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Mathematical Models for Fighting Zika Virus

By Carrie Manore and Mac Hyman

While the role of mathematics in comprehending and predicting certain sciences is obvious to most people, the usefulness of mathematical models in the field of epidemiology is often not so clear. Scientists (and laypeople) tend to greatly under or overestimate the state of the art and helpfulness of mathematics in understanding infectious disease spread. Among infectious diseases, mosquito-borne diseases are both ubiquitous and notoriously difficult to control.

When a little-known mosquito-borne pathogen such as Zika virus causes a sudden outbreak accompanied by potentially deadly or disabling side effects, like Guillain-Barré syndrome or microcephaly, public health workers and policy makers need a few basic questions answered: How many people will the outbreak potentially infect? How far and how quickly will the disease spread? What areas and people are at highest risk, and when are they most at risk? How can we best make use of limited resources? How can we best slow or prevent the outbreak and protect vulnerable populations?

Mathematical models that simulate the spread of mosquito-borne disease can provide important guidance and insight to all of these questions. They can help identify trends of where and how the infection will spread. The models can also determine the sensitivity of important epidemic quantities, such as total number of people infected, to input control parameters and thus inform best practices for control. Any known and broadly-held statistical correlations between disease incidence and demographic or weather data from past epidemics can often improve model-based estimates.

Forecasting even the most predictable diseases, such as seasonal influenza, is difficult and often met with little success. Accurate real-time prediction of mosquito-borne epidemics with mechanistic transmission models, particularly when outbreaks are sporadic, is currently beyond our capability. Even though present models forecast the number of infections with available data adequately at best, they can quantify the resulting "what-if" scenarios that may offer insight into disease control.

The first basic mathematical model addressing the spread of mosquito-borne disease was developed by Ronald Ross to model malaria [4]. He realized that disease spread depended on a few important factors, including the rate of contact between mosquitoes and humans, the number of times a female mosquito bites in her lifetime, the number of available susceptible humans, and the length of the infectious period in humans. An interesting and important tidbit: only female mosquitoes bite, as they need the protein in blood to produce eggs. Thus, the mosquito biting rate is intrinsically related to the egg-laying rate (gonotrophic cycle).

The Ross-MacDonald ordinary differential equation (ODE) model provided insight into how different aspects of the disease transmission cycle interact to cause outbreaks (see Figure 1 for a schematic of the mosquito-human transmission cycle). The model can be used to compute the basic reproduction number, R_0, that estimates the expected number of secondary cases resulting from a single infected person in a fully susceptible population. From a mathematical perspective, the stability or instability of the disease-free boundary equilibrium of the system is determined by the basic reproduction number. If $R_0 < 1$, then the disease-free equilibrium is locally asymptotically stable. If $R_0 > 1$, the disease-free equilibrium is unstable and introduction of an infected individual will result in an outbreak. In the early stages of an epidemic, R_0 is the key quantity of interest, and the goal is to identify mitigation strategies to reduce it below the threshold

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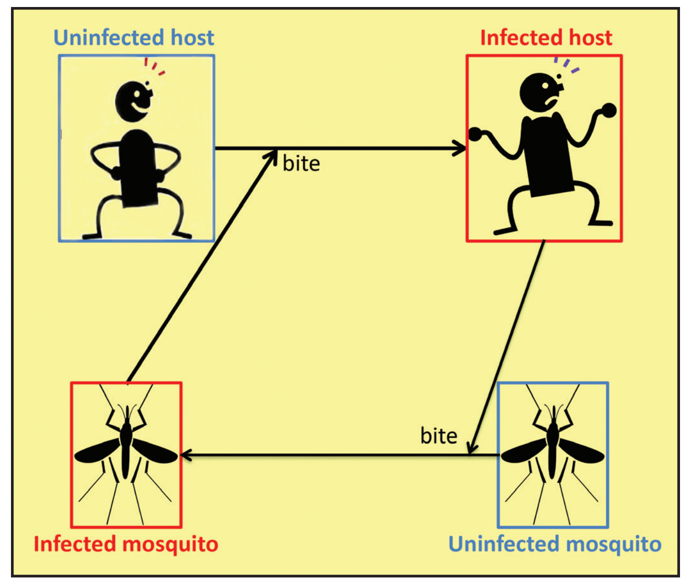

Figure 1. *The mosquito-human transmission cycle. An infectious mosquito bites a susceptible, uninfected human and transmits the virus via saliva. Once the human become infectious (usually accompanied by symptoms), the human can transmit the pathogen to an uninfected mosquito via the blood the mosquito ingests.*

Music Visualized by Nonlinear Time Series Analysis

By Miwa Fukino, Yoshito Hirata, and Kazuyuki Aihara

Music evokes emotion. Many musical factors, such as timbre, melody, harmony, meter, and beat, constitute musical structure and produce an emotional response in the listener. The relationship between emotion and the physical properties of music has been studied but is not fully understood. Previous research on the relation between music signal processing and physical properties has proposed the extraction of various musical features by assuming linear models. Multiple feature vectors are typically used to characterize musical factors and the structure of musical compositions. These methods perform very well when applied to extract physical and/or objective properties of the music, for example, chord estimation and beat analysis. However, such methods are not suitable for analyzing music's emotional effect since they mainly focus on amplitude information, discarding phase information among the features. We proposed a new method based on nonlinear time series analysis to solve this problem [2].

Recurrence Plots

A recurrence plot is an important analysis tool for nonlinear time series data. Eckmann et al. [1] originally proposed the plot to visualize a time series (for a review, see [7]). It is a two-dimensional plot whose axes both correspond to the same time axis. If two states at a pair of times are close to each other, we plot a point at the corresponding place; otherwise we do not plot a point there. Mathematically, letting $\{x_i \in R^m \mid i = 1, 2, ..., I\}$ be a time series and $d : R^m \times R^m \to \{0\} \cup R_+$ be a distance function, we can define a recurrence plot R as follows:

$$R(i,j) = \begin{cases} 1, & \text{if } d(x_i, x_j) < \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \qquad (1)$$

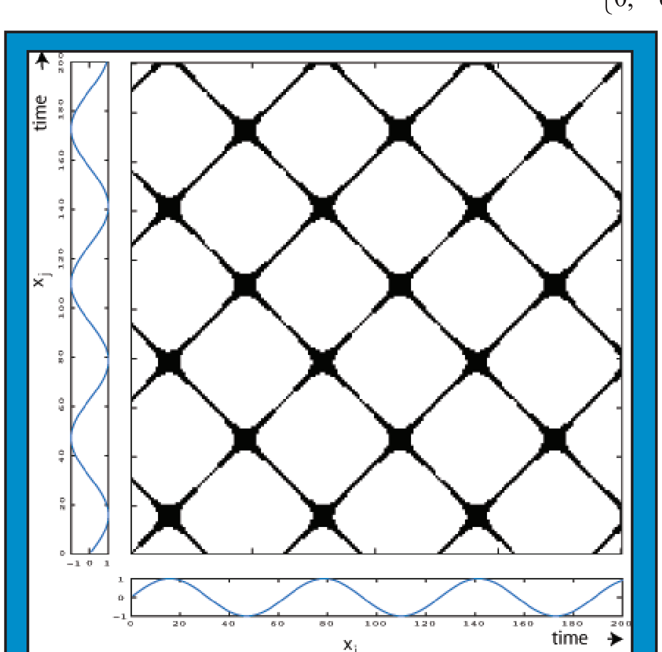

Figure 1. *A recurrence plot of a sine wave. Both the bottom* (x_i) *and the left* (x_j) *are waveforms along the same timeline. We used the Euclidean distance for* d *and set* $\varepsilon = 0.1$.

for $1 \le i, j \le I$. Here, $R(i,j) = 1$ means that a point is plotted at (i,j), and $R(i,j) = 0$ means that a point is not plotted at (i,j). The symbol ε is the threshold for obtaining a recurrence plot.

This simple graph can reveal a lot of things about the underlying dynamics of the data. For instance, a time series of white noise produces a recurrence plot where points are plotted almost randomly. A periodic time series produces a periodic pattern in the recurrence plot, as shown in Figure 1. A time series generated from deterministic chaos creates a recurrence plot containing many short diagonal segments. If two time series generate the same recurrence plot, their underlying dynamical rules are equivalent [8]. Therefore, by looking at a recurrence plot, one can discern the properties of the underlying dynamics, such as serial dependence [4] and consistency with Devaney's mathematical definition of deterministic chaos [3].

Given a multivariate time series with a fixed sampling frequency, one may use the Euclidean distance for the distance function. By replacing the distance function with another distance function, we can analyze various exotic data—time series of networks [6] and marked point processes [5, 9], for example—which are time series of events accompanied by supplementary information.

Recurrence Plot of Recurrence Plots

Conventional recurrence plots are usually applied to a time series of length up to 10,000, if we consider the plot's visibility. The length of musical data is too large to directly apply conventional recurrence plots: a musical piece of five minutes with 44.1 kHz 16-bit monaural linear pulse code modulation (PCM) has 13,230,000 time points. One cannot grasp global characteristics by such a large recurrence plot. On the other hand, our method [2] enables us to represent both local and global characteristics simultaneously by using recurrence plots hierarchically. In this method, we first divided a long time series into 4-second windows and calculated multiple short-term recurrence plots (short-term RPs), which represented local characteristics. Second, using distances between the short-term RPs, we calculated a recurrence plot—representing global characteristics—of the entire piece. We call this long-term recurrence plot a recurrence plot of recur-

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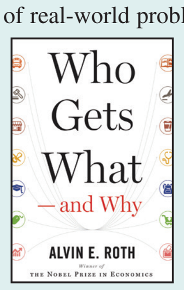
5 **Teaching Mathematical Modeling to Students**
Rachel Levy discusses the new national Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), a report co-published by SIAM. The report explores the benefits of teaching mathematical modeling from an early age.

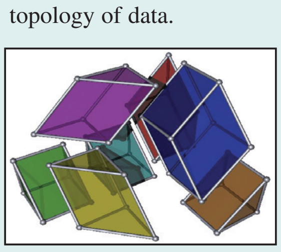
7 **Mathematical Matchmaking**
James Case reviews *Who Gets What – And Why* by Nobel Prize-winner Alvin Roth. Roth describes ways in which the Gale-Shapley matching algorithm can be applied to a variety of real-world problems.



8 **SIAM Meets With Congressional and Federal Agency Representatives**
The SIAM Committee on Science Policy recently met with representatives of U.S. federal science agencies. Topics for discussion included the Department of Mathematics budget and various proposed new initiatives.

9 **A Closer Look at SIAM Activity Groups**
Executive Director Jim Crowley and SIAM President Pam Cook offer an overview of SIAM activity groups (SIAGs), which are designed to allow members to exchange information and network with colleagues who share similar research interests.

12 **Applied Algebra and Geometry: A SIAGA of Seven Pictures**
In the second of three installments introducing the new *SIAM Journal on Applied Algebra and Geometry*, Anna Seigal interprets two images on the journal's cover illustrating polyhedral geometry and topology of data.



11 **Professional Opportunities and Announcements**

The Gambler's Fallacy

LETTER TO THE EDITOR

Response to *Hot Hands, Streaks and Coin-flips: How The New York Times Got it Wrong* (*SIAM News* March 2016):

I think I made it clear in my essay[1] for the Times (which began with a reference to Tom Stoppard's play "Rosencrantz and Guildenstern Are Dead") that the issue in Miller and Sanjurjo is not what the coin does but what the coin flipper perceives – and that the authors are suggesting how people might acquire a belief in the gambler's fallacy through what they *think* they experience. Here is how I put it:

"There is not, as Guildenstern might imagine, a tear in the fabric of space-time. It remains as true as ever that each flip is independent, with even odds that the coin will land one way or the other. But by concentrating on only some of the data—the flips that follow heads—a gambler falls prey to a selection bias."

Read in context, the paragraph that follows, describing how the Gilovich hot-hand paper might have been flawed by the same kind of misperception, also seems clear in its meaning and intent.

I certainly didn't say that there was a violation of the laws of probability. The whole point of the piece was to reflect on how easily the human brain, rebelling against the randomness inherent in life, can fool itself into perceiving patterns that do not exist. That is a theme of my book "Fire in the Mind: Science, Faith, and the Search for Order." The essay in the Times was keyed to the new 20th anniversary edition.

I knew when I was writing the piece that some specialists, parsing it line by line, might quibble with some of my wording. That is something we science journalists struggle with all the time – how to translate the precision of mathematics into words and metaphors that readers will understand. There are inevitably compromises.

