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Rational Functions and Beyond

By Lloyd N. Trefethen

Real analysis, complex analysis, and numerical analysis all start from polynomials. Since Newton at least, polynomials and their limits as Taylor series have captured the local behavior of functions, and the approximations become global with numerical methods such as data fitting, quadrature formulas, spectral methods, "proxy" rootfinding, and Chebfun. For smooth functions on bounded domains, this is often all you need — with the footnote that if a function is periodic, it is advantageous to switch from polynomials to Fourier series.

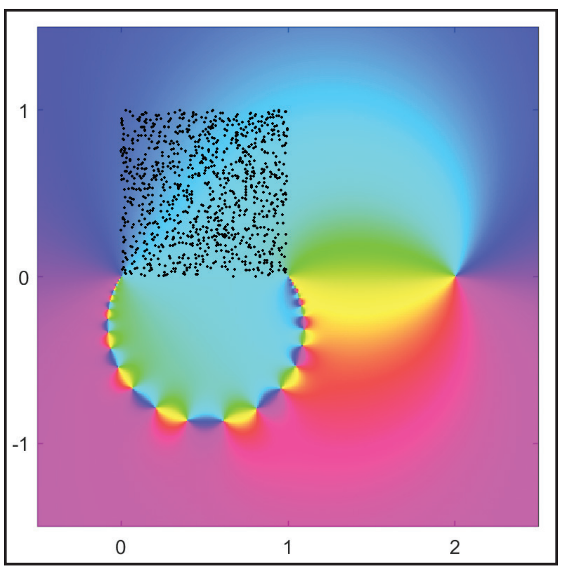

Figure 1. *Degree $n=20$ AAA rational approximation to $\sqrt{z-z^2}\,/\,(z-2)$ in 1,000 points in a square in the complex plane (black dots) with adaptively determined poles. This image is a phase portrait [5], with color marking the complex argument.*

A rational function is a quotient $r(z)=p(z)/q(z)$ of polynomials. We say that r is of degree n if it can be written in this way, where p and q are of degree at most n. Clearly r may have n zeros and n poles, real or complex, and a polynomial is nothing more than a rational function whose poles are constrained to all lie at $z=\infty$. But p/q is often not the best representation conceptually or numerically. A better start for many purposes is to write r in partial fractions:

$$r(z)=c_0+\sum_{k=1}^{n}\frac{c_k}{z-z_k}. \qquad (1)$$

This generic representation cannot treat poles of order higher than 1 exactly—or poles at infinity—but it can approximate them arbitrarily closely.

Formula (1) highlights the fact that a rational function can have its poles $\{z_k\}$ anywhere. In particular, if r approximates a function f with a singularity at $z=z_0$, it may achieve great accuracy by clustering its poles near z_0. This effect hit the headlines with Donald Newman's 1964 paper about the approximation of $f(x)=|x|$ on $[-1,1]$, or equivalently, the approximation (with half the degree) of $f(x)=\sqrt{x}$ on $[0,1]$. Newman showed that by clustering their poles and zeros exponentially near the singularity, rational functions can achieve *root-exponential convergence*: errors of order $O(\exp(-C\sqrt{n}))$ for some $C>0$. For six-digit accuracy in this example, a rational function of degree $n=26$ is enough, whereas a polynomial would need $n\approx 280,000$.

All of this is well established and extensively studied by rational approximation theorists. But can one apply these ideas to solve computational problems? Tradition regards rational functions as a computational minefield because of complications like "Froissart doublets" (pole-zero pairs), both theoretical and numerical. For example, in their work during the 1980s and 1990s, Richard Varga, Arden Ruttan, and Amos Carpenter used up to 200-digit precision. But in the past four years and in collaboration with Yuji Nakatsukasa, Abinand Gopal, and others, I have been part of some developments that appear to be changing this picture — all in ordinary floating-point arithmetic.

First came the *AAA algorithm* (adaptive Antoulas-Anderson), a method of unprecedented speed and robustness that finds near-best rational approximations [4]. The magic of AAA comes from its *barycentric* representation of rational functions in a third fashion, as a quotient of two partial fractions:

$$r(z)=\sum_{k=0}^{n}\frac{a_k}{z-t_k}\Bigg/\sum_{k=0}^{n}\frac{b_k}{z-t_k}. \qquad (2)$$

The numbers $\{t_k\}$ are not the poles of r, but rather a set of arbitrary *support points* that are chosen to enable numerical stability, even when the poles and zeros are exponentially clustered. For example, suppose that we execute the following commands in Chebfun:

```
Z = rand(1000,1) +
    1i*rand(1000,1);
F = sqrt(Z-Z.^2)./(Z-2);
[r,poles] = aaa(F,Z);
plot(Z,'.k'), hold on
phaseplot(r,[-0.5 2.5 -1.5
    1.5])
```

See **Rational Functions** *on page 3*

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Optimization Theory and Perspectives on the Field of Machine Learning

By Manuchehr Aminian

The work of Michael Jordan's (University of California, Berkeley) research group was a highlight of the 2020 SIAM Conference on Mathematics of Data Science (MDS20), which took place virtually earlier this year. The team's research focuses on the connections between optimization, geometry, probability, dynamical systems, and machine learning (ML). During his plenary talk at MDS20, Jordan presented an impressive array of theoretical results in optimization that were established by his group and motivated by ML. He also offered a broad vision of ML's past and present, and disclosed how he intends to orient his research towards what he sees as the field's future.

Gradient Descents and Saddle Points

Optimization is at the heart of model building in ML. One can broadly categorize optimization problems as either convex or nonconvex, depending on the type of function to be minimized and the space over which it will be optimized. Convex optimization problems are generally well understood and mathematically friendly; researchers can establish provable convergence results for algorithms, and the algorithms that solve them behave well in practice.

Most optimization problems in ML and data science are decidedly nonconvex. Objective functions are often nonlinear, and decision variables are frequently binary and/or integer-valued — especially when one wants to make a decision or recommendation. Jordan reviewed a range of proven results that relate to high-dimensional gradient descent (GD), which iteratively improves an initial guess x_0 and produces a sequence of approximations x_k that ideally converge to a global minimum of a function $f(x)$ by following the gradient "downhill."

First, Jordan highlighted a well-established result from Yurii Nesterov's 1998 lecture notes [2]. For unconstrained convex optimization, GD convergence rates do not depend on the space's dimensionality. In other words, if we measure a notion of how close we are to the global minimum, convergence rates depend on the function's properties and the number of descent steps; they are *independent* of whether a problem has 10 variables or 10 million. Readers might find this dimension-independence surprising, though perhaps there is a "catch" that practical problems may have a correlation between dimensionality and their Lipschitz constant.

To extend to nonconvex optimization, Jordan emphasized the main results of his 2017 study [1], which proves similar convergence rates with a few key differences. Nonconvex functions can have local minima and saddle points that trap and slow descent methods. He and his collaborators use a "perturbed" GD method—not to be confused with stochastic GD—to address this fact, occasionally adding a random jitter to the current iteration that may have otherwise trapped GD in a saddle point or local minimum. Expecting convergence to a global minimum in this general setting is too optimistic. But softening the statement to "strongly convex" minima produced a result that is only mildly worse than the convex case — with an extra factor of $\log^4(d)$ (with d as the dimension) that is conceptually a slowdown related to the escape from saddle points [1]. The group's proof technique is probabilistic and relies on geometry around the saddle points. Theoretically, aside from a narrow strip in a low-dimensional space around a saddle point, points that are initially in the highlighted region will escape after being perturbed away (see Figure 1).

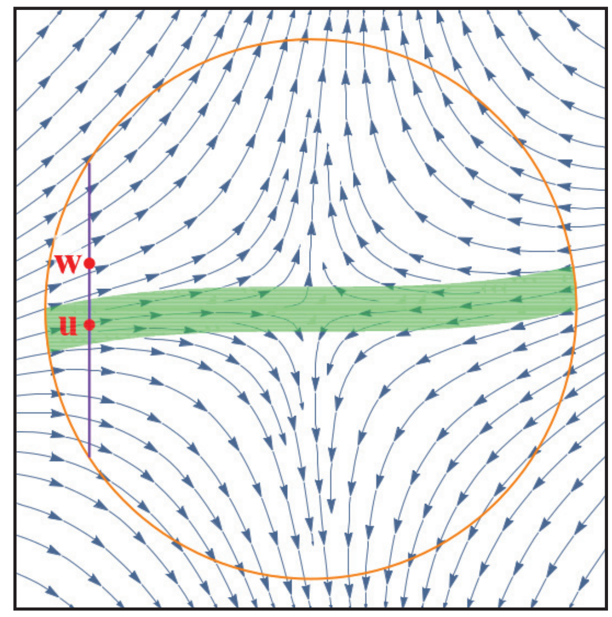

Figure 1. *Point u lies in the green region, where the gradient ∇f (represented by the blue arrows) will trap it near the saddle point in the center for many iterations. The perturbed point w will quickly escape. Figure courtesy of Michael Jordan [1].*

All else equal, the theory thus states that a problem in 10 million dimensions will converge slower—by a factor of 7^4, or 2,401—than a problem in 10 dimensions. Jordan believes that the exponent 4 is an artifact of the proof, which ought to be a plain $\log(d)$. However, he has been unable to improve upon it. Perhaps this is an opportunity for future collaboration.

Discrete and Continuous Time Dynamical Systems

Another aspect of Jordan's research involves understanding the connections between GD and other optimization algorithms, as well as the underlying mathematics

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Advancing SIAM's Mission with the COMAP Mathematical Contest in Modeling

By John Chrispell, Nathan Gibson, Kathleen Kavanagh, and Ben Galluzzo

The Consortium for Mathematics and Its Applications[1] (COMAP) has been promoting mathematical modeling and creative problem-solving through its undergraduate contest, the Mathematical Contest in Modeling[2] (MCM), since 1980. SIAM supports the MCM by awarding two "Outstanding" teams with the SIAM Award in the Mathematical Contest in Modeling.[3] Each student member of the winning team receives a cash prize of $500 and a one-year student membership with SIAM. Participating MCM teams, comprised of up to three students each, work over a long weekend to propose a solution to a broad, open-ended, and often messy real-world problem. Recent contest questions have focused on the opioid crisis, disaster relief, online marketing strategies, and the fishing industry.

