

Vibro-impact Energy Harvesting Via Computer-assisted Analysis

By Lanjing Bao, Rachel Kuske, Daniil Yurchenko, and Igor Belykh

Energy harvesting is the process of capturing some form of existing energy from the environment and converting it into electrical energy. One means of doing so is through vibration-based energy harvesting, which can source energy from human activities like walking and running and from the cyclic or random loading of

buildings, bridges, and other structures or machines. Energy harvesting devices typically include an automated component that captures mechanical energy and transforms it into electrical energy. While existing research suggests that nonlinear systems may harvest energy more efficiently than linear systems [6], relatively few studies have thoroughly explored the dynamic mechanisms that enable successful nonlinear energy harvesting.

The classical vibro-impact pair under external forcing offers an effective method for harvesting energy [7]. This system comprises a capsule and an inner ball whose impacts deform flexible dielectric polymer membranes that act as capacitors and are located at the capsule's ends (see Figure 1a). The impacts change the membranes' capacitance and allow the harvest of energy from periodic or random forcing. The power from a vibro-impact harvester is directly

proportional to the impact velocity, and the energy yield can be significantly higher—by an order of magnitude—than that of piezoelectric or electromagnetic techniques.

To design an efficient vibro-impact harvester and maximize its energy output, we must understand its global dynamics. Although local stability analysis of vibro-impact systems has garnered considerable attention in recent years [5], the exploration

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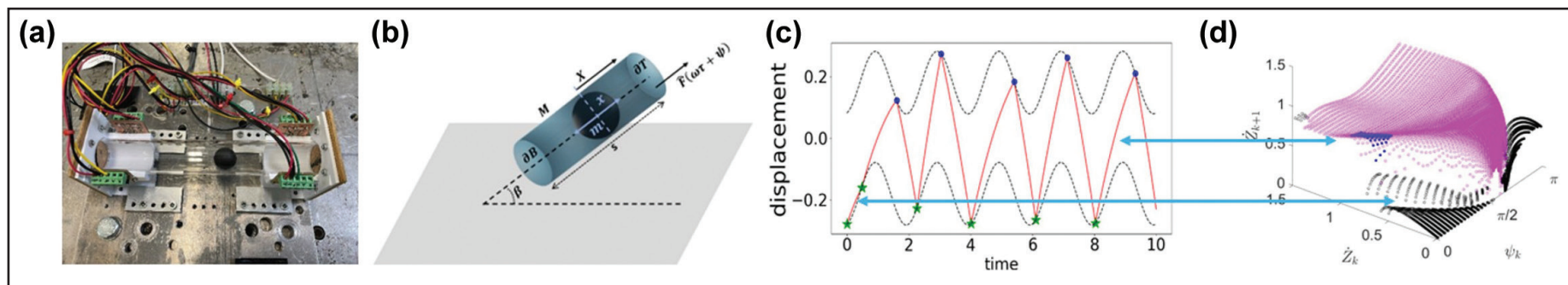


Figure 1. The vibro-impact energy harvesting device, its dynamical model, and return map construction. **1a.** Experimental setup. The transparent capsule is attached to a table that vibrates horizontally from left to right. **1b.** The corresponding vibro-impact model. The ball moves freely within the capsule, which is enclosed by deformable membranes on both ends. The capsule is positioned at an angle β relative to the horizontal plane and is excited by an external harmonic excitation $\hat{F}(\omega\tau + \psi)$. **1c.** Time series for the displacement of the top and bottom membranes (dashed black lines) and motion of the ball (red line). The green stars and blue dots respectively indicate the impacts at the bottom and top membranes. **1d.** Three-dimensional (3D) surfaces of the first return maps for the bottom-top-bottom impact sequence $\partial B \rightarrow \partial T \rightarrow \partial B$ (magenta) and bottom-bottom impacts $\partial B \rightarrow \partial B$ (black). The blue horizontal arrows between 1c and 1d relate the sequences to the corresponding 3D surfaces. Figure 1a courtesy of Henrik Sykora and Figures 1b-1d adapted from [1].

Deducing the Physics That Underlie Extreme Weather

By Matthew R. Francis

As the impacts of climate change continue to escalate, all regions of the globe are experiencing an uptick in extreme weather events: prolonged droughts, severe hot and cold spells, recurrent hurricanes (see Figure 1), and violent thunderstorms with attendant tornadoes. The scale and frequency of these occurrences defy our practices of language — not because thunderstorms, droughts, or hurricanes are unprecedented, but because we have no historical record of past incidents whose magnitudes and frequencies compare to present-day threats. The new normal is no normal.