— George Johnson, science writer[2] for The New York Times

[1] http://www.nytimes.com/2015/10/18/sunday-review/gamblers-scientists-and-the-mysterious-hot-hand.html

[2] http://talaya.net

Obituaries

By B.S. Ng and W.D. Lakin

Professor William "Bill" Hill Reid, a prominent physical applied mathematician, passed away on January 31, 2016, at age 89 with his loving family by his side. Bill is recognized worldwide for his many lasting contributions to the fields of fluid dynamics and hydrodynamic stability, especially his pioneering development of elegant asymptotic techniques for the analysis of the stability of shear flows. Bill was a fellow of both the Cambridge Philosophical Society and the American Physical Society. He was also a lifelong member of SIAM, and a member of the editorial board of the *SIAM Journal on Mathematical Analysis* from 1973 to 1981.

Bill was born on September 10, 1926, in Oakland, CA, to the late William MacDonald and Edna (Hill) Reid. He graduated from the University of California, Berkeley, in 1949 with a B.S. in electrical engineering, though his service as a U.S. Merchant Marine in the Pacific from 1945 to 1947 interrupted his undergraduate study. After receiving his M.S. from Berkeley in 1951, Bill embarked on his doctoral study at Cambridge University's Trinity College, where he joined a group of young researchers and graduate students from all over the world doing research in the then-emerging field of turbulence. He studied the geometrical and statistical theories of isotropic turbulence under the direction of Ian Proudman.

In 1954, after completing his doctoral thesis at Cambridge and before formally receiving his Ph.D., Bill was drafted by the U.S. Army and spent the next two years at the Aberdeen Proving Ground in Maryland. While there he continued his scientific research "in between KP duty" and lectured at Johns Hopkins University, at the invitation of Stanley Corrsin. During this time, he also decided that there were no more theoretical advances he could help make in the field of turbulence. As Keith Stewartson, one of the most distinguished British applied mathematicians of his generation, once remarked in 1980, "Bill Reid's work on isotropic turbulence dealt the field a body blow from which it never recovered." Bill's last paper on the subject, "Turbulent flow, theoretical aspects," which he co-wrote with C. C. Lin, was published in *Handbuch der Physik* in 1963.

In 1958, Bill began his academic career at Brown University after spending the previous year as an NSF Postdoctoral Fellow at Yerkes Observatory in Williams Bay, WI. His interaction with Subrahmanyan Chandrasekhar at Yerkes marked the beginning of his work on hydrodynamic stability. In 1963, Bill was recruited to the University of Chicago with a joint appointment in the departments of mathematics and geophysical sciences. He spent the next 26 years at Chicago where he did much of his pioneering research in the stability of shear flows.


William Hill Reid, 1926-2016.

The analytical study of shear flow stability has a long and illustrious history, beginning with Werner Heisenberg's 1924 doctoral thesis in which he attempted to find (for large values of the Reynolds number) asymptotic approximations to the solutions of the governing Orr-Sommerfeld equation. Subsequently, Heisenberg's work was improved upon by W. Tollmien in 1929 and further clarified by C. C. Lin in 1944. These analyses all shared a significant limitation – the obtained approximations lacked uniformity, which in turn led to considerable controversy on the validity of their use in stability calculations. Lin himself was well aware of this limitation and devoted a great deal of effort to this problem in the 1960s. In a series of groundbreaking papers published in the 1970s, Bill developed a systematic approach to obtain uniform asymptotic approximations to the solutions of the Orr-Sommerfeld equation as well as to the eigenvalue relation used in stability calculations. More importantly, his work provided an elegant framework for attaining uniform asymptotic approximation to the solutions of a large class of higher-order ordinary differential equations of the hydrodynamic type, of which the Orr-Sommerfeld equation is an important example. Bill's research also facilitated the development of the compound matrix method, now a widely-used shooting technique for the numerical solution of unstable eigenvalue and boundary value problems.

During his career, Bill published more than 70 research papers. His book, *Hydrodynamic Stability*, which he co-authored with the late P.G. Drazin of Bristol University in 1981, remains an authoritative classic of the subject.

Following his retirement from the University of Chicago in 1989, Bill accepted a position in the Department of Mathematical Sciences at Indiana University–Purdue University Indianapolis where he continued his teaching and research. He remained in Indianapolis until 2007, when he moved to Jacksonville, FL.

Bill is survived by his wife of 53 years, Elizabeth, and his daughter, Margaret F. Reid.

B.S. Ng is an emeritus professor and M. L. Bittinger Chair of Mathematical Sciences at Indiana University–Purdue University Indianapolis. W.D. Lakin is an emeritus professor of mathematics, statistics, and biomedical engineering at the University of Vermont.

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Modeling Seismic Waves for Hydrocarbon Exploration

By Alan Schiemenz

Nuclear test ban treaty monitoring, earthquake early warning systems, and knowledge of the earth's deep interior are all made possible by seismic wave analysis. Numerical modeling of seismic waves is also a well-practiced art within the oil industry, where construction of high-resolution seismic velocity models improves seismic imaging workflows, thus reducing drilling risk and enhancing recovery of hydrocarbons. Full-waveform inversion (FWI) [8] is now the state-of-the-art seismic velocity model-building algorithm, with a growing research community supporting commercial development at many oil and oil service companies. FWI is a nonlinear optimization problem which iteratively updates the velocity model to reduce misfit between recorded and synthesized seismic data via the adjoint method. The problem was originally conceived in the early 1980s [7], although it was not until the late 2000s [6] that computers caught up to the intense demands of industrial application. Although FWI resides broadly in the geophysical domain, applied mathematicians stand to make the most meaningful contributions to the community's unsolved problems. Among these contributions are improved wave equation solvers, accuracy of the inversion with respect to starting model, and robust strategies for multi-parameter inversion.

Seismic Waves

Elastic wave propagation is modeled by the equation of motion

$$\rho\, \partial_t^2 \mathbf{u} = \nabla \cdot \mathbf{T} + \mathbf{f}, \qquad (1)$$

where $\mathbf{u}$ is displacement of the seismic waveform, ρ is mass density, $\mathbf{f}$ represents body forces (e.g. earthquakes, explosions, air guns), and $\mathbf{T}$ is the elastic stress tensor. Hooke's law prescribes a linear stress-strain constitutive relationship: $\mathbf{T} = \mathbf{c} : \nabla \mathbf{u}$. The velocity model parameters are embedded within the tensor $\mathbf{c}$. In the simplified case of an isotropic medium (i.e. $\mathbf{c}$ is invariant to rotation), there are two independent parameters: P-wave (V_p) and S-wave (V_s) velocity.

Although the "speed of sound" in a medium is commonly conceptualized as a single scalar quantity, seismologists have long observed multiple types of wavefronts with greatly differing arrival time and amplitude behaviors. Primary (P) waves and secondary (S) waves are so named due to the order of their arrival $(V_p > V_s)$. S-waves do not exist in fluids, which led global seismologists to discover the fluid nature of the earth's outer core. Marine exploration seismologists tend to use acoustic modeling engines (i.e. P-waves only), which are far cheaper and typically of sufficient accuracy, as the ocean layer removes much sensitivity to S-wave structure anyway. After acoustic velocity, anisotropic parameters are generally of greatest concern, and are crucial to validating field models with vertical profiles obtained from well logs.

Numerical Modeling

The acoustic approximation to (1) is a scalar equation and can be written in a semi-discrete form as

$$M \partial_t^2 \phi = S\phi + f, \qquad (2)$$

where the mass and stiffness matrices (M,S) are spatial-differential operators applied to the potential ϕ, with $\mathbf{u} = \rho^{-1}\nabla\phi$. These operators are defined by the choice of spatial discretization deployed. Finite difference methods have for decades been the workhorse of the oil industry, owing to ease of implementation and efficient paralleliza-

See **Seismic Waves** *on page 5*

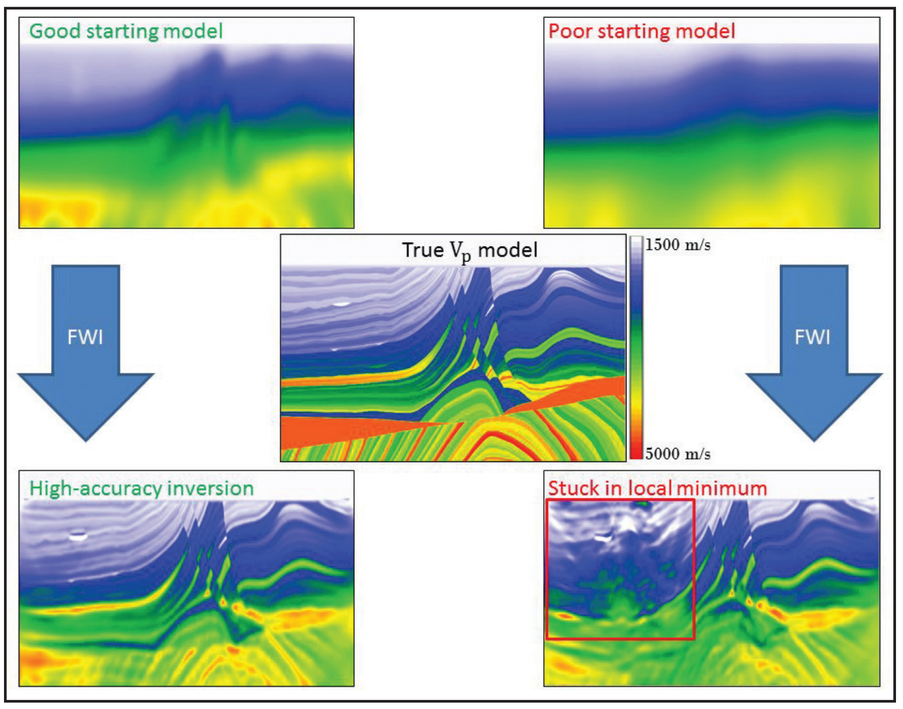


Figure 1. *Results after applying standard acoustic FWI workflow to the Marmousi2 velocity model. Initial models are constructed by smoothing the true model. Model dimensions are 17 km width x 3.5 km depth. Final model accuracy depends strongly on the accuracy of the starting model.*

Simon Levin and Michael Artin Receive the National Medal of Science

SIAM Fellows Simon Levin and Michael Artin were among nine distinguished scientists and researchers awarded the 2015 National Medal of Science, the highest award for scientific achievement in the United States. Prize recipients will be honored at a White House ceremony this month, Levin for his advancements in ecological complexity and Artin for his research in algebraic geometry.

Simon Levin, professor in the Department of Ecology at Princeton University.

Simon Levin is currently Princeton University's George M. Moffett Professor of Biology in the Department of Ecology and Evolutionary Biology, as well as director of Princeton's Center for BioComplexity. He is recognized as a leading scholar of systems analysis.

Levin earned his B.A. in mathematics from Johns Hopkins University in 1961 and his Ph.D. in mathematics from the University of Maryland, College Park in 1964. After joining Princeton's faculty in 1992, he has continued as an adjunct professor at Cornell and a distinguished visiting professor at the University of California, Irvine.

Levin received the National Medal of Science for his research in ecological complexity, which explores how small-scale evolutionary and behavioral elements (at the level of individual organisms) maintain macroscopic patterns in ecosystems and the biosphere. Levin uses mathematical models, empirical studies, and observational data to examine biological diversity in natural systems, the growth of diversification, and collective behavior. Throughout his career, Levin's use of theory in ecological complexity helped transform ecology into a more conceptual science.

Levin also studies infectious disease dynamics, antibiotic resistance, financial and economic systems, sustainable development, and the management of shared resources and public goods. He has examined similarities between ecological and economical/finance systems, including factors that make them prone to collapse; these concepts are outlined in his book, *Fragile Dominion: Complexity and the Commons*. Yet another of his interests is sustainable development and the link between socioeconomic and environmental systems.

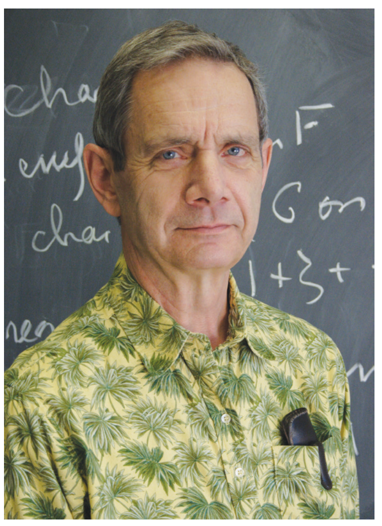
Levin is a member of the National Academy of Sciences and a fellow of both the American Academy of Arts and Sciences and the American Association for the Advancement of Science. He has served as chair and vice chair of the International Institute for Applied Systems Analysis, and is former president of the Ecological Society of America and the Society for Mathematical Biology. Currently Levin is a member of the advisory board for the SIAM Activity Group on Mathematics of Planet Earth (SIAG/MPE).

Over the years Levin received multiple awards acknowledging his research. Notable accolades include the Kyoto Prize in Basic Sciences in 2005; the American Institute of Biological Sciences' Distinguished Scientist Award in 2007; the Ecological Society of America's Eminent Ecologist Award in 2010; and the Tyler Prize for Environmental Achievement in 2014.

