

Biologically-inspired Linkage Design *Computing Form From Function*

By Jonathan D. Hauenstein and
J. Michael McCarthy

For several decades, engineers have been designing robots resembling the general structure of a human, called humanoid robots. In response to the nuclear disaster at Fukushima, Japan in 2011, the DARPA Robotics Challenge was created to accelerate the development of robots that could perform tasks in areas too dangerous for humans. A total of 23 teams participated in the finals of this challenge in June 2015, which was won, along with a \$2 million prize, by the humanoid DRC-HUBO (derived from “HUMANoid roBOT”) from Team KAIST (Daejeon, Republic of Korea). Coincidentally, the National Institute for Mathematical Sciences (NIMS), also in Daejeon, hosted the SIAM Conference on Applied Algebraic Geometry in August 2015. Although this Daejeon commonality is fortuitous, there is a long and storied history of the relationship between algebraic geometry and mechanical engineering, particularly in the area of kinematics, which studies the motion of systems such as a robotic arm.

The arm of the DRC-HUBO robot consists of four linkages coupled together: a hand linkage, a wrist linkage, an elbow linkage, and a shoulder linkage. In contrast to Rube Goldberg machines, biologically-inspired linkages should be minimalist based on the function required and thus follow an “Occam’s razor” mentality. That is, the form should be derived to yield the functionality required in a simple way, as unnecessarily complicated designs increase the chances of failure.

In addition to mimicking human motion, linkages may also be inspired by other organisms. One such example is the elegance of a bird flapping its wings. The development of this linkage [2], which received the A.T. Yang Memorial Award in Theoretical Kinematics at the ASME 2015 International Design Engineering Technical Conferences, provides an excellent case study of the symbiotic relationship between mechanism design and numerical algebraic geometry, the area of algebraic geometry that designs and implements numerical algorithms for solving and manipulating solution sets to systems of polynomial equations. *Numerically*

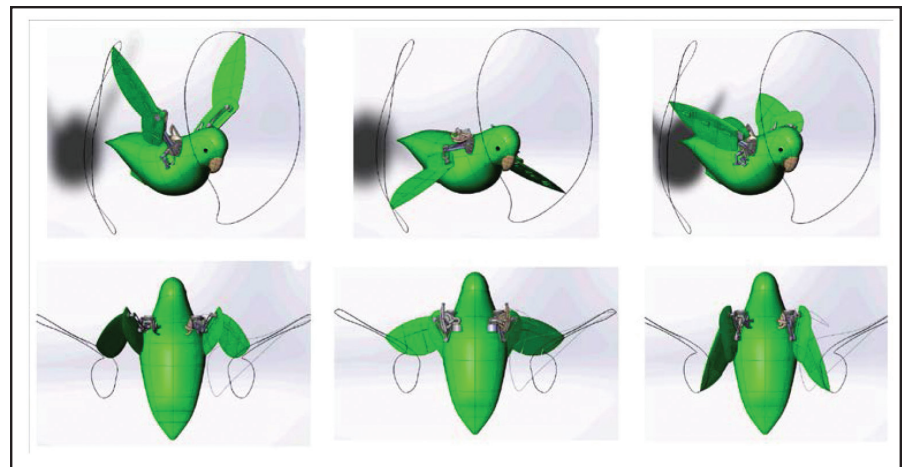


Figure 1. Four coordinated six-bar linkages designed to take a constant rotational input and transform it to angular outputs that match the wing movement of a bird.

Solving Polynomial Systems with Bertini, published by SIAM [1], provides an overview of numerical algebraic geometry with a focus on using the software Bertini.

This bird-flapping linkage, as shown in Figure 1 and in the video (<http://y2u.be/7aXmze9Ynis>), was designed to take a constant rotational input from a motor. From this functionality, the general form of the

linkage began to take shape, in this case, four coordinated six-bar linkages. Figure 2 (on page 3) shows a schematic drawing of a special kind of six-bar linkage used in the bird-flapping linkage, called a Stephenson II six-bar linkage.

Working with a family of four coordinated six-bar linkages, the next step for the *See Linkage on page 3*

Is There a Role for Mathematics in Food Security?

By Hans Kaper

What is “food security”? According to the World Health Organization, food security exists *when all people at all times have access to sufficient, safe, nutritious food to maintain a healthy and active life* [2]. Food security is a complex issue with important implications for public health, sustainable economic development, the environment, and trade. The Obama administration has declared food security a federal priority area, so here is another opportunity for our community to apply its modeling and computational expertise. The question is: Where is the mathematics, and if there is mathematics in food security, how can we participate?

To get the discussion started, and possibly define a research agenda, the American Institute of Mathematics (AIM) sponsored a workshop on Multiscale Modeling of the Food System at its San Jose facility in California, April 27–30, 2015 [3]. Organized by John Ingram (Environmental Change Institute, Oxford, UK) and Mary Lou Zeeman (Bowdoin College, US), the workshop attracted economists, social scientists, food and nutritional specialists, mathematicians interested in modeling complex systems, and data specialists.

The first thing to note is that food security involves more than food production; it also has to do with food availability, food accessibility, and food use. The entire set of activities by which calories and nutrients are grown, harvested, traded, processed, transported, stored, sold, prepared, and eventually consumed is called the food system, as illustrated in Figure 1. Clearly, it is a complex system: it has many components; all the components interact; the interactions are often nonlinear; and there are numerous feedback loops, most of which are poorly understood and which

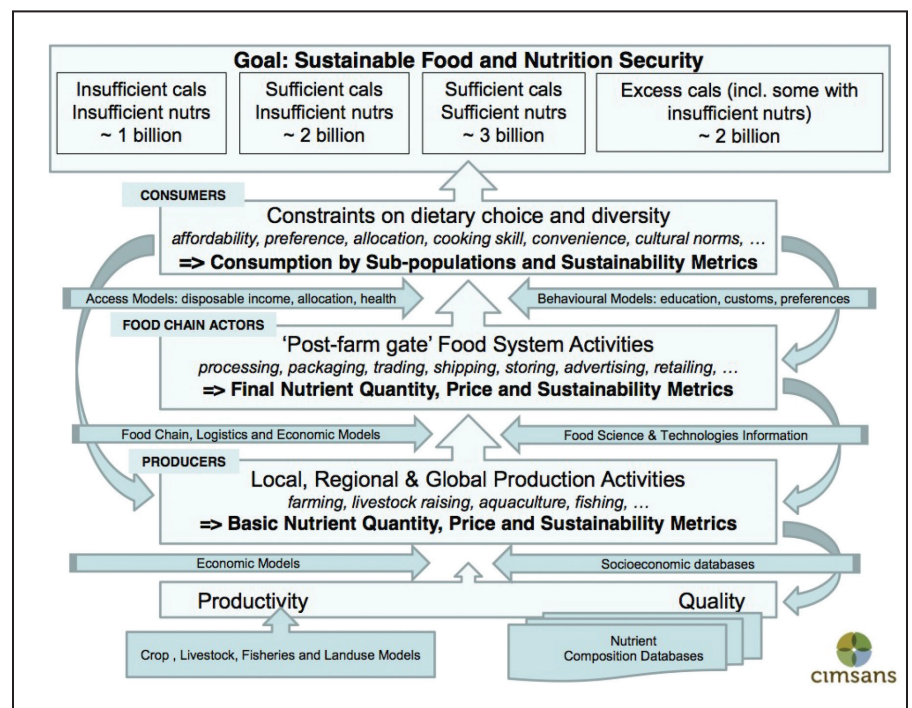


Figure 1. The Food System. The top box represents food system outcomes. The pale blue Consumers, Food Chain Actors, and Producers boxes represent food system activities. Darker blue boxes represent modeling approaches for separate components of the food system. Darker blue arrows represent inherent feedbacks. Reproduced from [1].

can be positive or negative, depending on the state of the system.

These observations suggest that the food system could be modeled as a network. For example, the nodes can represent nutrients and the edges (directed links) processes or activities along the food chain, with external forces (climate change, droughts, etc.) affecting the outcome of certain processes and activities. Bayesian networks are often used to model a domain containing uncertainty, and therefore provide a tool for reasoning under uncertainty. Uncertainties can arise due to an imperfect understanding of the system, incomplete knowledge of the state of the system, randomness in the mechanisms governing the behavior of the system, or any combination of these.

Mathematicians have developed techniques for studying the dynamics of networks, given its topology. These techniques can be applied or adapted to study food security models and could provide information on the relevance or irrelevance of certain components of the system.

The workshop at AIM offered a hands-on approach. Since most of the mathematicians had never been exposed to the problems of food security, the activities focused on three specific questions, each relevant to a particular aspect of food security in the US:

- (1) What currently drives dietary inequality in the US?
- (2) What are robust transformative strat-

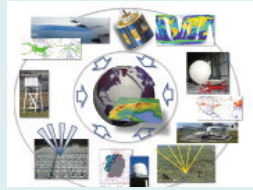
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8 Data Assimilation in Numerical Weather Prediction

Sebastian Reich and Andrew Stuart outline one of the central challenges for the mathematical sciences in the 21st century: the seamless integration of large data sets into sophisticated computational models. Watch for Part II of this article in the next issue.



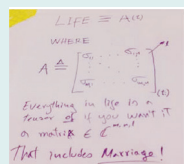
9 Computational Sciences in the Upstream Oil and Gas Industry: The ExxonMobil Experience

Thomas Halsey describes the growth of software spurred by the need to create computational models to understand the detailed structure of the subsurface, making the oil and gas industry one of the largest private sector users of high performance computing.



10 Life is a Tensor. . . Pilot Program Aims at Expanding SIAM Impact

Mary Ann Leung and Silvia Crivelli give an overview of a pilot mentoring program at the 2015 SIAM Conference on Computational Science and Engineering, which gave students a venue to present their research, and not only helped protégés connect with mentors, but also with opportunities to jobs, internships, collaborations, and much more.



3 Obituaries

10 Professional Opportunities

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Moser's theorem on the Jacobians

In one of his seminal papers [1], Moser proved a result, which in the simplest setting, still capturing the gist, states:

Given a positive continuous smooth function h on a compact, connected domain $D \subset \mathbb{R}^n$ with the average $[h] = 1$, there exists a diffeomorphism φ of D onto itself with the Jacobian h :

$$\det \varphi'(x) = h(x) \quad (1)$$

Solving this nonlinear PDE for the components of φ may seem like a difficult problem, but a physical analogy leads to a solution at once, as follows.

Interpreting h as the initial density of a chemical dissolved in a medium occupying the domain D , we imagine that the chemical diffuses, equalizing its density as $t \rightarrow \infty$ (the limiting density has to be 1 since $[h] = 1$). The map φ which sends each particle from $t=0$ to its position at $t \rightarrow \infty$ then satisfies (1).

In a bit more detail, let the density $\rho = \rho(x, t)$ evolve according to the heat equation

$$\rho_t = \Delta \rho \quad (2)$$

with Neumann boundary conditions (no diffusion through ∂D), starting with $\rho(x, 0) = h(x)$. Assume that each particle $z = z(t)$ diffuses according to

$$\rho \dot{z} = -\nabla \rho; \quad (3)$$

such evolution preserves the mass $\int_{\Omega_t} \rho dV$ of any region Ω_t .

Thus $h dV_0 = \rho(x, t) dV_t$,

$$\text{i.e.} \quad \frac{dV_t}{dV_0} = \frac{h}{\rho}.$$

In the limit $t \rightarrow \infty$ this turns into (1). The “diffusing particle” map φ solves the nonlinear PDE

1 indeed, the mass enters an infinitesimal patch dV at the rate $-\text{div}(\rho \dot{z}) dV = \Delta \rho dV$,

The missing details of this proof are not hard to fill in, or to find in [2].

There has been a lot of work on this problem since Moser's original paper, in particular on the regularity (references can be found in, e.g., [3]), but my modest goal here was to give a simple basic idea rather than a review of the latest results.

[1] Moser, J. *On the volume elements on a manifold*, Trans. Amer. Math. Soc. 120, 286–294 (1965).

[2] Levi, M. *On a problem by Arnold on periodic motions in magnetic fields*, Comm. Pure and Applied Mathematics. 56 (8), 1165–1177 (2003).

