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In Pursuit of Perfect Pinnacles

By Leif Ristroph, Jinzi Mac Huang, and Michael Shelley

Michelangelo described sculptures as captives that are held inside stone—or perhaps trapped in an artist's mind—and freed when one cuts away all of the excess material. Nature also seems to have preconceived notions of what her sculptures should look like, assigning recognizable and repeated shapes to landforms and other structures that wind and water currents chisel from rock, soil, and ice. The morphology and development of natural formations can have unexpectedly beautiful mathematical structures, as demonstrated by recent and ongoing investigations into sharply pointed pinnacles that form in soluble rock and melting ice.

Geomorphology—the discipline that explains and interprets geological forms—is often empirical due to the many complex and interactive processes that occur over widely varying length and time scales. But in some cases, researchers can identify the dominant processes and capture their physics with models. When this method is possible, employing a mathematical view of erosion, dissolution, melting, and so forth in the context of moving or free boundary problems is a powerful approach [4, 9].

Pinnacles are brashly defiant geological formations. While most structures tend to lose their edges and acquire softer features as they yield to erosion—rounded beach pebbles and the slumped slopes of older hills and mountains are prime examples—pinnacles seem to grow sharper and stand taller. Their prickly personalities are clearly visible in landscapes called stone forests (see Figure 1a). These so-called karst formations appear in soluble and porous minerals such as limestone, which dissolve over thousands of years. The most striking instances are Madagascar's *Tsingy*, an apt name in the indigenous language that roughly translates to "where one cannot walk barefoot." Ice can also melt into pinnacles, as evident when icebergs turn over and show their undersurfaces (see Figure 1b). Dissolving and melting are similar processes during which solute concentration plays much the same role as temperature.

Sharp spikes are quite unexpected in light of the classical Stefan problem for melting, wherein the thermodynamics of solid-to-liquid phase transitions dictate the interface dynamics. This classical problem considers thermal diffusion in the two phases, heat liberation at the boundary, and the Stefan condition for boundary motion. The latter is provided by Fick's law, which states that the local melt rate is proportional to the normal gradient of temperature in the liquid. These effects tend to suppress high curvatures. However, gravity is missing from this idealization and proves critical to pinnacle formation. In most earthly situations, melting (or dissolving) leads to density differences in the surrounding liquid that produce gravitational or buoyancy-driven flows. These flows strongly modify the temperature (concentration) field and hence the dynamics of the receding surface. Given initial conditions, this Stefan problem with gravitational convection seeks to determine the shape development.

Recent investigations into this issue illustrate the subtlety of shape dynamics problems and the value of combining

See **Perfect Pinnacles** *on page 3*

Figure 1. *Rock and ice pinnacles in nature.* **1a.** *Limestone formations from around the world reveal sharp spikes that often form in arrays called stone forests.* **1b.** *Melting ice displays similar features. Photographs courtesy of Steven Alvarez, Grant Dixon, Phillip Colla, and Stephen Nicol.*

Machine Learning for Multiscale Systems: From Turbulence to Climate Prediction

By Dhruv Balwada and Laure Zanna

Continued improvements in the prediction of our planet's future state—whether it be via weather reports on the order of days to weeks or climate forecasts for the coming decades—have been one of the great revolutions of the last century. This foretelling power relies on our ability to solve mathematical equations that describe the dynamics of natural systems. Such equations combine first principles with empiricism and often represent conservation and/or thermodynamic laws. These equations also comprise climate models, which are generally composed of interacting parts that represent distinct components like the ocean, atmosphere, ice, and/or land. The behavior of each component is highly nonlinear and turbulent, and the coupling results in emergent variability that is more complex than the sum of its parts (e.g., the dominant non-seasonal mode of variability on Earth that is called the El Niño-Southern Oscillation).

Climate as a Set of Coupled Multiscale Systems

The complexity and turbulent behavior of natural systems often stems from interactions between a wide range of spatial and temporal scales. For example, oceanic flows are frequently dominated by chaotic vortices that are approximately 100 to 200 kilometers (km) in diameter; the size of these structures is not a direct reflection of the forcing scales (e.g., atmospheric winds or solar heating that vary on scales larger than 1,000 km) or the instability (e.g., baroclinic instability with scales of 10 to 50 km). Instead, their size is a result of the natural propensity of vortices in flows with large aspect ratios to merge and form larger vortices. In the atmosphere, clouds develop from microscale processes and impact Earth's albedo and thus the planetary-scale energy balance. In fact, three researchers received the 2021 Nobel Prize in Physics[1] for their work towards understanding and modeling these complex nonlinear multiscale systems.

Identifying accurate and representative solutions to such multiscale systems requires the resolution of an extensive range of scales—from millimeters to thousands of km—which is impossible for any modern computer and likely implausible for any computational system that will arise in the near future. Climate scientists therefore solve the equations for the (resolved) scales that are computationally feasible and most useful for decision-making purposes, while also parameterizing the impacts of the unresolved scales (known as the closure problem in fluid dynamics).

We can conceptually solidify this parameterization challenge by considering the partial differential equations that describe the turbulent flows in the ocean or atmosphere:

$$\frac{\partial Y}{\partial t} = -\nabla \cdot (\mathbf{u}Y) + F.$$

Here, $\mathbf{u}$ signifies the velocity vector, Y corresponds to quantities like momentum (velocity) or tracers (e.g., temperature), and F represents forcings, sources, sinks, pressure gradients, dissipation, and so forth. The flux—i.e., the multiplicative term $(\mathbf{u}Y)$ on the right—usually encompasses the multiscale interactions that emerge in turbulent flows. However, solving these equations on a finite grid fails to resolve the scales that are close to or smaller than the grid resolution. We can thus mathematically represent the true solution (Y) as $Y = \underline{Y} + Y'$, where Y is written as a sum of the resolved $(\underline{Y})$ and unresolved (Y') components respectively. For the sake of

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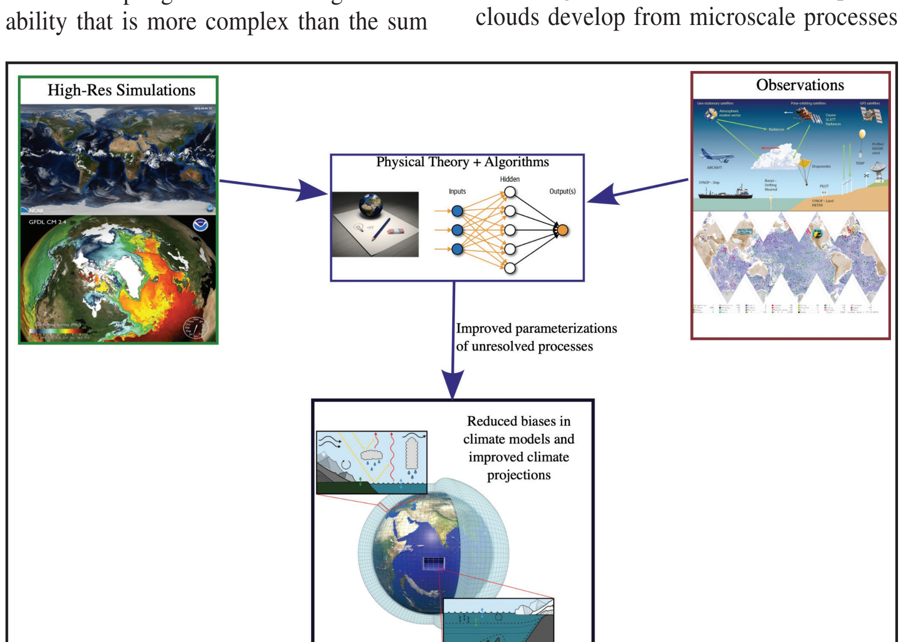