Over the years, the MCM has seen a dramatic increase in participation from China. In 2020, the nation hosted the vast majority of both participating and winning teams; only 264 of the total 13,753 teams were from the U.S. Although mathematical modeling is not predominantly integrated into most U.S. math curriculums, it is arguably one of the most powerful tools for workforce development (see Figure 1). COMAP's MCM can serve as a rewarding pathway for SIAM members and student chapters to provide undergraduate students with engaging math modeling opportunities.

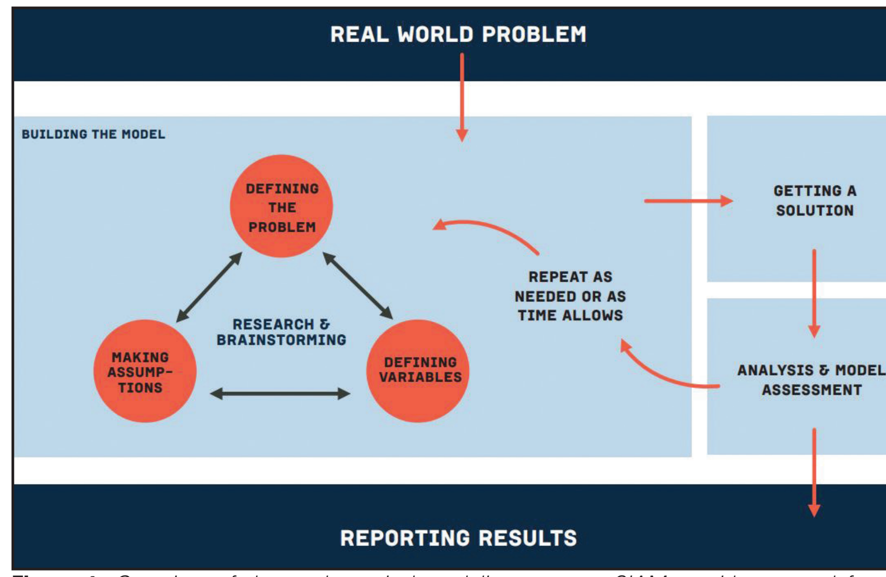

Figure 1. *Overview of the mathematical modeling process. SIAM provides several freely-available handbooks to help eager students get started. Figure courtesy of [1].*

Coaching strategies to prepare students vary widely across colleges and universities. At Clarkson University, where a team of sophomores earned a Mathematical Association of America prize for their solution in 2009, COMAP training takes place over roughly eight weeks. Students earn one course credit for completing the training, competing in the contest, and submitting a solution. The training focuses on the individual components of the modeling process—defining a concise problem statement, making assumptions, identifying variables, building a model, and analyzing the solution—before leading up to solving a full modeling challenge. Students also practice technical writing each week. Training begins in October, includes a longer collaborative modeling assignment over winter break, and continues for a few weeks in the spring semester until the MCM commences in February.

Clarkson often recruits potential participants from the Advanced Placement calculus II course (for first-year students) and the introductory math modeling course, as these early encounters can spark students' interest in applied mathematics. Some participating students have even added mathematics as a double major or proceeded to complete undergraduate research in mathematics. Many teams that are recruited from Clarkson's modeling class compete in subsequent years as well.

Students that have previously participated in the MCM generally serve as the most effective recruiters. At the Indiana University of Pennsylvania, the preceding year's competitors speak to potential participants each fall and describe both their experiences and the research topics from the last competition. Students often start to form teams during their sophomore year and frequently continue to partake until they graduate. To support students during the competition, the Department of Mathematical and Computer Sciences supplies keyed-access workspaces and ensures that the students have 24-hour access to these spaces throughout the contest. Faculty promote the event by discussing it in class and commending students who participate. They set aside a special mathematical modeling library, provide books that students can use during the MCM, and contribute ample amounts of food over the course of the weekend.

Students at Oregon State University (OSU) have been participating in the MCM since 2006 and achieved "Meritorious Winner" status for their first attempt. As with other schools, OSU's Department of Mathematics makes rooms available, issues after-hours passes, and offers written preparation materials[4] to supplement the resources on COMAP's homepage.[5] After the event, the Math Club hosts a special seminar during which participants can choose to present their solutions. All successful participants receive their official certificates during the year-end departmental awards ceremony.

OSU has no background requirements for MCM participation; students from all majors and years are welcome to partake. Past teams have consisted of freshmen through seniors whose majors ranged from math and engineering to other facets of science. The main requirements are a desire to compete and a commitment to working with teammates, often from different disciplines. These are of course the same characteristics that many employers seek, and the experience makes for excellent interview material. However, the secret to OSU's success—modest as it may be—is undoubtedly the university's writing-intensive mathematical modeling course. Repeated practice in writing about math, and applied math in particular, reinforces the notion of mathematics as a language into which real-world problems are translated for abstraction and efficient solution. Courses on the history of mathematics may also provide exposure to math modeling, as numerous fields of mathematics originated with attempts to describe and solve real-world phenomena.

After the competition and regardless of the outcome, many student teams use their work as the foundation for their first presented talks at regional conferences. In this way, the COMAP experience serves as a gateway into the academic research process.

We strongly encourage SIAM members to get involved with the MCM. At some universities, SIAM student chapters organize the training efforts — students can even be coaches! Registration[6] for the 2021 competition—which is scheduled for February 4-8, 2021—is already open, and teams can collaborate remotely. If you are interested in serving as a judge, more information about judging opportunities is available online.[7]

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[1] https://www.comap.com
[2] https://www.comap.com/undergraduate/contests
[3] https://www.siam.org/prizes-recognition/student-prizes/detail/siam-award-in-the-mathematical-contest-in-modeling
[4] https://math.oregonstate.edu/contests
[5] https://www.comap.com/undergraduate/contests/resources/index.html
[6] https://www.comap.com/undergraduate/contests/mcm/register.php
[7] http://www.comap-math.com/COMAP/judges/index.html

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SIAM Conferences, Reinvented

By Richard Moore

Like many of you, I am writing this article from a laptop perched on a kitchen island that is littered with the remains of my children's breakfasts and a jar of sourdough starter, while mentally counting the number of Zoom meetings I have today. My kids are "in class" on their computers in the neighboring room and my wife is participating in a department meeting from our den. I occasionally check my phone to see if my COVID-19 test result has come in yet. And I still feel truly fortunate for all of these things. This is 2020.

We have all adapted to the new, inconvenient, and sometimes tragic circumstances brought on by the pandemic, which has also disrupted many of SIAM's programs and services — none more so than its lineup of conferences. Meetings that took place in early 2020 through the end of February were all very successful. These included the ACM-SIAM Symposium on Discrete Algorithms (SODA20), the SIAM Symposium on Algorithm Engineering and Experiments (ALENEX20), the SIAM Symposium on Simplicity in Algorithms (SOSA20), the SIAM Symposium on Algorithmic Principles of Computer Systems (APOCS20), the SIAM Workshop on Combinatorial Scientific Computing (CSC20), and the SIAM Conference on Parallel Processing for Scientific Computing (PP20), as well as SIAM programming at the 2020 Joint Mathematics Meetings. However, COVID-19's spread throughout Asia, Europe, and North America led the World Health Organization to declare a global pandemic on March 11, which had immediate consequences for domestic and international travel. Since that date, all SIAM conferences have been cancelled, postponed, rescheduled, or pivoted to a virtual platform.

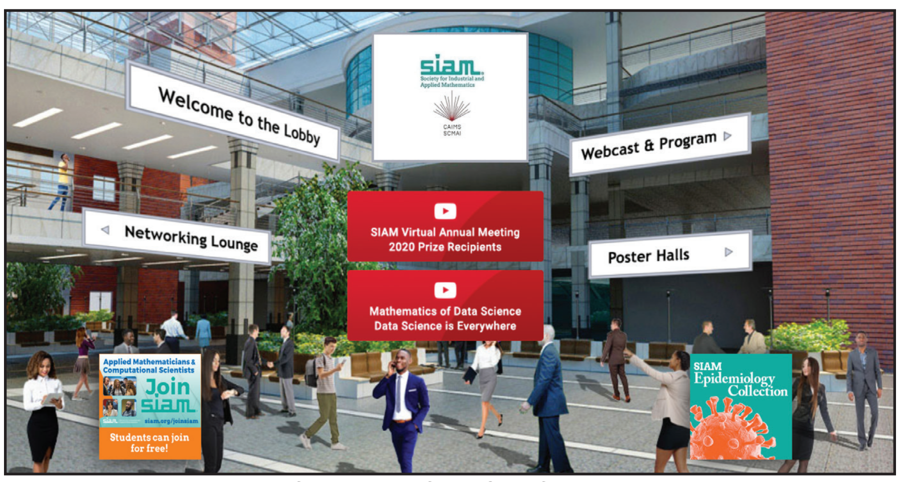

Figure 1. *Lobby space at the Second Joint SIAM/CAIMS Annual Meeting (AN20), which took place virtually this July.*

Throughout this unprecedented time, we have prioritized communication with conference organizers and the SIAM community. In early March, we quickly set up a COVID-19 response page[1] that summarizes the status of each conference and the measures that SIAM is taking to keep its attendees and presenters safe. SIAM senior staff and leadership engaged in several conversations with conference co-chairs and Activity Group officers to consider options and reassure them that we would work together to reach the right outcome for each conference while keeping everybody safe, addressing the community's needs, honoring the hard work of the co-chairs and organizing committees, and respecting the wide range of challenges that members of the SIAM community face as a result of the pandemic. The 2020 conference calendar[2] illustrates these varied outcomes.