Levin is grateful to accept the National Medal of Science. "It was of course exceptionally gratifying," he said, "both for me personally and as recognition of the importance of the issues that epitomized my career choices: developing a strong mathematical foundation for dealing with biological problems, and especially the ecological dimensions of sustainability. Nothing surpasses recognition in one's own country, and by one's own country."

Michael Artin is also greatly honored to receive the Medal. Artin is an emeritus professor of mathematics at the Massachusetts Institute of Technology (MIT). He is most well-known for his work in the fields of algebraic geometry and non-commutative algebra.

Born in Hamburg, Germany, to mathematician Emil Artin, Artin came to the United States as a child and earned an A.B. from Princeton University in 1955. He received his Ph.D. in mathematics from Harvard University in 1960. Artin joined the faculty of MIT's Department of Mathematics in 1963 and became a full professor in 1966. He has previously served as Norbert Wiener Professor and chaired both the Pure Mathematics Committee and the Undergraduate Committee at MIT.



Michael Artin, emeritus professor at the Massachusetts Institute of Technology.

Algebraic geometry is Artin's primary area of concentration. He has helped introduce and advance modern tools and theories in the field, such as algebraic spaces and stacks. Artin's work with surface singularities developed familiar concepts such as the fundamental cycle and rational singularity. He also furthered the theory of étale cohomology, and applied concepts of algebraic geometry to the study of diffeomorphisms of compact manifolds.

Artin's approximation theorem has advanced the understanding of moduli problems and deformation theory. He has also made noteworthy additions to classical algebraic geometry, including significant input on the proof of the Shafarevich-Tate conjecture for an elliptic K3 surface. Moreover, Artin's influence in the field of non-communicative algebra, particularly his introduction of algebro-geometrical methods, has been substantial. He has written several books, the most renowned of which is 1991's *Algebra*.

Artin has received several honors for his mathematical contributions. In addition to being a SIAM Fellow, he is a fellow of the American Mathematical Society (AMS), the American Academy of Sciences, and the American Association for the Advancement of Science. He is also a member of the National Academy of Sciences. Artin received the AMS's annual Leroy P. Steele Prize for Lifetime Achievement in 2002; the Harvard Graduate School of Arts and Sciences Centennial Medal in 2005; and the Wolf Prize in Mathematics in 2013. He is a past president of the AMS, and holds honorary doctoral degrees from the University of Hamburg and the University of Antwerp.

The NSF administers the National Medal of Science, which was created by Congress in 1959, and is awarded annually to those demonstrating both leadership and exceptional contributions to science and engineering. The Medal recognizes outstanding work in fields such as chemistry, mathematics, and biological, physical, and behavioral/social sciences. SIAM congratulates Levin and Artin for this prestigious honor.

To read more details and for information on other prize recipients, please visit the National Medal website.[1]

[1] http://www.nationalmedals.org/stories/new-class-of-laureates

Music

Continued from page 1

rence plots (RPofRPs) [2]. Given an original time series $\{x_i \in R^m \mid i = 1, 2, ..., IK\}$, where I and K denote the size of RPofRPs and short-term RPs respectively, we can write its definition [2] as

$$R(i,j) = \begin{cases} 1, & \text{if } d_r(X_i, X_j) < \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

for $1 \le i, j \le I$. Here, $d_r(X_i, X_j)$ presents a distance function between X_i and X_j, and X_i presents the ith unthresholded short-term RP whose (k,l) component is defined by

$$X_i(k,l) = d_s(x_{K(i-1)+k}, x_{K(i-1)+l}) \quad (3)$$

for $1 \le k, l \le K$. Here, $d_s(x_{K(i-1)+k}, x_{K(i-1)+l})$ shows a distance function between $x_{K(i-1)+k}$ and $x_{K(i-1)+l}$.

Roughly speaking, a short-term RP represents a local relation and a RPofRPs represents similarities between the local relations. This simple method can retain most of the information contained in the musical pieces, including the information of phases, which forms the very core of music.

Visualization

Visualization through the RPofRPs reveals fundamental physical aspects of musical pieces, as shown in Figure 2 (see also Figure 10 of [2]). For example, many nearly diagonal lines parallel to the main diagonal show nearly periodic regularity of phrases, and narrowing or widening the width of the two lines indicates gradual tempo transition. A boundary of graphical motifs is equivalent to a boundary of musical phrases. The RPofRPs can also reveal a similarity between phrases played in different key scales. Here, the diagonal lines of the RPofRPs show a succession of the same regularity, while such lines of conventional recurrence plots show a regularity. Thus, through the RPofRPs, many users can easily access and grasp the abstract image of the complicated nonlinear information in musical pieces. Therefore, while obtaining a RPofRPs may not be a goal in itself, it could be a keystone for researchers in other fields. For example, psychologists could explore how people are touched and/or healed by music.

Subsequent research will further analyze music and develop methods for nonlinear time series analysis. We hope that our methods will be used widely not only by mathematicians but also by non-mathematicians for analysis of complicated real big data.

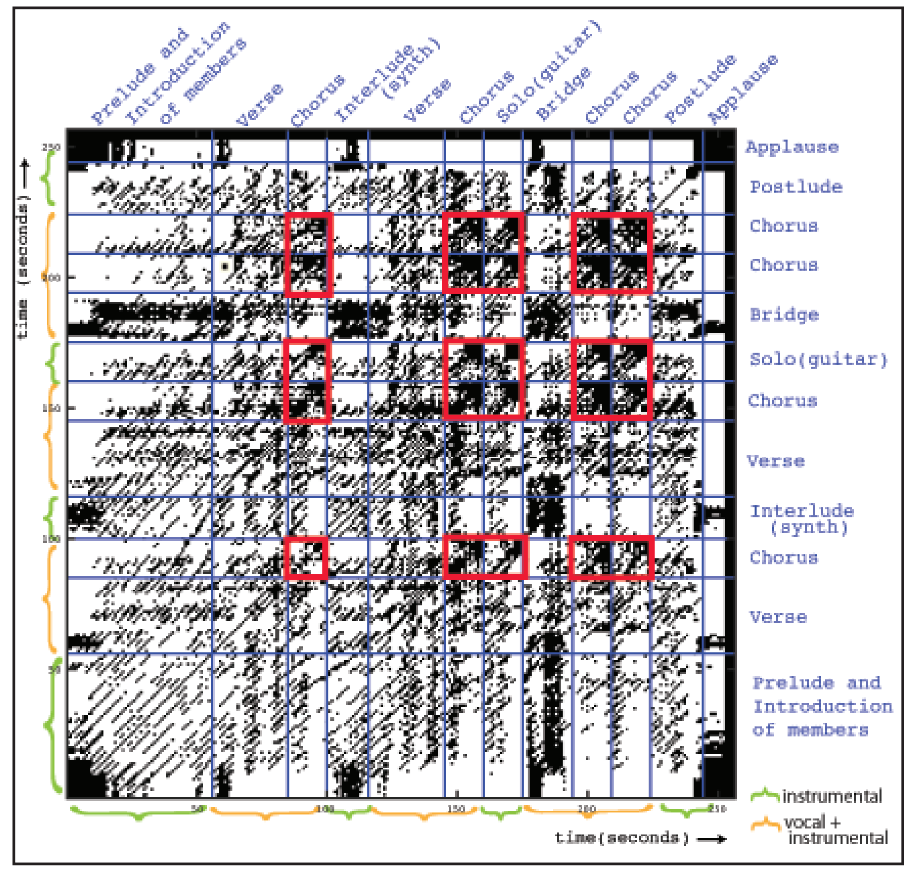


Figure 2. *An example of the thresholded RPofRPs. We calculated an amateur live pop tune that was played by drums, a chopper base, an electric guitar, an electric piano, two synthesizers, and a female vocal. We can find many diagonal lines parallel to the main diagonal line. Red squares show the intersections of the chorus section. Similar patterns of the figure represent similar sections of the music. We used modified Canberra distance for* d_r *and set* $\varepsilon = 0.2$.

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Zika Virus

Continued from page 1

$R_0 = 1$. For example, most mosquito-borne disease models can predict how decreasing the bites on humans or reducing the mosquito population below a certain level would impact the spread of Zika.

Local sensitivity analysis is another mathematical tool to quantify which parameters will be most able to reduce the severity of an outbreak. Sensitivity analysis uses the derivative of R_0—with respect to the parameters—to determine that in the many mosquito-borne epidemics, R_0 is most sensitive to the average number of bites of an infectious mosquito before it dies [1]. These bites depend on the mosquito biting rate, the extrinsic incubation period (EIP)—the time for an infected mosquito to become infectious—and the mosquito lifespan. Sensitivity analysis quantifies the way in which reducing the biting rate, say by using pesticides, can slow the epidemic. It explains how rising global temperatures, which could lower the EIP and increase the range of mosquito species, would increase R_0, allowing the disease to spread into new areas. The analysis helps predict the effectiveness of current mitigation efforts focused on reducing mosquito lifespan.

Figure 2 shows a diagram depicting disease transition and population dynamics used to derive the model's system of nonlinear coupled ODEs. In general, there is not enough data to parameterize transmission models with any confidence. The number of infections, as a function of time, in previous outbreaks can help estimate a few missing parameters and ranges for R_0. However, mathematical modelers must be careful not to over-fit the data by defining multiple free parameters in a simple mathematical model tailored to inaccurate, aggregated data. As John von Neumann famously stated, "With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." Because so little is known about so many model parameters, modelers must use uncertainty analysis to determine the identifiability of the model parameters and prevent over-fitting of the data.

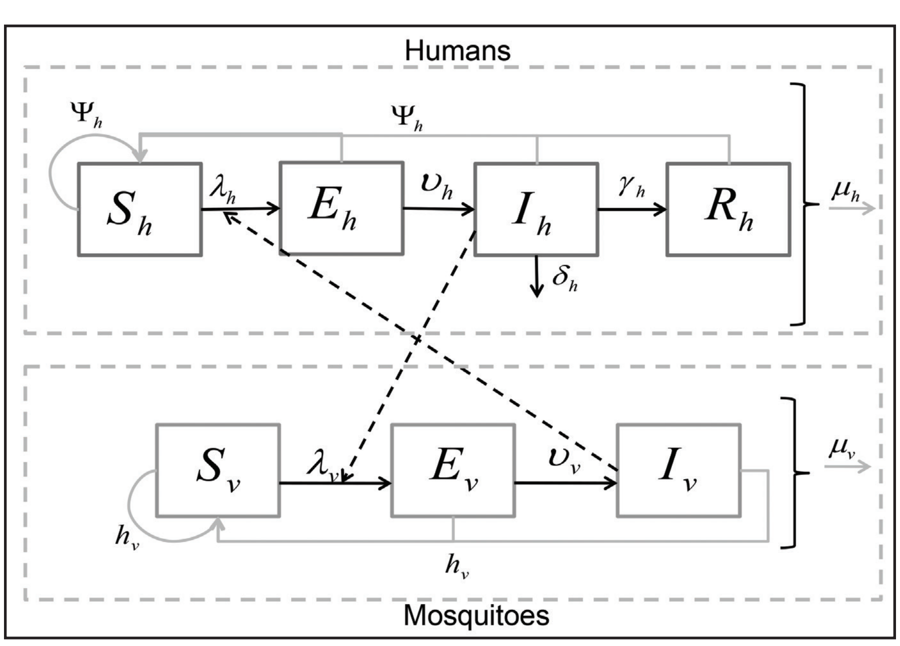

Figure 2. *Disease transition arrows are in black, the dashed arrows represent contacts between humans and mosquitoes, and population dynamics are in grey. Susceptible human hosts,* S_h*, can be infected when they are bitten by infectious mosquitoes. Infected humans become exposed (infected but not infectious),* E_h*, then infectious* I_h*. Infectious humans recover with a constant per capita recovery rate to enter the recovered,* R_h*, class. Susceptible mosquito vectors,* S_v*, can become infected when they bite infectious humans. The infected mosquitoes then move through the exposed,* E_v*, and infectious,* I_v*, classes. Population births and deaths are shown as well [1].*

Fitting an entire epidemic curve is different from fitting the beginning of a curve and predicting what will occur subsequently – a daunting task, particularly when evaluating a new outbreak. Currently, statistical and expert opinion models are better at predicting mosquito-borne disease outbreaks in real time than the more complex transmission models, as observed in the recent DARPA chikungunya challenge [2]. However, these statistical models cannot directly predict the impact of mitigation efforts such as the effect of mosquito spraying, release of genetically modified and sterile mosquitoes, or other changes in the underlying system, like differences in climate, infrastructure, culture, etc. Because the statistical models do not account for dynamic nonlinear correlations among the factors driving an epidemic, they are less capable of predicting the impact of changes beyond the data from which they have been derived.