Figure 1. *Physical theories and machine learning algorithms can work together to utilize information from high-resolution simulations and observations in the pursuit of better parameterizations. Such parameterizations can potentially reduce biases in climate models and improve climate projections. Figure adapted from [2] and [7].*

[1] https://www.nobelprize.org/prizes/physics/2021/summary

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Preparing Future Generations to Address Global Pandemics with Innovative Mathematical Thinking

By Padmanabhan Seshaiyer

While researchers continue to apply mathematics to increasingly complex real-world problems, sustainability challenges—as posed by the United Nations' Sustainable Development Goals[1] (UN SDGs)—remain of the utmost importance, especially in the context of COVID-19 (see Figure 1). For example, *Goal 3: Good Health and Well-being* aims to "Ensure healthy lives and promote well-being for all at all ages." This goal presents a major opportunity for mathematicians who wish to study the impact of COVID-19 and use their findings to tackle global challenges that relate to food production, health security, clean water, affordable and clean energy, climate change and ocean health, peace and justice, and so forth. To do so, they must utilize innovative approaches that integrate knowledge from various science, technology, engineering, and mathematics disciplines [4].

Goal 4: Quality Education seeks to "Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all." As we know, COVID-19 has affected education at all levels. By March 23, 2020, roughly 80 percent of learners around the world—from pre-primary to post-tertiary levels—faced the physical closures of their educational institutions. While the pandemic created numerous challenges, including the forced adoption of online learning for most educational establishments, it also offered creative opportunities for educators to exchange lessons and insights.

Here I share an example from a problem-solving course at George Mason University that transitioned to online instruction in 2020. The course, which was open to undergraduate students from multiple disciplines with a dual enrollment option for high school students, required participants to work in multidisciplinary teams and generate potential solutions for the UN SDGs. Students selected specific targets from the goals, applied global problem-solving principles, and considered future implications for research and development.

The course's design initially centered on face-to-face meetings, during which students worked on semester-long team projects that comprised the backbone of the class experience. This format introduced students to various skills and competencies, such as problem-solving, mathematical modeling, computational thinking, and communication techniques. During the course's second phase—which commenced in early March 2020 before pandemic-forced closures—self-assigned student teams chose global design challenges from the UN SDGs to pursue for the remainder of the course. However, COVID-19 constraints in mid-March forced us to rapidly contrive and implement novel, meaningful online team project experiences that allowed students to apply knowledge from previous exercises.

Many students were stressed by the unfolding situation and wanted to work on projects that pertained to COVID-19 and *Goal 4: Quality Education*. We thus devised assignments that encouraged creative thinking and required students to utilize previously learned skills and techniques. These assignments asked participants to employ data-driven decision-making tools that use simple mathematical ideas—like weighted means through a decision matrix framework—to identify the best vendor for delivering face masks. Students also utilized computational thinking via a *Fermi approach* to estimate the number of required ventilators given the daily growth of COVID-19 cases. The latter technique, which was difficult to solve with specific methods, could inspire similar approaches that estimate the necessary number of beds to support acute cases of COVID-19.[2] In addition, students learned to apply a human-centered *design thinking*[3] approach to develop ideas that addressed quality education and COVID-19.

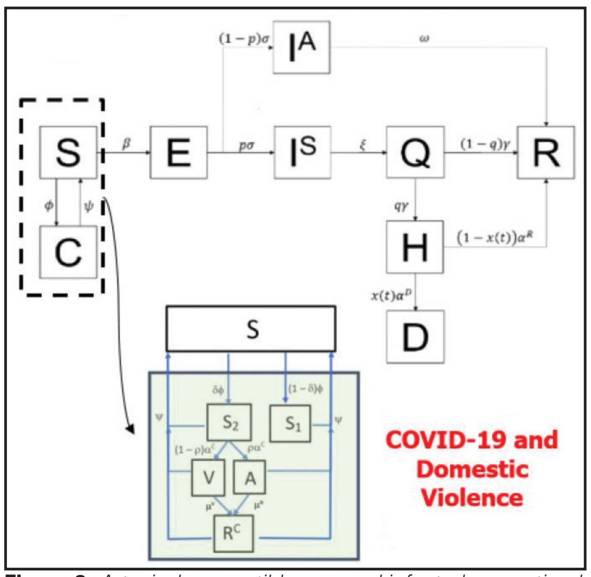

Figure 2. *A typical susceptible-exposed-infected-quarantined-hospitalized-recovered (modified SEIR) model. A confinement compartment accounts for domestic violence and further incorporates the dynamics of members who are susceptible to domestic violence, victims (V), abusers (A), and those who are removed. Figure courtesy of Padmanabhan Seshaiyer.*

An exposition of simple yet powerful mathematical approaches provides students with an uncomplicated introduction to a complex topic and prepares them for similar grand challenges. One way to continue this excitement is to engage students in research that fosters their own sense of agency [3]. Several course participants wanted to delve deeper into the mathematical research, so we exposed them to *graph-theoretic approaches* for understanding *contact tracing*, epidemiological modeling with *differential equations*, and machine learning techniques via *physics-informed neural networks* [2]. Students also learned about the importance of data collection, interpretation, visualization, analysis, and prediction, and discovered how to create user-friendly visualizations—such as *dashboards*—to understand disease spread. In response, they made connections between mathematical research and its capability to address specific challenges from the UN SDGs.

A sample project builds upon the analysis of Comfort Ohajunwa,[4] a current high school senior who has been working with me since July 2020 on a mentored research project to develop a new mathematical model that tracks the impact of social behavior during COVID-19 [1]. Ohajunwa's efforts have already led to four peer-reviewed journal publications; her most recent work helped generate novel models and ideas that pertained to *Goal 16: Peace, Justice, and Strong Institutions* of the UN SDGs, which intends to "Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable, and inclusive institutions at all levels."

Specifically, Ohajunwa identified a relevant global challenge that emerged due to the various lockdowns and confinement strategies throughout the world. While these interventions were implemented to save

See **Mathematical Thinking** *on page 3*

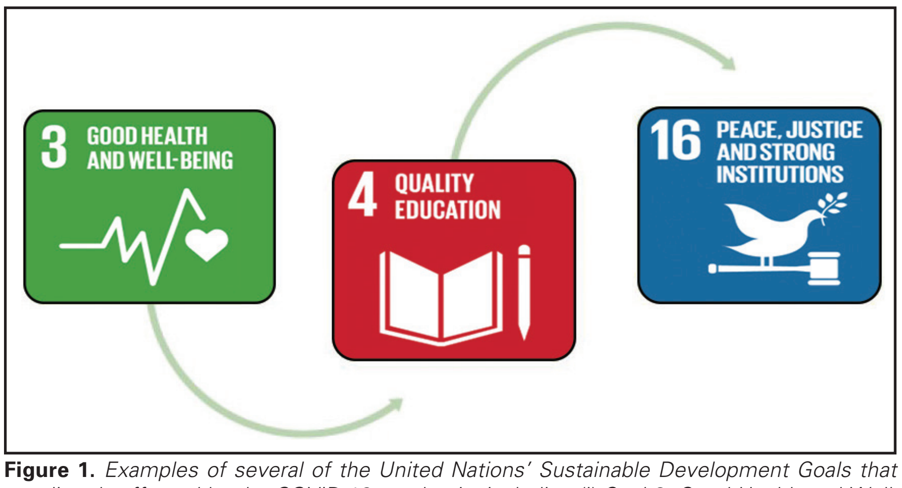

Figure 1. *Examples of several of the United Nations' Sustainable Development Goals that are directly affected by the COVID-19 pandemic, including (i) Goal 3: Good Health and Well-being, (ii) Goal 4: Quality Education, and (iii) Goal 16: Peace, Justice, and Strong Institutions. Figure courtesy of Padmanabhan Seshaiyer.*

