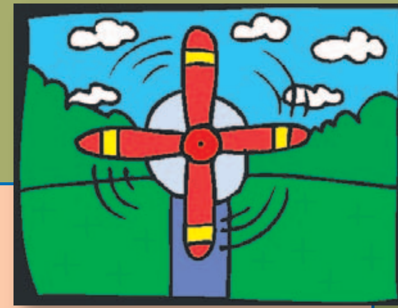
Math Matters

Apply It.



The math behind…
Renewable Wind Energy

Some technical terms used :

Dynamical systems, adaptive control, linear control, fatigue analysis, fluid dynamics

Uses and applications:

Increasing our use of renewable wind energy can reduce society's dependence on limited natural resources and fossil fuels. Wind turbines operate in harsh environments and often succumb to component fatigue. Characterizing and understanding the complex interactions of turbine components can help extend the operating life of these machines and lower the overall cost of energy.

How it works:

Modern wind turbines convert kinetic wind energy to electrical energy by transferring aerodynamic torque produced on the rotor to a generator. The total power available for conversion is proportional to the cube of the wind speed and also proportional to the square of the turbine blade length. To capture more power, wind turbines are made with longer rotor blades; to decrease overall weight, light and flexible materials are often used in large designs.

Dynamics in large, flexible wind turbines are very nonlinear and are modeled as a system of coupled nonlinear differential equations. Coefficients in these equations are often chosen based on known turbine characteristics (e.g. mass distribution, component natural frequencies, etc.).

The actual electrical power produced is determined by generator speed, which is controlled by adjusting the pitch of the blades to increase or decrease aerodynamic lift. When a turbine is operated in wind speeds below its rated speed (the wind speed at which the turbine produces its maximum rated power), nonlinear adaptive control is used to maximize generated power. For a turbine operating in wind speeds at or above its rated speed, linear feedback control methods are used to maintain a constant generator speed to produce the turbine's full rated power.

Interesting Fact:

In 1919, Albert Betz, a German physicist, published what is now known as the Betz limit. This limit suggests that the theoretical maximum power that a wind turbine can capture is only 59% of what is available in the wind. Betz based his derivation on simplified assumptions about mass flow rates of air and pressure differentials; critics say that these assumptions are too simplistic. However, even with the many technological advancements in real-time control and blade airfoil design since Betz's original paper, no wind turbine is known to have exceeded this limit.

Today's largest wind turbine is Germany's Enercon E-126, which produces enough power for over 1,700 average U.S. homes. The rotor diameter on this turbine is 126 meters (413 feet).

The U.S. Department of Energy is studying the possibility of obtaining 20% of the nation's energy from wind by the year 2030. Currently, only 1% of U.S. energy is obtained from wind.

For more information see:

www.nrel.gov/wind • www.20percentwind.org • www.enercon.de

Submitted by Glenn Parker, University of Alabama, Huntsville, third place Math Matters, Apply It! contest, January 2009

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