Therefore, the best approach combines these methods to both forecast the disease and quantify the effectiveness of different mitigation strategies. Confidence in these predictions acts as a strong function of the quality and quantity of available data. In a world of "big data," quality data necessary to understand mosquito-borne epidemics is surprisingly sparse and difficult to obtain. Researchers can combine mechanistic and statistical models to estimate missing data and quantify the uncertainty in the predictions as a function of the reliability of the available data. They can then use these methods to derive a defensible and reproducible framework within which to inform policy and predict risk.

Where do mathematicians go from here? There is rich theory for mosquito-borne disease models, but we need greater understanding of nonlinear, non-autonomous and heterogeneous systems to continue making progress [3]. Most importantly, mathematicians and statisticians must interact often and thoroughly with data collectors, laboratory and field biologists, doctors, mosquito biology experts, sociologists, and public health officials in order to move towards better models, more useful and sustainable mitigation and prevention, and prediction capabilities that will save lives and prevent serious illness. While we have a long way to go, we can derive inspiration from the weather modeling community, which has continued to improve and expand—despite the daunting complexity of the system—to provide accurate forecasts and communicate the uncertainty in these forecasts.

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Seismic Waves

Continued from page 3

tion. (2) is simulated for each of the seismic sources *f*, multiple times per FWI iteration, typically totaling $10^5 - 10^7$ wave equation solves for a standard industry acquisition. In recent years, the seismology community has investigated more exotic schemes, such as the continuous finite element [5] and discontinuous Galerkin [2] methods. Element-based methods allow for local mesh refinement and adaptivity to irregular geological structure. Improving mesh construction remains a prime challenge for such methods in the context of FWI, where updated seismic velocity models necessitate a new grid for wave propagation.

Multi-step, explicit time integration (e.g. classic Runge-Kutta) schemes are the industry standard approach to time evolution. Alternatively, one may apply a Fourier transform to (2), yielding the sparse linear system $B_\omega \tilde{\phi}(\mathbf{x},\omega) = \tilde{f}(\mathbf{x},\omega)$, where B_ω is the discrete impedance matrix for frequency ω. When a direct solve approach is tractable, the frequency-domain approach is superior, as all seismic sources can be cheaply simulated after inverting B_ω. However, large-scale, three-dimensional problems typically exceed memory capacities, necessitating iterative methods instead.

Inversion of Model Parameters

Conventional FWI uses a least-squares objective function, where the velocity model $\tilde{m}$ is derived by minimizing data misfit:

$$\tilde{m} = \min_m \frac{1}{2} \Sigma_{\text{source}\, i} \| d_i^{\text{syn}}(m) - d_i^{\text{rec}} \|^2 := \min_m \chi(m). \quad (3)$$

The synthetic data d_i^{syn} is computed by modeling (2) with a starting velocity model m_0, while d_i^{rec} is recorded data from a field survey. Gradient descent methods are used to calculate a model perturbation, where the steepest descent direction $-\frac{\partial \chi}{\partial(m)}\Big|_{m=m_0}$ is computed from the cross-correlation of forward and adjoint-wavefield solutions to (2). Additional wave equation solves are used to compute a step length search, deriving a new model for the next iteration. As is common with gradient-based methods, convergence of FWI is sensitive to local minima in the objective function χ. Denoted as "cycle-skipping" in the FWI community, the inversion will fail to converge to the global minimum when features of the synthetic waveform are more than half a wavelength out of phase with their recorded-data counterpart.

Although FWI delivers highly accurate results when applied to a suitable starting model, FWI models derived from poor-quality starting models contain spurious cycle-skipping artifacts (see Figure 1, on page 3). Better signal-to-noise ratio within the data (particularly at low frequencies) and longer acquisition geometries reduce the requisite starting model accuracy, but the fundamental problem remains. Quasi-Newton optimization schemes [4] may help by accounting for second-order scattering effects, at the expense of additional wave equation solves per iteration. Model-space strategies [e.g. 3] post-process the update by steering towards models of greater geological plausibility, while data-space strategies [e.g. 9] precondition the input to the objective function. Alternative objective functions [1] have likewise been explored in search of an error measure less sensitive to cycle-skipping.

Applied mathematics has played an integral role in the advancement of FWI technology, from the development of high-powered wave equation solvers to the innate nature of the optimization problem itself. The field is a promising one for applied mathematicians in search of multidisciplinary application, with a well-established foundation and many remaining open problems. Advancement beyond single-parameter inversion is a commercially lucrative proposition, but is currently restricted by the difficulty of mitigating parameter crosstalk and the cost involved to model additional physics. Avoiding convergence to local minima in the presence of poor data quality and/or starting model accuracy remains the top challenge in the field.

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Alan Schiemenz is a data scientist at Crown Castle International Corp. He has enjoyed a diversity of career experiences enabled by a foundation of applied mathematics, including time as a member of the seismic modeling and inversion team at Schlumberger, and prior postdoctoral appointments in the computational earth sciences.

Teaching Mathematical Modeling to Students

From Kindergarten Through College and Beyond

By Rachel Levy

Mathematical modeling can be taught at every stage of a student's mathematical education, from kindergarten to undergraduate school and beyond, as a basis for developing problem-solving skills and mathematical habits of mind.

That is the premise of the national Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), a new report co-published by SIAM and the Consortium for Mathematics and its Applications (COMAP) and in cooperation with the National Council of Teachers of Mathematics. The recommendation to create the report originated at two NSF-SIAM workshops on Modeling Across the Curriculum, held in August 2012 and January 2014, and was motivated in part by the American Statistical Association's highly successful GAISE report,[1] which promoted statistics education in K-16.

Written by faculty members as well as K-12 teachers engaged in teaching and applied/industrial mathematics research, the report was reviewed by K-12 teachers, mathematics teacher educators, mathematics education researchers, and mathematics faculty. The first release of the report was showcased in a panel and three workshops at this year's National Council of Teachers of Mathematics Annual Meeting and Exposition in San Francisco (April 2016). Future showcase locations include the SIAM Annual Meeting in Boston (July 2016), the SIAM Conference on Applied Mathematics Education in Philadelphia (September 2016), and the Joint Mathematics Meeting in Atlanta (January 2017).

In mathematics education, the word 'modeling' is used for many things, such as demonstrating a mathematical process (for example, solving an equation), using manipulatives (such as utilizing blocks to represent addition), and describing mathematical techniques (for example, repeated addition as a model for multiplication). The GAIMME report begins with a general chapter that describes mathematical modeling. The chapter emphasizes that mathematical modeling, both in school and in the workplace, employs mathematics to answer big, messy, reality-based questions by quantifying phenomena and analyzing relationships. The report shares experiences from the classroom that allow students to engage in genuine modeling activities. With appropriate facilitation from teachers, students can then use mathematics to answer meaningful questions that could help enhance their futures.

The report's opening chapter explains that merely adding a context to a mathematics problem, as with many traditional word problems, does not constitute mathematical modeling. However, transforming 2+3=5 to the elementary-level problem 2 apples + 3 apples = 5 apples moves on the continuum towards mathematical modeling. The question "How many slices of apple should be in your lunch?" moves much further. The most genuine modeling questions are generally open at the beginning (as not all necessary information is provided), open in the middle (so students have the opportunity to choose their mathematical tools), and open at the end (where many answers are possible and an answer's usefulness should be discussed).

The chapter ends with the following five guiding principles:

1. Modeling (like real life) is open-ended and messy. *It may seem like a good idea to help students by distilling a problem so they can immediately see which are the important factors to be considered. However, doing so prevents them from doing this on their own and robs them of the feelings of investment and accomplishment in their work.*

2. When students are modeling, they must be making genuine choices. *The best problems involve making decisions about things that matter to the students, and help them see how using mathematics can help them make good decisions.*

3. Start big, start small, just start. *After reading this report, you may feel ready to jump in and make big changes, and if so, that is great! However, even small changes to things you already do in your classroom can encourage students to engage in mathematical modeling.*

4. Assessment should focus on the process, not the product. *Mathematical models (and the results they produce) are intimately tied to the assumptions made in creating the models. Assessment should be in service of helping students improve their ability to model, which will, in time, translate to a better product.*

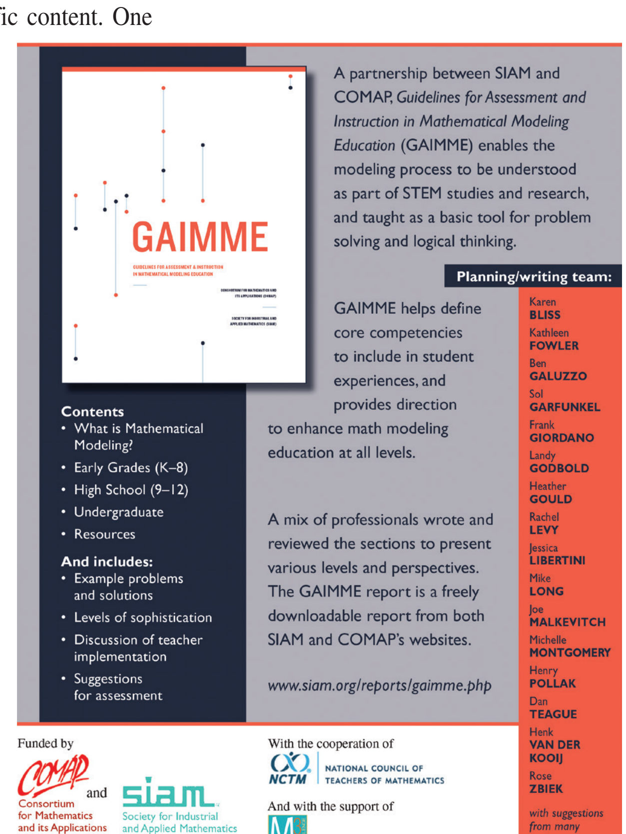
5. Modeling does not happen in isolation. *Whether students are working in teams, sharing ideas with the whole class, or going online to do research or collect data, modeling is not about working in a vacuum. The problems are challenging, and it helps to know you have support as you seek answers.*

The next three chapters of the report focus on more grade-specific content. One section focuses on early and middle grades, another on high school, and a third on college. The goal is to help people "look over the shoulders" of experienced modeling teachers and their students to see how modeling is enacted in the classroom. The final sections of the report provide tools for assessment and a set of further modeling resources.

The GAIMME report will exist as a living, growing document with an associated online repository of resources. Its writers hope that with the increasing awareness of math in everyday life—its continued need in academic/research environments, the growing prevalence of mathematics and data science in business, industry, and government (BIG) jobs, and the push for students to develop 21st century workforce skills like communication, collaboration, creativity, critical thinking, and citizenship/stewardship—teachers and faculty will view mathematical modeling as an effective way to prepare their students for the future. The GAIMME report and its associated materials will help educators facilitate and develop modeling abilities in their students, as well as pave the way for the inclusion of modeling as part of the established curriculum in a balanced mathematics education.

View and download the full report here: http://www.siam.org/reports/gaimme.php

Rachel Levy is SIAM VP for Education, as well as an associate professor in the Department of Mathematics and Associate Dean for Faculty Development at Harvey Mudd College.



The Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) offer suggestions for incorporating mathematical modeling in classrooms.

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1 http://www.amstat.org/education/gaise/

Announcing the 2016 Class of SIAM Fellows

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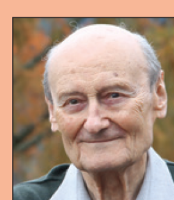

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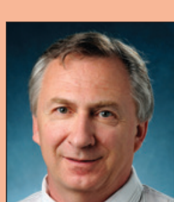

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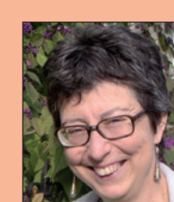

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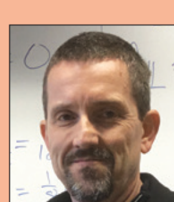

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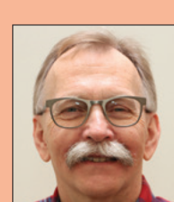

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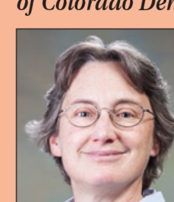

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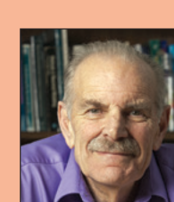

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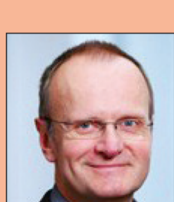

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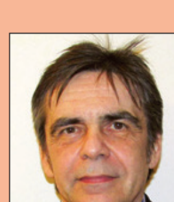

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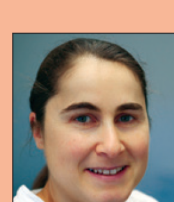

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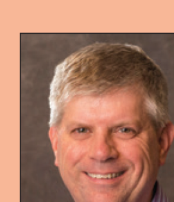

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Mathematical Matchmaking

Algorithms That Address Real-World Problems

BOOK REVIEW
By James Case

Who Gets What—And Why. *By Alvin E. Roth, Houghton Mifflin Harcourt, New York, 2015, 272 pages, $28.00.*

In this exciting new book, Alvin Roth—McCaw Professor of Economics at Stanford University, joint winner of the 2012 Nobel Prize in Economics, and one of the world's leading experts on market design and game theory—discusses the ongoing proliferation of electronic markets. In particular, he describes the several ways in which he and his colleagues have extended the Gale-Shapley matching algorithm to apply to a growing variety of real-world problems.