[1] https://sustainabledevelopment.un.org

[2] https://sinews.siam.org/Details-Page/covid-19-models-mathematics-and-myths

[3] https://sinews.siam.org/Details-Page/applying-design-thinking-to-mathematics-research

[4] https://www.gmu.edu/news/2021-07/governors-school-innovation-park-student-comfort-ohajunwa-leads-exceptional-stem

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Perfect Pinnacles

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mathematical modeling and analysis with laboratory experiments and numerical simulations. A review of this pursuit begins with a 2015 study that poses the question, "Do dissolving objects converge to a universal shape?" [6]. Experiments on upright cylinders of amorphous or noncrystalline solids that were immersed in initially quiescent water revealed the formation of sharp-tipped spires that approach a quasi-paraboloidal form at long times. Further experiments used hard candy as a "mock rock" and indicated that such spires sharpen indefinitely — or at least for as long as the ever-thinning shape can be accurately measured [1]. These first experiments thus suggested that the late-stage dynamics involve a quasi-paraboloidal form whose curvature grows continuously in time. Figures 2a and 2b offer some typical images and data from the aforementioned experiments.

Subsequent works then established the mathematical foundations [5, 7]. The full dynamical equations include the incompressible Navier-Stokes equations for flow, an advection-diffusion equation for the solute concentration c in the fluid, and Fick's law $V_n \propto n \cdot \nabla c$ or the recession velocity of the interface with outward normal n. Since the driving flows are stably attached to the surface (see Figure 2d), one can utilize boundary layer theory to derive a moving interface model. The key dynamical equation

$$V_n = -a \frac{r(s,t)^{1/3} \cos^{1/3} \theta(s,t)}{\left[\int_0^s r(s',t)^{\frac{4}{3}} \cos^{\frac{1}{3}} \theta(s',t) ds' \right]^{1/4}}$$

is applicable to three-dimensional axisymmetric bodies and specifies the normal velocity at any boundary point at radius r from the axis of symmetry, arc length s from the tip, and tangent angle θ of the surface (see Figure 2e). Fluid and solid material properties determine the constant a. The numerator intuitively indicates that steeper slopes dissolve more quickly due to faster flows, while the integral in the denominator captures the lower rate of dissolution due to the accumulated solute from points upstream. This reduced system takes the form of a nonlinear integro-partial differential equation that is completed by the geometric evolution equation $\frac{\partial \theta}{\partial t} = \frac{\partial V_n}{\partial s} + V_s \frac{\partial \theta}{\partial s}$. An artificial tangential velocity of $V_s = \int_0^s V_n \frac{\partial \theta}{\partial s} ds'$ is convenient for separating s and t as independent variables [2, 3, 5].

Even with this framework in hand, understanding the shape dynamics—especially the behavior of the apex—has proven challenging. Numerical solutions of a Cartesian formulation of the model demonstrated nonuniversal behavior; different initial shapes yield different terminal dynamics [7, 8]. Sufficiently slender initial shapes, such as quasi-cylindrical forms, were shown to sharpen over time via power law growth of tip curvature. Contrarily, another study presented a local analysis of the tip region that suggests the tip curvature blows up via a finite-time singularity [5]. The addition of a curvature regularization term produced numerical solutions that yielded good agreement with the shape dynamics, which were measured experimentally. Although these studies differ in quantitative details, they do concur that hydrodynamics drive the formation of an ever-sharpening needle. They also agree on the physical origin of this effect. Higher dissolution rates near the top come from thin boundary layers that entrain the fresh outer fluid, while lower rates further down the body result from the insulating effect of thicker layers, which are laden with the solute that was released from upstream points.

Recent work paints a different picture of the longtime dynamics [3]. Numerically propagating the characteristics of the previous system causes the apex to sharpen dramatically before eventually saturating to a finite value for the tip curvature as the object approaches a terminal shape. Furthermore, the authors present an elegant analysis of the model that furnishes exact solutions for boundaries that preserve their shapes as they recede. One can express these equilibrium shapes as the distribution of curvature over the tangent angle

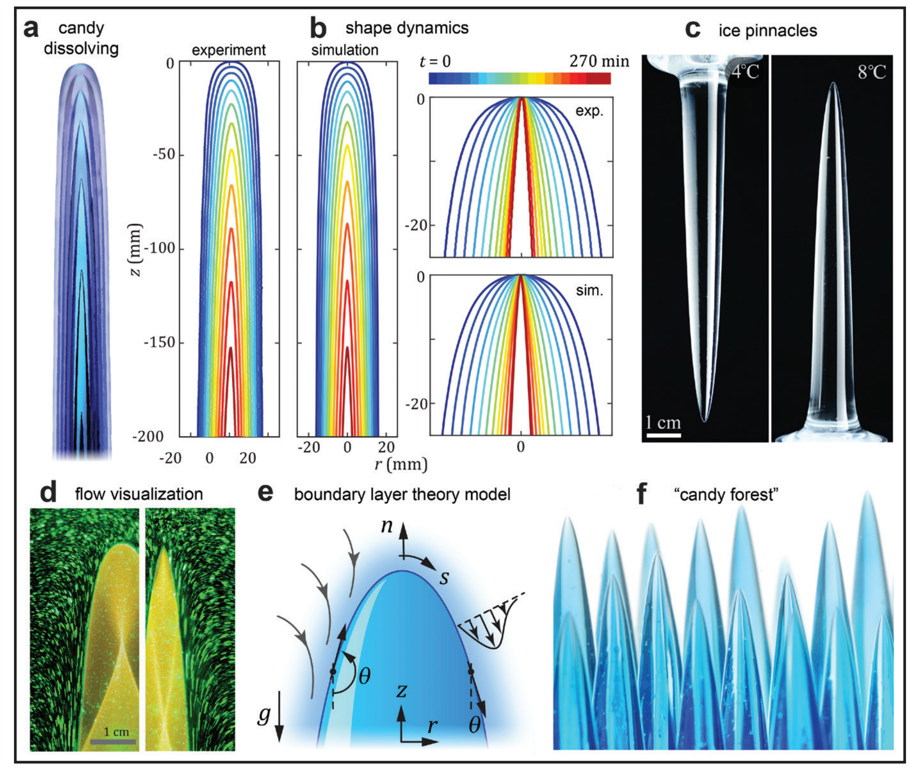

Figure 2. *Shape dynamics of pinnacles.* **2a.** *Experiments on pillars of hard candy that dissolve in water yield needle-like shapes with sharpening tips.* **2b.** *Simulations closely match the measured shape dynamics.* **2c.** *Inverted and upright pinnacles form in melting ice at low and higher temperatures respectively.* **2d.** *Boundary layer flows are induced by density differences in the fluid.* **2e.** *A moving interface model serves as the basis of simulations and shape analysis.* **2f.** *A "forest" or array of pinnacles emerges in experiments on dissolving blocks of porous candy. Figures 2a, 2b, 2d, 2e, and 2f courtesy of [5]; Figure 2c courtesy of [10].*

$\kappa / \kappa_0 = \sin^5 \theta / (1 + 2\cos^2 \theta)$, where the tip curvature κ_0 parametrizes the family of such "perfect pinnacles." Treating κ_0 as a fitting parameter attained a convincing comparison to the longtime shape from numerical simulations. Interestingly, the finite-time singularity of previous work [5] disappears upon the consideration of higher-order terms in the analysis of the apex region.