The phrase "pivoting to virtual" is new for SIAM, and we used the period from May to August to experiment with different online formats. The co-chairs of the inaugural SIAM Conference on Mathematics of Data Science (MDS20),[3] which was originally planned for early May in Cincinnati, Ohio, decided to offer the invited plenary talks and minitutorial sessions via Zoom webinars throughout both May and June.[4] These sessions went very well, with a peak attendance of over 500 listeners. Minisymposium organizers were given the opportunity to schedule virtual sessions through their own institutional videoconferencing accounts and post these events on a spreadsheet that viewers could access from the conference website. 64 sets of organizers did so, with reported attendance as high as several hundred participants. SIAM used a similar model for the SIAM Conference on the Life Sciences (LS20)[5] and the SIAM Conference on Mathematics of Planet Earth (MPE20),[6] both of which were originally scheduled for early June in Garden Grove, Calif., but instead deployed in virtual form. LS20 occurred over the month of June and MPE20 transpired in mid-August. SIAM staff once again ran the invited plenary talks and minitutorials on Zoom, while minisymposium organizers set up their own sessions. However, coding changes in SIAM's conference management system allowed

See **SIAM Conferences** *on page 5*

1 https://sinews.siam.org/Details-Page/covid19
2 https://www.siam.org/conferences/archives
3 https://www.siam.org/conferences/cm/conference/mds20
4 https://sinews.siam.org/Details-Page/2020-siam-conference-on-mathematics-of-data-science-a-meeting-goes-virtual
5 https://www.siam.org/conferences/cm/conference/ls20
6 https://www.siam.org/conferences/cm/conference/mpe20

Rational Functions

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In $1/50$ seconds (s) on my laptop, AAA computes a degree-20 rational approximation that matches F to 14 digits at the 1,000 random data points. Figure 1 (on page 1) shows how the poles and zeros of r simulate a circular branch cut that connects the branch points 0 and 1. There is also a pole at $z = 2.00000077$, whose six-digit agreement with the pole of f demonstrates the power of rational functions for extrapolation and analytic continuation. In fact, most existing methods for estimation of poles and acceleration of convergence (Padé, Aitken, eta, epsilon, etc.) are based on rational functions.

Next came *lightning approximation* [2]. AAA has free poles that are adaptively determined, and there is no guarantee that they will not fall in a region where one wants analyticity. Moreover, we do not know how to use AAA to approximate harmonic functions, i.e., *real parts* of analytic functions, which is what one needs to solve Laplace problems via function approximation. The new idea is to leverage our understanding of exponential clustering at singularities by prescribing the poles $\{z_k\}$ a priori — simply placing them in a configuration with exponential clustering at the corners, which are likely to be branch points of the solution. A polynomial term to handle "the smooth part of the problem" is included as well. Now one has the linear problem of finding good coefficients c_k for an approximation (1) with known points $\{z_k\}$, which is readily solved by least-squares fitting. If you want to fit the real part to solve a Laplace boundary value problem, this makes no significant difference to the calculation. Figure 2 depicts the solution to a Laplace problem on an octagon computed by this method. In 0.13 s on a laptop, the code `laplace.m`[1] has computed a solution with 138 poles that is accurate to six digits, all the way up to the corner singularities. Being just the real part of a rational function, the solution can be evaluated in 11 μs per point. For 10-digit accuracy, these figures change to 0.9 s, 286 poles, and 26 μs. Similar computations are possible for biharmonic problems, which are reducible to harmonic functions, and Helmholtz problems, where the poles of a rational function become center points of shifted Hankel functions [1].

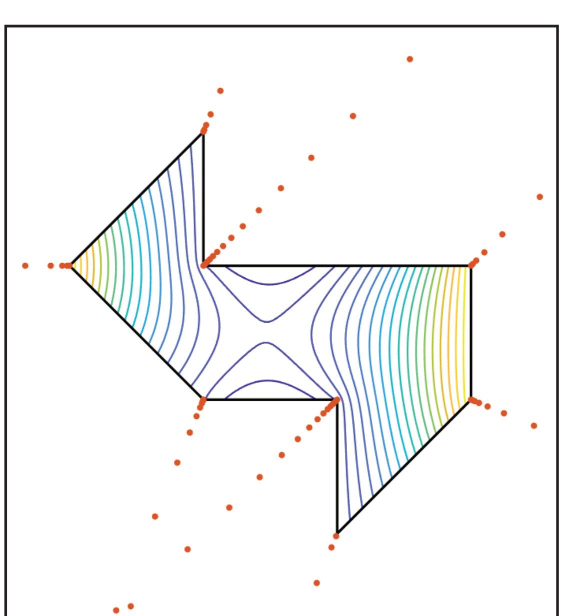

Figure 2. *Lightning solution to a Laplace problem in an octagon by least-squares fitting of boundary data by the real part of a rational function with prescribed, exponentially clustered poles (red dots). The name "lightning" alludes to the exploitation of the same mathematics that leads lightning to strike at sharp points.*

I want to finish with a third development that turned up unexpectedly just a few months ago [3]. What about the use of functions other than rationals, with singularities other than the poles of (1)? In a new development, we have found that *reciprocal-log* or *log-lightning* approximations of the form

$$r(z) = c_0 + \sum_{k=1}^{n} \frac{c_k}{\log(z - z_k) - s_k} \qquad (3)$$

can speed up convergence from root-exponential to exponential or exponential-minus-log, i.e., $O(\exp(-Cn / \log n))$. The approximations take advantage of analyticity on a Riemann surface and can be used for analytic continuation to other Riemann sheets beyond the branch cuts. Figure 3 shows an approximation to the function $z^{1/3}$ of this kind with

$$n = 30, \quad z_k = 0, \quad \text{and} \quad s_k = \frac{1}{2} n (1 + i t_k)^2,$$
$$t_k = -\pi + 2\pi (k - \tfrac{1}{2})/n, \quad 1 \le k \le n.$$

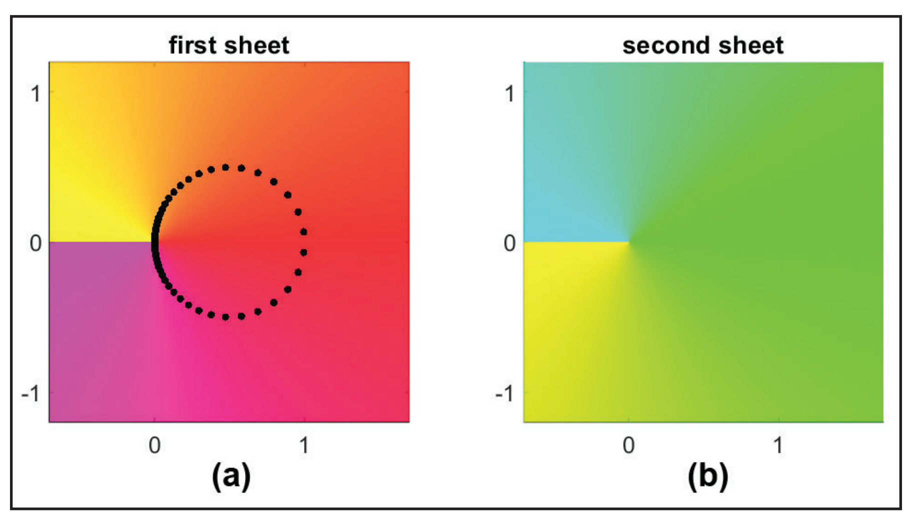

Figure 3. *Phase portraits of a degree $n = 30$ reciprocal-log approximation of $z^{1/3}$, based on least-squares fitting in 1,000 exponentially clustered points on the circle $|z - \frac{1}{2}| = \frac{1}{2}$ (black dots).* **3a.** *On the first Riemann sheet, the maximum error in the region is 2×10^{-9}.* **3b.** *On the second sheet, the error is 2×10^{-6}.*

It seems that a new era of numerical computation with rational functions and other functions with singularities is arriving. This short essay is confined to scalar problems, but there are also exciting ongoing developments that involve rational functions in large-scale linear algebra. Some key names are Athanasios Antoulas, Christopher Beattie, Bernhard Beckermann, Peter Benner, Vladimir Druskin, Serkan Güğercin, Stefan Güttel, Leonid Knizhnerman, Eric Polizzi, Valeria Simoncini, Alex Townsend, Heather Wilber, and Karen Willcox.

This article is based on Nick Trefethen's John von Neumann Prize Lecture at the 2020 SIAM Annual Meeting,[2] which took place virtually this July. Trefethen's presentation is available on SIAM's YouTube Channel.[3]

The figures in this article were provided by the author.

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Nick Trefethen is Professor of Numerical Analysis at the University of Oxford. He served as president of SIAM during 2011-2012.

1 https://people.maths.ox.ac.uk/trefethen/lightning.html
2 https://www.siam.org/conferences/cm/conference/an20
3 https://go.siam.org/KthZxi

Optimization Theory

Continued from page 1

that connects the algorithms' transitions between discrete and continuous time. Jordan explored this area and compared observations about some of the proven results. For instance, while GD has an asymptotic convergence rate $O(1/k)$ for unconstrained convex optimization (see Figure 2), Nesterov demonstrated that "accelerated" GD—a two-term method—has a faster convergence rate of $O(1/k^2)$.

To unify these and other descent methods, researchers have been studying the corresponding continuous-time flows that are associated with a discrete descent method. For example, one can understand GD as the time discretization of so-called *gradient flow* when taking the stepsize $\beta \to 0$; here, one keeps the corresponding "rate" of $O(1/t)$. Jordan then observed that previous work has utilized the same concept to identify a second-order differential equation that corresponds to accelerated GD, which has a peculiar form [3]. The analogous rate of $O(1/t^2)$ is again kept.

Given these observations, a motivating question for Jordan was whether the results could be extended. This was the topic of his 2016 paper with Andre Wibisono and Ashia Wilson [4]. The group approached this query from the perspective of variational calculus and defined the so-called Bregman Lagrangian, from which one can build a family of differential equations that include gradient flow, accelerated gradient flow, and a continuum of other differential operators. Many interesting details are available in this work and subsequent studies; for instance, while one can achieve *exponential* convergence rates in continuous time—compared with algebraic rates for GD and accelerated GD—the same convergence rate *provably* cannot retain its speed after discretization. To provide some intuition into this phenomenon, Jordan remarked that a bound of $O(1/k^2)$ for discrete convergence rates stems from the fact that GD methods will inevitably slow down in regions of high curvature.

