gap between local and global stability is more subtle than one might initially realize. The proof contained in [9] makes use of a Lyapunov function, which is strictly convex on the strictly positive orthant $\mathbb{R}^m_{>0}$. All trajectories consequently descend for all times through the contours of the Lyapunov function toward the positive steady state. This led the authors of the original paper [9] to errantly conclude that they had proved not only the above theorem, but the GAC as well! They only later realized that, since the Lyapunov function is bounded along $\partial\mathbb{R}^m_{>0}$, trajectories could possibly approach $\partial\mathbb{R}^m_{>0}$ instead of the positive steady state. The claim was consequently retracted in 1974 in [8], which finally correctly states the conjecture as an open problem.

The GAC has experienced a resurgence of interest since the 2000s. Published papers affirm the GAC in a number of special cases, including 2-dimensional systems [5], networks with a single linkage class [2], 3-dimensional systems [6], systems with 3-dimensional stoichiometric subspaces [10], and strongly endotactic networks [7]. In 2015, Gheorghe Craciun (University of Wisconsin-Madison) proposed a general proof, valid in all dimensions and without special conditions.

The purpose of the Global Attractor Conjecture Workshop was to bring together mathematicians to discuss the proof in detail and learn more about the ideas surrounding this significant breakthrough. The workshop began with an opening talk by Craciun. For the remainder of the weekend, participants gave presentations on the sections of [4], working through the proof step-by-step. The 41-page manuscript [4] introduces and utilizes sophisticated ideas from dynamical systems, polyhedral geometry, differential inclusions, and other areas. A main idea involves embedding toric dynamical systems into toric differential inclusions. Polyhedral geometry also plays an interesting role. The necessary differential inclusions are constructed by looking at the dual cones in the polyhedral fan formed by the reaction vectors. This application of polyhedral geometry allows the proof to very naturally extend to over four dimensions.

The GAC is related to several open problems about persistence and permanence of dynamical systems on the positive orthant. Most notably, the so-called Permanence Conjecture involves a much larger class of dynamical systems on the positive orthant. The GAC also has strong connections to open problems in thermodynamics and statistical mechanics, and in particular to global convergence problems derived from discrete approximations of the Boltzmann equation. So, while Craciun's proof may bookend decades of work on global dynamics in chemical reaction network theory, the ideas of his proof will serve as the opening pages to new volumes of exploration.



Gheorghe Craciun discusses his construction of zero-separating surfaces, which are hypersurfaces in the positive orthant that play a key role in his proof of the Global Attractor Conjecture. Photo credit: Nida Kazi Obatake.

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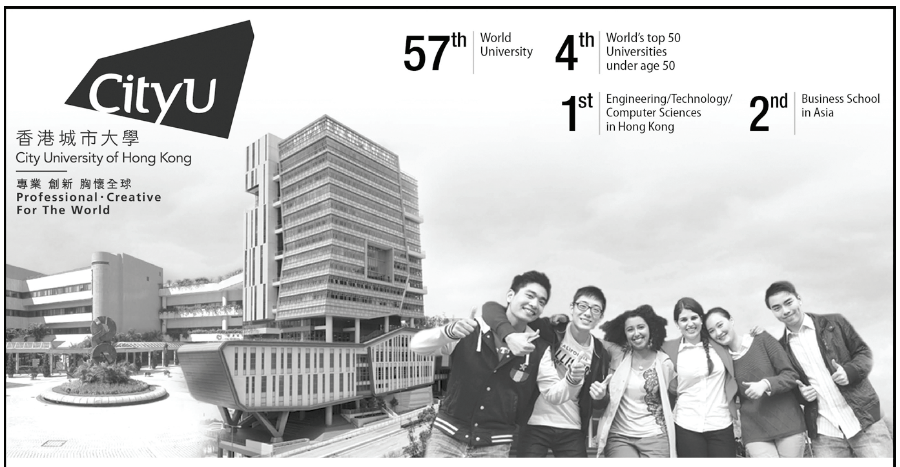