These findings show the subtleties behind the original motivating question about shape universality in dissolving [6]. Some initial shapes indeed seem to converge to a member of an infinite family of terminal shapes, but it is not easy to predict which member will ultimately be selected based on the initial form. It is also not clear whether all initial shapes eventually become members of these perfect pinnacles. As for next steps, obtaining evidence for a terminal shape in experiments and implementing numerical simulations of the full dynamical equations—which may require new methods to resolve the extremely fine scales at late times—seem to be of paramount importance. Additionally (or alternatively), future studies could prescribe a member of the predicted equilibrium family as the initial form and test this shape's preservation thereafter. One might also wish to employ stability analysis to address whether such equilibria are stable within the model, and explore shape perturbations in experiments and simulations. The latter could provide insight into the selection problem regarding which aspects of the initial form are most important in determining the final form. This knowledge would be useful in engineering applications that exploit dissolutive reshaping to manufacture ultra-fine structures [5].

Returning to natural pinnacles, researchers have both suggested that dissolutive sharpening is responsible for karst pinnacles and proposed a hypothesis for the formation of a "forest" or array of such structures [5]. An experimental test involves the dissolution of a block that was seeded with vertical pores as a simple model of porous and fissure-riddled stone. A dramatic reshaping unfolds; the pores widen as they act as conduits for downward flows and eventually merge, leaving an array of interstitial pillars that then sharpen to produce the bed-of-nails morphology in Figure 2f. Furthermore, although ice pinnacles are very much expected based on dissolution studies, they still promise to yield more surprises. A recent paper demonstrated conventional pinnacles in ice [10], but only for melting that occurs in sufficiently warm water (see Figure 2c). The very cold far-field temperatures that are typical of melting in nature yield inverted or downward-pointing pinnacles—which could explain why icebergs often form pinnacles on their underbellies—while intermediate temperatures produce more exotic shapes. These outcomes stem from liquid water's anomalous density-temperature profile, whose maximum at four degrees Celsius drastically alters the flows that are generated.

Looking ahead, pinnacles and other natural formations will clearly continue to serve as muses that inspire further investigations. Applied and computational mathematics provide the necessary tools that allow scientists to chip away at geomorphology problems and reveal their fascinating geometries and shape dynamics.

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Mathematical Thinking

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lives and mitigate the spread of COVID-19, multiple unfortunate consequences affected the economy, education, health, and even the basic blocks of society: homes. Pandemic-related fears, financial stress, and isolation all detrimentally affect mental health, thereby disrupting peace in one's place of living. If a home is susceptible to domestic violence, confinement under a deteriorating mental health climate and abnormally high levels of home interaction can create a perfect storm — particularly for intimate partner violence. While most mathematical models focus on the nature of COVID-19's spread, Ohajunwa's model provides deeper insights into other adverse impacts of COVID-19 of which many people are not aware. Figure 2 (on page 2) illustrates a typical susceptible-exposed-infected-quarantined-hospitalized-recovered (a modified SEIR) model with additional complexities in a confinement compartment that further incorporates the dynamics of members who are susceptible to domestic violence, victims (V), abusers (A), and those who are removed. These types of models and tools can allow researchers to study the relationship between lockdowns, confinement strategies, COVID-19, and domestic violence to ultimately mitigate the social problems that accompany such drastic measures.

Although students initially struggled when they switched to virtual learning in the middle of the semester, they still developed a sense of ownership and produced solutions that addressed grand challenges with design thinking principles and mathematical problem-solving. They tackled real-world societal problems that were inspired by the UN SDGs and applied mathematics in a meaningful and impactful way. I encourage other instructors and institutions to implement similar projects to encourage critical thinking and mathematical modeling applications. These projects can provide students like Ohajunwa with opportunities to not only make significant contributions to science and engineering, but also develop a system that will change lives and positively impact society.

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Climate Prediction

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simplicity, we assume that this decomposition is akin to a low-pass filtering, which we accomplish with a Reynolds operator. Under this conceptual decomposition, the flux in the equation for the resolved scales becomes $\underline{\mathbf{u}Y} = \underline{\mathbf{u}}\,\underline{Y} + \underline{\mathbf{u}'Y'}$, with contributions from resolved components $(\underline{\mathbf{u}}\,\underline{Y})$ and unresolved or unknown small-scale components $(\underline{\mathbf{u}'Y'})$. The parameterization (or closure) problem refers to the estimation of the unresolved contribution $(\underline{\mathbf{u}'Y'})$ solely as a function of resolved variables.

Researchers often frame the parameterization of unresolved scales in terms of estimating a dependence between the small scales' impact on the large scales as a function of the large scales. They have traditionally achieved this outcome through purely physics-based approaches, which combine semi-empirical techniques with intelligent guesswork. For example, G.I. Taylor's seminal 1922 study parameterized the impact of small-scale turbulent motions on the large-scale dispersion of a passive tracer—like smoke from a chimney—as an *eddy* diffusion [5]. In this formulation, one would have to empirically determine the parameter—the eddy diffusivity—which would be many orders of magnitude larger than the molecular diffusivity of air. Scientists have employed similar reasoning to develop a number of parameterizations that are currently used in modern climate models; they often utilize observations of turbulence in the natural world to constrain the structure and parameters in these schemes [1, 4]. In such scenarios, additional sophistication may account for physical constraints and the parameters (like eddy diffusivity) might depend on large-scale variables with tunable coefficients. While these parameterizations form the backbone of modern-day climate models, inaccuracies in the parameterizations' structural forms—or even the parameters themselves—result in biases or systematic errors in the solutions.

Machine Learning as a Potential Avenue for Parameterization Improvement

An alternative route to parameterization involves using a statistical/machine learning (ML) algorithm for regression to determine the functional form of the small-scale impacts on the large scales, rather than scientists prescribing it themselves. Doing so requires the availability of data that pertains to the small scales, which we can procure through limited high-resolution simulations and observations with resolved small scales. It also necessitates computational technologies that can handle these large datasets, like graphics processing unit clusters on computational clouds — which are fortunately becoming more accessible.

The simplest data-driven approach that directly learns functional dependence assumes little to no prior knowledge and uses traditional "out of the box" ML algorithms, including neural networks (NNs), convolutional neural networks (CNNs), Gaussian processes, and random forests — all of which have shown potential in many domains. For example, deep NNs—which include an increasingly large number of layers and trainable parameters—have displayed a high degree of skill in image recognition and game play. These purely data-driven approaches have also exhibited promise for climate science and are now leading rapid research advancements in "physics-aware" ML methods that combine data-driven approaches with physical knowledge (see Figure 1, on page 1).

Broadly speaking, scientists are currently investigating three approaches for physics-aware ML methods. The first category is parameter estimation, which addresses parameterizations wherein the structure is based on physical principles and the ML algorithm estimates some unknown free coefficients. In the second category, the loss function that trains the ML algorithm has a penalty or regularization term that incorporates certain known physical constraints. And in the third category, the structure of the neural network or another ML algorithm is modified to preserve some known symmetries or conservation properties of the system.