	Discrete	Rate	Continuous
Gradient Descent (GD)	$x_{k+1} = x_k - \beta \nabla f(x_k)$	$O(1/k)$	$\dot{X} = -\nabla f(X)$
Accelerated GD	$x_{k+1} = (1-\lambda_k) y_{k+1} + \lambda_k y_k$	$O(1/k^2)$	$\ddot{X} + \frac{3}{t}\dot{X} = -\nabla f(X)$

Figure 2. *Comparisons between gradient descent (GD) and accelerated GD. The accelerated approach has provably faster convergence and limits to an interesting continuous-time flow. The differential equation for accelerated GD is courtesy of [3].*

The results that Jordan presented during this portion of his talk went well beyond this concept and included approaches for obtaining rates, backward error results, and explicit schemes via the power of numerical analysis, symplectic geometry, differential manifolds, Hamiltonians, and dissipative systems.

Machine Learning and Chemical Engineering

Jordan had quite a lot to say about his vision of ML as a budding field. His first observation pertained to recommendation systems, wherein a machine offers a user a recommendation based on two or more choices. However, the ideal approach to recommendation varies with different applications. Suppose the system recommends the "best" option to every user. If it recommends the "best" driving route from point A to point B or the "best" restaurant to everyone, issues will quickly arise. Jordan argues that microeconomics will play an upcoming role in these types of settings. Traditional markets—while not a direct algorithm and not without their own issues—are adaptive, robust, and scalable in a way that already handles these difficulties; therefore, he believes that researchers should consider them as alternatives to artificial intelligence (AI).

Jordan was also concerned with the increasingly common use of AI-driven decisions in medicine. Suppose that a doctor feeds patient data into an AI model that is trained to detect a disease and receives a numerical score of 0.71, just above threshold 0.70 for positive detection. Should the patient receive treatment? Jordan contends that one should not utilize such thresholds in isolation. Instead, the doctor should account for factors that relate to the data that trains the algorithm. Frameworks must also be established for practitioners to quickly understand how and why the AI system in question made its decision, including error bars, applicability of the associated study data that drives the decision, and so on.

Jordan classified the past decade of ML as a "pattern recognition" era, with a focus on tasks like speech recognition, computer vision, and natural language processing. Such algorithms are significantly impacting billions of people around the world. The applications to which Jordan alludes, as well as others in the area of pattern recognition, are most transparent in our daily lives. However, algorithms that handle loan and job applications, predict recidivism, and so forth arguably have equal—if not more long-term—impacts on society. As humanity strives towards an increasingly just and equitable future, holistic methods will likely be an ongoing research priority.

This article is based on Michael Jordan's invited talk at the 2020 SIAM Conference on Mathematics of Data Science (MDS20),[1] which occurred virtually earlier this year. Jordan's presentation is available on SIAM's YouTube Channel.[2]

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[1] https://www.siam.org/conferences/cm/conference/mds20

[2] https://www.youtube.com/watch?v=Z4u3EA2k8vg

SIAM Conferences

Continued from page 3

the minisymposium links to be displayed through SIAM's familiar online program.[7]

SIAM partnered with a virtual conference vendor (see Figure 1, on page 3) to offer a more complete experience for the Second Joint SIAM/CAIMS Annual Meeting (AN20)[8] and concurrent SIAM Conference on Imaging Science (IS20),[9] both originally scheduled for early July in Toronto, Canada. The platform allowed us to feature 30 invited plenary, prize, and minitutorial talks; two career-related panel discussions; 154 minisymposia; and 75 posters, as well as student orientations, a student mixer, panel breakout sessions, and the Workshop Celebrating Diversity "luncheon." Exhibitors occupied booths in the virtual exhibit hall (see Figure 2), and the annual SIAM Career Fair took place as a one-day virtual event, drawing strong international participation from job seekers and a solid list of recruiters despite an obviously difficult hiring season. In total, AN20 and IS20 attracted over 4,000 registrants and nearly 3,000 attendees — a record for SIAM conferences. In recognition of the challenging circumstances that many members of our community are currently facing, including reduced access to customary sources of travel funding, SIAM and its leadership decided to forgo registration fees for virtual conferences in 2020.

As we look towards an uncertain 2021, it has unfortunately become clear that the COVID-19 pandemic will continue to disrupt travel and gatherings well into the new year. As I write this, SODA21, SOSA21, ALENEX21, APOCS21, the SIAM Conference on Computational Science and Engineering (CSE21), the SIAM International Conference on Data Mining (SDM21), the rescheduled SIAM Conference on Mathematical Aspects of Materials Science (MS21), and the SIAM Conference on Applications of Dynamical Systems (DS21) have already pivoted to virtual formats.[10] The SIAM Conference on Applied Linear Algebra (LA21) will be either virtual or hybrid. In anticipation of a long string of online conferences, SIAM has taken the lessons from its 2020 offerings and applied them in a new platform that will promote increased interactions and networking opportunities, along with fully-featured live sessions, poster forums, and exhibit halls. Virtual conferences will no longer be free in 2021, but SIAM will provide as much support as possible to allow students, early-career researchers, and others who are suffering hardship to apply for fee waivers. We encourage virtual conference attendees with children in need of care to apply for SIAM's child care grants.

Despite its challenges, this year has once again reminded me of the wonderful and dedicated nature of the people who comprise the SIAM community. To the conference co-chairs, SIAM leadership, and heroic staff with whom I have the pleasure of working: you have made a year of uncertainty and difficult transitions feel like a time of growth and opportunity. Given these new modalities of conference participation, SIAM will emerge from the era of COVID-19 with an expanded global reach and a greater capacity to welcome new voices to the field of industrial and applied mathematics.

I look forward to seeing you, online for now, at the next SIAM conference!

Richard Moore is the Director of Programs and Services at SIAM.

[7] https://meetings.siam.org/program.cfm?CONFCODE=Is20
[8] https://www.siam.org/conferences/cm/conference/an20
[9] https://www.siam.org/conferences/cm/conference/is20
[10] https://www.siam.org/conferences/calendar

Figure 2. *Exhibitors at the virtual Second Joint SIAM/CAIMS Annual Meeting (AN20), which took place this July, occupy space in the exhibit hall.*

Take Advantage of SIAM's Visiting Lecturer Program

Hearing directly from working professionals about research, career opportunities, and general professional development can help students gain a better understanding of the workforce. SIAM facilitates such interactions through its Visiting Lecturer Program (VLP), which provides the SIAM community with a roster of experienced applied mathematicians and computational scientists in industry, government, and academia. Mathematical sciences students and faculty—in particular, SIAM student chapters—can invite SIAM VLP speakers to talk about topics that are of interest to developing professional mathematicians. The VLP is a valuable resource now more than ever. The current climate has caused many departments to rethink their procedures, so why not host a SIAM visiting lecturer for a virtual talk?

The SIAM Education Committee sponsors the VLP and recognizes the need for all members of our increasingly technological society to familiarize themselves with the achievements and potential of mathematics and computational science. We are grateful to the accomplished applied mathematicians who have graciously volunteered to serve as visiting lecturers.

Things to consider in advance when a department decides to host a visiting lecturer include the choice of dates, speakers, topics, and any additional or related activities, such as a follow-up discussion. Organizers must make sure to address these points when communicating with a potential lecturer. It is important to familiarize speakers with their audience—including special interests or expectations—so that they can refine the scope of their talks, but just as crucial to accommodate speakers' suggestions so that the audience can capitalize on their expertise and experience.

Read more about the program and view the list of speakers online.[1]

[1] https://www.siam.org/students-education/programs-initiatives/siam-visiting-lecturer-program

Who Was Frank Ramsey?

BOOK REVIEW
By James Case

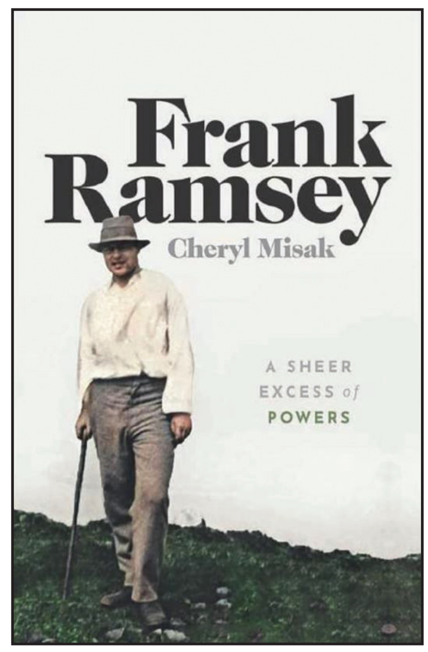
Frank Ramsey: A Sheer Excess of Powers. *By Cheryl Misak. Oxford University Press, Oxford U.K., March 2020. 544 pages, $32.95.*

Frank Plumpton Ramsey was born in February 1903 and died in January 1930, leaving behind a wife and two infant daughters. He is remembered as a towering genius—compared by some to Isaac Newton—who made significant contributions to mathematics, economics, and philosophy during his short life. In *Frank Ramsey: A Sheer Excess of Powers*, Cheryl Misak explores his experiences and career.

Ramsey was born into the Cambridge branch of what is known as the British intellectual aristocracy. His father Arthur was a recognized mathematician who served as master (second in command) at Cambridge University's Magdalene College for many years. Ramsey's mother, Agnes Wilson Ramsey, was a university graduate at a time when few women attended college. She was politically liberal and active in feminist causes, the latter putting her in contact with the mother of John Maynard Keynes. Keynes was quick to recognize young Ramsey's remarkable talent and made every effort to advance his career.