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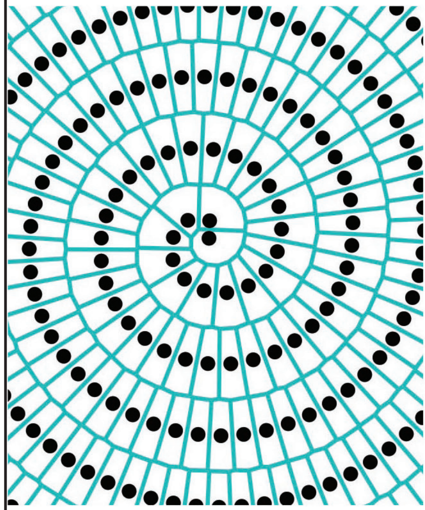

Figure courtesy of:
John Carlsson and Raghuveer Devulapalli

Some Light Geometry

MATHEMATICAL CURIOSITIES

By Mark Levi

Of the two geometrical curiosities below, the first involves zero work and the second (almost) zero words.

1. This physical “proof” of the Pythagorean theorem involves no work – mechanical work, that is. Figure 1a shows a right triangle, its hypotenuse held vertical. We take a point mass of the same weight c as the length of the hypotenuse,[1] so that c plays a double role of the length and of the weight. Lifting the mass along AB requires the same work as dragging it up the slippery ramps AC and CB:

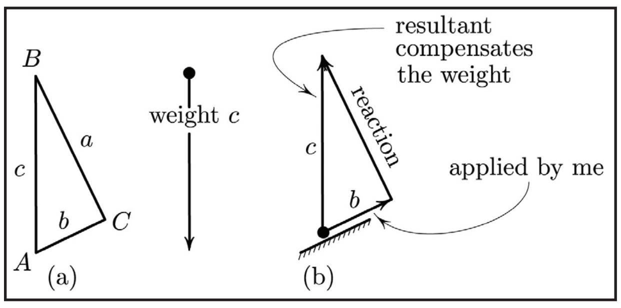

Figure 1. *Forces felt by the mass as one drags it up AC (with no acceleration) add up to zero. Thus, the resultant of the reaction of the ramp and the supplied force compensates the gravitational force. The triangle formed by these two forces is congruent to* ΔABC, *and thus the force one must apply is b. In short,* ΔABC *plays two roles, a geometrical one and a force one.*

$$W_{AB} = W_{AC} + W_{CB}. \quad (1)$$

Indeed, had the left-hand side been, say, smaller, we could have cycled the weight along the closed path $ABCA$, extracting more energy on the way down than we spent on the way up – a functioning perpetual motion machine.

Now (1) gives the theorem, since $W_{AB} = c \cdot c = c^2$, $W_{AC} = b \cdot b = b^2$, and $W_{CB} = a^2$, as explained by Figure 1 for W_{AC}. The Pythagorean theorem is thus one consequence of the constant vector field’s conservativeness.

2. A wordless proof of the formula $\sin(\beta - \alpha) = \sin\beta\cos\alpha - \sin\alpha\cos\beta$, referring to Figure 2, expresses the same area in two different ways:

$$A = 1 \cdot 1 \cdot \sin(\beta - \alpha) = \begin{vmatrix} \cos\alpha & \cos\beta \\ \sin\alpha & \sin\beta \end{vmatrix} = \sin\beta\cos\alpha - \sin\alpha\cos\beta.$$

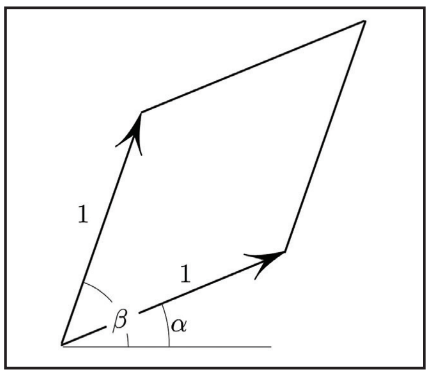

Figure 2. *A is the area of the parallelogram.*

All figures in the article are provided by the author.