Recent work [3, 6] has demonstrated the promise of these physics-aware ML approaches for parameterizations of subgrid momentum and heat fluxes that arise in ocean turbulence (see Figure 2). Researchers used two physics-aware approaches that incorporated the known physical constraints into the architecture of the ML algorithm. The first approach utilized a CNN with a modified final layer that included physical conservation laws, and the second approach employed relevance vector machines that discover equations by combining basis functions that were selected based on physical knowledge about the problem. Both approaches showed superior skill over traditional parameterizations that are purely physics-based; they produced better pointwise predictions of subgrid fluxes (offline evaluation), and the evolution of their parameterized coarse scale model more closely agreed with the high-resolution model (online evaluation). Along with other studies, this work has provided an exciting proof of concept and a potential way to accelerate improvements in climate models with data-driven ML techniques.

Is a Major Upgrade to Climate Models on the Horizon?

ML's arrival in recent years has promised accelerated advancement in many areas of science, including the understanding and parameterization of turbulent processes in climate models. Initial attempts to utilize these technologies have hinted at the possibility of potential breakthroughs that could provide a major upgrade to the current generation of climate models, ultimately reducing bias, improving the skill of predictions, and hopefully translating to better resource management and preparedness for the future. Only time and research will tell whether this promise comes to fruition.

Many exciting challenges related to implementation, generalization, and interpretation lie on the path towards these goals. The aforementioned developments have galvanized the climate science community, as evidenced by the formation of the Multiscale Machine Learning In Coupled Earth System Modeling[2] (M^2LInES) international collaborative team and many other centers and institutes. Researchers are addressing multiple specific questions that advance critical thinking and move progress forward. For instance, how can we generalize (extrapolate) to regimes that are not part of the training data (for example, a future climate with warmer temperatures)? Do we have to use ML primarily as a black box, or can it be interpretable and aid in scientific discovery? What are the best ways to learn from both high-resolution simulations and observational noisy and/or sparse datasets? How should we approach the practical challenges that accompany the combination of disparate technologies and scientific domains? How can we make these scientific advancements more reproducible and accessible?

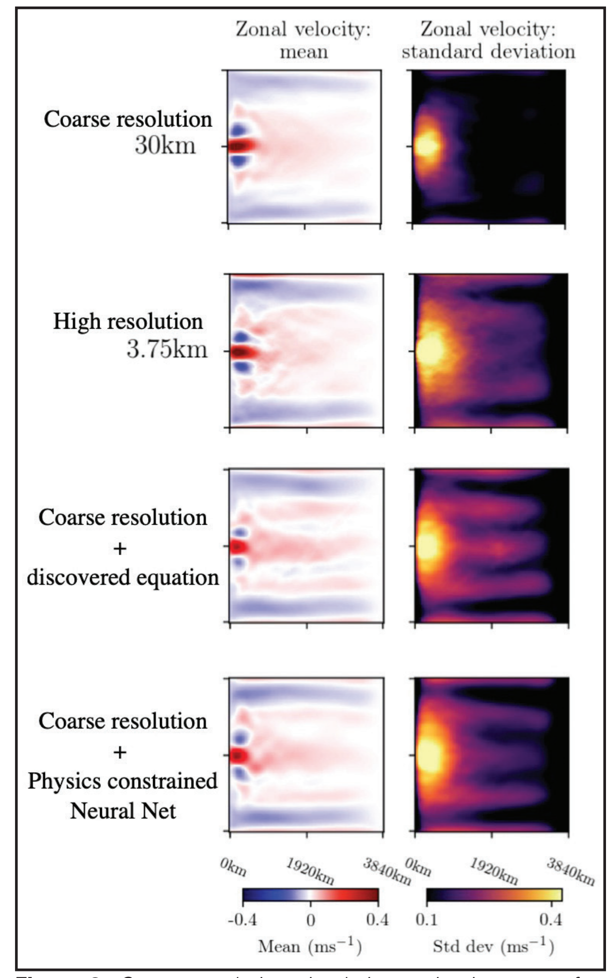

Figure 2. *Coarse-resolution simulations miss important features that high-resolution simulations can capture. The addition of physics-aided, machine-learning-based parameterizations to the coarse-resolution models significantly improves simulation skill. Figure adapted from [7].*

We currently find ourselves at the beginning of an era with more questions than answers, similar to the early days of numerical fluid dynamics in the 20th century. While some of these problems will initially be solved with brute force empiricism, the development of a fundamental understanding that is based on a solid theoretical, mathematical, and numerical foundation is essential for long-term success.

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Dhruv Balwada is an associate research scientist at the Lamont-Doherty Earth Observatory of Columbia University. He uses ocean observations and numerical simulations to understand the operation of ocean turbulence at scales of 1-100 kilometers and its impact on ocean circulation and climate. Laure Zanna is a professor in mathematics and atmosphere/ocean science at New York University's Courant Institute of Mathematical Sciences. Her research focuses on the ocean's role in climate on local and global scales through the analysis of observations and a hierarchy of simulations.

[2] https://m2lines.github.io

NumFOCUS: A Fiscal Sponsor of Scientific Software

By Andy R. Terrel

NumFOCUS[1] was founded in 2012 with the simple goal of helping to fund the maintenance burden of scientific software. Over the past two decades, the community has recognized a gap between the organizational practices of open source communities and the practices that raise funding. We can address this gap with fiscal sponsorship: a financial tool for sharing fiscal and legal entity status between groups. Today, NumFOCUS promotes open practices in research, data, and scientific computing. It is the fiscal sponsor for more than 100 projects, many of which are used every day by academic and industrial researchers. Here I review the necessity of our work, the role of the fiscal sponsor in applied mathematics research, and the ways in which scientists can engage with our efforts.

Software Libraries: The Backbone Between Industrial and Applied Mathematics

Nearly all of the software that powers applied mathematics research has shifted to open source distribution models, which allow researchers the freedom to create derived works and distribute their own software. As a result, open source scientific software has become the backbone for the delivery of applied mathematics libraries to the wider community. The community then employs these libraries in a broad array of applications that range from space launches to internet businesses, home appliances, and so on. Unfortunately, the research groups that develop these software libraries have difficulty procuring reliable funding for maintenance.

A software's popularity is both a blessing and a curse. The credit for innovative applied mathematics research is several degrees removed from commercial applications, and profits are not shared with the software communities. Because software maintenance is not the primary goal of any organization, it becomes an unfunded burden that is left to the mathematical researcher. As a result, open source scientific software is rarely maintained to professional standards.

SOFTWARE AND PROGRAMMING

Researchers constantly seek new funding opportunities, ultimately creating a complex "funding fabric" that is comprised of various grants, contracts, book royalties, event fees, and other sources. Managing this fabric is therefore an administrative challenge that university mathematics departments are not equipped to handle — especially since mathematicians are not isolated within a single entity or country. Many commercial solutions have emerged over the years, such as the use of consulting agencies (e.g., Kitware[2] for ParaView[3]), value-added resellers (e.g., Red Hat[4] for Linux[5]), and internal groups within companies. However, these groups have different goals than the software community. Fiscal sponsorship is thus a solution that offers the community a sense of control.

Fiscal Sponsorship

At face value, fiscal sponsorship simply serves as a way for numerous like-minded groups to share legal and fiscal status. But even the mere act of accepting a check as a project—rather than as an individual—changes the relationship between the funder and the contributor. Fiscally sponsored software projects, on the other hand, can professionally manage their assets and liabilities.

Furthermore, a project's legal status allows it to manage contracts. This ability opens the door for hiring, insuring events, and owning trademarks and copyrights. However, every asset class almost certainly comes with an accompanying liability. Figure 1 (on page 7) presents a modest list of the types of assets and liabilities that may arise within an open source software community.