Ramsey was a schoolroom prodigy who stood at the top of virtually every class, despite being three years younger than most of his classmates. He won prizes at Winchester College (read: prep school) in Latin, German, and math, the latter of which was taught by L.M. Milne-Thomson, an emerging expert on fluid mechanics and aerodynamics. Ramsey considered him to be a "bad teacher and bad explainer," but a generous soul who loaned him books like Louis Couturat's *Die Philosophischen Prinzipien der Mathematik* and Hermann Weyl's *Raum, Zeit, Materie*. According to his diary, those were but two of almost 50 books that Ramsey read between January and March during his final year at Winchester.

Even before his last year at the school, Ramsey was well acquainted with *Principia Mathematica*, a multi-volume treatise by Alfred North Whitehead and Bertrand Russell on the foundations of mathematics. Fascinating though it was, it seemed to raise more questions than it answered. However, it did ignite Ramsey's lifelong interest in the foundations of mathematics, economics, probability, and knowledge itself. The latter topic belongs to philosophy, and it is as a philosopher that Ramsey is best remembered today.



Frank Ramsey: A Sheer Excess of Powers. By Cheryl Misak. Courtesy of Oxford University Press.

Upon graduating from Winchester in 1920, Ramsey enrolled in Trinity College, Cambridge to study mathematics. At Keynes' behest, he was soon invited to join the Apostles, a somewhat elite debating society at the university. He also explored several other undergraduate debating societies that dotted the Cambridge landscape, including the famed Cambridge Union. It was their debates, which primarily consisted of aspiring politicians, that Ramsey found uninteresting; Keynes' Political Economy Club and the so-called "Heretics Society" were more to his liking. He spoke and/or "read papers" at a number of the meetings before confiding to his diary that he loathed his "perverted ambition" to excel at debating merely for "recognition." The purpose of debate, Ramsey felt, was to generate insights that might lead listeners a bit closer to whatever truth they were seeking. He also attended almost every meeting of a socialist organization known as CUSS during his first year at school.

Ramsey met most of his lifelong friends at Cambridge, including his eventual wife. She was the treasurer of the Heretics Society, and he thought her both beautiful and "nice" upon their first meeting. They did not get together right away but remembered each other well when their paths crossed again.

When he first arrived at Cambridge, Ramsey was seriously considering a career in economics. However, he was soon persuaded that there was more glory—and more lasting glory—in the venerable Mathematical Tripos. While a mind like his might exhaust a subject like economics in just a few years, math would always provide appropriately daunting challenges. As such, Ramsey chose to pursue a degree in—and later teach—mathematics, all while dabbling in economics, probability, and whatever else caught his fancy. But his passion was always philosophy, and he had hoped to complete his *magnum opus* in this subject by the age of 30.

As an undergraduate, Ramsey was asked to translate Ludwig Wittgenstein's all but incomprehensible *Tractatus Logico-Philosophicus* into English. Upon seeing the result, Wittgenstein declared Ramsey to be the only person in the world who understood his book. After finishing the Tripos in 1923, Ramsey travelled to Vienna. There he met Wittgenstein, was befriended by several members of the wealthy Wittgenstein family, and underwent psychoanalysis (by an associate of Sigmund Freud) to relieve him of his anxieties concerning sex. Ramsey became a fellow of Kings College in 1924, was appointed as a university lecturer in mathematics in 1926, and later became Director of Mathematical Studies.

Misak, a professor of philosophy at the University of Toronto, is eminently qualified to comment on Ramsey's philosophy, his life and times, the circumstances surrounding his untimely death, and his on-again-off-again relationship with Wittgenstein. She does all of this in welcome detail. However, she is not a mathematician, economist, or expert in probability and has therefore enlisted several specialists in Ramsey's fields to provide brief summaries of his more lasting

See **Frank Ramsey** *on page 9*

Scale-bridging with Machine Learning to Characterize Brittle Damage and Failure

By Gowri Srinivasan, Daniel O'Malley, and Maria Giselle Fernandez

Many engineering applications utilize brittle materials—such as glass, ceramics, graphite, concrete, and some metals like beryllium—for their stiffness, lightweight properties, and ability to maintain their shapes at extreme temperatures. For example, beryllium is particularly useful in defense and aerospace applications for building lightweight instruments with high precision and controlling fission reactions. Concrete, on the other hand, is widely used in civil applications as the primary substance for buildings and bridges. Though brittle materials have many desirable properties, they are also prone to catastrophic failures that arise because the materials can handle only some elastic deformation and almost no plastic deformation. Instead of bouncing back (elastic behavior) or permanently deforming (plastic behavior), brittle materials fail with little warning. Understanding the mechanisms that cause them to fail is imperative for avoiding accidents that can jeopardize the safety and security of people and systems.

Brittle materials fail through the nucleation, evolution, and coalescence of microcracks. Figure 1 displays the aftermath of several experiments wherein one beryllium disc crashed into another at an extreme speed [1], revealing a handful of large, readily-visible cracks. The finite-discrete element method (FDEM) is ideally suited for modeling crack dynamics at the mesoscale, since the finite elements describe intact cells in the material and the discrete elements characterize the cracks (void space). Researchers at Los Alamos National Laboratory (LANL) have developed an FDEM solver—the Hybrid Optimization Software Suite (HOSS)—that models the evolution of many cracks and generates an accurate prediction of the failure path and failure time. Unfortunately, the cost associated with these simulations can be enormous. For example, performing just a single simulation may be unmanageable at large spatial or temporal scales, even with high-performance computing resources. And performing many simulations for optimization or uncertainty quantification can be problematic at more modest scales. Continuum model simulations provide a relatively inexpensive alternative. However, these simulations sacrifice the ability to track each crack for computational efficiency by homogenizing microcrack information through upscaled parameters. This upscaling process is scientifically challenging, as it tends to lose crucial information that is related to the structure and evolution of cracks.

Machine learning (ML) is well-suited for modeling the dynamics of systems where phenomena at smaller length scales significantly impact the larger scales of interest [6]. Access to sufficient ground truth data for validation—in terms of experimental evidence and mechanistic, lower-length scale models—renders ML a powerful tool for scale-bridging applications. Here, ML offers the tantalizing possibility of learning about crack evolution and coalescence via time series data from high-fidelity FDEM simulations. This data can inform a variety of ML models that predict damage characteristics in brittle materials—such as where and when materials will fail—and deliver additional information about stress, strain, and individual cracks.

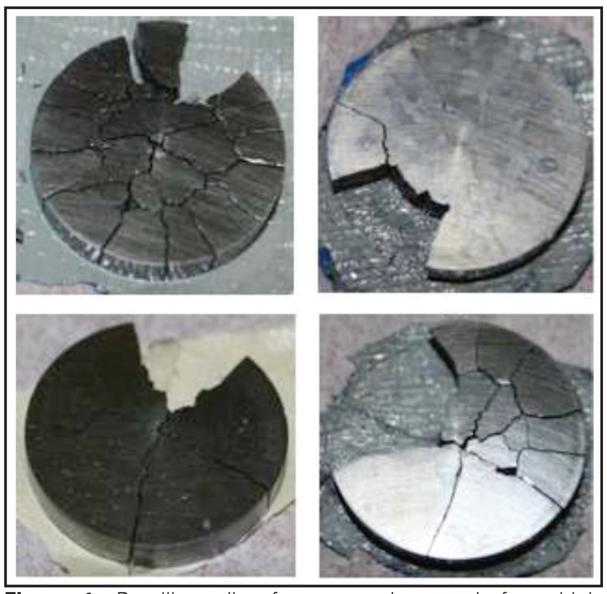


Figure 1. *Beryllium disc fragments that result from high-velocity impact. Figure courtesy of [1].*

Recent research has utilized ML to model brittle materials that undergo low strain rate dynamic tensile loading [3, 5]. Tensile loading refers to a load that pulls an object apart (like the rope in a tug-of-war game), and low strain rate conditions result from the slow application of tensile loading. We simulated a concrete plate ($2\,\mathrm{m} \times 3\,\mathrm{m}$) whose upper edge moves upward at a speed of $0.3\,\mathrm{m/s}$, resulting in a low strain rate of $0.1\,\mathrm{s}^{-1}$. The simulation, which we performed in two dimensions using the FDEM code, has 20 initial cracks with identical length ($30\,\mathrm{cm}$), random uniform location, and three different possible orientations. We then explored multiple conceptual approaches to predict the path of failure when compared with the expensive FDEM code (HOSS). The best of these approaches, called Microcrack Pair Informed Coalescence (McPIC), treats each crack as a node in a dynamic graph where the presence of edges indicates the coalescence of two cracks. The model predicts two things: (i) whether two cracks will coalesce, and (ii) the timing of the coalescence if they do. The development of McPIC involved an investigation of several supervised ML algorithms, including decision trees, random forests, and neural networks. The neural networks provided a marginally better performance than the random forest approach, and decision trees performed the worst.

Moving beyond these initial models, we developed a workflow in which ML-based upscaling methods feed into a continuum-scale Lagrangian simulator called FLAG [2, 4, 7]. While the upfront cost of generating the training data for this type of ML model can be expensive, every continuum-scale simulation that utilizes the ML upscaling experiences computational savings afterwards. This workflow has produced a continuum-scale simulator that is both accurate and faster by three to four orders of magnitude [2].

See **Scale-bridging** *on page 11*

A Simple Derivation of Heron's Formula

Heron's formula gives the area A of a triangle with sides a, b, c:

$$A=\sqrt{s(s-a)(s-b)(s-c)}, \quad (1)$$

where $s=\frac{1}{2}(a+b+c)$ is the semiperimeter. Most proofs hide the simple reason for this result. This reason is twofold:

1. An observation that A^2 is a polynomial of degree 4 in a, b, c.
2. $A^2=0$ if the triangle degenerates into a point or a segment, i.e., if $a+b+c=0$ or if $a+b-c$ or any of its cyclic permutations vanish.

Taking (1) for granted for the moment, (2) implies that

$$A^2=k(a+b+c)(a+b-c)(b+c-a)(c+a-b), \quad (2)$$

where the unknown constant k is independent of a, b, c. To find k, we apply (2) to a right triangle with sides 1, 1, $\sqrt{2}$, thus obtaining

$$\left(\frac{1}{2}\right)^2=k(2+\sqrt{2})(2-\sqrt{2})\sqrt{2}\sqrt{2},$$

or $k=1/2^4$. With this value, (2) becomes Heron's formula (1).