[3] Dacorogna, B and Moser, J. *On a partial differential equation involving the Jacobian determinant*. Ann. Inst. H. Poincaré Anal. non linéaire. 7(1), 1–26 (1990).

precisely in agreement with (2). Formally, differentiating the mass integral gives two terms which cancel each other.

Food Security

continued from page 1

egies to promote urban food production, healthy diets, and social capital?

(3) How can we design a market system so that food prices embody the externalities (social and environmental costs) of food choices?

These questions were selected through a voting process from a longer list of 14 questions suggested by the participants on the first day of the workshop (see sidebar).

The (often lively) discussions yielded insight into the nature of the questions, relevant metrics, the availability of quantitative data (or lack thereof), and the modeling options. Clear cross-cutting themes emerged: interdisciplinary research, hybrid modeling, data mining, etc. Also, it soon became apparent that there is no hope for a one-size-fits all approach to the questions. For example, the discussions on the incorporation of externalities in food prices (question 3) brought to light the fact that products as diverse as corn and shrimp pose very different challenges: while the government has significant authority to regulate the price of a domestic staple product like corn, it has very little control over the externalities of shrimp, which is mostly produced abroad and imported.

Most interesting, workshop participants identified a long list of exploratory projects suitable for research with graduate and undergraduate students, including summarizing and visualizing data sets; construct-

ing heat maps representing particular indices from nutrition databases; designing agent-based models to simulate behavior and choice processes; coupling conceptual dynamical-systems models, agent-based models, and Bayesian network models; and designing an object-oriented framework for modeling the food system.

The AIM workshop was the start of an effort to bring a new area of applications to the attention of the mathematics and computational science communities. More needs to be done. Food security will be one of the themes at the inaugural conference of the newly formed SIAM Activity Group on Mathematics of Planet Earth (SIAG/MPE), which will be held in Philadelphia, September 30–October 2, 2016.

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[1] T. Acharya et al., *Assessing Sustainable Nutrition Security: The Role of Food Systems*, The International Life Sciences Institute, Research Foundation, Center for Integrated Modeling of Sustainable Agriculture and Nutrition, Washington, DC, June 2014; <http://goo.gl/gEyQ1F>.

Notes

[2] <http://www.who.int/trade/glossary/story028/en/>

[3] <http://aimath.org/pastworkshops-foodsystem.html>

Hans Kaper, founding chair of SIAG/MPE and editor-in-chief of SIAM News, is an adjunct professor of mathematics at Georgetown University.

14 Questions Related to Food Security

(1) What drives dietary inequality in the US?

(2) What strategies can we implement to create a more self-sustaining highly urbanized population with few immediate agricultural resources in the environment?

(3) How do we make market prices for food reflect the true cost?

(4) How do we maintain adequate water supply for all stakeholders?

(5) What can the US do to preemptively protect against disaster or attack of the US food distribution system?

(6) How can agriculture in California represent food supply nationally by growing suitable crops that are wanted and needed in the diet?

(7) If the US diet transitions to align with the USDA healthy eating guidelines, what policies would facilitate that transition without increasing environmental impacts?

(8) How would national regulation of greenhouse gas emissions impact food security in the US?

(9) What is the role of US policies and programs in contributing to a healthy food system that operates within planetary boundaries?

(10) How can we inform and empower consumers so they can make informed decisions with their dollars about the systemic impact of their dollars?

(11) How do we ensure equitable access to and availability of nutritious food for all in a growing population?

(12) How does one capture the ethical dimension of hunger in modeling the food system?

(13) What are the technologies and resources (use of land, water, etc.) needed to sustainably and resiliently maintain and improve food security (of food produced by US)?

(14) How will emerging economies (such as China) affect the US food system?

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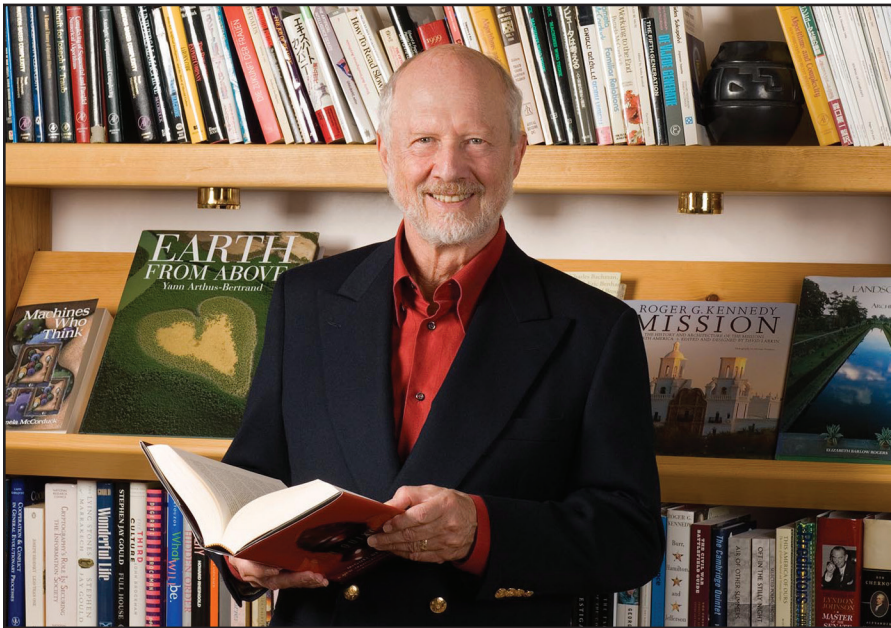
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Obituaries



Joseph F. Traub. Photo credit: Columbia University, Computer Science Department

Joseph F. Traub passed away on August 24, 2015, in his home in Santa Fe, New Mexico. He was 83 years old. His death was sudden and unexpected. Joe will be missed by his many friends and colleagues.

I knew Joe for 42 years. After I finished my PhD thesis from the University of Warsaw in 1973, Joe invited me to

Carnegie Mellon University in Pittsburgh as a visiting assistant professor. Since then, we have collaborated and written many papers and books together. We were close friends and I still find it hard to believe that Joe is no more.

Joe had a long and distinguished career as a pioneering computer scientist. In 1959,

he received his Ph.D. in theoretical physics from Columbia University. His Ph.D. thesis was based on six months of programming to calculate the ground energy state of a helium atom. Soon after receiving his doctorate, Joe started to work at Bell Labs, where his research focused on zero finding. In 1964, he wrote his first book, *Iterative Methods for the Solution of Equations*, which was published by Prentice Hall. In 1971, he became chair of computer science at Carnegie Mellon University and helped make it one of the strongest computer science departments in the world. He founded the computer science department at Columbia University in 1979, where he was the Edwin Howard Armstrong Professor of Computer Science until his passing.

Joe's main research area was solving continuous problems for which information is partial, priced, and contaminated. Today, this field is called information-based complexity (IBC). The first research monograph on IBC, entitled *Information-Based Complexity*, was written by Joe along with Grzegorz Wasilkowski and myself, and published by Academic Press in 1988. Today there are eight books and literally hundreds of papers on IBC. Throughout his career, Joe organized multiple conferences and workshops with IBC as the main

research subject. The last of such meetings that Joe helped organize, the Dagstuhl Seminar, was held in Dagstuhl, Germany last month, sadly without Joe.

In 1985, Joe founded the *Journal of Complexity*, and continued as the journal's editor-in-chief for the past three decades. Joe also founded the Computer Science and Technology Board of the National Research Council, serving twice as its chairman.

Joe was the author of over 120 papers, as well as the author or editor of ten monographs. His last few papers were on quantum computing. He received numerous awards and honors.

Joe is survived by his wife Pamela McCorduck and his two daughters from his first marriage, Claudia Traub-Cooper and Hillary Spector.

Henryk Wozniakowski is Emeritus Professor of Computer Science at Columbia University and Emeritus Professor of Applied Mathematics at the University of Warsaw.

Joseph F. Traub was a loyal member of SIAM for well over 30 years. An oral history with Traub was conducted by computer historian Thomas Haigh in March of 2004. View it on SIAM's history project here:

<http://history.siam.org/oralhistories/traub.htm>

Linkage

continued from page 1

engineer is to synthesize the mechanism, that is, find a member of the family that accomplishes specific tasks. Starting with data from the flight of black-billed magpies and pigeons [3] (see Figure 2 (right)), a set of tasks was developed to impose constraints on the family. In this case, the constraints are represented as polynomial equations that must be satisfied, thereby providing the "link" between mechanical engineering and algebraic geometry. This is commonly referred to as algebraic kinematics [4].

One common synthesis approach is to specify task points where the correspond-

ages, the engineer culls through to locate a linkage that suitably accomplishes the prescribed function. That is, selecting a linkage that is free of branch and circuit defects and evaluating other transmission characteristics. In the case of the bird-flapping linkage, this produced 7,363 candidate designs, one of which is shown in Figure 1, which passes through 8 of the 11 selected task points as seen in Figure 2 (right). This is a rich haul for the designer to select a proper linkage, particularly since current optimization methods yield a single design.

This synthesis calculation of the form of a six-bar linkage to achieve a given function was simply not possible before, and demonstrates the opportunities for innovation that

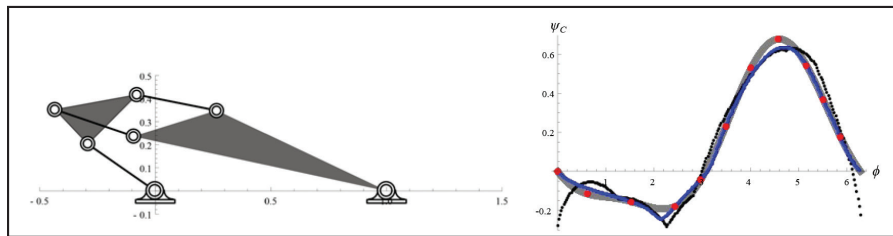


Figure 2. (left) A Stephenson II six-bar linkage designed to transform uniform rotation to a specific joint angle function. (right) Comparing the angle of the elbow joint of a bird-wing [3] (black dots), fitted Fourier function (gray), and the function obtained from the synthesized six-bar linkage (blue) passing through 8 of the 11 task points (red dots). Both figures reprinted from [2] with permission.

ing curve generated by the linkage, called a coupler curve, must pass through. To yield finitely many six-bar linkages, 15 task points are needed, yielding a system of 154 quadratic equations in 154 variables. Since computing all solutions of this polynomial system was beyond the current reach of available computing resources, a simplification was made to produce a polynomial system consisting of 70 quadratic equations in 70 variables specifying up to 11 task points. Although such a polynomial system could have $2^{70} \approx 10^{21}$ isolated solutions, algebraic geometry in the form of the multihomogeneous Bézout theorem shows that there can be at most roughly 3×10^8 isolated solutions. Solving such a system is within reach and all of its isolated solutions were obtained using Bertini running for over 300 hours on a cluster consisting of 256 processing cores. After solving this once, numerical algebraic geometry via a parameter homotopy permits one to solve other synthesis problems on this family of mechanisms in 2 hours on a typical desktop workstation or under 30 seconds on the cluster described above.

After computing the full solution set, in this case consisting of over 1.5 million link-

ages can arise from the collaboration of numerical algebraic geometry and mechanical design.

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- [3] B. Tobalske and K. Dial, *Flight kinematics of black-billed magpies and pigeons over a wide range of speeds*, *J. Exper. Biol.*, 199:2 (1996), 263–280.
- [4] C.W. Wampler and A.J. Sommese, *Numerical algebraic geometry and algebraic kinematics*, *Acta Numer.*, 20 (2011), 469–567.

Jonathan D. Hauenstein is an associate professor in the Department of Applied and Computational Mathematics and Statistics at the University of Notre Dame. J. Michael McCarthy is a professor in the Department of Mechanical and Aerospace Engineering at the University of California, Irvine.

Gil Strang Celebration



Gil Strang, (right), at his 80th birthday celebration at ICIAM 2015, with the organizer of the event, Alan Edelman, posing in front of a birthday cake designed based on the cover of Gil's SIAM book "Introduction to Linear Algebra, Third Edition."