The Fiscal Sponsor's Role in Applied Mathematics Research

For several decades, the standard in scientific software was that a university or government research laboratory would own the assets and liabilities for open source projects. This trend has changed over the last 10 years due to the recent explosion of software teams in both number and size. These teams have also crossed more borders and organizational boundaries, making it even harder to share costs.

We can measure the growth of software complexity in numerous ways, including with the number of lines of code, modules, contributors, and supported systems. NumPy[6] and Jupyter[7]—prominent software libraries that mathematical researchers regularly use—serve as two illustrative examples. NumPy seems like a straightforward array library, except for its deployment and downstream dependency requirements — it supports a dizzying number of compilers and plug-ins on laptops, servers, embedded systems, and so forth. The project currently has more than 300,000 lines of code,[8] 850 contributors,[9] and 90 million downloads per month.[10] Financial support for this library stems from contributors' salaries at universities and various companies around the world, but the first grant to work directly on the software came about only recently. As for the second example, Jupyter's contributors are spread across Asia, Europe, and North and South America. We hence see that an open source foundation can work directly with professional programmers throughout the world.

Few organizations are willing to take on this common effort for software support. Hiring and organizing people becomes a challenge even at a single institute, and the specialized skillsets of numerical

See **NumFOCUS** *on page 7*

1 https://numfocus.org
2 https://www.kitware.com
3 https://www.paraview.org
4 https://www.redhat.com/en
5 https://www.linux.org
6 https://numpy.org
7 https://jupyter.org
8 https://github.com/numpy/numpy
9 https://libraries.io/pypi/numpy
10 https://pypistats.org/packages/numpy

The Hazards of Counting: Is 2+2 Always 4?

Uncountable: A Philosophical History of Number and Humanity from Antiquity to the Present. *By David Nirenberg and Ricardo L. Nirenberg. The University of Chicago Press, Chicago, IL, October 2021. 432 pages, $30.00.*

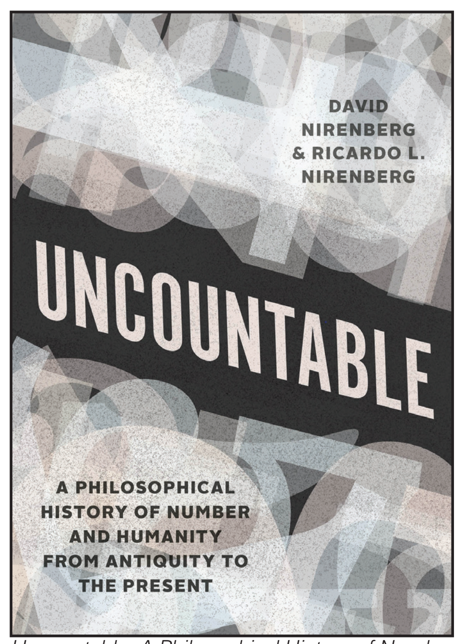
BOOK REVIEW
By Ernest Davis

In many cases, a thoughtful response to a question of the form "How many Xs are there?" begins with "It depends on how you count." The number of entities in a given category X is often not merely unknown, but indeterminate. The answer depends on what counts as an X and how the inquisitor individuates one entity from another; such decisions can be ill-defined and arbitrary. For example, consider the following question: "How many languages are currently spoken?" We must first decide on the definition of a legitimate language and the qualifications of "currently spoken," then identify the relevant distinctions between languages. Are Klingon and Pig Latin legitimate languages? Are Moroccan Arabic and Iraqi Arabic different languages or different dialects? How about Swedish and Norwegian or French and Québécois? The problem becomes even more complicated if we look across time at continuously changing entities and ask questions such as "How many languages have been spoken in Europe since 1000 BCE?" These types of queries require determinations about whether medieval French and modern French are different languages.

Counting—as well as operations like addition that build on counting—is thus inherently dependent upon individuation, which is frequently problematic. Nonetheless, philosophers have often taken arithmetic statements such as "$2+2=4$" to be the *ne plus ultra* of indubitable, necessary truths. *Uncountable: A Philosophical History of Number and Humanity from Antiquity to the Present*—written by father-son team David (a medieval historian) and Ricardo L. Nirenberg (a mathematician and philosopher)—addresses this paradox and traces it through the history of philosophy and literature.

Uncountable has two stated goals. The first is "to convince you that it *is not* true of all things that two and two make four." The second is to serve as "an exhortation to interrogate more self-consciously the consequences of extending laws of thought derived from one domain to others where the necessary conditions of 'sameness' may not apply."

Chapter one opens with an account of the "crisis" that gripped the European intellectual world in the early 20th century. According to Oswald Spengler in his influential book *The Decline of the West*, a titanic struggle existed between two world views: one was organic and intuitive while the other was mechanical and mathematical. The authors of *Uncountable* recount how mathematicians like Bertrand Russell, L.E.J. Brouwer, and David Hilbert; physicists like Albert Einstein and Erwin Schrödinger; philosophers like John Dewey and Edmund Husserl; and poets like Paul Valéry addressed this dichotomy. The book then goes back in time; the next four chapters are a historical survey of philosophical and literary discussions about unity and number, sameness and difference, and the necessity of mathematical truths. Chapter two covers the pre-Socratic philosophers, chiefly Pythagoras, Parmenides, and Heraclitus. Chapter three includes Plato, Aristotle, and other classical philosophers, and chapter four examines philosophers from monotheistic religions — including Philo, Augustine of Hippo, the Epistles of the Brethren of Purity, Ibn Tufayl, and Simone Weil (Weil is described as "still fashionable;" the phrase is rather grating, particularly since she is the only woman who is discussed at length). Chapter five offers a whirlwind tour of several 17th- and 18th-century philosophers: René Descartes, Gottfried Wilhelm Leibniz, John Locke, David Hume, and Immanuel Kant.



Uncountable: A Philosophical History of Number and Humanity from Antiquity to the Present. By David Nirenberg and Ricardo L. Nirenberg. Courtesy of the University of Chicago Press.

The text summarizes the main contributions of each of these thinkers and provides a few quotes for every individual (which are generally very hard to understand). One by one, David and Ricardo Nirenberg then weigh the philosophers in the balances and find them wanting. They condemn most of them for exaggerating the scope and certainty of mathematics and a few for the opposite failing; none had seemingly found the proper happy medium.

In chapter six, the authors present two arguments to support the notion that $2+2$ is not always 4. The first argument closely pertains to the history of the philosophy of sameness and difference, which I mentioned previously; if individuation is unreliable, then counting and addition are too. However, the Nirenbergs are much more interested in a second, much weaker argument; they appear to contend that counting involves *physically gathering all things together*. In some cases, doing so may be impossible — i.e., if the objects are too heavy or vulnerable to change as a result of the physical action of motion.

The authors pursue this second argument to the point of silliness. For instance, they write that philosopher David Lewis "creates the category [of 'cats'] by taking the 'fusion' or 'sum' of all cats, consisting in putting together—into the same sack, as it were—all the cats there are in such [a] way that all the parts of cats—whiskers, tails, front halves, back halves, right sides, left sides, even each molecule in any cat—will be in this fusion or sack.'' They then continue that "[I]n Reality, if those cats were thrown into the same sack, there would be not just fusion but violent confusion from which no cat would emerge unchanged." Note that it is the Nirenbergs, not Lewis, who introduce the metaphor of the "sack," which they then proceed to take literally.