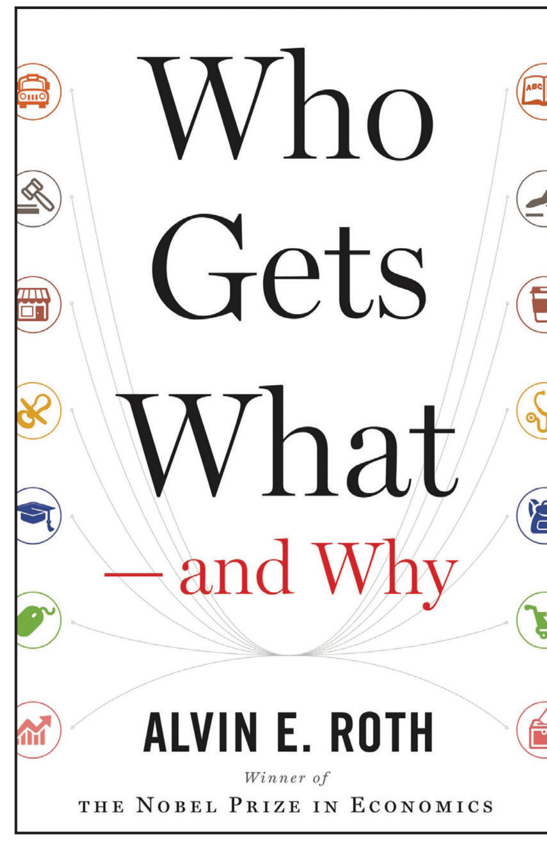
Roth seems proudest of his life-saving efforts to match voluntary kidney donors with transplant candidates. Though the first kidney exchange took place in the year 2000, the procedure was seldom performed in the months and years that followed. Previously, patients had often been obliged to wait a long time for a compatible donor, and more than a few died before a donor could be found. Family members were frequently willing to donate but were not always compatible. In such cases, a donor exchange in which A's willing donor gives a kidney to B and B's willing donor gives one to A can sometimes save lives. However, all four individuals must be located in nearby operating rooms at (more or less) the same time, which isn't always easy to arrange. Additional lives can be saved with three-way exchanges, though these are trickier to organize since the cycle $A \to B \to C \to A$ has to close, and all three patient-donor pairs must be in the same place at the same time. Four-way exchanges have also taken place.

Transplant candidates may receive kidneys from altruistic donors or deceased organ donors. When a patient A who is paired with a willing but incompatible donor receives a kidney from a deceased or altruistic donor, A's designated donor becomes free to donate to a compatible B and so on, setting up the possibility of a (potentially) endless chain and eliminating the need for multiple simultaneous transfers.

Working with a pair of graduate students, Roth devised an algorithm that identifies potential cycles and chains within a pool containing both altruistic donors and patient-donor pairs. This work helped found the New England Program for Kidney Exchange (NEPKE) in 2004, which organized the fourteen kidney transplant centers in New England into a consortium for helping incompatible patient-donor pairs find matches. There is also now a National Kidney Registry, designed to increase the size of the pool that identifies potential chains and cycles of viable kidney transplants.

As Roth notes, some 17,000 kidney transplants are now performed each year in the United States alone. But, because there are more than 100,000 people awaiting transplants, much remains to be done. At present, each of the 11,000 deceased-donor kidneys per year produces only a single transplant. If they were used instead to start chains, the number of transplants could be increased significantly.

Roth entered the matching algorithm business in 1995, and joined the faculty of Harvard University a few years later. Here he learned of the Boston Pool Plan for matching incipient medical school graduates with hospital residencies. The Plan had performed admirably for many years, but was having difficulty managing the growing number of married couples among the new graduates demanding same-city appointments. Roth discovered that the existing plan utilized a matching algorithm equivalent to the Gale-Shapley algorithm—published in 1962, exactly ten years after the Boston Pool Plan's initiation—for producing *stable matchings*.



Who Gets What—And Why. By Alvin E. Roth. Courtesy of Houghton Mifflin Harcourt.

In an article entitled *College Admissions and the Stability of Marriage*, written before the days of same-sex marriage, David Gale and Lloyd Shapley[1] defined a matching of prospective brides and grooms to be *stable* if none of the assigned brides (seekers) could be reassigned to a groom (seekee) she would prefer, and who would in turn prefer her to his assigned bride. That definition extends, in a more or less obvious manner, to colleges and college applicants, jobs and job seekers, etc. According to Roth, there now exists a considerable body of empirical evidence that stable matchings are longer lasting, and on the whole more satisfactory, than unstable ones.

To produce a stable matching, both the Gale-Shapley algorithm and the Boston Pool Plan require each prospective seeker and seekee to submit a list of potential partners in order of preference. Working with Elliott Peranson, the engineer who had previously computerized the Boston Pool Plan and was intimately acquainted with its inner workings, Roth was able to produce an algorithm applicable to situations in which some of the seekees are located in different cities, while some pairs of seekers must be assigned to the same city. Now known as the Roth-Peranson algorithm, it employs the Gale-Shapley algorithm to produce a preliminary matching of newly minted doctors and residency programs, before trying to placate dissatisfied pairs one by one. Though examples exist in which no stable matching is possible, the possibility is rarely encountered in practice. It is "virtually never the case that a stable matching cannot be found," writes Roth.

In 2003, Roth was called in to streamline the complicated, paper-based process that assigned some 90,000 eighth graders to New York City's high schools. Although the number of applicants was roughly the same as the number of openings, the process was not working well. Whereas some 17,000 applicants were receiving multiple letters of acceptance from schools on their preferred list of up to five (and exhausting the allotted time before making a decision), as many as 30,000 were receiving no letters at all from the schools they had listed. As a result, these "unsuccessful applicants" were often not assigned to a school until late August. Both parents and children were reporting high levels of anxiety.

Roth, together with two co-workers, increased the number of schools a student was allowed to list, designed an applicable version of the Roth-Peranson algorithm, and set out to explain the new system to all interested parties. In the first year of operation, the number of students unassigned to one of the schools on their preferred list declined from 30,000 to 3,000. And in the first three years of operation, the number of students who got their first choices increased steadily, as did the number who got their second through fifth choices. Today, "New York's high school choice system is holding up well," writes Roth. The algorithm is also being imitated, with varying degrees of success, in several other cities!

Throughout the book, Roth is careful to describe matchmaking venues—in which nothing is either bought or sold—as "markets," and to inquire after the conditions likely to promote either success or "market failure." He identifies five critical factors; markets should be thick, uncongested, safe, simple, and efficient. By "thick," Roth implies that markets should contain numerous potential parties and counterparties. Participants in "uncongested" markets should be able to avoid interfering with one another, and those in "safe" markets have nothing to lose by revealing their true preferences to the "yenta" in the computer. "Simple" markets allow the entire process to be understood clearly and quickly, and "efficient" markets do not expend undue quantities of energy – psychic or otherwise.

In closing, it should be mentioned that by classifying matchmaking venues as markets, Roth gives the impression that economists are uniquely qualified to design and modify them, though nothing in the traditional economics curriculum seems particularly relevant. On the contrary, students of combinatorics, computer science, and/or operations research (in which Roth himself received his doctoral degree) would seem at least as well prepared for the task. The authors of *Freakonomics* and *SuperFreakonomics* are guilty of similar territorial aggression, seeking to coopt consulting opportunities from the statisticians upon whose methods they almost exclusively rely. Nevertheless, *Who Gets What—And Why* presents an interesting account of the way algorithmically-driven electronic markets are beginning to eliminate the barriers that so often impede desirable commerce.

James Case writes from Baltimore, Maryland.

[1] Lloyd Shapley, who received the Nobel prize for economics in 2012, died recently at the age of 92: http://www.economist.com/blogs/freeexchange/2016/03/matchmaker-heaven

SIAM Meets with Congressional and Federal Agency Representatives

A Discussion of Programs Related to Applied Mathematics and Computational Science

By Jim Crowley and Miriam Quintal

In a time of generally tight budgets, SIAM's Committee on Science Policy (CSP) held its regular spring meeting on March 28 and 29. The committee met with key decision makers at federal agencies and in Congress to better understand the environment for research, influence congressional legislation and federal programs related to applied mathematics and computational science, discuss new research initiatives, and provide input on issues of concern to the SIAM community.

From left to right: Sven Leyffer, David Levermore (chair), Philippe Tondeur, and Hans Kaper of the SIAM Committee on Science Policy in Washington D.C. for the spring meeting.

Among the guests were Steve Binkley, associate director of the Department of Energy Office of Science for Advanced Scientific Computing Research (ASCR); Michael Vogelius, director of the National Science Foundation (NSF) Division of Mathematical Sciences (DMS); Jim Kurose, assistant director of NSF for Computer and Information Science and Engineering (CISE); Melissa Flagg, Deputy Assistant Secretary of Defense for Research; Chuck Romine, director of the National Institute of Standards and Technology (NIST) Information Technology Laboratory (ITL); and Ron Boisvert, head of the Applied and Computational Mathematics Division of ITL. The first day included presentations by and discussions with the invited guests, along with a general presentation on background by SIAM's representatives at Lewis-Burke Associates LLC. The second day featured visits with key staffers on Capitol Hill for further discussions.

As an interdisciplinary organization of members interested in applied mathematics and computational science, SIAM routinely holds discussions with several groups within the NSF at its CSP meetings. This meeting featured both mathematics and computer science.

It is interesting to note that mathematics is currently a division (DMS) within the Directorate for Mathematical and Physical Sciences (MPS), while computer science has a full directorate encompassing all of CISE. Thirty-five years ago, mathematics and computer science were two sections within a single division (the Division of Mathematical and Computer Sciences). Around 1983, then-division head Ettore (Jim) Infante laid the plans for separating mathematics and computer science. Today, the computer science directorate has a budget of $936 million, whereas mathematics remains a division with a $234 million budget, with applied and computational mathematics receiving a fraction of that figure. Given the interdisciplinary nature of SIAM—roughly 60% of SIAM's academic members are in mathematical sciences departments—SIAM's CSP must cast a wide net to learn about programs of interest to members.

As background, it should be noted that the Obama administration proposed a fiscal year (FY) 2017 President's Budget Request with an unusual feature – the request included both a proposed amount to be funded through the usual discretionary funds which would need to pass in appropriations legislation, and a proposed increment from mandatory sources (i.e. money that would require Congressional approval but would bypass the usual appropriations process). While interesting projects may have been proposed for the mandatory funding, it seems unlikely that mandatory-type funding will be provided.

DMS Division Director Michael Vogelius spoke about the budget situation for mathematics and the plans for FY 2017. Noting that recent budgets have spread increases fairly evenly among the MPS divisions, Vogelius said that not including mandatory funding, the proposed budget for FY 2017 would still leave the DMS budget less than its 2012 budget. Along with DMS, MPS has placed an emphasis on core research (applied mathematics and computational mathematics are among the DMS core programs). It was noted that mathematics does play a role in various Foundation-wide initiatives but focuses on fundamental research.

Vogelius said the DMS seeks suggestions from the community for initiatives that would be natural fits. Data science is one such area, and it is hoped that mathematics and statistics can play a larger role in big data initiatives at the NSF, for example, in algorithmic foundations of data science.

Vogelius also noted that the DMS remains committed to the institutes program and to "math+x" institutes that reach out to other disciplines; the DMS is seeking ways for biology and mathematics to mutually fund efforts, especially in areas of biology where mathematics has transformative potential.

Other mentioned initiatives include cybersecurity (mostly in CISE) and optics and photonics. SIAM is hosting an optics and photonics workshop[1] with the NSF at the SIAM Annual Meeting this July to help familiarize a wide range of researchers with the NSF "Optics and Photonics" call for proposals, and to encourage collaborations among researchers in this field. Further topics of discussion included the math institutes and joint programs, and the role of mathematics in the Strategic Computing Initiative. Vogelius also spoke about internships for non-academic career tracks, noting as an example the Enhanced Doctoral Training (EDT) program, which includes elements to better prepare participants for non-academic careers. Additional discussions regarding internships may take place within the DMS, following the NSF-IPAM Mathematical Sciences Internship workshop[2] held last September.