Given the severity of such events, climate researchers and applied mathematicians have developed tools to better understand the statistics of weather extremes, provide probabilities, and predict their frequency. However, these methods do not typically

address the underlying physical drivers of severe episodes, i.e., the specific climate-driven atmospheric conditions that make them possible. Recognizing this deficiency, an array of climate scientists, physicists, mathematicians, meteorologists, and other researchers have compiled a suite of mathematical techniques to address the gap. These experts use dynamical systems theory, climate models, and statistical physics to calculate the probability of extreme events directly from geophysical principles and meteorological data, ultimately improving the science of climate attribution.

“Climate and geophysical extremes are very complex,” climate researcher Gabriele Messori of Uppsala University in Sweden said. “They are naturally rare, and that poses data challenges. They are [also] associated with dynamics on a very broad range of scales, and they often result from multiple drivers.”

As Messori and his coauthors noted in their 2024 review paper [2], conventional statistical approaches to rare events cannot effectively handle spatial inhomogeneity—i.e., strong differences in conditions from place to place—or storms that arise from underlying conditions that have not previously occurred in recorded history. “Rather than [ask] questions like ‘How rare is this or how often would it occur?’, we could ask, ‘Given its extremity, is the physics behind this event unusual, or is it a typical physical pathway to something that is this extreme?’” Messori said. “We are not really doing modeling in the sense of modeling the climate. We are more developing tools to interpret data [that] can come from climate models, observations, or climate models constrained by observation. It can be any sort of data.”

Fluctuations in Space and Time

Weather models are a classic example of chaotic systems, as exemplified in meteorologist Edward Lorenz’s famous 1963 paper [3]. Small tweaks to initial conditions—including approximations in numerical simulations—can yield drastically different outcomes given enough runtime. The developers of data-informed simulations tend to utilize stochastic dynamical systems, whose equations incorporate nondeterministic factors (much like statistical physics) to account for the lack of precise knowledge of all relevant variables. Additionally, atmospheric models cannot neglect turbulence (see Figure 2, on page 3), which involves self-similar vortices on a wide range of length scales and is not treatable with ordinary dynamical modeling; in fact, turbulence is still a major unsolved problem in physics.

Even without human activity, weather is a nonequilibrium system with a fluctuating energy input from the sun that is coupled

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Figure 1. Hurricane Isabel in 2003, as seen from the International Space Station. Climate change is increasing the frequency and severity of hurricanes, but the exact mechanisms that drive these escalations are still unknown. Figure courtesy of Mike Trenchard, Earth Sciences and Image Analysis Laboratory, NASA Johnson Space Center.

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6 The Outreach Component of My NSF CAREER Award

In 2019, Arvind Saibaba received the U.S. National Science Foundation's Faculty Early Career Development Program (CAREER) award, which supports early-career researchers and educators. Saibaba overviews his CAREER project, which produced educational materials for high school classrooms, and discusses the CAREER application process.

7 openCARP: Personalized Computational Model of the Heart Examines Cardiac Rhythm

Axel Loewe, Aurel Neic, Gunnar Seemann, Edward Vigmond, and Gernot Plank present the openCARP software package, which uses cardiac bidomain equations to simulate the electrophysiology of the heart. This software marks a significant step forward in cardiac research and may facilitate the development of personalized therapies.

9 A New Biography of a Great Ancient Scientist

Archimedes is widely considered to be the greatest inventor of classical antiquity, having introduced many mathematical concepts that are still in use today. Ernest Davis reviews Nicholas Nicastro's newest book, *Archimedes: Fulcrum of Science*, which explores Archimedes' work as both an engineer and mathematician.

10 Virtual Panel Dives Into Data Science Careers in Industry and Government

As the wide-reaching relevance of data science continues to grow, new opportunities are unfolding for applied mathematicians and computational scientists across many industries. In late 2024, the SIAM Industry Committee organized a virtual panel that explored data science career opportunities outside of academia.

11 Robust Poster Sessions at MDS24 Highlight Diverse Applications of Data Science

The 2024 SIAM Conference on Mathematics of Data Science featured a vibrant five-day sequence of evening poster sessions to encourage broader engagement between attendees. Manuchehr Aminian highlights a random sample of the roughly 550 posters to showcase the variety among the featured research topics.