More than one hundred people gathered at the 8th International Congress on Industrial and Applied Mathematics (ICIAM 2015) in Beijing to honor Gil Strang and celebrate his 80th birthday. The organizers of the event (Alan Edelman and Pavel Grinfeld) had invited select Congress attendees to join in the festivities. Guests were treated to a sumptuous buffet dinner. After a champagne toast to Gil, Alan Edelman invited participants to come to the microphone and share a brief story about Gil. It was a wonderful way to celebrate one of SIAM's most esteemed members and a most memorable event.



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ICIAM 2015 in Beijing



Jean-Michel Coron of the Université Pierre et Marie Curie receives the 2015 ICIAM Maxwell Prize from China's Vice-President Li Yuanchao.



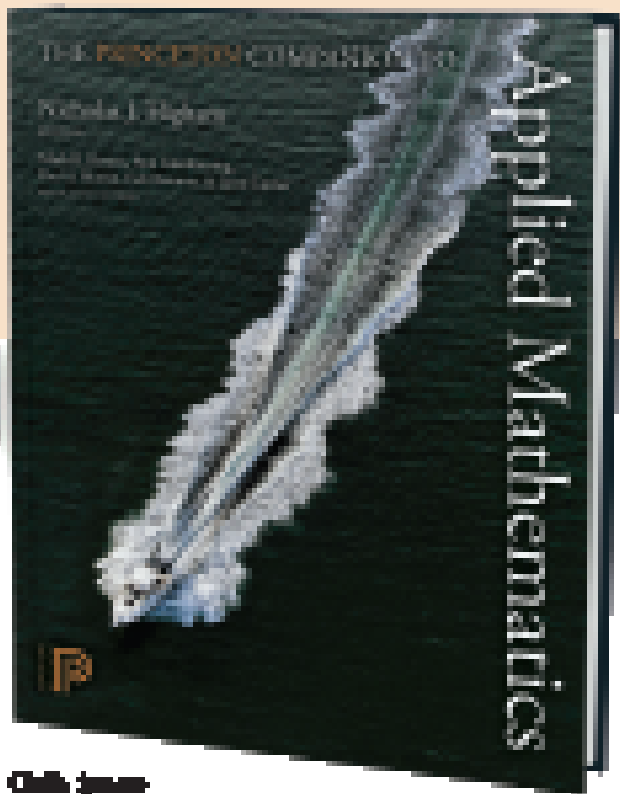
ICIAM President Barbara Keyfitz greets China's Vice-President Li Yuanchao at the ICIAM 2015 opening ceremony.

The 8th International Congress on Industrial and Applied Mathematics (ICIAM 2015) held in Beijing, China, in August was a great success, drawing over 3,400 attendees. A stellar lineup of invited speaker sessions and minisymposia, prize lectures, and poster sessions contributed to the meeting's success. Recipients of the five ICIAM prizes for 2015 received their awards and delivered prize lectures: Annalisa Buffa (Institute for Applied Mathematics and Information Technologies, Pavia-Genoa-Milan section) received the Collatz Prize; Andrew J. Majda (Courant Institute at New York University) received the Lagrange Prize; Jean-Michel Coron (Université Pierre et Marie Curie) received the Maxwell Prize; Björn Engquist (The University of Texas at Austin) received the Pioneer Prize; and Li Tatsien (Fudan University) received the Su Buchin Prize.



2015 ICIAM Prize winners with China's Vice-President Li Yuanchao and ICIAM President Barbara Keyfitz. (From left): Collatz Prize winner Annalisa Buffa; Lagrange Prize recipient Andrew Majda; Maxwell Prize winner Jean-Michel Coron; China's Vice-President Li Yuanchao; ICIAM President Barbara Keyfitz; Pioneer Prize recipient Bjorn Engquist; and Su Buchin Prize winner Li Tatsien.

Look for more photos and updates in subsequent issues, including items on SIAM prizes.



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The Jungle of Stochastic Optimization

By Warren B. Powell

There is a vast range of problems that fall under the broad umbrella of making sequential decisions under uncertainty. While there is widespread acceptance of basic modeling frameworks for deterministic versions of these problems from the fields of math programming and optimal control, sequential stochastic problems are another matter.

Motivated by a wide range of applications, entire fields have emerged with names such as dynamic programming (Markov decision processes, approximate/adaptive dynamic programming, reinforcement learning), stochastic optimal control, stochastic programming, model predictive control, decision trees, robust optimization, simulation optimization, stochastic search, model predictive control, and online computation. Problems may be solved offline (requiring computer simulation) or online in the field, which opens the door to the communities working on multi-armed bandit problems¹. Each of these fields has developed its own style of modeling, often with different notation (x or S for state, $x/a/u$ for decision/action/control), and different objectives (minimizing expectations, risk, or stability). Perhaps most difficult is appreciating the differences in the underlying application. A matrix K could be 5×5 in one class of problems, or $50,000 \times 50,000$ in another (it still looks like K on paper). But what really stands out is how each community makes a decision.

Despite these differences, it is possible to pull them together in a common framework that recognizes that the most important (albeit not the only) difference is the nature of the *policy* being used to make decisions over time (we emphasize that we are only talking about *sequential problems*, consisting of decision, information, decision, information, ...). We start by writing the most basic canonical form as

$$\min_{\pi \in \Pi} \mathbb{E}^{\pi} \sum_{t=0}^T C(S_t, U_t^{\pi}(S_t)) \quad (1)$$

where $S_{t+1} = S^M(S_t, u_t, W_{t+1})$.

Here, we have adopted the notational system where S_t is the state (physical state, as well as the state of information, and state of knowledge), and u_t is a decision/action/control (alternatives are x_t , popular in operations research, or a_t , popular in operations research as well as computer science). We let $U_t^{\pi}(S_t)$ be the decision function, or policy, which is one member in a set Π where π specifies both the type of function, as well as any tunable parameters $\theta \in \Theta^{\pi}$. The function $S^M(S_t, u_t, W_{t+1})$ is known as the transition function (or system model, state model, plant model, or simply "model"). Finally, we let W_{t+1} be the information that first becomes known at time $t+1$ (control theorists would call this W_t , which is random at time t).

Important problem variations include different operators to handle uncertainty; we can use an expectation in (1), a risk measure, or worst case (robust optimization), as well as a metric capturing

system stability. We can assume we know the probability law behind W_t , or we may just observe S_{t+1} given S_t and u_t (model-free dynamic programming).

While equation (1) is well-recognized in certain communities (some will describe it as "obvious"), it is actually quite rare to see (1) stated as the objective function with anything close to the automatic writing of objective functions for deterministic problems in math programming or optimal control. We would argue that the reason is that there is no clear path to computation. While we have powerful algorithms to solve over real-valued vector spaces (as required in deterministic optimization), equation (1) requires that we search over spaces of functions (policies).

Lacking tools for performing this search, we make the argument that all the different fields of stochastic optimization can actually be described in terms of different classes of policies. In fact, we have identified four fundamental (meta) classes, which are:

1. Policy function approximations (PFAs) – These are analytical functions that map states to actions. PFAs may come in the form of lookup tables, parametric, or non-

parametric functions. A simple example might be

$$U^{\pi}(S_t | \theta) = \sum_{f \in F} \theta_f \phi_f(S_t) \quad (2)$$

where F is a set of features, and $(\phi_f(S_t))$, $f \in F$ are sometimes called basis functions.

2. Cost function approximations (CFAs) – Here we are going to design a parametric cost function, or parametrically modified constraints, producing a policy that we might write as

$$U^{\pi}(S_t | \theta) = \arg \min_{u \in U_t^{\pi}(\theta)} C_t^{\pi}(S_t, u | \theta) \quad (3)$$

where $C_t^{\pi}(S_t, u | \theta)$ is a parametrically modified set of costs (think of including bonuses and penalties to handle uncertainty), while $U_t^{\pi}(\theta)$ might be a parametrically modified set of constraints (think of including schedule slack in an airline schedule, or a buffer stock).

3. Policies based on value function approximations (VFAs) – These are the policies most familiar under the umbrella of dynamic programming and reinforcement learning. These might be written as

$$U_t^{\pi}(S_t | \theta) = \arg \min_{u \in U_t^{\pi}(\theta)} C(S_t, u) + \mathbb{E}^{\pi} \{ \bar{V}_{t+1}^{\pi}(S^M(S_t, u, W_{t+1}) | \theta) | S_t \} \quad (4)$$

where $\bar{V}_{t+1}^{\pi}(S_{t+1})$ is an approximation of the value of being in state $S_{t+1} = S^M(S_t, u, W_{t+1})$, where π captures the structure of the approximation and $\theta \in \Theta^{\pi}$ represents any tunable parameters.

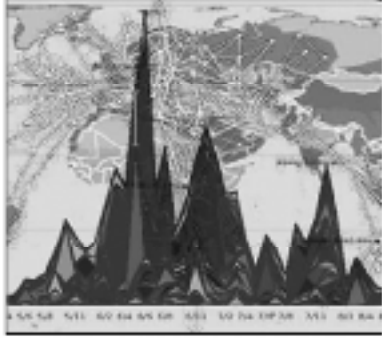
4. Lookahead policies – Lookahead policies start with the basic observation that we can write an optimal policy using

$$U_t^{\pi}(S_t | \theta) = \arg \min_u \left(C(S_t, u) + \min_{\pi \in \Pi} \mathbb{E}^{\pi} \left\{ \sum_{t'=t+1}^T C(S_{t'}, U_{t'}^{\pi}(S_{t'})) | S_t, u \right\} \right) \quad (5)$$

The problem is that the second term in (5) is not computable (if this were not the case, we could have solved the objective function

See *Stochastic Optimization* on page 12

INSTITUTE FOR PURE AND APPLIED MATHEMATICS
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Culture Analytics

March 7 - June 10, 2016

ORGANIZING COMMITTEE: Tina Elfassi-Rad (Rutgers University), Mauro Maggioni (Duke University), Lev Manovich (CUNY), Vwani Roychowdhury (UCLA), Timothy Tangherlini (UCLA)

Scientific Overview

The explosion in the widespread use of the Internet and social media and the ubiquity of low cost computing have increased the possibilities for understanding cultural behaviors and expressions, while at the same time have facilitated opportunities for making cultural artifacts both accessible and comprehensible. The rapidly proliferating digital footprints that people leave as they crisscross these virtual spaces offer a treasure trove of cultural information, where culture is defined as expressions of the norms, beliefs and values of a group. This program encourages the exploration of the unsolved mathematical opportunities that are emerging in this cultural information space. Many successful approaches to the analysis of cultural content and activities have been developed, yet there is still a great deal of work to be done. In this program, we aim to promote a vigorous collaboration across disciplines and devise new approaches and novel mathematics to address these problems of culture analytics, by bringing together leading scholars in the social sciences and humanities with those in applied mathematics, engineering, and computer science.





Workshop Schedule

- Tutorials: March 7-11, 2016.
- Workshop I: Culture Analytics Beyond Text: Image, Music, Video, Interactivity and Performance. March 21-24, 2016.
- Workshop II: Culture Analytics and User Experience Design. April 11-15, 2016.
- Workshop III: Cultural Patterns: Multi-scale Data-driven Models. May 9-13, 2016.
- Workshop IV: Mathematical Analysis of Cultural Expressive Forms: Text Data. May 23-27, 2016.
- Culminating Workshop at Lake Arrowhead. June 5-10, 2016.

Participation

Most participants, including senior and junior researchers, will be in residence at IPAM for the entire period. Between the workshops there will be a program of activities involving the long-term and short-term participants, and visitors. Applications will be accepted through December 7, 2015, but decisions will be made starting in July. We have funding especially to support the attendance of graduate students and researchers in the early stages of their career, but we welcome applications from researchers at all levels. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications.

For more information: www.ipam.ucla.edu/ca2016

¹ <http://www.castlelab.princeton.edu/jungle.htm>

Discrete-Time Markov Jump Linear Systems

By O.L.V. Costa and M.D. Fragoso

When associated with unexpected events that cause losses, abrupt changes are extremely undesirable. Such changes can be due, for instance, to environmental disturbances, component failures or repairs, changes in subsystem interconnections, changes in the operation point of a nonlinear plant. These situations can arise in economic systems, aircraft control systems, solar thermal plants with central receivers, robotic manipulator systems, communication networks, large flexible structures for space stations.