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[1] in some chosen units

Car Parts, Neutrons, and Bridges

By William Regli

Not long ago, I was attending a birthday party for a friend of my then six-year-old son. The father of the birthday girl was a friend and colleague of mine in the Department of Mathematics at Drexel University. Several newer department members whom I did not know were also in attendance, so we struck up a conversation around the natural question, “So, what do you work on?” One worked in Lie theory, another in the area of differential geometry, yet another in real analysis, etc. When it came to my turn I said, “Well, my most recent project involved creating decision aides for signal corps officers, a role the Army calls the S6, to make better network management decisions.” Suddenly, conversation ceased and the looks on their faces changed. Something was clearly wrong, wrong in a third-arm-growing-out-of-my-head kind of way. Was it something I said?

Realizing my faux pas was the human users and their scruffy human problems, I quickly backpedaled and blurted out, “Optimization. I work in network optimization algorithms.” All was well. Conversation resumed. Songs were sung, cake was consumed, and network management on the battlefield was reduced to a mathematical optimization problem.

This was, on my part, a terrific oversimplification of my project, which had demanded structured interviews of several S6 officers about their current cognitive workflows and development of a task model for their activities across different network and mission software systems. All of this involved novel elements of mathematics, computing, software engineering, human-machine interfaces, and, yes, optimization. Sure, there were some interesting algorithmics, but this was about much more than an optimization problem. The real excitement in the project lay in our effort to take a fuzzy, human-driven process and accelerate it with cognitive models and algorithmics. Why couldn’t I better convey the intellectual excitement of trying to use formal techniques to make this scruffy problem more tractable?

My birthday party conversation was a reminder of how natural it is for us to self-identify with our academic sub-disciplines – our tribe. But describing problem-driven and use-inspired research is still a hard task, and sometimes results in a combination of curiosity and derision. I’ve been wondering how to change this perception.

As we look around the science and engineering landscape, there are many such opportunities for mathematics. In most of these cases, the nature of the opportunity is the same; people are desperate for new tools and techniques that can transform a scruffy, ill-posed problem into one that is more precise, and can ideally then be fully captured analytically or attacked with computational techniques.

In many ways, we are back at the beginning. The origin story of mathematics often centers on how the ancients developed primitive accounting techniques and tools for everything from documenting recipes for beer and handling agricultural inventories to performing land management. Mathematics emerged from the need to solve these then-muddled, real-world problems. What new vistas can we open for exploration today?

In the Defense Sciences Office (DSO) at DARPA, we try to break assumptions about use-inspired research, and mathematics is one of our main enablers. The approach followed by several of our program managers is to make the mathematics codependent with a problem-specific question. Here are just a few examples.

Mathematicians in our Enabling Quantification of Uncertainty in Physical Systems (EQUiPS) program[1] are working on new ways to manage the curse of dimensionality in the context of physical systems, such as marine vehicle design. The use of physical phenomena to focus and constrain the fundamental mathematical inquiry yields new insights for both members of the mathematical community and the applied sciences domains they study. Members of SIAM’s Activity Group on Uncertainty Quantification[2] are actively involved in this multi-year program.

Figure 1. *Grumman F6F-3 Hellcats in tricolor camouflage. The Grumman F6F Hellcat was developed during World War II. Photo credit: Wikimedia Commons.*

Graph-theoretic Research in Algorithms and the PHenomenology of Social Networks (GRAPHS)[3] is a program in theoretical computer science and combinatorial algorithms intended to create new techniques for processing graph-based data. In this project, each performer brought a domain context within which to test and evaluate the algorithms they were developing. The program, which is nearing completion, has shown how to improve social media analysis, reduce hospital readmissions, and optimize resource allocation in adversarial games.