More seriously, the authors present a third goal for the book at the end of chapter six. They coin the technical term "pathic" and its opposite "apathic," which they define as follows: "We will call objects, items, things, categories, concepts, beings *apathic* if and only if whenever collected together or separated they remain the same." The Nirenbergs then make two claims: (i) Being pathic or apathic is not an absolute feature of a thing but rather a contingent matter of perspective and context, and (ii) one can only properly apply mathematics to things if viewing them apathically.

Though I do not know for certain, I would guess that most of the aforementioned thinkers would have concurred that both of these claims are valid as general principles. However, some of them would certainly not have agreed that the claims are important or significant limitations on the assertion that $2+2=4$. Plato would have probably viewed the pathic aspects of things as a limitation that is not shared by the ideal Forms.

The remainder of *Uncountable* examines a variety of intellectual issues through the lens of the contrast between pathic and apathic. Chapters seven and ten are discursions as to how these themes play out in 20th-century physics and literature; they mention W.H. Auden, Rainer Maria Rilke, Jorge Luis Borges, and Franz Kafka, among others. Chapter eight constitutes a critique of mathematized economics and social science, which the authors view as frequently violating the prior second principle. Chapter nine tackles the problem of time by contrasting the accounts of mathematics and physics with human experience.

Although the text discusses a large number of philosophers and writers, there are some glaring omissions. The most egregious is Ludwig Wittgenstein's[1] *Remarks on the Foundations of Mathematics*, which contains some observations that are quite similar to those of David and Ricardo Nirenberg:

> This is how our children learn sums; for one makes them put down three beans and then another three beans and then count what is there. If the result at one time were 5, at another 7 (say because, as we should now say, one sometimes got added and one sometimes vanished of itself), then the first thing we said would be that beans were no good for teaching sums. But if the same thing happened with sticks, fingers, lines and most other things, that would be the end of all sums.
>
> "But shouldn't we then still have $2+2=4$?" — This sentence would have become unusable.

Another omission is George Orwell's *1984*, which is surely the most widely known literary discussion of whether $2+2=4$:

> "Freedom is the freedom to say that two plus two makes four."
>
> "Sometimes, Winston. Sometimes they are five. Sometimes they are three. Sometimes they are all of them at once. You must try harder. It is not easy to become sane."

Though Orwell was not a philosopher, I would think that this dramatic scene—in which the denial of mathematical truth becomes a tool for tyranny—deserves a disquieting place among the many literary references in the opposite direction that the Nirenbergs adduce. Finally, I hope that I am not being too self-serving in pointing out that Philip Davis (my father) and Reuben Hersh discussed these issues at length in their jointly authored books: *The Mathematical Experience* and *Descartes' Dream*.

I found the literary discussions in *Uncountable* often beautiful and even moving, the history of philosophy generally difficult but frequently eye-opening, and the authors' explanations of their personal philosophy sometimes insightful but occasionally willfully obtuse. Ironically, the dichotomy between the pathic and apathic itself becomes a somewhat rigid framework into which the Nirenbergs cram a wide range of different issues. But the book is always thought-provoking, which is paramount in philosophy. Anyone who is interested in the difficult questions that surround abstract mathematics' application to the real world and the accompanying risks and opportunities will find much to ponder in *Uncountable*.

Ernest Davis is a professor of computer science at New York University's Courant Institute of Mathematical Sciences.

[1] Wittgenstein is mentioned in the book, but not his work on mathematics.

NumFOCUS

Continued from page 5

software developers make them some of the most expensive hires in industry. Company employment requires numerous confidentiality agreements to protect trade secrets and intellectual property; these agreements are often vague in terms of the software's contribution to the public. Even universities greatly differ in their technology commercialization policies for work of this nature. Furthermore, promotion and recognition usually depend on results that are produced for the commercial entity — not on the success of outside projects. Laws that pertain to employment also vary widely in different countries. As such, bringing open source software teams under a single institutional banner is certainly not without its challenges.

The open source fiscal sponsor has become an archetypical organization in the software business world. These types of organizations are growing in popularity and act as a natural place to land the intellectual and communal assets of open source projects. Other fiscal sponsorship organizations have also become commonplace. The Apache Software Foundation[11]—born out of the need to support the Apache HTTP Server[12] when many web servers were becoming commercial—maintains software for the public good. The Linux Foundation[13] supports the Linux ecosystem and provides a neutral, trusted hub where developers can code, manage, and scale open technology projects. The Free Software Foundation[14] was established to bolster the GNU community. While many fiscal sponsors exist, NumFOCUS is differentiated by its sole intention to support scientific software and the needs of the common ecosystem.

How to Engage with Our Work

NumFOCUS engages with software projects through its sponsorship programs, to which projects can apply each quarter. It has three basic criteria: be scientifically oriented, be open, and be kind. We require projects to have a scientific orientation because that is our mission. To be open, a project must be published under a standard open source license and seek engagement with the public as an intrinsic part of its development. Finally, participants must be kind because NumFOCUS aims to foster a constellation of healthy, productive communities. We believe that we can only achieve this goal if every project actively works towards making their communities friendly, inclusive, and respectful of each participant. Further details are available on our website.[15]

A list of NumFOCUS-sponsored projects,[16] as well as further details about the mission,[17] is accessible online. Each project has its own guidelines for development contributions, but the organization at large houses several programs that interested persons can support. These include the Open Science Champions program[18] that promotes our mission of making scientific computing more open and accessible. NumFOCUS depends on advocates and volunteers like our Board of Directors,[19] PyData committee members, and other committees such as our Small Grants Committee, Infrastructure Committee, and Diversity and Inclusion Committee.

Through fiscal sponsorship, NumFOCUS manages the funding fabric that is required for mathematical software projects to realize a broader impact for industry. We invite readers to join us on our mission to serve the scientific community.

Andy R. Terrel was on the NumFOCUS Board of Directors from 2012 to 2021 and served as president from 2014 to 2021. He is currently the Vice President of Infrastructure. Terrel received his Ph.D. from the University of Chicago in 2010 and has since led data science teams at Anaconda, REX Homes, and now Xometry.

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15 https://numfocus.org
16 https://numfocus.org/sponsored-projects
17 https://numfocus.org/projects-overview
18 https://numfocus.org/your-support
19 https://numfocus.org/community/people

Assets	Liabilities
Registered Copyrights	Protected Debt
Accounts Receivable	Maintenance Tracking and Reporting
Registered Trademark	Salaries / Dividends
Business Plan	Administration Overheads
Endorsement Contracts	Endorsement Obligations
Brand Awareness	Public / Media Relations
Event Profits	Event Management

Figure 1. *Assets and liabilities that may arise within an open source software community.*

SIAM's United Kingdom and Republic of Ireland Section Hosts 26th Annual Meeting

By Jennifer Scott and Kirk M. Soodhalter

The 26th Annual Meeting[1] of the SIAM United Kingdom and Republic of Ireland (UKIE) Section[2] took place on January 7, 2022. SIAM UKIE president Jennifer Scott (University of Reading and the Science and Technology Facilities Council), vice president Kirk M. Soodhalter (Trinity College Dublin), and secretary/treasurer Francesca Arrigo (University of Strathclyde) organized the conference, and Trinity College Dublin served as the meeting's virtual host. Though we originally intended for the festivities to commence in person, the sudden arrival of COVID-19's Omicron variant and the resulting spike in cases meant that we regretfully had to switch to a virtual event. However, the online format widened access to the conference and enabled the participation of attendees who otherwise would not have been able to travel to Dublin.