To justify (1), we write

$$A^2=a^2b^2\sin^2\theta=a^2b^2-a^2b^2\cos^2\theta,$$

where by the theorem of cosines $4a^2b^2\cos^2\theta=(c^2-a^2-b^2)^2$. Observation (2) allowed us to avoid the algebra of factoring A^2.

MATHEMATICAL CURIOSITIES

By Mark Levi

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

Frank Ramsey

Continued from page 6

achievements. Perhaps the most interesting of these, at least for SIAM members, was written by the late Ron Graham[1] about "Ramsey theory," which builds on a theorem that Ramsey proved in a paper published posthumously in 1931. One of the theory's consequences concerns a complete graph on n-vertices, each edge of which is colored either red or blue. If $n \geq 6$, one of the triangles in the graph must be either all red or all blue. Thus, 6 is the "Ramsey number" for the monochromatic triangle property; $n \geq 18$ for the monochromatic quadrilateral and $43 \leq n \leq 48$ for the monochromatic pentagon. Graham asserts that there seems to be little hope of anyone ever finding the Ramsey number for monochromatic hexagons. Other Ramsey numbers, associated with additional graph-theoretic properties, have been found. The subject lay largely dormant until 1947, when Paul Erdős introduced his powerful "probabilistic method" for establishing bounds for Ramsey numbers on graphs.

Another summary concerns Ramsey's paper on "Truth and Probability," also published posthumously in 1931. Ramsey argued that one need not infer probabilities from a record of past events when placing bets on the occurrence of one or more inherently unpredictable eventualities, such as the potential orders of finish in a horse race. Subjective probabilities are simply numbers that are meant to reflect a person's degree of confidence that a certain event or combination of events will occur. A carelessly formulated (i.e., internally inconsistent) list of probabilities is all but certain to include the ingredients of a Dutch book — a combination of wagers that guarantee the bettor a sure profit. There will be no such book only if the assigned probabilities obey Bayes' rule $P(A\&B)=P(A|B)P(B)$ for each pair (A, B) of potential outcomes. Although Bruno de Finetti is usually credited for this observation, careful scholarship reveals that Ramsey got there first!

Ramsey's two contributions to economic theory were just as pathbreaking and are equally well described by guest commentators. The first concerned an optimal tax code—one that raises a target amount of revenue with minimal damage to public utility (also known as satisfaction)—while the second identified an optimal allocation of current national income between present consumption and future investment. Both ideas were too mathematical to be understood by economists of the day (the first utilized a Lagrange multiplier and the second employed the calculus of variations). However, thanks to later economists who rediscovered them in the aftermath of World War II, the concepts are now recognized precursors of presently active subdisciplines.

To address Ramsey's findings in the field of philosophy, Misak quotes several living philosophers, all of whom arrived at some illuminating new insight — only to find that Ramsey had beaten them to it. She has crafted the sort of in-depth intellectual biography that, regrettably, will likely never be written about John von Neumann. Having been born in the same year, the two were almost exact contemporaries and were launched on parallel career tracks by the time of Ramsey's death in 1930. Ramsey was a don at age 21 and von Neumann was the youngest *privatdocent* in German history. It was possible for Misak to write such a book because interviews that were recorded more than 40 years ago for a never-written biography of Ramsey remain intact, and because many of the notes he exchanged with friends and colleagues are preserved. It seems implausible that one could assemble a similar trove of material on von Neumann.

It is interesting to speculate about the path that history might have taken if von Neumann had died in 1930 while Ramsey lived on. They shared an interest in the foundations of mathematics, and Ramsey likely would have been just as quick as von Neumann to appreciate the significance of Kurt Gödel's theorems. He would also have known of Alan Turing's solution to the *Entscheidungsproblem*, on which he himself had already begun to work. Ramsey would almost certainly have been assigned to the code-breaking unit at Bletchley Park—along with Turing, his teacher Max Newman, and several Cambridge colleagues—after the outbreak of World War II. While there, he would surely have been cognizant of Colossus, the world's first fully programmable computer. But unlike von Neumann, he would not have been at Los Alamos to work on the atom bomb, and his interest in computer development might or might not have survived the war.

All that aside, Misak has written a splendid biography of a rare genius who might have accomplished far more had he been granted his biblical three score years and ten.

James Case writes from Baltimore, Maryland.

[1] https://sinews.siam.org/Details-Page/obituary-ronald-lewis-graham

At Last, Our Hindsight is Truly 2020

By Ken Boyden

The generosity of the SIAM community has never been clearer—or more needed—than during the challenges that we have faced together in 2020. Since its incorporation in 1952, SIAM members have led with the vision that applied mathematics and computational science are vital to the advancement of humankind, both through research and solving problems in our society and in business, industry, and government. As we continue to navigate our way through difficult times in the present day, your work has never been more important — and your support of SIAM has never been more crucial.

Charitable donations in 2020 have been impressive, impactful, and inspiring. The dedication of SIAM's members and friends remains at the very foundation of our community, as evidenced by the numerous members who reliably make annual contributions to support SIAM's greatest needs. We are also exceedingly grateful for the outpouring of support after the Board's recent establishment of the *James Crowley Endowed Fund for Student Support*,[1] which honors the extraordinary career of recently-retired SIAM executive director Jim Crowley.[2]

As we approach the new year prepared to face new realities and challenges, your support remains just as essential to our mission, members, students, and society. SIAM is an organization that is built to tackle both important issues and the complex conditions that encompass them. 2021 also celebrates the beginning of the exciting tenure of Suzanne Weekes as SIAM's third executive director.[3] It is remarkable to note that despite SIAM's nearly 70-year history, Dr. Weekes now joins the staff as only the third leader of our organization. Clearly the future is bright.

Mark Twain is frequently credited with the succinct observation that "History doesn't repeat itself, but it often rhymes." This past year has reminded us that the world suffered through a 1918 influenza pandemic that was in many ways similar to COVID-19. However, the ongoing pandemic threat yet again reiterates the fact that humankind is resilient, robust, and resurgent. In these times of great change and challenge, a profound truth gives us hope: the collective SIAM community has an extraordinary impact on society. SIAM remains a valuable resource to our members through its achievements in education, publications, collegiality, the promotion of research, and more. 2020 has proven that SIAM also provides our members with certainty in uncertain times.

As we look forward to 2021 with enthusiasm, gratitude, hope, and the conviction that SIAM will continue to fulfill our mission, we express thanks to our community. Our conferences, publications, geographic sections, activity groups, networking and collaboration opportunities, prizes and award programs, and student support—including travel, scholarship, and mentorship opportunities—distinguish SIAM from other organizations. While anticipating 2021 and reflecting upon SIAM's value to your own career, please consider supporting SIAM by making a charitable gift to *your* society. Gifts can be made at **www.SIAM.org/donate**. If you prefer, you can of course also mail a check addressed to SIAM at 3600 Market Street, 6th Floor, Philadelphia, PA 19104 U.S.

However, if you decide that you would like to speak confidentially about making an impact on the future wellbeing of SIAM and our numerous programs through an estate gift, please email me directly at **boyden@SIAM.org**. Gifts that are made through bequests, charitable trusts or annuities, and transfers of real estate or appreciated securities are impactful and important to SIAM and our community. 2020 demanded that we all make countless sacrifices in many different ways; SIAM itself adjusted throughout the course of the year to best ensure that our values, service, and member support remained resilient.

If you appreciate the impact that SIAM's strategic international initiatives have made to strengthen both your career and the global landscape of applied mathematics and computational science, please make a gift before the close of 2020 to support our work in 2021.

At long last, our hindsight is finally 2020. Yet because of our members and our unyielding commitment to lead, 2021 promises to be a year of renewed success and leadership. On behalf of the SIAM Board, Council, staff, members, and student beneficiaries, thank you for your considered financial support and association.

Ken Boyden, Esquire is the Director of Development and Corporate Relations at SIAM.

[1] https://sinews.siam.org/Details-Page/new-james-crowley-endowed-fund-for-student-support

[2] https://sinews.siam.org/Details-Page/executive-director-jim-crowley-retires-after-25-years-of-siam-leadership

[3] https://sinews.siam.org/Details-Page/dr-suzanne-l-weekes-named-siam-executive-director

Mathematics and the Social and Behavioral Sciences

By Donald G. Saari

The following is a short reflection from the author of Mathematics Motivated by the Social and Behavioral Sciences, which was published by SIAM in 2018 as part of the CBMS-NSF Regional Conference Series in Applied Mathematics.

The title of my 2018 book, *Mathematics Motivated by the Social and Behavioral Sciences*, may seem like an oxymoron to many readers. But given that serious problems in the social and behavioral sciences confront us on a daily basis, the title actually reflects an important invitation for more mathematicians to get involved with this area of research. My book, which captures portions of my Conference Board of the Mathematical Sciences (CBMS) lectures on this general topic, identifies several relevant challenges. As I state in the preface:

> "The mathematics needed to advance the social and behavioral sciences most surely differs from what has proved to be successful for the physical sciences. Remember, a strong portion of contemporary mathematics reflects a fruitful symbiotic relationship enjoyed by mathematics and the physical sciences over a couple of millennia: Advances in one area motivated advances in the other. As it must be expected, this intellectual relationship shaped some of our mathematics and influenced the way in which certain physical sciences are viewed. Centuries of experimentation in the physical sciences, for instance, led to precise measurements and predictions, which motivated the creation of mathematical approaches, such as differential equations, that allow precision predictions.
>
> As fully recognized, it is unrealistic to expect "precise" predictions for many issues in the social and behavioral sciences. But researchers have access only to limited number of mathematical approaches, where favorite choices for theoretical models tend to involve methods designed for precision predictions — not much else is available. This comment underscores the need to develop appropriate mathematical tools that, rather than designed for exactness, reflect the current status for much of the social and behavioral sciences, which requires qualitative predictors."