The Complex Adaptive System Composition and Design Environment (CASCADE) program[4] addresses the mathematics of an emerging class of design problems: systems of systems. If systems design (i.e., design of a new aircraft or satellite) was not hard enough, designing a “systems of systems” in which the functions are disaggregated across many simpler elements is positively baffling. Such design requires formal means that consider how to best break down the collective objectives into more primitive behaviors and then coordinate the entities producing these behaviors to achieve shared objectives, all the while adapting to changing states. Most existing approaches to this problem space (e.g., the design of communications networks) are empirical or based on simulations and Monte Carlo methods. In seeking a more analytic framework, the CASCADE program looks toward emerging areas like category and sheaf theory to provide breakthrough insights and become the foundations to design tools.

Our TRAnsformative DESign (TRADES) program[5] aims to leverage the huge advances in materials science and manufacturing technologies over the past decade. In spite of these advances, the mathematical structures that underlie current computer-aided design systems are based on work by mathematicians like Bezier, Braid, Riesenfeld/Cohen/Lyche, and Voelcker/Requicha in the 1960s and 1970. Members of SIAM’s Activity Group on Geometric Design[6] have made many contributions to this body of work as well. However, we have reached a crisis point where the materials structures we can conceivably produce are far beyond the mathematical modeling tools and software systems we use to design things. The TRADES program attempts to create new mathematical foundations for these possibilities, but will require mathematicians to embed themselves with both materials scientists and traditional engineers.

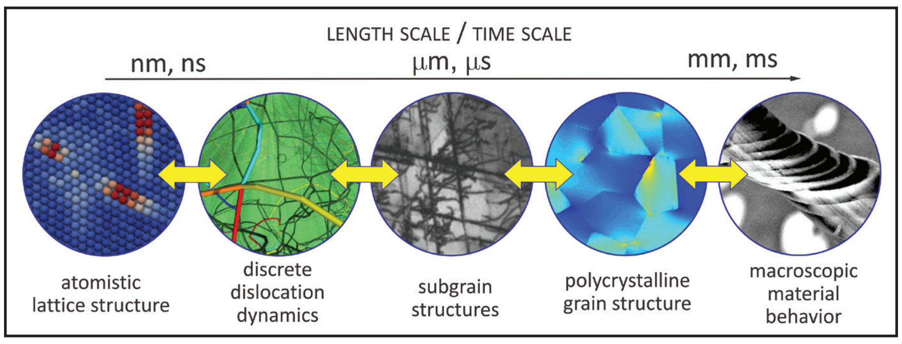

Figure 2. *Information technologies now make it possible to design configurations of matter across* 10^6 *to* 10^9 *orders of magnitude. Various computational techniques can describe and predict the mechanics and physics of materials on many different length and time scales. Figure credit: Dennis Kochmann.*

In each of these cases, the mathematics and computation do not exist in a vacuum. Rather, they are wholly situated within the context of the larger problem. This is hardly a new idea; these programs are a continuation of the use-inspired scientific and research culture in the United States that emerged from the problem-driven needs of our nation during World War II. It is worth remembering that Bezier’s curves are based on the shape of the hoods of Renault cars, Monte Carlo techniques were the Manhattan Project’s means of simulating the movement of neutrons during atomic detonation, and graph theory originated in response to the need to plan a nice Sunday stroll across the bridges of a Prussian town on the Baltic Sea.

Embedded in the car parts, neutrons, and bridges are mathematical realms that we have yet to fully explore. We are wrestling with issues such as the implications for machine learning and deep learning technologies, the role of computation and data as a potential accelerator for scientific discovery, the appearance of the human-machine innovation team, and our understanding of what makes us human – from the workings of our neurons and our individual behaviors to our social systems. In this current world, with its data-rich and increasingly complex problems, what other mathematical frontiers are waiting to be discovered?

William Regli is Deputy Director of the Defense Sciences Office at the Defense Advanced Research Projects Agency (DARPA). He can be contacted at regli@darpa.mil.

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[1] http://www.darpa.mil/program/equips

[2] https://www.siam.org/activity/uq/

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[5] http://www.darpa.mil/program/transformative-design

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