It is important to have efficient tools to deal with the effects of abrupt changes. To that end, we must be able to model the changes adequately. In a control-oriented perspective, attempts to carve out an appropriate mathematical framework for the study of dynamical systems subject to abrupt changes in structure (switching structure) date back at least to the 1960s.

In this scenario, a critical design issue for modern control systems is that they should be capable of maintaining acceptable behavior and meeting certain performance requirements even in the presence of abrupt changes in the system dynamics. Within this context lies a particularly interesting class of models: discrete-time Markov jump linear systems (MJLSs). Since its inception the models in this class have been closely connected with systems that are vulnerable to abrupt changes in their structure, and the associated literature surrounding this subject is fairly extensive (see, for example, [2, 3, 13], and references therein).

To introduce the main ideas, we consider the simplest homogeneous MJLS, defined as:

$$\begin{aligned} x(k+1) &= A_{\theta(k)} x(k), \\ x(0) &= x_0, \theta(0) = \theta_0, \end{aligned} \quad (1)$$

where $A_i \in \mathbb{R}^{n \times n}$ and $\{\theta(k)\}$ represents a Markov chain taking values in $\{1, \dots, N\}$, with transition probability matrix $P = [p_{ij}]$. Here, $\{\theta(k)\}$ accounts for the random mechanism that models the abrupt changes (this is sometimes called the “operation mode”). Although an MJLS seems, *prima facie*, to be a simple extension of a linear equation, it carries a great deal of subtleties that distinguish it from the simple linear case, and it provides us with very rich structure.

A first analytical difficulty is that $\{x(k)\}$ is not a Markov process, although the joint process $\{x(k), \theta(k)\}$ is. Because stability is an important bedrock of control theory, a key issue was to work out an adequate stability theory for MJLSs. In earlier work stability was sometimes considered for each mode of the system, but it soon became clear that this approach could not adequately deal with the many nuances of MJLSs. This issue was adequately settled only after the introduction of the concept of mean-square stability for this class of systems.

To illustrate how MJLSs can surprise us and run counter to our intuition, we present three examples that unveil some of these subtleties in the context of stability. Of the several different concepts of stochastic stability, we simplify the presentation here by considering only the following: The homogeneous MJLS is mean-square stable (MSS) if for any initial condition (x_0, θ_0) , $E(\|x(k)\|^2) \rightarrow 0$ as $k \rightarrow \infty$. It is shown in [2] that mean-square stability is equivalent to the spectral radius of an augmented matrix \mathcal{A} being less than one or to the existence of a unique solution to a set of coupled Lyapunov equations, which can be written in four equivalent forms. This augmented matrix \mathcal{A} is defined as $\mathcal{A} = \mathcal{CN}$, where $\mathcal{C} = P' \otimes I$ and $\mathcal{N} = \text{diag}[A_i \otimes A_i]$ (with \otimes representing the Kronecker operator). Our three examples illustrate only the equivalence between mean-square stability and the spectral radius of \mathcal{A} .

Example 1

Consider the following system with two operation modes, defined by matrices $A_1 = 4/3$, $A_2 = 1/3$ (mode 1 is unstable, mode 2 stable). The transitions between these modes are given by the transition probability matrix

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.$$

It is easy to verify that for this transition probability matrix we have

$$\mathcal{A} = \frac{1}{2} \begin{bmatrix} 16 & 1 \\ 9 & 9 \\ 16 & 1 \\ 9 & 9 \end{bmatrix}$$

and $r_\sigma(\mathcal{A}) = 17/18 (< 1)$, and so the system is MSS. Suppose now that we have a different transition probability matrix, say

$$\bar{P} = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{bmatrix},$$

the system will most likely stay longer in mode 1, which is unstable. Then

$$\mathcal{A} = \begin{bmatrix} 144 & 1 \\ 90 & 10 \\ 16 & 1 \\ 9 & 9 \end{bmatrix},$$

$r_\sigma(\mathcal{A}) = 1.61 (> 1)$ and the system is no longer MSS. This evinces a connection between mean-square stability and the probability of visits to the unstable modes, which is translated in the expression for \mathcal{A} .

Our next two examples, borrowed from [9], illustrate how the switching between operation modes can play tricks with our intuition. As shown in these striking examples, an MJLS composed only of unstable modes can be MSS, and, alternatively, an MJLS composed only of stable modes can be unstable in the mean-square sense.

Example 2

Here we consider a non-MSS system with stable modes. The two operation modes are defined by matrices

$$A_1 = \begin{bmatrix} 0 & 2 \\ 0 & 0.5 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 0.5 & 0 \\ 2 & 0 \end{bmatrix}$$

and the transition probability matrix

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.$$

Both modes are stable. Curiously, $r_\sigma(\mathcal{A}) = 2.125 > 1$, which means that the system is not MSS. A brief analysis of the trajectories for each mode helps to clarify the matter.

We begin by considering only trajectories for mode 1. For initial conditions given by

$$x(0) = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

the trajectories are given by

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 2(0.5)^{k-1} x_{20} \\ 0.5(0.5)^{k-1} x_{20} \end{bmatrix} \quad \text{for } k = 1, 2, \dots$$

With the exception of the point $x(0)$, the whole trajectory thus lies along the line $x_1(k) = 4x_2(k)$ for any initial condition. This means that if, in a given time, the state is not on this line, mode 1 dynamics will transfer it to the line in one time step and it will remain there thereafter. For mode 2, it is easy to show that the trajectories are given by

$$\begin{aligned} x(k) &= \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ &= \begin{bmatrix} 0.5(0.5)^{k-1} x_{10} \\ 2(0.5)^{k-1} x_{10} \end{bmatrix} \\ &\quad \text{for } k = 1, 2, \dots \end{aligned}$$

Much as in the case for mode 1, if the state is not on the line $x_1(k) = x_2(k)/4$, mode 2

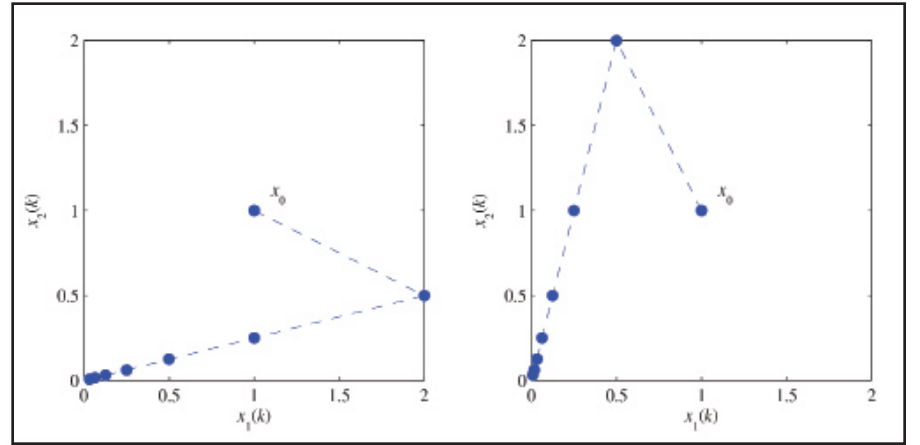


Figure 1. Trajectories for operation modes 1 (left) and 2 (right).

dynamics will transfer it to the line in one time step. The equations for the trajectories also show that the transitions make the state switch between these two lines. Notice that transitions from mode 1 to mode 2 cause the state to move away from the origin in the direction of component x_2 , while transitions from mode 2 to mode 1 do the same with respect to component x_1 . Figure 1 (left) shows the trajectory of the system with mode 1 dynamics only, for a given initial condition; Figure 1 (right) does the same for mode 2. Figure 2 shows the trajectory for a possible sequence of switches between the two modes, an indication of the instability of the system.

Example 3

For our final example we consider an MSS system with unstable modes in which

$$A_1 = \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

and the transition probability matrix

$$P = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}.$$

Although both modes are unstable, $r_\sigma(\mathcal{A}) = 0.4 (< 1)$.

■■■

The general conclusion we extract from these examples is that the stability of each operation mode is neither a necessary nor a sufficient condition for the mean-square stability of the system. Mean-square stability depends on a balance between the transition probability of the Markov chain and the operation modes. These and many other examples, in the context of stability illustrate peculiar properties of these systems, which can be included in the class of complex systems (roughly defined as systems composed of interconnected parts that as a whole exhibit one or more properties not obvious from the properties of the individual parts).

Other features that set MJLSs outside classical linear theory include the following: (i) The filtering problem is associated with more than one scenario. In the harder case of partial observations of $(x(k), \theta(k))$, the filter is infinite-dimensional; a separation principle for this setting is an open problem. (ii) In view of a set of coupled Riccati equations, which appears in some filtering and control problems, a fresh look at such concepts as stabilizability and detectability was necessary, giving rise to a mean-square theory for these concepts. (iii) With the various possible settings of the state-space of the Markov chain (e.g., finite, infinite countable, Borel space), the analytical complexity of the problem can change. In a nutshell, we can say that an MJLS differs from the linear case in many fundamental issues.

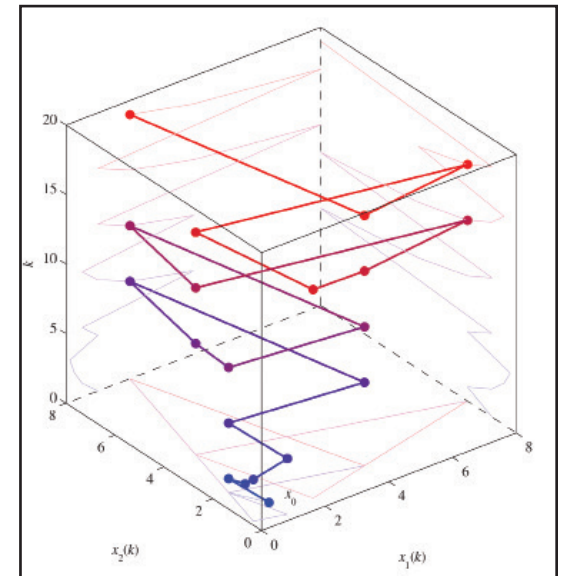


Figure 2. One possible trajectory for the Markov system. The trajectory tends to move away from the origin. The time $k = 1, \dots, 20$ is presented on the z-axis. The colored lines are trajectory projections.

Other interesting instances and a compilation of ideas about MJLSs can be found in [1, 2, 3, 5, 13]. Due, in part, to an adequate set of concepts and mathematical techniques developed over the last decades, MJLSs have a well-established theory that provides systematic tools for the analysis of many dynamical systems subjected to abrupt changes, yielding a great variety of applications.

Since the specialized literature on applications of the theory of MJLS is very large and rapidly expanding, we provide here only some representative references, including [16], on applications in robotics; [6] and [18], on problems of image enhancement (e.g., tracking and estimation); [4] and [19], on mathematical finance; [8, 14, 15] and [20], on communication networks (packet loss, fading channels, chaotic communication); [10], on wireless issues; [7], on flight systems (including electromagnetic disturbances and reliability; see also [17], for control of wing deployment in aircraft); [11, 12], on issues related to electrical machines. Additional references are given in [2] and [3].

Last, but not least, we round out this note by mentioning that some MJLS-control problems belong to a select group of solvable stochastic control problems and are therefore of great interest in any course on stochastic control. In addition, despite the notable abundance of relevant reference materials on the subject, MJLSs stand firmly as a topic of intense research.

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Markov Jump

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The Personality of Numbers

Single Digits: In Praise of Small Numbers.

By Marc Chamberland, Princeton University Press, Princeton, NJ, 2015, 240 pages, \$26.95.

Almost sixty years ago, the New Mathematical Library was set up to publish a series of monographs that would appeal to a wide audience. Guided by an editorial panel of a dozen distinguished mathematicians, the series had as its technical editor Anneli Lax at New York University.

Lax tapped me to be one of the first to contribute a volume to the series. I agreed, provided that my text could contain some material that was exceedingly elementary. She said OK to that, and so *The Lore of Large Numbers* (Yale University Press, 1961) was born.