The Impact of Generative Models and Social Choice Theory in Elections

By Manuchehr Aminian

A generally accepted principle of democracy is that the political institutions that make impactful decisions should adequately represent the will of the people. During an invited talk¹ at the 2024 SIAM Conference on Mathematics of Data Science² (MDS24), which took place last October in Atlanta, Ga., Moon Duchin of Cornell University employed a data science lens to analyze several efforts that seek to preserve and uphold that will within society.

Through a series of vignettes about her research and work as a political consultant, Duchin explored the concepts of *electoral districting* and *social choice theory*. Electoral districting involves the fair definition of geographic boundaries of individual subsets within a state, each of which has elected representatives who express their constituencies' interests in higher-level legislative processes. Social choice theory, in turn, examines the challenges that are associated with making collective decisions about important issues from a list of options (e.g., selecting a mayor from a ballot or passing constitutional amendments). This theory is based on the idea that different voting rules—i.e., algorithms that aggregate many opinions into a single decision—can yield qualitatively different outcomes.

The Mathematics of Elections

Because political representation in the U.S. is both multiscale and heterogenous, the details of political processes are remarkably complex. One prominent issue is *gerrymandering*, which refers to a political

party's manipulation of electoral district boundaries to maintain power or minimize the influence of particular factions (e.g., political or racial groups). Growing interest in historical and current incidences of gerrymandering has inspired a bevy of mathematical models that map two-dimensional shapes to numerical scores of compactness. Although this methodology is mathematically rigorous, it often struggles to translate into political impact. However, Duchin's recent efforts to quantify gerrymandering have successfully contextualized hand-drawn maps; Figure 1 provides an ensemble of graph partitions that she generated based on the adjacency of small political units.

An interrelated source of tension in the U.S. is the plurality voting system, wherein voters select only one option on their ballot per race and the candidate in each category with the largest aggregated proportion of votes ultimately wins. In this system, elections with three or more choices/candidates can cause "vote splitting" among mostly aligned voting groups. In short, options that share certain similarities draw votes away from either other and eventually lead to two-party systems in a phenomenon that is known as *Duverger's law* [3, 5]. Given this suboptimality in U.S. electoral processes, voters frequently express frustration at the misalignment of political strategy with the needs of the population. The study of alternative voting schemes has its own mathematical history that is often steeped in axiomatic statements of system features and "impossibility" theorems.

Sampling and Evaluating Electoral District Maps

A major focus of Duchin's work is the automated generation and evaluation of feasible electoral district maps that rep-

resent consistent geographical regions. A common way to communicate the boundaries of geographic regions is through nontopological data storage formats called *shapefiles*, which describe boundaries with sequences of longitude and latitude coordinates. However, the federated, heterogeneous nature of the U.S. means that data curation and management practices can vary wildly on the county, tract, and census block level within a single state; some data may also be stored in unexpected formats. These inconsistencies manifest as a complicated data synthesis problem. In order to build accurate maps and enforce explainable and trustworthy evaluations of the political ecosystem, researchers must have access to accurate data [2].

Using a consistent dual adjacency graph, Duchin illustrated the objective of Markov-chain-based map generation during her MDS24 talk. Given a desired number of precincts n , the goal is to quickly partition a graph into n disjoint subgraphs—i.e., by labeling every vertex from the choices $\{1, \dots, n\}$ —subject to a list of constraints (e.g., connectedness and legality). The ability to capture feasible districts relies on the performance of iterative processes on these subgraphs. One approach hinges on classical Ising models from statistical physics, wherein "flips" are performed one vertex at a time to slowly evolve from the initial condition. A simplified Ising-based "flip" iteration occurs in two steps. First, one must identify an edge k that connects vertices i and j with different labels l_i and l_j . The next step is to randomly choose a vertex and replace its label with that of the other vertex; modifications can be made to account for the preservation of size or other constraints.

To improve upon the Ising method, Duchin and her collaborators proposed the so-called recombination (ReCom) algorithm (see Figure 2, on page 5) [1]. The ReCom algorithm restructures two adjacent districts/subgraphs I and J that are only constrained by size preservation. The vertices of these subgraphs—associated with labels l_i and l_j —are used to take a "union" of the subgraphs. ReCom then constructs a spanning tree of this new subgraph. Next, one must identify a balanced edge of this spanning tree, i.e., an edge whose removal would produce two (approximately) equal-size subgraphs. The vertices of these new subgraphs are reassigned labels l_i and l_j .

Duchin explained that the ReCom approach is provably fast because the computational time of an iteration is dominated by the computation of spanning trees—a classical graph theoretic problem with runtime $O(E \log(E))$, where E is the number of edges in the subgraph. On both rectangular grids and realistic geometries, several hundred iterations of ReCom can sufficiently generate a well-mixed districting map from an arbitrary initial condition.