Jim Kurose of CISE initiated the discussion on programs and plans within CISE with a presentation outlining several initiatives. Current projects include big data, robotics, and the BRAIN initiative, which encompasses several CISE programs. New efforts—or ones growing substantially in FY 2017—include the National Strategic Computing Initiative, Smart and Connected Communities, Data for Scientific Discovery and Action (D4SDA), and Computer Science for All, the computer science education initiative. Kurose noted that in contrast to mathematics, where roughly 60% of research funding comes from the NSF, computer science receives 82% of federal academic funding from the NSF. The request for FY 2017 is $995 million for CISE, representing a balance among core, cross-cutting initiatives, and education.

Of course, core programs remain important in CISE. Kurose concluded with, "Submit your best ideas. It's the heart of what we do."

Another major area for applied mathematics and computational science is the DOE Office of Science, which is undergoing major budgetary changes. Steve Binkley discussed the Advanced Scientific Computing Research (ASCR) program, specifically focusing on applied mathematics and exascale. Binkley noted that DOE is the dominant funder of basic research in the physical sciences, and the applied mathematics program is a non-trivial source of funding in that area. The ASCR budget is proposed to increase by 7% ($42 million). Roughly half of the increase would go to basic research, and half would go to facilities.

A transformation is expected in the exascale program's organization. Current exascale efforts within the applied mathematics program and across other ASCR programs will be pulled out into a single Exascale line called the Office of Science Exascale Computing Project (SC-ECP). As a result, $10 million will move from applied mathematics to SC-ECP. Funded programs will presumably go with this money in the short term, but this will clearly represent a shift towards programs that are more applied in the longer term. The hope is that applied mathematics and other research programs will be built back up in the future.

Binkley made an interesting observation about the role of software and algorithms in improving computer performance. He noted that software (parallelism) has contributed increasingly more than hardware (Moore's law) to improvements from petascale to exascale. This trend will only continue, given the difficulties in moving to 10-nanometer technology and beyond.

The DOE is also considering alternative computing models, including quantum computing, with a focus on scientific computing. These models will cause this relatively small part of the budget to grow in coming years.

Chuck Romine presented information on NIST's relevant programs, most of which are intramural research programs within ITL rather than university research-funding programs. ITL has seven divisions, one of which is Applied and Computational Math (others include two in cybersecurity and one each in core computer science, advanced networking, statistics, and information science). NIST hosts many guest

See **Agency Representatives** *on page 10*

[1] http://www.siam.org/meetings/an16/workshops.php#optics

[2] https://sinews.siam.org/DetailsPage/tabid/607/ArticleID/763/NSF-IPAM-Workshop-Tackles-Workforce-Issues.aspx

A Closer Look at SIAM Activity Groups

In many ways, SIAM activity groups are the heart of SIAM. That statement is not intended to diminish the significant role of SIAM journals, but rather to indicate the importance of activity groups for individual members; they offer members an opportunity to network and share information on topics of mutual interest.

SIAM activity groups (SIAGs) are comprised of smaller subsets of SIAM members. Each group is focused on a technical subarea such as life sciences, algebraic geometry, discrete mathematics, or geometric design. SIAGs may run periodic conferences or workshops, recognize their members' achievements with prizes, and provide media to facilitate the exchange of information. Media preferences vary among activity groups; some host a website or web-based platform while others prefer a simple e-mail correspondence among members. Each method offers some benefits to the SIAG's members, and the types of benefits and activities depend largely on the members themselves and the officers they elect to represent them.

How does an activity group form?

Groups of members may petition the SIAM Council and Board to form an activity group. A standard protocol for such requests, which requires signatures from interested members, is detailed on the SIAM website.[1] The Council and Board review such petitions and decide whether to approve the requested activity group.

A packed lecture hall at the 2015 SIAM Conference on Applied Algebraic Geometry (AG15), sponsored by the SIAG on Algebraic Geometry and held last August in Daejeon, South Korea.

On what is the decision based?

The Council and Board look to see whether there appears to be sustained interest in the proposed activities, then determine if there are a sufficient number of SIAM members who might support these activities. They also consider whether the proposed activities fit within the mission of SIAM – to promote mathematics *and* its application and to support applied mathematics and computational science.

How does a SIAG operate?

After the Council and Board allow the creation of a new SIAG, the group must propose a preliminary set of officers and create its rules for operation. Often proposers submit these items with the initial petition. Once the Council and Board approve the rules—formally called Rules of Procedure—and officers, the officers run the SIAG with support from the SIAM office.

SIAGs hold contested elections for new officers every few years, a process intended to provide opportunities to appoint new members with fresh ideas.

Most SIAGs run a periodic conference or workshop. Typically the officers appoint co-chairs (one of whom may be Program Director of the SIAG) who then form an organizing committee. The organizing committee picks the themes and invited speakers, with assistance from the SIAM conference staff and approval from the VP for Programs.

SIAGs may also propose prizes to the SIAM Major Awards Committee. If the committee approves a prize, the SIAG officers appoint a prize selection committee and supply money for the award. The SIAM office assists with the promotion and collection of nominations and the administration of the prize.

There are certain responsibilities that a SIAG must observe. For example, every SIAG is expected to participate in the SIAM Annual Meeting during off years (when the SIAG does not have a conference) by organizing minisymposia. SIAGs should also periodically (every six or seven years) have a track at the SIAM Annual Meeting, consisting of an invited speaker and a critical mass (at least four) of minisymposia.

Is there a size constraint for SIAGs?

SIAGs vary in size from about a hundred members to over a thousand members (e.g. the SIAG on Computational Science and Engineering). The Council and Board evaluate activity groups on their activities and whether they serve a need within the SIAM membership.

Do SIAGs live forever?

SIAGs are chartered for a fixed period of time, typically for one conference, which in most (but not all) cases is two years. The officers must petition the Council and Board for charter renewal if they wish to continue beyond the period for which they were approved. The charter renewal petition[2] is a form with nine questions; officers provide the Council and Board with a sense of how active the field has been, how well the SIAG has performed its promised activities, and what plans the group has for the future. The form also provides an opportunity for the officers to say how they feel SIAM can support the SIAG in the future and what the SIAG can do to help SIAM.

SIAGs can discontinue or be terminated. A topic that was a hot area of research at one time may have fewer active researchers at a later date. The number of members may decline and those who remain may show less interest in group activities. An activity group's direction can also drift away from SIAM's mission. And so there may come a time when a SIAG must cease, perhaps with the hope that its remaining members will find homes in other groups.

A SIAG conference is also quite sensitive to size. When conference attendance falls below a critical mass (well below 200, for example), it becomes financially difficult for SIAM to run the conference in the standard style of a SIAM conference (a standalone conference in a hotel or similar venue with full support of the SIAM conference staff). When attendance decreases, SIAM must find other ways for the SIAG to hold meetings (such as meeting with the SIAM Annual Meeting or with another SIAG).

Ultimately, of utmost importance to the Council and Board is whether the SIAG is representing and serving both SIAM members and the SIAM mission.

— Jim Crowley and Pam Cook

[1] http://www.siam.org/activity/start.php

[2] http://www.siam.org/activity/renewal.php

Newtonian Dynamics = "Spring Theory"

A simple but illuminating equivalence exists between Newtonian mechanics of a particle and the *statics* of Hookean springs. In light of this equivalence, some basic facts of Hamiltonian mechanics become truly transparent. One can see, with very few formulas, why energy is conserved in Hamiltonian systems and why these systems preserve Poincaré's integral invariants, among other things. Here is a brief sketch of this equivalence; more details can be found in [1].

We look at a point mass m with potential energy $P=P(x)$, $x\in\mathbb{R}^n$ in some conservative force field. Hamilton's principle in this setting says that the motions $x(t)$ are stationary functions of the action

$$\int_0^T\left(\frac{m\dot{x}^2}{2}-P(x)\right)dt\equiv\int_0^T(K-P)dt \quad (1)$$

with fixed ends $(0,x_0)$ and (T,x_T) in the time–space (t,x).

Generations of students of the subject have been bothered by the depressing lack of a physical interpretation of the *difference* $K-P$. Einstein made an absolutely striking observation that (1) is the first significant term in the relativistic expression $\int E\,dt^*$, where t^* is the particle's proper time and E is the energy; so, (1) does have an independent meaning – a relativistic one. Setting this gem aside, however, I would like to stick to classical mechanics.

The action (1)—which comes from dynamics—admits a *static* interpretation. As an Archemedian thought experiment, let's hang an idealized spring by two ends (see Figure 1). Consider the child's toy "slinky," which we treat as a one–dimensional object, perhaps thinking of it alternatively as a heavy rubber band that stretches non–uniformly, as shown in the same figure. There is no resistance to bending, and our spring satisfies linear Hooke's law. The spring has mass, and we denote Hooke's constant of the unit mass of the spring by m, suggestively.[1]

We parametrize the position $x(t)$ of the spring's particle by the mass t between the particle and the spring's end, as seen in Figure 1. The spring's total potential energy then turns out to be

$$\int_0^T\left(\frac{m\dot{x}^2}{2}+U(x)\right)dt, \quad (2)$$

where $U(x)$ is the potential energy of a unit mass; the term $\frac{m\dot{x}^2}{2}$ is the stretching energy per unit mass.

Now (2) coincides with (1) if $P=-U$. We endowed the dynamic action (1) with a static meaning: *the action (1) is the potential energy of a slinky placed in the force field with potential* $-P$. Since I used no special features of the gravitational field, this statement applies to any potential P.

And so the Newtonian dynamics is equivalent to statics of springs. A table of equivalence can be found on page 45 of [1], and lists static equivalents of velocity, momentum, etc.

MATHEMATICAL CURIOSITIES
By Mark Levi

Here are some examples and consequences of the aforementioned equivalence:

1. Connect the two ends of the slinky, making a loop, and place it in the repelling Coulomb field with potential $U=\frac{1}{r}$ in $\mathbb{R}^3$ (see Figure 2). What are the possible equilibrium shapes of the loop? This is a static equivalent of Kepler's problem (in the potential $-U$). We conclude that the equilibria are ellipses (assuming that the mass T of the spring is finite, which excludes parabolas and hyperbolas).

2. In the preceding example, what is the static equivalent of the conservation of angular momentum? It is the following: *for any arc of the spring in the equilibrium in a central potential, the torques of the two tension forces exerted on the ends of the arc by the rest of the spring cancel each other out* (torques are computed relative to the origin). Conservation of the angular momentum is thus a consequence of Newton's *first* law in its rotational version: zero angular acceleration means zero torque.

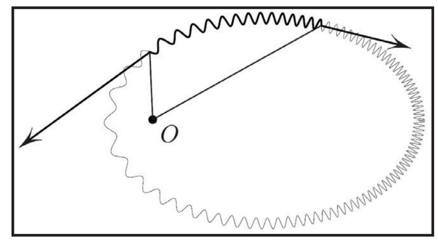


Figure 2. *An equilibrium shape in the repelling Coulomb field is an ellipse. Conservation of the angular momentum in Kepler's problem is equivalent to the torque balance for any segment. Loops count the time.*

3. Conservation of total energy $K+P$ in dynamics has a static counterpart. What is it? By asking this question I discovered something I hadn't realized before but is obvious in retrospect: *the difference* $K-U$ *between the energy densities (per unit mass) of stretching and of "elevation" is constant along the spring in equilibrium.* As an example, consider the hanging spring in Figure 3. The higher the loop, the more it is stretched, i.e. both energy densities increase with height – interestingly, by the same amount! This remark applies to any potential field, for instance to the Coulomb field of the previous example.

4. This is a (literally) hand–waving explanation of Poincaré's integral invariance. Imagine holding both ends of the spring in Figure 1 and moving the two hands cyclically and slowly, so as not to excite any vibrations. In doing so we perform zero net work:

$$\oint F_0\cdot dx_0+\oint F_T\cdot dx_T=0, \quad (3)$$

with F_0, F_T denoting the forces we exert. Putting this formula through the statics-dynamics dictionary leads to

$$\oint_{C_0} p_0\cdot dx_0=\oint_{C_T} p_T\cdot dx_T, \quad (4)$$

where $p=m\dot{x}$ is the momentum, C_0 is a curve of initial data in phase space $\{(x,\dot{x})\}$, and C_T is the image of this curve under the phase flow after time T. Therefore, the Poincaré integral invariance follows from the general principle that a perpetual motion machine cannot work. Note that it was not even necessary to write the Hamiltonian equations of motion to obtain (4).

5. A quick derivation of the Euler-Lagrange equation for (1), bypassing the (more general) method of Euler and Lagrange, is to observe that one can interpret the minimizer of (1) as a minimizer of the potential energy of a spring in the field with potential $-P$. But a spring in such an energy-minimizing state is in equilibrium, and thus the sum of forces on each infinitesimal element vanishes. There are three of these forces: two tensions on the ends, $m\dot{x}(t+dt)$ and $-m\dot{x}(t)$, and the force due to the ambient field, $P'(x(t))dt+O(dt^2)$. Dividing the sum of all these by dt and setting $dt\to 0$ gives $m\ddot{x}+P'(x)=0$. We derived the Euler–Lagrange equation for (1) from Newton's first law.