I titled a section of this book “The Personality of Numbers,” and by that I meant simply such arithmetic features of numbers as odd, even, square, triangular, prime, perfect; after discussing various tests for divisibility, with trumpets blaring, I wound up with Wilson’s theorem: A number N is prime if and only if it divides $(N - 1)! + 1$.

Marc Chamberland, a professor of mathematics at Grinnell College, has produced a book that I would sub-subtitle “The Personality of the Numbers One to Nine.” Devoting a separate chapter to each of these numbers, he extends my notion of “personality” far beyond mere divisibility. Here are examples from some of the chapters (each of which contains much, much more):

In Chapter Three, Chamberland discusses, among other things, Morley’s theorem; Poincaré and the three-body problem; the Lorenz attractor and chaos (“Period Three Implies Chaos”); Ramanujan sequences; and Monge’s three-circle theorem.

In Chapter Five, we meet the platonic solids, the difficult quintic equation, and the Miquel five-circles theorem.

In Chapter Eight, we find the pizza theorem, the game of life, and Lie groups.

In Chapter Nine, among much else, we learn about two circle packings and casting out nines. But I hoped to see the famous nine-point circle theorem, of which Dame Mary L. Cartwright once told me she needed to know two proofs in order to advance her career.

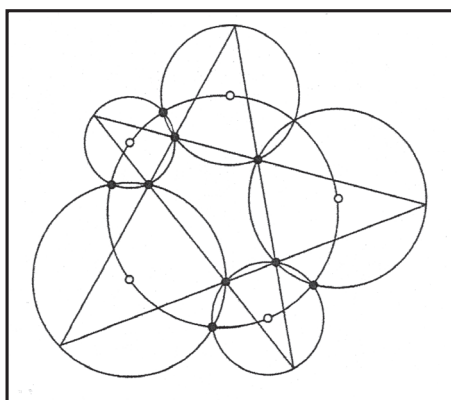
But where is the number zero—about which a whole book could, should, and perhaps has been written? Some authorities have claimed that zero is not a number at all.

Chamberland’s writing is lucid, interesting, often amusing, and informative. The reader will find numerous conjectures and unsolved problems. I came away from the book knowing of many things I had never encountered.

One of the pleasures I find in reviewing comes when something an author mentions—however remotely—connects up with something I had thought about or carried out. An example in

BOOK REVIEW

By Philip J. Davis



Miquel’s Five Circle Theorem

Chamberland’s book in Chapter Two shows the Apollonian packings of a circle by smaller circles. This reminded me of a proof I once gave, using interpolation theory, of the visually obvious result that a circle cannot be filled out with a finite number of non-overlapping smaller circles. Now, if something is visually obvious, what is the need for proof? Call up Mr. Euclid and ask him about the role intuition should play in the math biz.

*SIAM News published reviews of two books about zero in the September 2000 issue: *The Nothing That Is: A Natural History of Zero* by Robert Kaplan, and *Zero: The Biography of a Dangerous Idea* by Charles Seife.

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Data Assimilation in Numerical Weather Prediction

By Sebastian Reich and Andrew M. Stuart

[A second part to this article will be published in the November issue of SIAM News.]

The seamless integration of large data sets into sophisticated computational models provides one of the central challenges for the mathematical sciences in the 21st century. When the computational model is based on dynamical systems and the data set is time ordered, the process of combining models and data is called data assimilation. The assimilation of data into computational models serves a wide spectrum of purposes, ranging from model calibration and model comparison, all the way to the validation of novel model design principles.

Historically, the rise of numerical weather prediction (NWP) in the 1950s played a major role in germinating the field of data assimilation. The computational models that were employed subsequently demanded algorithms for determining initial model states from available observations. Such a task falls naturally within the realm of ill-posed inverse problems [2], with the important caveat that Tikhonov-type regularizations have to be consistent with the underlying model dynamics [1]; indeed it was discovered that forecast skill could be dramatically improved by explicitly including the NWP models into the data assimilation cycle [12]. The data assimilation technique associated with this viewpoint

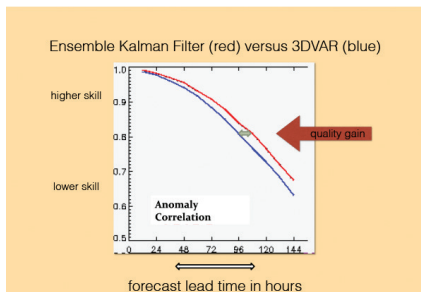


Figure 1. Improvement of forecast skills for temperature in the southern hemisphere through the use of an ensemble Kalman filter data assimilation system at the German Meteorological Service (DWD). Image credit: Roland Potthast (German Meteorological Service & University of Reading)

is still widely used in operational weather forecasting, and, collectively, the methods are referred to by the synonym 4DVAR (standing for four dimensions—three space plus time—and a cost functional to be

minimized). The 4DVAR methodology fits into the framework of Tikhonov regularized inverse problems where the regularization term on the initial condition is balanced by a term reflecting faithful reproduction of the model dynamics.

A second class of algorithms widely used by the NWP community are the Kalman filter-type methods emerging from the control community [6, 7]. Kalman's work has been enormously influential, constituting an early systematic development of a methodology to combine models and data for dynamical systems; it is applicable to linear problems subject to additive Gaussian noise.

An early suggestion to use the Kalman filter in the solution of linear PDEs arising in the atmospheric sciences is [4]. Early extensions of the classic Kalman filter to nonlinear systems include the extended Kalman filter [5]. However, computational expense, together with the strong nonlinearity of atmosphere-ocean dynamics, prevented an operational implementation of the extended Kalman filter. Instead, operational weather centers implemented a greatly simplified version of the Kalman update equations by cycling the 3DVAR methodology [9, 11]. In this method, the data is incorporated sequentially at each fixed time so it is optimized over three space dimensions. Structurally this cycled 3DVAR looks like an extended Kalman filter-type update, but with a fixed covariance structure to weight the model reliability versus that of the data.

Steadily increasing computer power eventually allowed for an extension of this cycling approach by combining it with ensemble prediction, which became prevalent in the NWP community in the 1980s [8]. In this approach, rather than making a single best weather forecast, an ensemble of forecasts is made and their variability is used to weight the reliability of the model in comparison with the reliability of the data. Current ensemble-based data assimilation methodologies rely on linear regression to combine forecast uncertainties and observations and are collectively termed ensemble Kalman filters (EnKFs) [3]. Employing EnKFs in operational data assimilation systems has led to improved forecast skill compared to the simplified 3DVAR approach (see Figure 1). While EnKFs provide an elegant extension of the classic Kalman filter to the highly large-scale non-Gaussian and nonlinear NWP models in use today, the underlying linear

regression ansatz also places limitations on their ability to predict, for example, extreme meteorological events.

Current research activities in data assimilation for NWP focus on expanding the range of observational systems (see Figure 2) and on merging 4DVAR with ensemble prediction systems on the one hand, and sequential Monte Carlo methods with EnKFs on the other. Practical challenges for such extensions arise, for example, from the relatively small affordable ensemble sizes (on the order of 100) and the presence of spatially and temporally correlated data and model errors that cannot be easily represented by standard stochastic processes.

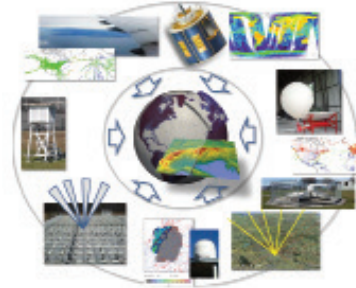


Figure 2. Range of observational systems that deliver data to numerical weather prediction systems. Image credit: Roland Potthast (German Meteorological Service & University of Reading)

The field of petroleum reservoir simulations has also led to innovation in the development of data assimilation methods; in that context, there is a strong focus on combined model state and parameter estimation [10]. With reservoir modeling parameters (eg. permeability) often being hugely uncertain, data assimilation and uncertainty quantification become even more challenging for petroleum reservoir engineering.

[In part II to be published in the next issue, the authors explain why data assimilation is ripe for development by the mathematics community.]

Acknowledgments: Andrew Stuart is grateful to EPSRC, ERC, and ONR for financial support that led to the research underpinning this article. Sebastian Reich acknowledges support under the DFG Collaborative Research Center SFB1114: Scaling Cascades in Complex Systems.

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Sebastian Reich is a professor of numerische mathematik at the Institut für Mathematik at the Universität Potsdam, Germany, and the Department of Mathematics and Statistics, University of Reading, UK. Andrew Stuart is a professor of mathematics at the Mathematics Institute, Warwick University, UK. This article is based, in part, on a lecture delivered by Stuart at the 2015 SIAM Conference on Applications of Dynamical Systems, held in Snowbird, Utah.

40 Years of Preconditioning

By Wil Schilders

Eindhoven University of Technology, The Netherlands

The 9th International Conference on Preconditioning Techniques for Scientific and Industrial applications was held at Eindhoven University of Technology in June. About 100 participants and 8 invited speakers shared ideas and had lively discussions on a wide variety of topics. Information about the lectures and invited speakers, as well as some downloadable presentations are available on the conference website: <http://www.win.tue.nl/precon2015/>.

An evening lecture by Koos Meijerink and Henk van der Vorst about “40 years of preconditioning” at the Van Abbemuseum of modern art was a special feature, marking the speakers’ seminal paper about Incomplete Choleski Conjugate Gradient published in 1977 in *Mathematics of Computation*, the results having been available already around 1975. The location was chosen to reflect Henk’s current interest: [art](http://www.henkvandervorst.nl)¹.



Henk van der Vorst during the opening of the evening lecture

DMS Mathematical Sciences Research Institutes Update

The NSF Division of Mathematical Sciences (DMS) has published an update on the DMS Mathematical Sciences Research Institutes program. See insert for full article.



Computational Sciences in the Upstream Oil and Gas Industry: *The ExxonMobil Experience*

By Thomas C. Halsey

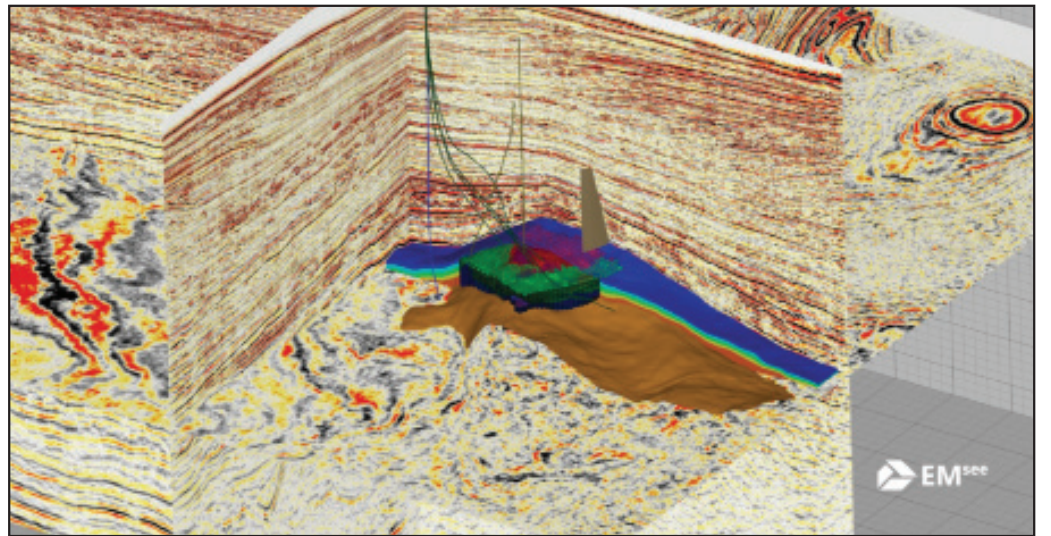
In the upstream oil and gas industry our aim is to identify and produce subsurface accumulations of oil and gas. From the earliest beginnings of our industry, our key scientific and engineering challenges have been related to our ignorance of the detailed structure of the subsurface. Our abilities to image the subsurface, thereby locating oil and gas fields, and to predict the flow of these fluids through subsurface porous rocks are often unsatisfactory, particularly in view of the large capital investments required in our business. While we do apply the most advanced technologies available to these problems, much uncertainty inevitably remains—one proverb of the industry is that we only truly understand an oil or gas field the day we close it down (and perhaps not even then!).