More than 60 registered participants attended the conference talks, which highlighted advances in deep learning, weather simulation, numerical linear algebra, data assimilation, and inverse problems. The five invited speakers were David Barrett (Google Deepmind), Sarah Dance (University of Reading), Marco Marletta (Cardiff University), John Pearson (University of Edinburgh), and Valeria Simoncini (University of Bologna). Rather than host our usual lunchtime poster event, six Ph.D. students from different universities around the U.K. and Ireland delivered 10-minute talks about their respective projects; these talks afforded the students important early-career experience in presenting and facilitated networking opportunities. Another nice feature of the day was the use of informal, randomly assigned Zoom breakout rooms during coffee breaks, which allowed attendees to chat and network in brief increments. Additionally, a pre-recorded statutory business meeting and financial discussion of the SIAM UKIE Section was made available to all section members.

Barrett's talk—intriguingly titled "The Geometric Occam Razor Implicit in Deep Learning"—outlined new mathematical insights that explain the surprising effectiveness of several deep learning algorithms in domains as diverse as image classification and language translation. Barrett used backward error analysis to reveal a hidden built-in mechanism called implicit gradient regularization that guides deep learning towards solutions with low geometric model complexity.

Dance spoke about "Making the Most of Observations in Numerical Weather Prediction: A Large Nonlinear Least-squares Problem." She discussed the ability of mathematical weather simulations to update real-time weather data and improve forecasts by solving a large nonlinear weighted least-squares problem, wherein the model and observational data are weighted by their respective uncertainties. She also presented recent research about the characterization and treatment of observation uncertainty in assimilation systems.

Marletta's presentation on "An Inverse Problem in Electromagnetism with Partial Data" addressed the feasibility of determining unique permeability, permittivity, and conductivity in a domain's time-harmonic Maxwell system by taking certain measurements of electric and magnetic fields on only a small, open subset of the boundary.

Pearson considered "Numerical Methods and Linear Algebra for PDE-Constrained Optimization Problems." These problems arise in a huge range of practical applications, including fluid flow control, medical imaging, biological and chemical processes, and electromagnetic inverse problems. Pearson introduced general computational strategies for the solution of such problems with an all-at-once approach; these types of strategies lead to large-scale linear systems that one can solve via Krylov subspace methods that are paired with robust levels of preconditioning.

Finally, in a talk entitled "Computational Methods for Large-scale Matrix Equations and Application to PDEs," Simoncini discussed the numerical treatment of large-scale Lyapunov and Sylvester equations and their generalizations via projections onto Krylov subspaces. These equations are important in dynamical systems, control theory, and eigenvalue computation, but they have recently been recognized in the discretization of partial differential equations (PDEs) — particularly stochastic PDEs.

We hope that Trinity College Dublin can host the 2023 SIAM UKIE Annual Meeting in person next year, provided that the COVID-19 pandemic allows such an event to occur. However, we will plan to include a virtual component for those who cannot travel to Dublin in order to bolster conference access for prospective attendees.

Jennifer Scott is a professor of applied mathematics at the University of Reading and Director at Reading of the Mathematics of Planet Earth Centre for Doctoral Training. She is also a Research Fellow at the Science and Technology Facilities Research Council and president of the UKIE Section of SIAM. Kirk M. Soodhalter is the Ussher Assistant Professor in Numerical Analysis at Trinity College Dublin. He is vice president of the UKIE Section of SIAM.

[1] https://www.siam.org/conferences/cm/conference/ukie22

[2] https://outreach.mathstat.strath.ac.uk/siamukie

COMSATS University Islamabad SIAM Student Chapter Holds Inauguration Ceremony

By Fatima Sehar

It is with great pride that we announce the formation of the COMSATS University Islamabad[1] (CUI) SIAM Student Chapter in Pakistan. This chapter seeks to bring CUI students into the mainstream of industrial and applied mathematics as well as the research-oriented track of the larger SIAM community. It aims to promote student interest in applied and computational mathematics while simultaneously highlighting industrial applications. The chapter will also create exciting research opportunities and collaborations for undergraduate and graduate students, both locally and abroad.

The inauguration ceremony for the CUI Student Chapter took place in December 2021 in CUI's Junaid Zaidi Library. The ceremony introduced students to SIAM and its activities and sparked valuable conversations about the use of mathematics in real-world scenarios. Hina Tariq (a human resources officer for the CUI Student Chapter) and Areeba Liaqat (head of the chapter's Design Team) delivered the opening remarks and invited Mr. Sohaib (a Ph.D. scholar) onstage for the recitation of the Holy Quran. Next, chapter president Sania Ejaz offered a detailed introduction to SIAM activities around the world and explained the perks of SIAM membership. She expressed the CUI Student Chapter's intention to represent SIAM by collaborating with various national and international institutes and organizations. After Ejaz's informative presentation, Rutaba Saleem (secretary of the CUI Student Chapter) encouraged attendees to join SIAM and walked them through the membership process. She also reviewed the chapter's rules and regulations and discussed the election procedures for new executive committee members.

Abdullah Shah, an associate professor at CUI and faculty advisor of the CUI Student Chapter, then addressed the crowd. Shah's energetic speech conveyed his passion for collaboration between institutions and organizations. He recalled his journey to establish the first SIAM student chapter in Pakistan and explained the hurdles that he and other SIAM members faced along the way. The resulting CUI Student Chapter is the only one of its kind amongst Pakistani universities and educational institutions, and Shah was beyond grateful to finally attend the inauguration ceremony after a year of hard work.

"CUI is a community of students who hold a passion for learning," Shah said. "They are delighted to start this new journey of growth and learning with one of the biggest and undoubtedly the best society for applied and computational mathematics." His dynamic and active demeanor brought forth the same spirit in the CUI students, and many of them joined SIAM upon the ceremony's conclusion. During his speech, Shah shared his plans to uphold strong relations with SIAM and collaborate with the Society on research and technical projects. He also acknowledged the support of Sohail Iqbal, Imran Parvez Khan, and Saad Raza in establishing the CUI Student Chapter.

Next to speak was Mehmood ul Hassan, head of CUI's Department of Mathematics. He expressed his gratitude and happiness on behalf of all students and faculty for the start of a new SIAM student chapter, and encouraged the attending students to dedicate time for similar ventures and ignite their creativity to explore the real-life applications of different mathematical fields.

Sohail Iqbal and Salman Amin, assistant professors in CUI's Department of Mathematics, shared their insights as well. Amin spoke about SIAM's peer-reviewed research journals and emphasized their value within the fields of applied mathematics and computational science. Iqbal urged attendees to work for the overall growth of SIAM and advised them to give back to the community. Shamsul Qamar, chairperson of Incharge Academics, was the ceremony's chief guest and gave a concise address.

The CUI SIAM Student Chapter has been working hard since the inauguration ceremony by breaking into multiple teams that are collaborating for the purpose of innovation. The chapter wishes to provide an intellectual platform for students to share ideas, harbor an environment of creativity and development, and ultimately represent SIAM in the best possible way.

Acknowledgments: The author wishes to recognize Rutaba Saleem and Hina Tariq for their contributions to this piece.

Fatima Sehar is a B.S. student of mathematics at COMSATS University Islamabad (CUI). She is the content team head for the CUI SIAM Student Chapter.

[1] https://www.comsats.edu.pk

Inauguration ceremony participants, SIAM members, and officers of the COMSATS University Islamabad (CUI) SIAM Student Chapter pose with faculty members of the Department of Mathematics at CUI. Photo courtesy of Kamran Ashiq.

Shamsul Qamar, the inauguration ceremony's chief guest, addressed the students of COMSATS University Islamabad and motivated them to work hard. Photo courtesy of Kamran Ashiq.