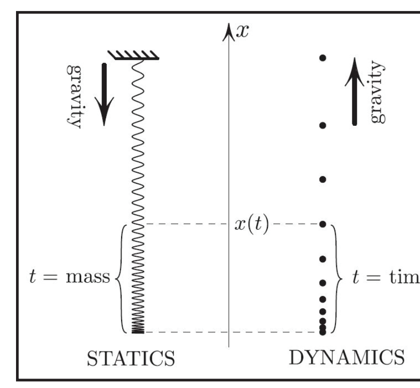


Figure 3. *The fleeting time t equals the static mass t. Equal times correspond to equal masses. A hanging spring gives a static view of the entire history (for $0\le t\le T$) of the falling body (note the opposite directions of gravity, however). Position $x(t)$ of a spring's particle is parametrized by the mass t lying between the particle and the spring's end.*

[1] More details on all of this can be found in [1].

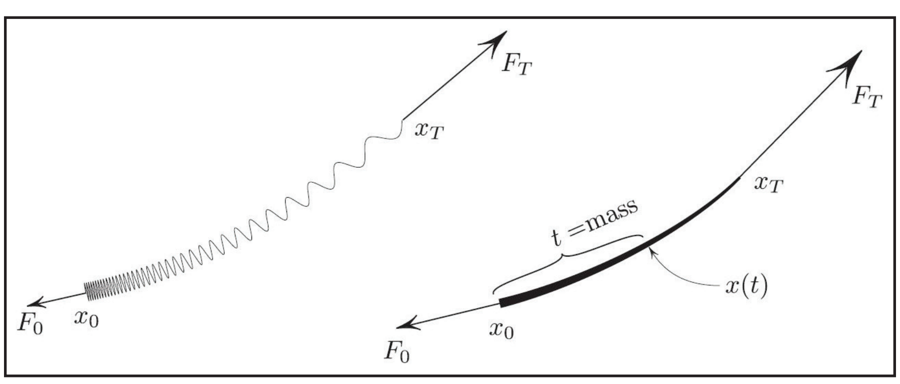


Figure 1. *A Hookean spring hanging by two ends, thought of either as a spring or an elastic latex band.*

References

[1] Levi, M. (2014, March). Classical Mechanics with Calculus of Variations and Optimal Control: an Intuitive Introduction. AMS, 42-49 & 274-280.

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Agency Representatives

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researchers who work (without remuneration) with lab scientists.

NIST will play a role in the National Strategic Computing Initiative. Some of the thrusts within NIST include math of metrology, high-performance computing and visualization, advanced materials, and quantum information science. Probably the piece most well-known to the mathematics community is the Digital Library of Mathematical Functions (DLMF).

Finally, Melissa Flagg talked about basic research at the DoD, emphasizing the major role of modeling and verification. She asked how we establish trust in our projections, noting that this is a big problem in affordability of systems, for instance.

Much of what she described as research needs and thrusts concern autonomous systems and human-machine collaboration. The goal is to make faster decisions at the speed of a machine while maintaining trust, ultimately resulting in better decisions and allowing humans to do what they do well.

Flagg discussed the DoD's future reliance on mathematics in areas such as enhancing cybersecurity defenses, modeling and simulation, computational methods for data analysis, and new analysis techniques to harness the power of big data. The DoD is looking for new ideas and big areas where mathematics can play a major role.

To support FY 2017 appropriations and robust federal investment in applied mathematics and computational science, CSP members met with congressional staff from their state delegations and key congressional committees on the second day. CSP members provided information on the importance of research funding at the NSF, DOE, DoD, and the National Institutes of Health (NIH), and underscored the value of mathematics and computational science research as well as related education and workforce issues.

The CSP will meet with additional agency representatives and discuss SIAM priorities and advocacy strategy for 2017 this fall. We encourage you to lend your voice in support of federal investments in math and computational science by meeting with congressional officers for your state. If interested, please contact Miriam Quintal, SIAM's Washington liaison, at miriam@lewis-burke.com.

Jim Crowley is the executive director of SIAM. Miriam Quintal is SIAM's Washington liaison at Lewis-Burke Associates LLC.

International MPE Competition

A second international MPE competition for science museum exhibits (modules) was announced at the Next Einstein Forum in Dakar, Senegal, on March 10, 2016.

The competition is organized jointly by Mathematics of Planet Earth (MPE), the International Basic Science Program (IBSP) at UNESCO, the International Mathematical Union (IMU), the International Commission of Mathematical Instruction (ICMI), and IMAGINARY.

The modules submitted to the competition will enrich the virtual Open Source MPE Exhibition. They can be downloaded and adapted by science museums and schools and can serve as examples for inputs for the new competition.

The deadline for the new MPE competition is June 30, 2017, and the competition will be judged by an international jury.

The modules submitted can be of four types. (1) Program: An interactive software exhibit to be used either on the web or in a museum. (2) Hands-on: A module explaining how to realize a physical module in a museum. (3) Film: A short video that can be shown in an exhibition. (4) Image gallery: A collection of images accompanied by their mathematical description.

Prizes are as follows: first prize: 5,000 USD, second prize: 2,000 USD, third prize: 1,000 USD. Special prize for African module: 2,000 USD.

For more information, visit www.mathofplanetearth.org/competition.

Launch of the Center for Mathematical Modeling "Carlos Castillo-Chavez" in El Salvador

By Byong Kwon and Rachel Levy

The Universidad Francisco Gavidia (UFG) in El Salvador, in collaboration with Arizona State University (ASU), held its First Congress of Mathematical Modeling in San Salvador from February 22–26, 2016. The event coincided with the inauguration of the UFG Center for Mathematical Modeling, named in honor of SIAM Fellow Carlos Castillo-Chavez. Castillo-Chavez is a Regents' Professor and director of the Simon A. Levin Mathematical, Computational and Modeling Sciences Center at ASU. At the Congress, participants from Argentina, Colombia, El Salvador, and the United States presented talks and held discussions concerning health, security, and education. The Center will address crime and insecurity in Salvadorian society, along with other real-world problems facing Central America.

Founded in 1981, UFG was named for a prominent Salvadoran humanist and focused on social sciences and humanities. In 2012, UFG expanded its offerings and research activities to science, technology, engineering, and mathematics (STEM) fields. Dr. Óscar Picardo Joao, director of the Institute of Science, Technology and Innovation at UFG, said he was inspired to launch the Center after meeting Castillo-Chavez in 2015 and reading *Designing the New American University* by Michael Crow and William Dabars. "We are convinced that the Center…[will] participate in designing solutions to the main problems of insecurity, violence, health, and education [in El Salvador]. In addition, the [Center's] results should impact the design of public policies and government programs," said Picardo.

The Congress and Center inauguration received extensive Salvadoran media coverage concerning how the Center can address crime and insecurity in El Salvador. In addition to attending the Congress, Castillo-Chavez and keynote speaker Edward Kaplan, President of INFORMS from Yale University, attended a private meeting between the government of El Salvador Security Council and Salvadoran civil society. "The presence of the Vice Minister of Science and Technology Dr. Erlinda Hándal [at the Congress] and the support of the Minister of Education Carlos Canjura again stressed the central role that the mathematical sciences play in decision making and policy planning in the Americas," Castillo-Chavez said of the event.

Besides crime and insecurity, other prominent themes during the week were STEM education and vector-borne diseases, such as Zika virus. Castillo-Chavez met with Salvadoran health officials in response to concerns about Zika in El Salvador. Along with other ASU applied mathematicians, he also made a special presentation about STEM education to a select panel at the Salvadoran Ministry of Education. The panel included Minister Canjura, Vice Minister of Science & Technology Hándal, directors of national institutes, and other guests.

When asked about the naming of the Center, Castillo-Chavez said, "Being born and raised in Mexico, educated and empowered in the USA, married to a Colombian, and mentored by a Ph.D. advisor from Canada makes me a citizen of the Americas. Being honored by the Universidad Francisco Gavidia in San Salvador was unexpected and humbling. Martin Luther King's words, 'Life's most persistent and urgent question is, 'what are you doing for others?'' and Nelson Mandela's belief that 'Education is the most powerful weapon which you can use to change the world' became even more present in my life after this recognition."

Byong Kwon is a Ph.D. student in the applied mathematics for the life and social sciences program at Arizona State University. Rachel Levy is SIAM VP for Education, as well as an associate professor in the Department of Mathematics and Associate Dean for Faculty Development at Harvey Mudd College.

Salvadoran TV news interviewing SIAM Fellow Carlos Castillo-Chavez in San Salvador. Photo courtesy of Universidad Francisco Gavidia.

Math Congress of the Americas

The Math Congress of the Americas, better known to some as MCofA, will hold its second Congress in Montreal, Canada, on July 23-28, 2017 – starting nine days after the conclusion of the SIAM Annual Meeting in Pittsburgh, Pennsylvania (July 10-14, 2017).

The Math Congress of the Americas is governed by the Math Council of the Americas (also known as MCofA), of which SIAM is a member. MCofA is an organization of professional math society and research institutes whose goal is to promote mathematics (generally) throughout the Americas, with a special focus on Latin America. A further goal is to foster the scientific integration of all mathematical communities in the Americas.

The Congress in Montreal follows the first Math Congress of Americas held in Guanjuato, Mexico, in 2013. The MCofA is held once every four years.

The MCofA covers the breadth of the mathematical sciences, and SIAM has some representation on the committees responsible for MCofA. As such, applied mathematics is a part of MCofA.

The SIAM Annual Meeting, which precedes the MCofA, embraces mathematics, computational science, and their applications in multidisciplinary settings. It is hoped that people with interest in mathematics and its applications who intend to go to the MCofA will consider attending the SIAM Annual Meeting beforehand.

Professional Opportunities and Announcements

*Send copy for classified advertisements and announcements to: **marketing@siam.org**; For rates, deadlines, and ad specifications visit **www.siam.org/advertising**.*

*Students (and others) in search of information about careers in the mathematical sciences can click on "Careers and Jobs" at the SIAM website (**www.siam.org**) or proceed directly to **www.siam.org/careers**.*

Argonne National Laboratory

Mathematics and Computer Science (MCS) Division Director

Argonne invites applications for the position of Director of the Mathematics and Computer Science Division, one of the four divisions and three joint institutes within the Computing, Environment, and Life Sciences (CELS) directorate. The CELS directorate integrates Argonne's research in the life sciences with the environmental sciences and the computing sciences, and has established a leadership computing facility essential for advanced simulation and data intensive science. The Mathematics and Computer Science (MCS) Division aims to increase scientific productivity in the 21st century by providing intellectual and technical leadership in the computing sciences, computer science, and applied computational mathematics.

The MCS Division Director will provide leadership and long-range vision for major science and technology areas consisting of multidisciplinary activities. They will serve as a key member of the directorate leadership team working directly with the Associate Laboratory Director, senior management, and external stakeholders, including program managers in DOE/ASCR. The Division Director will also provide a combination of strategic, operational, and technical/functional leadership to staff.

Qualifications:

• A Ph.D. degree or equivalent research experience in a relevant scientific/technical discipline is required.

• Demonstrated management and leadership skills in a high-tech organization and in planning, conducting, and administration of relevant complex scientific programs is required.

• Broad knowledge and high scientific reputation in areas of applied mathematics, computer science, or computational science of current or future interest to the division is required.

• Experience in managing and maintaining effective interpersonal relationships in a large matrixed environment of internal contacts, as well as with external contacts made up of government, academic, scientific, or industrial institutions is required. Experience in forming ad-hoc working groups from diverse and cross-organizational collections of individuals is preferred.

• Experience developing, managing, and motivating a high-performing and diverse population of multi-disciplinary technical and professional support teams is preferred.

• Experience or ability to discern and influence promising research directions from a national and DOE Office of Science perspective and to build complementary partnerships is preferred.

• A long-term strategic view of computer science and computing technology that extends beyond exascale and the end of Moore's Law scaling is preferred.

• Experience in the strategic incorporation of talent management and diversity and inclusion programs in overall workforce planning is preferred.

Chartered in 1946 as the nation's first national laboratory, Argonne enters the 21st century focused on solving the major scientific and engineering challenges of our time: sustainable energy, a clean environment, economic competitiveness, and national security. Argonne is home to a staff of over 3,300 employees, including over 1,500 scientists, engineers, and postdoctoral scholars. With world-renowned scientific facilities, it hosts over 6,500 facility users annually.

We offer a competitive salary and an excellent benefits package. For consideration, please apply via our website at http://www.anl.gov/careers, and refer to requisition #400930.