Because of our inability to directly measure important features of the subsurface, much of our decision-making is based on models. We use a variety of models to reconstruct seismic data into three-dimensional images of the subsurface, and we construct large reservoir models based on these images and other information to predict the flow of multiphase fluids through the reservoir during the production phase of an asset. Successful construction of these models requires the skills of expert

units that develop standalone or plug-in software used by technical decision makers throughout their businesses. While equipment and hardware advances remain critical to the industry, much of the innovation has moved into the software space, with research methods moving seamlessly into software used throughout our businesses.

The size of the computational problems we face has made the oil and gas industry one of the largest private sector users of high-performance computing. Numerous oil and gas operating and service companies have advertised their rapidly improving capabilities in petascale computing, and these companies have developed sophisticated technical teams to deploy, maintain, and develop specialized code for these systems.

Within ExxonMobil, we often respond to the need to develop and maintain technical excellence in an area by forming a functional organization dedicated to that area. Thus, in the last few years we have formed internal functional organizations dedicated to computational sciences in both our research and information technology organizations. These organizations



Specialized software allows oil and gas professionals to visualize, analyze and simulate subsurface phenomena. Photo courtesy of ExxonMobil.

bined to succeed in computational sciences in our company. These are

- 1) Modeling physics: the construction and validation of practical multi-physics models covering phenomena that are key to the performance of our business.
- 2) Computational and applied mathematics: the design and implementation of robust, accurate algorithms to solve our models on the necessary time scales.
- 3) Technical software engineering: the development of computer code suitable for solution of scientific and engineering problems within the constraints posed by our IT environment.
- 4) High-performance computing: the design, procurement, maintenance, and specialized code development required to solve industry grand challenge problems on leading high-performance computing architectures.

These skills, combined with more traditional industry geoscience and engineering skills, have enabled us to advance several major innovations based on

new computational approaches to seismic imaging as well as reservoir modeling and simulation since we increased our focus on the computational sciences a few years ago.

In addition to the subsurface element in our business, we also manage complex logistical operations and global supply chains in both our upstream and our refining and petrochemicals businesses. Business analytics and optimization are

important tools to improve our profitability in this aspect of our business as well.

Our industry is currently experiencing a cyclical downturn in oil and gas prices, which is driving an urgent need to improve business performance. One of the key methods to do so is to ensure that we are getting as much production as possible from our current assets, which requires optimization of a complex system including the subsurface reservoir, wells, pipelines, and surface facilities.

A new wave of innovations is emerging in the industry to help with this optimization; many of these innovations are based on data analytics methods that have been very successful in the finance, marketing, and social media industries. The oil and gas industry differs from these industries in the richness of the physics and geology that is embedded in our legacy modeling approaches, which must be effectively combined with the newer data analytics methods to enable our continued success. I am certain that the oil and gas industry will emerge as a leader in developing new science and mathematics that effectively combine data-driven and physics- or geoscience-driven methods in powerful ways.

Thomas Halsey is currently Chief Computational Scientist at ExxonMobil Upstream Research Co. in Spring, Texas. He has held a number of research, management, and staff positions in ExxonMobil in New Jersey and Texas. Prior to joining ExxonMobil in 1994 he was on the faculty of the University of Chicago.



Many oil and gas operating and service companies have invested in petascale computing over the past few years. Photo courtesy of ExxonMobil.


applied physicists, mathematicians, geoscientists, and engineers; these are among the most advanced modeling challenges, in scope and technical depth, to be found in any industry.

These models are invariably embedded in software. A large service industry has grown up to provide the software needed to construct and solve these models; in addition, many oil and gas operating companies have internal software development

drive rapid sharing of best practices among teams working diverse business problems, as well as assisting in key skills assessment, development, and maintenance. Of course, it is important to maintain strong links between computational work and the geoscientists and engineers with whom we must collaborate closely to impact our business.

We have concluded that there are four key skills areas that must be closely com-


The French Applied and Industrial Mathematical Society



SMAI Events :

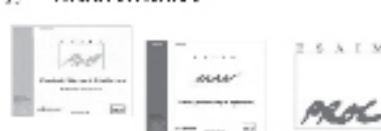
- > Conferences
- > Industrial Workshops
- > Summer Schools
- > Math job fair

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electronic publication (codram)
<https://ojs.math.cnrs.fr/index.php/SMAI-JCM>

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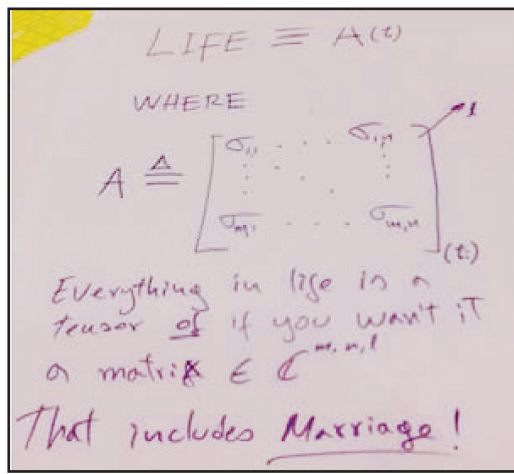
Life is a Tensor. . . Pilot Program Aims at Expanding SIAM Impact

By Mary Ann Leung and Silvia Crivelli

The 2015 SIAM Conference on Computational Science and Engineering (CSE15) broke attendance, fund raising, and other records. Adding to the list of accomplishments was a pilot mentoring and Broader Engagement (BE) program organized by Sustainable Horizons Institute (SHI), a nonprofit organization founded by Mary Ann Leung, a longtime SIAM member and member of the CSE15 Organizing Committee.

When Silvia Crivelli, a computational biologist at Lawrence Berkeley National Laboratory (LBNL), contacted Leung about a venue for students to present their research from the WeFold program, an immersive and intensive learning experiment she organized last summer at LBNL, CSE15 was the ideal choice. The program was part of the WeFold “coopetition” that brings together scientists worldwide to compete and collaborate with the goal of advancing research in protein folding.* By the end of the program, students, mostly undergraduates, who had never written computer code, and were unfamiliar with basic UNIX commands, had written and run code on a supercomputer, and created a database with protein models, among other things. At the end of the summer, the students—who had never been to or presented at a conference before—submitted abstracts for a minisymposium at CSE15. From this plan grew the idea of starting a Broader Engagement (BE) and mentoring program, with the vision of broadening SIAM’s impact by supporting the attendance of students, faculty, and professionals from diverse backgrounds, and encouraging their participation in CS&E research. The goals of the mentoring program were to not only connect protégés with mentors during the conference, but also to influence their careers long after the meeting. The mentoring program was open to all BE members as well as a small number of CSE15 attendees who expressed

*<http://wefold.nersc.gov>



Johnny Corbino, a doctoral student at San Diego State University, created a drawing positing that life, both professional and personal, is a tensor, expressing that there are trade-offs and decisions to be made with life stresses. Photo by Mary Ann Leung.

an interest in finding a mentor.

Through funding provided by NSF, SIAM, Argonne National Laboratory (ANL), Google, LBNL and individual donors, 40 students, faculty, and professionals from 32 institutions and 21 states, Puerto Rico, and Saudi Arabia received travel grants and participated in the program. The inaugural cohort consisted of nearly 50% female and 3% American Indians/Alaska Natives, 16% Asian, 28% Black/African American, 11% multiple race, 39% white, and 31% percent Hispanic. In addition, 3% indicated having a disability. It was a multidisciplinary group including computational biology; medicine; physics; earth sciences; computer science; computer engineering; numerical analysis/simulations; operations research; and stochastic research. Many were first-generation college scholars.

Mentors were recruited through the Workshop Celebrating Diversity, CSE15 registration system, as well as a campaign led by Leung and Elizabeth Leake from STEM-Trek who also served on the CSE15 BE committee. Other CSE15 BE committee members came from the SIAM and Supercomputing BE communities. Christine Harvey of Mitre Corporation organized

icebreaking, team building, and professional development activities. Other committee members included Richard Coffey of ANL, Ken Craft from Intel, and Veronica Vergara Larrea from ORNL.

One of the common challenges with mentoring programs is facilitating meaningful and lasting relationships between strangers. Two unique aspects of the CSE15 BE mentoring program: the “Pathways to Success” workshop and the “Wilderness” craft project, were specifically designed to tackle this challenge. In a “Pathways to Success” activity led by Leung, mentors and protégés were asked to draw and discuss a pathway through their past, present, and future. Janette Garcia, an undergraduate student at University of Texas Pan American, said “This workshop made me realize that there are many people that have overcome difficult situations in life, and still never gave up. I also realized how incredible my life has been, coming from a very low income family without enough money to buy food, and now having the opportunity to improve our way of life by being the first one in my family pursuing a bachelor’s degree in computer science.” Melissa Romanus, a doctoral student at Rutgers had this to say, “This workshop helped me realize that the route to your goals and dreams does not have to be perfect. In fact, it can be REALLY ‘hilly’ with ‘sharp and unexpected turns.’ But even if the path is non-traditional, you can still get to that endpoint as long as you keep pushing forward.”

A protégé described the “Wilderness” craft project, devised and led by Larisse Voufo of Google, as an effective icebreaker that gave the pair an opportunity for conversation to evolve naturally as they designed and built their project. Vicky Fondjo, a faculty member at Langston University, said of the project, “It had a good impact on my mentoring experience. It helped me make a

better connection with my mentor, and made us see similarities in our personal/professional lives.” Romanus observed, “I am not very good at art projects in general, but it was really therapeutic to be creating artwork and to be able to give it to someone else. While we were all working on our projects, we were also chatting about our fields and our experiences. It was amazing to find out how much we had in common, even though our backgrounds were so diverse.”

It was not all fun and games: the WeFold students also used the BE room to practice for their minisymposium. Seeing their accomplishments and watching them pres-



Ami Radunskaya, left, Associate Professor of Mathematics, Pomona College (mentor) and Vicky Fondjo, Assistant Professor of Mathematics, Langston University (protégé) in the BE room wearing their wilderness project artifacts. Photo by Mary Ann Leung.

ent their research with confidence and poise inspired and motivated other BE participants. Daily lunchtime sessions organized by Tony Drummond (LBNL) and Silvia Crivelli highlighted the next 24 hours of activities, supporting the technical focus of the BE program. To encourage participation in the poster session, Sally Ellingson of University of Kentucky discussed the importance of presenting a poster, preparation, and presentation skills. Protégé Iftikhar Ali, an applied mathematics doctoral student, spoke of the impact the program had on his experience, “Attending a SIAM meeting was always a dream for me. Initially, I thought it was going to be

See **Mentoring** on page 12

Professional Opportunities

Send copy for classified advertisements to: Advertising Coordinator, SIAM News, 3600 Market Street, 6th Floor, Philadelphia, PA 19104-2688; (215) 382-9800; fax: (215) 386-7999; marketing@siam.org. The rate is \$3.00 per word (minimum \$375.00). Display advertising rates are available on request.

Advertising copy must be received at least four weeks before publication (e.g., the deadline for the December 2015 issue is October 30, 2015).

Advertisements with application deadlines falling within the month of publication will not be accepted (e.g., an advertisement published in the December issue must show an application deadline of January 1 or later).

Students (and others) in search of information about careers in the mathematical sciences can click on “Careers and Jobs” at the SIAM website (www.siam.org) or proceed directly to

www.siam.org/careers

Institute for Advanced Study

School of Mathematics

The School of Mathematics at the Institute for Advanced Study has a limited number of memberships with financial support for research during the 2016–17 academic year. Applicants must give evidence of ability in research comparable at least with that expected for the PhD degree but otherwise can be at any career stage. Successful candidates will be free to devote themselves full time to research.

About half of the institute’s members will be postdoctoral researchers within five years of their PhD, and the IAS expects to offer some two-year postdoctoral positions. Up to eight von Neumann Fellowships will be available for each academic year. To be eligible for a fellowship, applicants should be at least five, but no more than fifteen, years following the receipt of their PhD.

