MATH MATTERS

Apply It.

The math behind... GPS Navigation

Technical terms used:

Least squares, Kalman filter, mathematical model, dynamical system

Uses and applications:

The mathematical methods used in GPS navigation are also used in weather prediction, seismology (earthquake studies), and oceanic and atmospheric modeling.

How it works:

A GPS unit uses a mathematical model to calculate the position of a moving vehicle, based upon previous measurements of its position. However, these measurements are inaccurate, due to factors such as: atmospheric conditions that interfere with instrument readings, and noise in the wireless signals. In addition, the mathematical model used to find the car's position, given its velocity, is also inaccurate and incomplete; for example, it does not take into account wind speed. Therefore, all the unit can do is find a good estimate of the position of the vehicle.

There are many different estimates that could be calculated, but it is desirable to find the "best" estimate, given the measurements and model. The "best" estimate is often defined as the least squares estimate, which minimizes the errors (distances) between the calculated estimates and the original measurements.

The more measurements that are collected and used, the better the least squares estimate becomes. However, as additional data is added to the calculation of the estimate, the calculations and storage become unwieldy, resulting in a unit that cannot produce an estimate quickly enough to be of any use. The Kalman filter addresses this problem. It generates a least squares estimate of the current position of the vehicle, but instead of using all of the previous measurements, it uses just the estimate of the position from the previous time and the current measurement. This reduces the computational time significantly, allowing GPS systems to operate close enough to real time to be useful to the vehicle operator.

Interesting fact:

Use of the least squares method on a dynamical system dates back to Gauss, who used it to estimate the orbit of the planet Ceres. Gauss first began to work on this problem in 1795, when very few measurements of the orbit were available, and the instruments used to make those measurements were highly inaccurate. However, his method still managed to produce results that were accurate enough for astronomers to locate Ceres a year later in the area he had predicted.

References:

[1] H.W. Sorenson. Least Squares Estimation: from Gauss to Kalman. *IEEE Spectrum* **7**, 63-68 (July 1970).

[2] K. Gupta, Apurva, P. Jindal, V. Snehi. Implementing Kalman Filter in GPS Navigation. *Int. Journal of Innovative Tech. and Exploring Eng* **2**(4), 21-25 (March 2013).

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