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INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES

The *Institute for Computational Engineering and Sciences (ICES)* at *The University of Texas at Austin* is searching for exceptional candidates with expertise in computational science and engineering to fill several Moncrief endowed faculty positions at the Associate Professor level and higher. These endowed positions will provide the resources and environment needed to tackle frontier problems in science and engineering via advanced modeling and simulation. This initiative builds on the world-leading programs at ICES in Computational Science, Engineering, and Mathematics (CSEM), which feature 16 research centers and groups as well as a graduate degree program in CSEM. Candidates are expected to have an exceptional record in interdisciplinary research and evidence of work involving applied mathematics and computational techniques targeting meaningful problems in engineering and science. For more information and application instructions, please visit: www.ices.utexas.edu/moncrief-endowed-positions-app/. This is a security sensitive position. The University of Texas at Austin is an Equal Employment Opportunity/Affirmative Action Employer.

THE UNIVERSITY OF TEXAS AT AUSTIN

Applied Algebra and Geometry: A SIAGA of Seven Pictures

By Anna Seigal

SIAM's brand-new journal, the SIAM Journal on Applied Algebra and Geometry, will feature exceptional research on the development of algebraic, geometric, and topological methods with strong connections to applications. The cover of the new journal shows seven pictures. By describing these pictures and discussing the topics they represent, we hope to give readers a glimpse into the world of algebraic, geometric, and topological problems of interest to applied mathematicians.

This article is part II of a three-part series. Stay tuned for the final part in our next issue.

3. Polyhedral Geometry

The Context

A polyhedron is a combinatorial object that crops up in many places. For example, it is the shape of the feasible region for a linear optimization problem. It is a convex shape in d-dimensional space $\mathbb{R}^d$ described by an intersection of finitely many closed half-spaces

$$P = \{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} \le b\},$$

where A is a $d \times n$ matrix describing the angles of the half spaces and $b \in \mathbb{R}^d$ encodes their translational information.

A polyhedral complex is an object consisting of a compatible collection of polyhedra. The association of a polyhedral object to an algebraic variety paves the way for the use of combinatorial tools to gain understanding. A typical way to do this is via toric geometry. This approach has been used in many areas of applied mathematics, including phylogenetics, economics, integer programming, biochemical reaction networks, and computer vision (from where this particular figure arose).

Tropical geometry offers one method, called tropicalization, for obtaining a polyhedral complex from an algebraic variety. Features of the new object provide useful information. For example, the dimension of the original variety equals that of the new polyhedral complex, and the dimension of the polyhedral object is far easier to compute.

The Figure

Figure 1 depicts one example of the use of polyhedral tools to understand an algebraic-geometric object. This provides us with an understanding of the application from which the variety arose.

In the field of *computer vision,* and in the real world, 'taking a photo' maps the three-dimensional world to a two-dimensional photograph. As any good photographer knows, the resulting features of the photo depend heavily on the angle and location of the camera.

We consider our pictures to be photographs of three-dimensional projective space $\mathbb{P}^3$. Each camera, A, is a 3×4 matrix which determines a map $\mathbf{x} \mapsto A\mathbf{x}$ to two-dimensional projective space $\mathbb{P}^2$. This map tells us where each point of the original world ends up in the photograph.

Considering multiple cameras (A_1, A_2, A_3) at different locations yields more information. Now we have a map:

$$\phi : \mathbb{P}^3 \dashrightarrow (\mathbb{P}^2)^3$$
$$\mathbf{x} \longmapsto (A_1\mathbf{x}, A_2\mathbf{x}, A_3\mathbf{x}).$$

The closure of the image of this map is an irreducible variety. For example, if A_i are the coordinate projections, this variety in $(\mathbb{P}^2)^3$ is cut-out by the Gröbner basis

$$\{z_0y_2 - x_0z_2, z_1x_2 - x_1z_2, z_0y_1 - y_0z_1, x_0y_1x_2 - y_0x_1y_2\}.$$

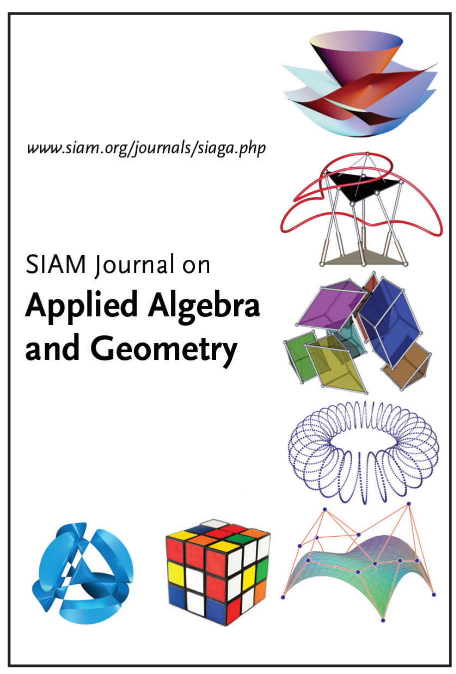
When we take the initial monomials of these generators, their zero-set decomposes into seven pieces: one copy of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ and six copies of $\mathbb{P}^1 \times \mathbb{P}^2$. To illustrate these three-dimensional spaces, we identify each projective space $\mathbb{P}^i$ with the i-simplex.

For example, $\mathbb{P}^2$ corresponds to the two-dimensional simplex Δ_2, the triangle, under the map

$$\mathbb{P}^2 \ni (x_0 : x_1 : x_2) \longleftrightarrow \frac{1}{x_0 + x_1 + x_2}(x_0, x_1, x_2) \in \Delta_2,$$

and we identify each copy of $\mathbb{P}^1$ with the one-dimensional simplex Δ_1, the unit-length line.

So our initial ideal represents a collection of polytopes which are faces of $(\Delta_2)^3$. Our copy of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ corresponds to the polytope $\Delta_1 \times \Delta_1 \times \Delta_1$. This is the dark blue cube. The remaining six pieces $\mathbb{P}^2 \times \mathbb{P}^1$ correspond to copies of $\Delta_2 \times \Delta_1$. These six triangular prisms are the remaining pieces of the figure. The pieces are separated a little so they are easier to see, but the adjacent parallel faces show how the different pieces meet. Meeting along a triangle Δ_2 means the projective spaces meet at a copy of $\mathbb{P}^2$. If their shared facet is a square $\Delta_1 \times \Delta_1$, then the projective spaces meet at a copy of $\mathbb{P}^1 \times \mathbb{P}^1$. The original figure, along with similar ones, can be found in [1]. The figure was created using Michael Joswig's software 'Polymake' [3].



The new SIAM Journal on Applied Algebra and Geometry (SIAGA).

4. Topology of Data

The Context

Topology offers a set of tools that can be used to understand the shape of data. The techniques detect intrinsic geometric structures that are robust to many common sources of error, including noise and arbitrary choice of metric. For an introduction, see [2, 4].

Say we have noisy data points arriving from some unknown space X, which we believe possesses an interesting shape. We are interested in using the data to capture the *topological invariants* of the unknown space; these are its holes of different dimensions, unchanged by continuous squeezing and stretching.

The holes of different dimensions are the homology groups of the space X. They are denoted by $H_k(X)$, where k is some nonnegative integer. The zeroth homology group discloses the number of zero-dimensional holes, or more intuitively, the connectedness of the space. For a space X with n connected components, the zeroth homology group is

$$H_0(X) = \mathbb{Z}^n,$$

the free abelian group with n generators. One-dimensional holes are counted by $H_1(X)$. For example, a circle $X = S^1$ has a single one-dimensional hole, so $H_1(S^1) = \mathbb{Z}$.

The connectedness properties of sampled data reveal a lot about the underlying space from which they are sampled. In some situations, such as for structural biological information, knowing the structure of the holes is also indispensable. These structures remain unchanged regardless of the metric we use or the space into which we embed the points. The higher homology groups $H_k(X)$ for $k \ge 2$ similarly offer such summarizing features.

But there's a problem: sampling N points from a space gives us a collection of zero-dimensional pieces, which—unless two points land in exactly the same place—are all unconnected. Let's call this data space D_0. The space D_0 has homology groups

$$H_k(D_0) = \begin{cases} \mathbb{Z}^N & k = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Usually many points are very close together, and should be treated as coming from the same connected component. To measure this we use *persistent homology*. We take balls of increasing size centered at the original data points, and measure the homology groups of the space occurring as the union of these balls. We call this space D_ϵ, where ϵ is the radius of the balls. The important structural features are those that *persist* for large ranges of values of ϵ.

The Figure

Figure 2 shows data points sampled from a torus, which we imagine to live in three-dimensional space. It was created by Dmitriy Morozov of Lawrence Berkeley National Lab. He applies topological methods to cosmology, climate modeling, and materials science.

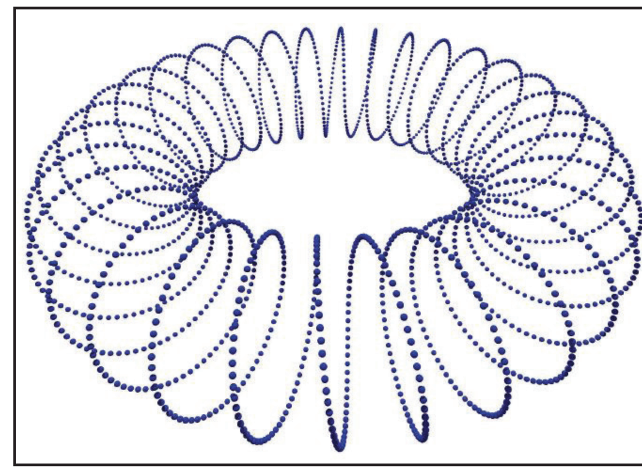


Figure 2. *Topological features from data points.*

The sampled points in Figure 2 lie on the torus, specifically in a more specialized, slinky-shaped zone. Topological methods capture this important feature of the shape.

The original data consists of 5,000 points, and our persistent homology approach involves taking three-dimensional balls $B_\epsilon(d_i)$ of radius ϵ centered at each data point d_i. When the radius ϵ is extremely small, none of the balls will be connected, and the data's shape is indistinguishable from any other collection of 5,000 points in space.

Before long, the radius will exceed half the distance to all the points' nearest neighbors. The 5,000 balls connect to form a curled-up circular piece of string. Topological invariants do not notice the curling, so topologically, the shape obtained is a thickened circle with a one-dimensional hole $H_1(D_\epsilon) = \mathbb{Z}$. When the radius is large enough for the adjacent curls of the slinky to meet, but not large enough to reach the opposite side of each curl, we get a hollow torus with $H_1(D_\epsilon) = \mathbb{Z}^2$ and $H_2(D_\epsilon) = \mathbb{Z}$. Finally, the opposite sides of each curl of the slinky will meet up, and they will meet with the slinky-curls on the other side of the torus. The shape then becomes three-dimensional with no holes, and $H_1(D_R) = 0$.

In this example, we can visualize the data points, confirming that our intuition for the important structure of the shape agrees with the homological computations. For higher-dimensional examples, where visualizing the data is not possible in this way, it is the persistent features that will guide our understanding of the data's shape.

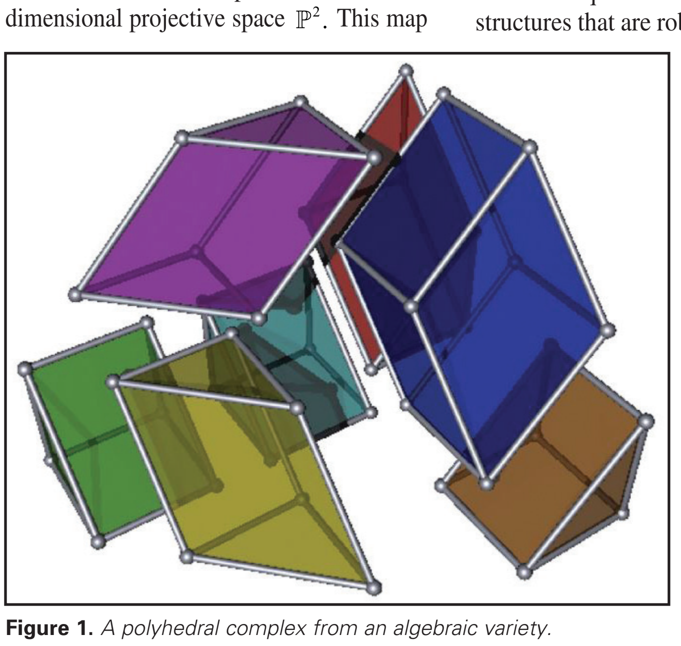


Figure 1. *A polyhedral complex from an algebraic variety.*

References

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[3] Gawrilow, E., & Joswig, M. (2000). Polymake: a Framework for Analyzing Convex Polytopes. *Polytopes–Combinatorics and Computation, 29*, 43-73.

[4] Ghrist, R. (2014). *Elementary Applied Topology* (1st ed.). Createspace.

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