The Veblen Research Instructorship is a three-year position in partnership with the department of mathematics at Princeton University. Three-year instructorships will be offered each year to candidates in pure and applied mathematics who have received their PhD within the last three years. Usually the first and third year of the instructorship will be spent at Princeton University and will carry regular teaching responsibilities. The second year is spent at the Institute and dedicated to independent research of the instructor’s choice. Applicants interested in a Veblen instructorship position may apply directly at the IAS website (<https://application.ias.edu>) or through MathJobs. If applying at MathJobs, applicants must also complete the application

form at <https://applications.ias.edu> but do not need to submit a second set of reference letters. Questions about the application procedure should be addressed to applications@math.ias.edu.

In addition, there are also two-year postdoctoral positions in computer science and discrete mathematics offered jointly with the following institutions: The Department of Computer Science at Princeton University (<http://www.cs.princeton.edu>); DIMACS at Rutgers, The State University of New Jersey (<http://www.dimacs.rutgers.edu>); and the Simons Foundation Collaboration on Algorithms and Geometry (<https://www.simonsfoundation.org/mathematics-and-physical-science/algorithms-and-geometry-collaboration/>).

School term dates for the 2016–17 academic year are as follows: Term I, Monday, September 19 to Friday, December 16, 2016; Term II, Monday, January 16, 2017, to Friday, April 14, 2017.

The School frequently sponsors special programs. However, these programs comprise no more than one-third of the memberships so that each year a wide range of mathematics is supported. During the 2016–17 academic year, the School of Mathematics will have a special program on homological mirror symmetry, and Paul Seidel from MIT will be the Distinguished Visiting Professor. Maxim Kontsevich from IHES will be attending the program for one month during each of the fall and spring terms (from mid-October to mid-November) and for the month of February. Denis Auroux from UC Berkeley will be attending for term II.

Homological mirror symmetry (HMS) was initiated by Kontsevich. It benefits from a close relation with string theory and has developed into a powerful and versatile idea. During the program, participants will consider the core conjectures of HMS and its role as a framework within which wider questions from mirror symmetry and other parts of mathematics can be studied. This is still a developing subject, and the program is open to a variety of approaches and viewpoints.

The intention is that the fall term will have a greater focus on the core building blocks of HMS as currently understood: the A-model theory (Lagrangian submanifolds, holomorphic curves and their generalizations), the B-model theory (derived categories in algebraic geometry) and mathematical interpretations of the Strominger–Yau–Zaslow approach, including the Gross–Siebert program. Specific questions of interest include: the role of singular Lagrangian submanifolds (such as Lagrangian skeleta), the effect of instanton corrections on the construction of mirror manifolds, and the structure of wrapped Fukaya categories. Participants will also consider the interplay between the various algebraic notions that appear in HMS.

The second term would widen the focus, allowing space for emerging interactions between HMS and other areas. Examples are the theory of special Lagrangian submanifolds, tropical geometry and non-Archimedean analytic geometry, as well as sheaf-theoretic methods. Also investigated will be applications of ideas from HMS to specific classes of manifolds, such

as complex symplectic manifolds and cluster varieties.

There will be two workshops during the special program. The term I workshop “Homological Mirror Symmetry: Methods and Structures” will be held November 7–11, 2016. The term II workshop “Homological Mirror Symmetry: Emerging Developments and Applications” will be held March 13–17, 2017.

University of Nebraska—Lincoln

Department of Mathematics

The Department of Mathematics at the University of Nebraska—Lincoln invites applications for the following positions:

■ Milton Mohr Professor of Mathematics, at the associate or full professor level. Review of applications will begin December 15, 2015, and continue until a suitable candidate (or candidates) is found.

■ Two tenure-track assistant professor positions in mathematics. Review of applications will begin November 13, 2015, and continue until suitable candidates are found.

Each of these positions begins August 2016. For more information about these positions and information on how to apply for them, please go to: <http://www.math.unl.edu/department/jobs/>. The University of Nebraska is committed to a pluralistic campus community through affirmative action, equal opportunity, work–life balance, and dual careers.

See **Professional Opportunities** on page 11

Professional Opportunities

continued from page 10

Williams College

Department of Mathematics and Statistics

The Williams College Department of Mathematics and Statistics invites applications for two tenure-track positions in statistics, beginning fall 2016, at the rank of assistant professor. (In an exceptional case, a more advanced appointment may be considered.)

The department seeks highly qualified applicants who have demonstrated excellence in teaching and research and who are committed to working with an increasingly diverse student body. The successful candidates will become the fourth and fifth tenure-track statisticians in the department, joining a vibrant and active statistics group with a newly established statistics major. The teaching load is two courses per 12-week semester and a winter term course every other January. In addition to excellence in teaching, the chosen candidate will be expected to have an active and successful research program. He or she should have a PhD by the time of appointment. The department welcomes applications from members of groups traditionally underrepresented in the field.

Applicants may apply at interfolio.com by uploading a curriculum vitae and having three letters of recommendation on teaching and research uploaded to <http://apply.interfolio.com/30206>. Teaching and research statements are also welcome. Evaluations of applications will begin on or after November 15 and will continue until the position is filled. All offers of employment are contingent upon completion of a background check. Further information is available upon request. For more information on the Department of Mathematics and Statistics, applicants should visit <http://math.williams.edu/>.

Williams College is a coeducational liberal arts institution located in the Berkshire Hills of western Massachusetts. The college has built its reputation on outstanding teaching and scholarship and on the academic excellence of its approximately 2,000 students. Please visit the Williams College website (<http://www.williams.edu>). Beyond meeting fully its legal obligations for non-discrimination, Williams College is committed to building a diverse and inclusive community where members from all backgrounds can live, learn, and thrive.

Northwestern University

Department of Engineering Sciences and Applied Mathematics

The Department of Engineering Sciences and Applied Mathematics at Northwestern University (<http://www.mccormick.northwestern.edu/applied-math/>) invites applications for two full-time, tenure-track faculty positions to begin in September 2016. Hiring will be focused on the assistant professor level, although exceptional applicants at a more senior level will be considered.

Requirements include a PhD and demonstrated ability to conduct high-impact interdisciplinary research in applied mathematics. The department seeks candidates who can enhance the breadth of its research activities. Applicants from all areas of applied mathematics are encouraged to apply. Areas of particular interest include, but are not limited to, (i) stochastic modeling of

physical, biological, or financial systems and (ii) scientific computing. Duties involve teaching and research.

The application package should include a curriculum vitae, a statement of research accomplishments and interests, and a statement of teaching experience and philosophy. The application should be submitted online at: <https://facultysearch.mccormick.northwestern.edu/apply/index/NzM=>.

In addition, applicants should arrange for at least three—but no more than four—letters of recommendation. Recommendation letters will be solicited automatically from the professional references by email after their names are entered in the online application system. Questions may be sent to esam-facultysearch@northwestern.edu (subject line: 2016 Faculty Search). Review of applications will begin November 1, 2015, and will continue until the positions are filled.

Northwestern University is an equal opportunity, affirmative action employer of all protected classes, including veterans and individuals with disabilities. Women and minorities are encouraged to apply. Hiring is contingent upon eligibility to work in the United States. For further information, see: <http://www.mccormick.northwestern.edu/applied-math/>.

University of Michigan

Center for the Study of Complex Systems

The Center for the Study of Complex Systems (CSCS) at the University of Michigan invites applications for a tenure-track position of assistant professor of complex systems. Applicants at a more senior-level will also be considered. The appointment will begin September 1, 2016. This is a university-year appointment. Information about the Center can be found at: <http://www.lsa.umich.edu/cscs>.

Applicants must have a demonstrated research agenda focused on complex systems. This may involve theoretical or applied research on complexity, including (but not limited to) mathematical and computational models in areas such as networks, computation, emergence, large events and robustness, or applications where complexity lies at the core, such as quantitative modeling of social systems, soft condensed matter physics, evolutionary or ecological dynamics, epidemiology and disease transmission, artificial life, neuroscience, and cognition. Preference will be given to applicants with a track record of working across disciplines. All application materials must be uploaded onto this website: <https://complexsystems-lsa.applicantstack.com/x/apply/a2guio5y9crr>.

The position is based in CSCS but will be a joint appointment with another department. In the cover letter, applicants should identify one or more partner departments at the University of Michigan suitable for such a joint appointment. Applicants must submit a current curriculum vitae, a statement of current and future research plans, a statement of teaching philosophy and experience, evidence of teaching excellence (if any), and one writing sample. At least three letters of recommendation are required and must be uploaded onto the same website. Applications will be reviewed starting October 1, 2015. Applications will be accepted until the position is filled.

Women and minority candidates are encouraged to apply. The University of Michigan is an

equal opportunity/affirmative action employer and is supportive of the needs of dual-career couples.

Colorado State University

Department of Mathematics

The Department of Mathematics in the College of Natural Sciences at Colorado State University has an open search for the chair of the department. Applicants who hold a PhD in mathematics or an allied field, with (1) a proven record of excellence in research and teaching, (2) evidence of effective leadership, and (3) eligibility to hold the rank of full professor at a Carnegie RU/VH institution are encouraged to apply. This is a twelve-month, full-time position with salary and other forms of support commensurate with qualifications.

For full consideration, applications should include a letter of intent/cover letter that contains a statement of leadership experience and skills pertaining to research, teaching, and outreach; a current curriculum vitae; a statement of research; and list of references by November 15, 2015. For the complete job description and to apply visit: <http://jobs.colostate.edu/postings/16878>. Application materials, including letters of recommendation of semifinalist candidates will be made available for review by the mathematics faculty.

CSU is an EO/EA/AA employer. The university conducts background checks on all final candidates. Inquiries should be addressed to Darrell Whitley, Chair, Mathematics Chair Search Committee, College of Natural Sciences, Colorado State University, Fort Collins, CO 80523-1801, email: whitley@ColoState.edu.

Indiana University, Bloomington

Department of Geological Sciences

Applications are invited for a postdoctoral research assistant in the Atmospheric Group of the Department of Geological Sciences at Indiana University. The group's research is aimed at understanding the dynamics of hurricane-like vortices. The successful candidate is expected to possess mathematical skill in compressible fluid and nonlinear dynamical systems. Knowledge and/or experience with numerical modeling is desirable. Applicants should be self-motivated; be prepared to interact with atmospheric scientists and geophysicists; and have a PhD in applied mathematics, physics, atmospheric sciences, or related sciences. The successful candidate is expected to communicate results at scientific meetings and publish in peer-reviewed journals.

Initial appointment for the position is one year and can be extended to the second year upon successful performance. Salary is commensurate with experience. The selected candidate will be jointly supervised by Dr. Chanh Kieu (Department of Geological Sciences) and Dr. Shouhong Wang (Department of Mathematics). Applications should include a statement of research, curriculum vitae, and the contact information for three references. Interested candidates should review the application requirements and submit their application at: <https://indiana.peopleadmin.com/postings/1719>. Questions regarding the position or application process can be directed to: Dr. Chanh Kieu (ckieu@indiana.edu) or Dr. Shouhong Wang (showang@indiana.edu). The submission deadline is December 15, 2015, or until filled.

Indiana University is an equal employment and affirmative action employer and a provider of ADA services. All qualified applicants will receive consideration for employment without

regard to age, ethnicity, color, race, religion, sex, sexual orientation or identity, national origin, disability status, or protected veteran status.

Michigan State University

Department of Mathematics

The Department of Mathematics at Michigan State University will have one tenure-track position to begin Fall 2016. It is expected that the successful candidate will be appointed at the rank of assistant professor, but truly outstanding applicants for appointment at a higher rank will be considered. Applicants will be evaluated on their merits in research and teaching, and excellence is essential in both. Preference will be given to applicants with at least two years of experience beyond the PhD. Applications from all areas of mathematical research will be considered.

Applicants should include a cover letter, curriculum vitae, research statement, and teaching statement, and should arrange for at least 4 letters of recommendation to be sent by the letter writers through www.mathjobs.org/jobs/jobs/7551. One of the letters of recommendation must specifically address the applicant's teaching ability. Review of applications will begin on October 15, 2015, and will continue until the position is filled.

Michigan State University is an affirmative action, equal opportunity employer and is committed to achieving excellence through cultural diversity. The university actively encourages applications and/or nominations of women, persons of color, veterans, and persons with disabilities. Job applicants are considered for employment opportunities and employees are treated without regard to their race, color, religion, sex, sexual orientation, gender identity, national origin, disability, or veteran status.