These sentiments lead to chapter one: "Evolutionary Game Theory." The social and behavioral sciences are dominated by change. Everything changes: opinions, economics, politics, and preferences. Because the best way to model this phenomenon is unclear, researchers rarely examined "change" — until recently. Much like the story of a drunk who is searching around a streetlight for the keys he lost elsewhere because "the light is better here," we tend to emphasize things that can be analyzed with currently available techniques. We seek results where there is sufficient "light," such as attempting to find equilibria without any knowledge or exploration of the associated dynamics.

Multiple factors—including a lack of reliable information—hinder our understanding of how to model change. In many cases, that which is best known reflects behavior in specialized, local settings. The qualitative approach that I develop in the first chapter thus emphasizes the way in which one can connect local information with a global dynamic. Because we know so little about the dynamics, we should keep the emphasis on qualitative modeling — wherein refinements must come from the host area data.

FROM THE SIAM BOOKSHELF

Adam Smith's invisible hand metaphor, which is a supply-and-demand aggregation process that combines the economic agents' preferences and resources, serves as another example. The key word of "aggregation" is central across the social and behavioral sciences. Statistical methods, probabilistic predictions, migration, social movements, political processes, and so on all involve aggregations for which even reasonably correct assertions require sound methods. But to the best of my knowledge, there is no complete and general mathematical analysis that describes the potential pitfalls of aggregation rules and explores what can go right or wrong.

An obvious obstacle is the overwhelming number of dissimilar aggregation approaches that cloud the issue. One can handle this problem by embracing Occam's razor, which in contemporary terms is the KISS philosophy ("Keep it simple, stupid"). My initial emphasis in chapter two was therefore to examine a particular aggregation class: voting methods. These methodologies are often linear aggregations, meaning that we can reasonably expect the successful transfer of any resulting lessons to more complicated settings. This is because a standard mathematical way of identifying a system's characteristics involves the use of linear approximations of derivatives, tangent spaces, and so forth.

An examination of voting systems might seem mathematically trivial. After all, commonly used voting rules just sum ballots; what can go wrong? This question reflects my seriously mistaken initial attitude. A clue should have come from actual events with pundits wondering, "How did so-and-so win the election?" In fact, a telling measure of the intricacies of paradoxical outcomes is that one can use the complexity of chaotic dynamics to identify the characteristics, number, and types of these mysteries! The difficulties are mind boggling (a more detailed description is available in section 2.3 of my book); using a thousand of the fastest computers, it would be impossible to count (not even list) the plurality vote paradoxical outcomes that arise with only eight candidates — even if the counting had started at the Big Bang.

These troubling, unanticipated behaviors help identify unexpected properties of other aggregation tactics. Understanding paradoxical behavior in voting provides guidelines for the discovery of similar actions in aggregation methods, ranging from bizarre features of the aggregate excess demand function in Adam Smith's supply-and-demand story to puzzling behavior in nonparametric statistics. Chapters three and four address a selection of these topics.

Now I'll move to a different subject. When introducing vectors or eigenvectors to my students, I always confess that there are far too many vectors—even in just two dimensions—for one to intimately know them all. A convenient approach is to become acquainted with carefully selected choices, such as $\mathbf{i}, \mathbf{j}$ or the eigenvectors, and then describe all other vectors in terms of their relationship to our newly acquired friends.

This commentary reflects the common mathematical methodology of dividing a construct into component parts to clarify the analysis. Although researchers apply aspects of this useful approach to differentiate features of observations in areas like psychology, it has not been generally adopted to address mathematical concerns in the social and behavioral sciences. The strong advantages of doing so are themes of chapters four and five. To ensure consistency in the described topics, my illustrating choices come from the first two chapters; I demonstrate how one can use symmetries to decompose voting rules and games (many of the game theory results involve joint work with Dan Jessie). Both decompositions significantly simplify the discovery of new conclusions.

The final chapter of my book addresses the customary reductionist approach; all readers are likely familiar with this whole-parts system analysis. This realistic approach handles a complex problem by dividing it into tractable parts, solving the questions that each part poses, and assembling the answers into a solution for the whole. Although it is widely used, the reductionist approach can suffer serious, unexpected problems. As outlined at the end of the concluding chapter, many of the complexities that I describe in my book reflect unanticipated consequences of this method. The positive angle is that understanding the source of the difficulties helps us identify the causes of many complexities that the social and behavioral sciences face. This description of "what can go wrong" extends to shed light even on problems from engineering and the physical sciences, such as the compelling dark matter mystery of astronomy. Comprehending the causes of problems focuses our attention in a search for resolutions.

My hope is that readers of this book will join me in exploring the mysteries of the social and behavioral sciences.

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Scale-bridging

Continued from page 8

At LANL, our workflow is of particular interest for high strain rate scenarios that occur during explosions. High-impact velocities (sometimes on the order of $1\ \mathrm{km/s}$) produce a high strain rate. Figure 2a depicts the setup for a flyer plate experiment wherein the flyer plate crashes into the target plate with an impact velocity of $721\ \mathrm{m/s}$.

A critical insight during the development of the upscaling ML model was the observation that a small set of features could accurately inform the continuum-scale model. In our case, the two key features were the maximum stress and length of the longest crack. We took this into account while also acknowledging that the full crack length distribution, crack orientations, and stress fields at the microscale do not seem to play a substantial role in this setting. The combination of ML upscaling and the continuum scale model FLAG, referred to as FLAG-ML, was able to make predictions as accurately as HOSS. FLAG-ML also showed good agreement with experiments that utilized an impact velocity that was approximately 73 percent faster than the HOSS simulations that we used for training. Figure 2b shows the ML prediction for the shock wave velocity at the bottom of the target plate as a function of time. The ML emulator was trained with the HOSS simulation data of a $0.721\ \mathrm{km/s}$ flyer plate impact, and FLAG-ML was validated against experiments for which the flyer plate impact velocity was $1.2\ \mathrm{km/s}$. This powerful result could enable transfer learning — the application of an ML model that is trained for a particular problem to a similar but somewhat different problem.

The relentless growth of computational power over the past several decades has enabled the use of detailed, high-fidelity simulations. Now it employs a large amount of data to drive the training of fast ML models. Scientists often exploit data from high-fidelity simulations to train these ML models; this dynamic can influence the study of material behavior under extreme loading conditions, as the fast ML models are able to bridge scales. While significant progress has occurred in this field, the work is representative of a new beginning in upscaling and scale-bridging. Researchers traditionally had to perform years of in-depth studies to upscale complex phenomena, sacrificing accuracy for efficiency through approximations that neglect known physics. ML provides an alternative that can exchange people-years for central-processing-unit-years. Although this assertion does not downplay the importance of the traditional tactics, there are grounds for optimism that a data-centric ML approach will offer solutions to challenging problems.

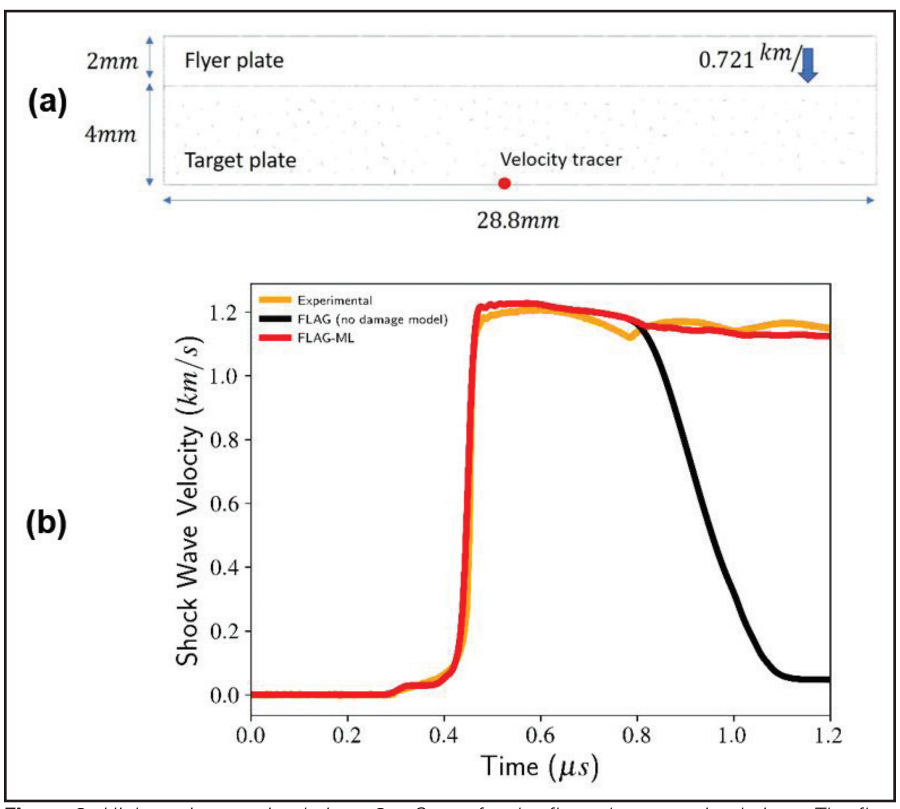

Figure 2. *High strain rate simulations.* **2a.** *Setup for the flyer plate test simulations. The flyer plate has a starting velocity of* $0.721\ \mathrm{km/s}$ *and initially has contact with the target plate.* **2b.** *Evolution of the shock wave velocity at the middle rear of the target plate for a second experiment with an impact velocity of* $1.246\ \mathrm{km/s}$. *FLAG-ML improves the predictions obtained by FLAG without an imposed damage model. Figure courtesy of [2].*

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A Simple Proof of Fermat's Last Theorem?

Some time ago, while working on a very difficult math problem, it occurred to me that one could also apply the strategy that I was using to find a possible simple proof of Fermat's Last Theorem (FLT). In brief, the strategy is as follows:

Assume that a counterexample $x^p + y^p = z^p$ to FLT exists, where x, y, z, and p are positive integers and p is a prime — the smallest such prime, in fact.

Write this equation as $x^p + y^p - z^p = 0$.