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Old Dominion University

Graduate Programs in Computational and Applied Mathematics

The Department of Mathematics and Statistics at Old Dominion University invites outstanding students to apply for graduate assistantships in its vigorous applied mathematics and statistics MS and PhD programs for the 2016–2017 academic year. Old Dominion faculty are active in various areas of applied and computational mathematics, scientific computing, analysis, and numerical analysis; they play a key role in the university's multidisciplinary modeling and simulation graduate program, and the Center for Computational Science. Old Dominion faculty enjoy an excellent funding record, including grants from NSF, NIH, NASA, AFOSR, the U.S. Army, and other state and federal funding agencies. For further information, applicants should see <http://ww2.sci.odu.edu/math/academics/grad.shtml>.

NC STATE UNIVERSITY

ASSISTANT/ASSOCIATE/FULL PROFESSOR
Data Driven Science Cluster

As one of the leading land-grant institutions in the nation, North Carolina State University is proud to announce the continuation of its Chancellor's Faculty Excellence Program, a major initiative of the University's 2011-2020 strategic plan, "The Pathway to the Future." This research cluster in Data-Driven Science invites all nationally recognized scholars conducting high-impact research and education in large-scale data-enabled sciences to apply. The position is open rank and all exceptional candidates will be considered. Because this hire will be part of an interdisciplinary cluster-hiring program, it is expected that the successful candidate bring a vibrant collaborative research team that attracts visiting faculty with strong skills in complementary areas. There is particular interest in growing NC State's expertise in the following areas: computational geometry and topology, applied graph theory, compressed sensing, mathematical signal processing, machine learning, mathematical network analysis, including the internet, social networks, and analysis of massive data sets.

The candidate should have an earned doctorate in mathematics or a related field and preferably at least three years of experience conducting high-impact research in a university, private sector, government or not-profit organization. Priority will be given to candidates that have demonstrated interdisciplinary collaborations that cut across academic units. Successful candidates will be recognized as a pioneering leader in the development of new approaches for large-scale data-enabled sciences and have experience in leading, growing and managing research efforts and a record of substantial external funding.

For more information and to apply, visit <https://jobs.ncsu.edu/postings/33623> (position number: 081R2878).

Requests for additional information should be addressed to the Cluster Coordinator, Alyson Wilson, awilson@ncsu.edu.

NC State University is an equal opportunity and affirmative action employer. Persons with disabilities requiring accommodations in the application and interview process please call (919) 515-3148. We welcome the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners.

INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES

The Institute for Computational Engineering and Sciences (ICES) at The University of Texas at Austin is searching for exceptional candidates with expertise in computational science and engineering to fill several Moncrief endowed faculty positions at the Associate Professor level and higher. These endowed positions will provide the resources and environment needed to tackle frontier problems in science and engineering via advanced modeling and simulation. This initiative builds on the world-leading programs at ICES in Computational Science, Engineering, and Mathematics (CSEM), which feature 16 research centers and groups as well as a graduate degree program in CSEM. Candidates are expected to have an exceptional record in interdisciplinary research and evidence of work involving applied mathematics and computational techniques targeting meaningful problems in engineering and science. For more information and application instructions, please visit: www.ices.utexas.edu/moncrief-endowed-positions-app/. This is a security sensitive position. The University of Texas at Austin is an Equal Employment Opportunity/Affirmative Action Employer.

THE UNIVERSITY OF TEXAS
— AT AUSTIN —

Stochastic Optimization

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in (1) directly). For this reason, we create a lookahead model which is an approximation of the real problem. Common approximations are to limit the horizon (e.g. from T , which might be quite long, to $t+H$ for some appropriately chosen horizon H), and (most important) to replace the original stochastic information process with something simpler. The most obvious is a deterministic approximation, which we can write as

$$U_t^\pi(S_t | \theta) = \arg \min_{u_t, \tilde{u}_{t+1}, \dots, \tilde{u}_{t+H}} \left(C(S_t, u_t) + \sum_{i=t+1}^{t+H} C(\tilde{S}_i, \tilde{u}_i) \right). \quad (6)$$

To make the distinction from our original base model in (1), we put tildes on all our variables (other than those at time t), and we also index the variables by t (to indicate that we are solving a problem at time t), and t' (which is the point in time within the lookahead model).

A widely used approach in industry is to start with (6) and then introduce modifications (often to the constraints) so that the decisions made now are more robust to uncertain outcomes that occur later. This would be a form of (hybrid) cost function approximation.

We may instead use a stochastic lookahead model. For example, the stochastic programming community most often uses

$$U_t^\pi(S_t | \theta) = \arg \min_{u_t, \tilde{u}_{t+1}, \dots, \tilde{u}_{t+H}} \left(C(S_t, u_t) + \sum_{\omega \in \Omega_t} p(\omega) \sum_{i=t+1}^{t+H} C(\tilde{S}_i(\omega), \tilde{u}_i(\omega)) \right) \quad (7)$$

Here, we would let θ capture parameters such as the planning horizon, and the logic for constructing $\tilde{\Omega}_t$.

Other variations include a robust objective (which minimizes over the worst outcome rather than the expected outcome), or a chance-constrained formulation, which approximates the costs over all the uncertain outcomes using simple penalties for violating constraints.

All of these policies involve tunable parameters, given by θ . We would represent the policy π as the policy class $f \in \mathcal{F}$, and the parameters $\theta \in \Theta^f$. Thus, the search over policies π in equation (1) can now be thought of as the search over policy classes $f \in \mathcal{F}$, and then over the tunable parameters $\theta \in \Theta^f$.

No, this is not easy. But with this simple bit of notation, all of the different communities working on sequential stochastic optimization problems can be represented in a common framework.

Why is this useful? First, a common vocabulary facilitates communication and the sharing of ideas. Second, it is possible to show that each of the four classes of policies can work best on the same problem, if we are allowed to tweak the data. And finally, it is possible to combine the classes into hybrids that work even better than a pure class.

And maybe some day, mathematicians will figure out how to search over function spaces, just as Dantzig taught us to search over vector spaces.

Warren B. Powell is a faculty member of the Department of Operations Research and Financial Engineering at Princeton University.

Mentoring

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a usual conference such as listening to lectures and meeting people. But being a part of the BE and Mentor–Protégé program, I learned many different things by attending diverse gatherings of students. I received personalized advice from my mentor Prof. Susan Minkoff and useful recommendations

to start a chapter of the Association of Women in Mathematics at her campus. She plans to start a mentoring program modeled after the BE program. Joseph Emelike, an undergraduate at Bowie State University, was so excited about his experience that he is starting a SIAM student chapter.

Using at times unconventional techniques like drawing and crafts, the BE/mentoring pilot program appears to have had a posi-



The BE Program participants and organizers.

on how to benefit from the conference. I appreciated the excellent opportunities for students to enhance their communication skills and polish their talent." Mentors also gained benefits as indicated by Jay Bardhan, Assistant Professor at Northeastern University, "I would absolutely volunteer again, and I'd recommend everyone do so if they have an opportunity! My mentee, and the other students who participated in the program, brought an infectious enthusiasm and curiosity to their work, and to possible career paths," Bardhan said. "My immediate takeaway from watching the students' talks at CSE15 was that similar programs should be started across the country, in all branches of STEM and CS&E."

Pre- and post-conference surveys indicate that the level of participant confidence in research, mentoring, internships, careers, and exploring scientific interests all increased after CSE15 BE. Yeonjoo Yoo, Vice President of the SIAM Student Chapter at Indiana University, has received approval

tive impact on participants and their career trajectories. Post-conference activities continue to facilitate mentoring, and while lasting impacts are still unknown, initial indications are promising.

"The Broader Engagement program at SIAM was hands-down the best conference program I have ever participated in," Romanus reflected on her experience "This conference helped me rediscover my passion. It forced me to hone my "research elevator pitch" and think about my long term goals in graduate school. . . In talking to my new BE friends, a lot of us have found that this conference has opened the door to full-time jobs, internships, collaborations, friendships, and much more."

Mary Ann Leung is president of the Sustainable Horizons Institute. Silvia Crivelli is a computational biologist at Lawrence Berkeley National Laboratory.

Inaugural Meeting of the SIAM Activity Group on Mathematics of Planet Earth

Climate, climate change, sustainability, ecology, biodiversity, energy, food, and water are among the areas of greatest global concern. The newly formed SIAM Activity Group on Mathematics of Planet Earth (SIAG/MPE, <http://www.siam.org/activity/mpe/>) is organized to provide a forum in SIAM for mathematicians and computational scientists who are engaged in these critical priority areas. The interests of SIAG/MPE span the range from developing quantitative techniques to providing policy makers with tools for qualitative decision support.

SIAG/MPE will hold its inaugural meeting (MPE16) at the DoubleTree by Hilton Hotel Philadelphia Center City in Philadelphia, Pennsylvania, Friday, September 30-Sunday, October 2 (morning only), 2016.

- MPE16 will have four themes:
- Planet Earth as a physical system (e.g., earth and space, climate dynamics, oceans, atmosphere, cryosphere)
 - Planet Earth as a system supporting life (e.g., mathematical ecology, biodiversity, carbon cycle, sustainability)
 - Planet Earth as a system organized by humans (e.g., natural resources, communication, transportation, socio-economics)
 - Planet Earth as a system at risk (e.g., food security, epidemics, extreme events, risk management)

The program will feature invited speakers on each of the themes, as well as a public lecture on the general theme of MPE.

Parts of the program will be organized in collaboration with other SIAM Activity Groups and organizations outside SIAM. In addition, there will be minisymposia, poster sessions, and contributed paper sessions. Talks focused on work in progress are especially encouraged. Abstracts will be solicited on the conference web site (<http://www.siam.org/meetings/mpe16/>) in November 2015.

The activity group is eager to engage students and junior researchers in any of the four themes listed above. To facilitate participation, some travel awards for students and early career academics will be available.

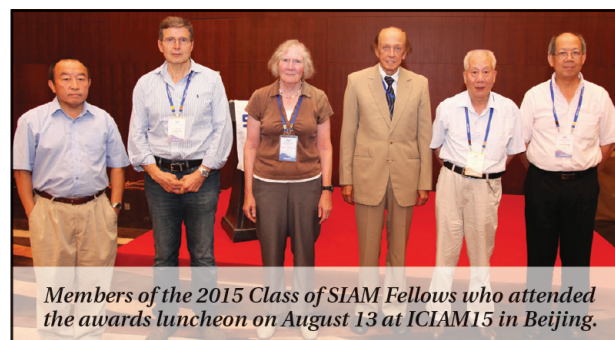
The Program Committee of MPE16 is chaired by the Program Director of the Activity Group, Hans Engler (Georgetown University). Co-chairs are the officers of the activity group: Hans Kaper (Argonne National Laboratory), Antonios Zagaris (Wageningen UR Central Veterinary Institute, the Netherlands), and Mary Lou Zeeman (Bowdoin College). The twelve members of the SIAG/MPE Advisory Board (listed on the SIAG's web site, <http://www.siam.org/activity/mpe/>), serve on the meeting's Organizing Committee.

MPE16 will be held at the same place and at the same time as the inaugural meeting of the SIAM Activity Group on Education (SIAG/ED). One registration fee will allow conference participants to attend sessions at both meetings. The meetings will have separate programs, but some joint activities are being planned.

Nominate a SIAM Fellow

NominateFellows.siam.org

SIAM members can nominate up to two colleagues who have made distinguished contributions to the disciplines of applied mathematics and computational science to be considered for the SIAM Fellows Class of 2016. Up to 30 SIAM members will be selected for this honor in 2016.



Members of the 2015 Class of SIAM Fellows who attended the awards luncheon on August 13 at ICIAM15 in Beijing.

Nominations will be evaluated based on excellence in research, industrial work, educational activities, or other activities related to the goals of SIAM.

Class of 2016 nominations will be accepted until November 2, 2015.

Support your profession by helping SIAM identify those members who have made the most significant contributions to our fields.

For more information please visit www.siam.org/prizes/fellows.



SIAM SOCIETY for INDUSTRIAL and APPLIED MATHEMATICS