One can then express this equation as the inner product $<(x^{(p-k)}, y^{(p-k)}, z^{(p-k)}), ((x^k, y^k, -(z^k)) >= 0$, where $1 <= k <= (p-1)$. Exploit some of the facts that follow.

I welcome comments on "First Approach" in the "…Inner Products" section referenced in "Important Note" on the first page of the first part of "Is There a 'Simple' Proof of Fermat's Last Theorem" on **occampress.com**. Each approach uses only freshman-math-major mathematics. I guarantee complete confidentiality in all communications.

My degree is in computer science, and for most of my career I have been a researcher in the computer industry. Wiles' proof in the early 1990s of the part of the Shimura-Taniyama Conjecture that implies FLT was well over 100 pages of some of the most sophisticated mathematics of its time. I hence believe that it is natural to wonder if there might not be a simpler proof.

— *Peter Schorer,* peteschorer@gmail.com

When Data Meets Diversity

By Brianna C. Heggeseth and Chad M. Topaz

In 2017, social media discussions about the opening of a new wing at the Massachusetts Museum of Contemporary Art first alerted us to the dearth of works by women and people of color in major museum collections. When we asked Steven Nelson—now Dean of the Center for Advanced Study in the Visual Arts at the National Gallery of Art in Washington, D.C.—about the magnitude of underrepresentation, he noted that no researchers had ever gathered a data set with the size and completeness required to address this issue in a systematic way.

Together, we built the first collaborative research effort to quantify certain axes of demographic diversity among artists with works in the permanent collections of major art museums [3]. The lack of publicly available museum collections data, as well as the lack of data and consistency standards across museum curatorial databases, complicated our research efforts. Though we were eventually able to locate and standardize our data, the process was cumbersome and time-consuming.

The response of major museums to the COVID-19 pandemic has since shifted the terrain. Museums are increasingly putting their collections online,[1,2] thereby presenting a golden opportunity for data scientists to ask and answer questions that were previously inaccessible. To this end—and noting that many areas of applied mathematics are now inextricably dependent on data—we present some opportunities and complexities of museum collections data.

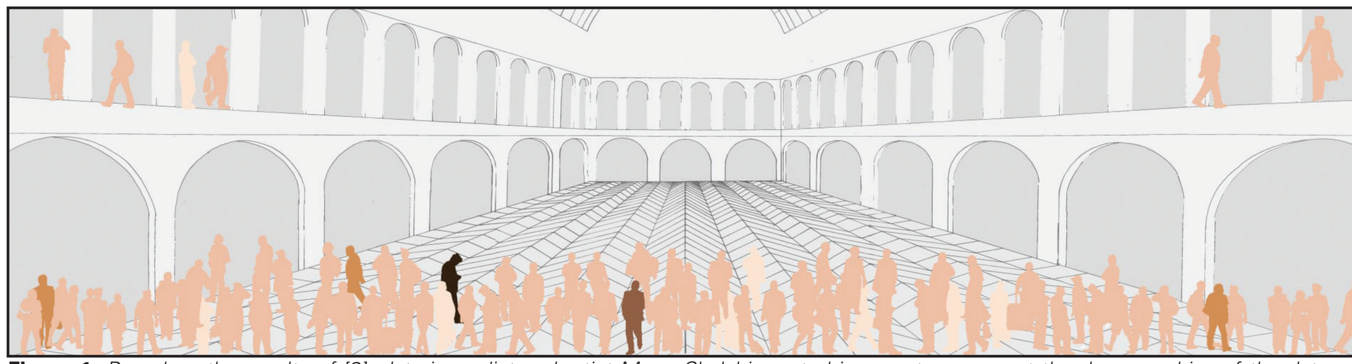

Figure 1. *Based on the results of [3], data journalist and artist Mona Chalabi created images to represent the demographics of the data set, scaled down from over 10,000 artists to 100 for the purpose of visualization. In this case, 88 of the artists would be men (75 White, eight Asian, three Latinx, one Black, and one man of another race/ethnicity). Figure courtesy of Mona Chalabi [2].*

Accessing Museum Data

Museum collections data and metadata are curated predominantly in an industry standard software package called The Museum System (TMS). TMS allows museums to record acquisitions; the movement of art objects and their separable or non-separable components; and object metadata such as titles, artists, dates, and dimensions. The data formats' internal consistency depends on the historical implementation of data content standards, which were established by museum registrars and database managers. In 2006, the Cataloguing Cultural Objects [1] standards were published to "move toward shared cataloging and contribute to improved documentation and access to cultural heritage information." While these data standards are gaining traction in the U.S., vocabularies (e.g., object classifications and artist nationality) and data entry formats (e.g., date formats and artist name) are still inconsistent. Missing data is also an issue, as some information has been lost in the transition to digital cataloguing systems; this is especially true of early acquisitions.

Until the last decade, only museum staff could access this data. The rise of social media and digital communication inspired major art museums to engage their audiences with online collections on their websites. While this move increases data accessibility, the collections' HTML formatting is often inconsistent. Data scraping is possible, but it is cumbersome since it must be customized for each museum.

1 https://www.nytimes.com/2015/01/30/arts/design/art-museums-are-increasingly-adding-their-collections-online.html

2 http://www.getty.edu/foundation/initiatives/current/osci

Analyzing Museum Data

Coding artist data for gender, ethnicity, and other demographics can be difficult, as these metadata are not often stored in TMS. This leaves any data scientist who is interested in anti-racism and other forms of social justice with three primary options for coding the relevant data.

First, one could manually code artist profiles. For example, living artists can self-identify or art historians can infer demographic characteristics based on primary sources. Second, researchers might decide to consult data sets such as the Getty Research Institute's Union List of Artist Names Online (ULAN),[3] a large database of artists that includes some information about gender and other identities. However, linking ULAN to art museum website data requires that one manually add the static URL from Wikipedia or the ULAN ID to each piece on a museum's website. As an additional challenge, the available race and ethnicity information appears within a variable called "nationality," which, perhaps surprisingly, "contains reference to the nationality, culture, ethnic group, religion, or sexual orientation associated with the person."

Finally, large-scale coding through Human Intelligence Tasks on web-based crowdsourcing platforms like Amazon Mechanical Turk provides a mechanism that handles sizable data sets at scale, especially when the aforementioned two options are infeasible or impractical. This crowdsourcing approach requires that laypersons read secondary sources online and make informed guesses about ethnicity and gender on a random sample of artists. We expect more erroneous inferences from non-experts, so statistical techniques ensure inter-rater reliability and provide confidence intervals on aggregate museum-level statistics.

It is important to remember that all demographic data, with the exception of information that the artists provide themselves, are *inferred* data. An individual's characteristics, such as gender and ethnicity, can be reliably stated only by the individual themself.

3 https://www.getty.edu/research/tools/vocabularies/ulan

Our Projects

Our first deep dive into museum data sought to measure the (under)representation of female artists and artists of color in 18 major American art museums [3]. Within this group of museums, we estimate that 85 percent of artists are White and 87 percent are men (see Figure 1). Some people believe that the large proportion of White people and men "makes sense" because museum collections sometimes focus on time periods and geographic regions in which those two groups dominated artistic production. However, even putting aside issues pertaining to whose work is considered art and valued by society, our results support a different conclusion.

To reach our conclusion, we clustered the 18 museums in two different ways. First we clustered them by their "collection habits" — the time periods and geographic regions in which their art was created. Next we clustered them by their estimated demographic percentages for gender and ethnicity. Quite simply, these two clustering schemes are uncorrelated. For instance, the Museum of Fine Arts in Boston and the Detroit Institute of Arts both have catalogs in which the average artist birth year is around 1800 and roughly 30 percent of artists are of North American origin. However, we estimate that 95 percent of identifiable artists in the Detroit catalog are White, in contrast to only 80 percent of the artists in Boston. Of course, our study represents one snapshot in time. Collections are subject to change as museums make acquisitions and loan, sell, or gift various pieces.

In response to our work, the National Gallery of Art invited us to participate in a two-day datathon,[4] during which they allowed us full access to their internal data stores. Our major focal point for this event was the representation of women and artists of color on public view in the gallery spaces and exhibitions curated by the National Gallery, as differentiated merely from their representation within the catalog. More specifically, we sought to answer the question, "Whose art is being seen by the public as they visit the museum?" Using metadata from TMS and location history data for the individual pieces, we found that over 75 percent of the art objects in public view at the National Gallery are attributable to an identifiable male and/or White artist.

However, the last five years have seen an increase in female and Black representation, primarily in the renovated East Gallery. Contemporary photographs, prints, and drawings by female and Black artists have driven this shift. The gallery staff informed us that while these media are more financially accessible to new artists, they are also more physically sensitive and can only be on public view for short periods of time. We created an online interactive visualization tool[5] for gallery staff, the public, and other researchers that explores representation across the gallery space and over time.

4 https://www.nga.gov/audio-video/audio/datathon.html

Next Steps

Based on the results of our study, we recommend that museums immediately focus on standardizing data and metadata practices to allow for greater transparency. These improvements would enable easier and more rigorous data collection and analysis, thus helping researchers identify the extent of the underrepresentation of minoritized artists. The time is long past for major art museums to become activist collectors, emphasizing the work of women and artists of color in their collection practices to address historical underrepresentation and bias. Some museums have already begun to engage in this practice.[6]

This area of study presents numerous opportunities for the applied mathematics, statistics, and data science communities. When subject to the right disciplinary and data cleaning expertise, museum databases can serve as looking glasses into museums' degrees of success in living up to their missions, including their attention to diversity. Local art museums may have limited in-house resources and could be open to collaboration with data scientists and applied mathematicians. For example, we are currently working with the Minneapolis Institute of Art to analyze their collection's accession and deaccession history, apply natural language processing to facilitate tag creation, and plan for data improvements and consistency. Greater communication between and within art and the mathematical and computational sciences could help improve museum data management systems, ultimately enabling greater progress towards diversification within the collections.

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5 https://bheggeseth.shinyapps.io/DiversityOnDisplay

6 https://bit.ly/3jSIpH5