After illustrating the concept on a rectangular grid, Duchin applied the fast ReCom algorithm to assess a set of eight traditionally drawn proposed maps for the state of Pennsylvania. She employed a function that assigns a political bias score to a proposed district map and constructed a histogram across an ensemble of ReCom-generated maps, ultimately assisting with the Pennsylvania Supreme Court's selection of a map in 2022 as part of Carter v. Chapman³ [4]. Duchin also served as an expert witness and *amicus curiae* in the Wisconsin Supreme Court in 2021 during Johnson v. Wisconsin Elections Commission⁴ and in the U.S. Supreme

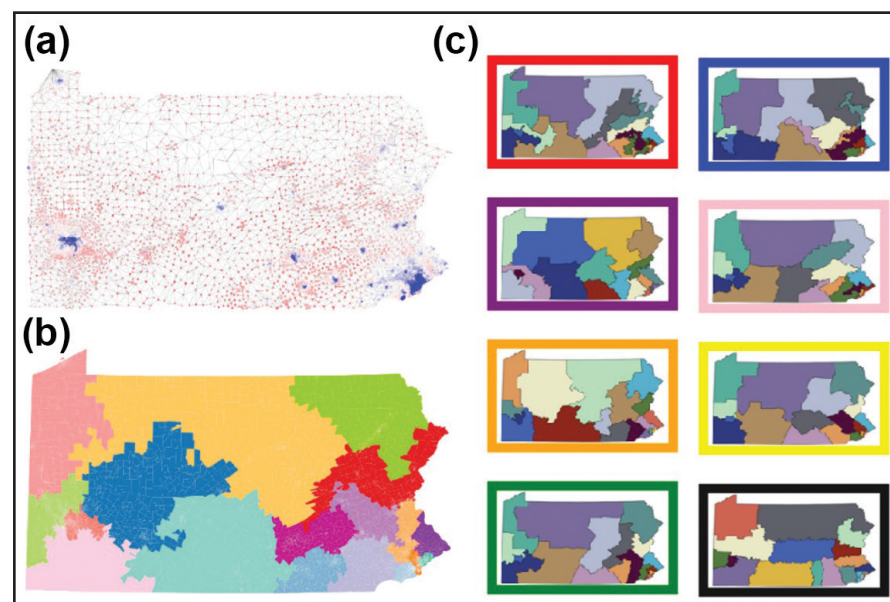


Figure 1. Practical electoral district sampling in the state of Pennsylvania based on Markov chain methods. **1a.** Adjacency graph of census tracts in Pennsylvania. The colors indicate historical voting patterns; blue represents votes for Democratic candidates and red represents votes for Republican candidates. **1b-1c.** Electoral maps that were generated from 1a with the recombination (ReCom) algorithm, which does not use past voting trends to influence districting. Figure courtesy of Moon Duchin.

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³ <https://thearp.org/litigation/carter-v-chapman>

⁴ <https://thearp.org/litigation/johnson-v-wisconsin-elections-commission>

Data Science 4 Everyone Seeks to Revitalize K-12 Mathematics Education

By Mahmoud Harding

K-12 mathematics educators are currently charting a new course to redefine the purpose and methodology of their lesson plans. Modern workforce demands—coupled with a push for improved student understanding of math’s utility in real-world situations—are actively shaping discussions about the teaching and assessment of mathematical concepts. At the same time, educators continue to emphasize the importance of interdisciplinary, project-based classroom activities that incorporate science, technology, and mathematics. In today’s world, these efforts are often driven by the widespread importance of *data*, which plays a major role in decision-making and innovation.

Given data’s profound impact on many areas of industry and academic research, the path forward is clear: *the integration of data science into K-12 classrooms will equip all students with the necessary tools to thrive in an increasingly data-driven world*. This vision aligns with the mission of Data Science 4 Everyone¹ (DS4E)—to catalyze the adoption of data science as an integrated component of K-12 education by 2030—and is echoed in DS4E’s *State of the Field: Data Science and Data Literacy Education in US K-12* report [5].

Since its inception, DS4E has worked to raise awareness of the importance of data in K-12 education. The idea for a national coalition stemmed from a 2019 episode of the podcast *Freakonomics*

¹ <https://www.datascience4everyone.org>

Radio,² in which Steven Levitt (co-author of the book *Freakonomics*) remarked, “I believe that we owe it to our children to prepare them for the world that they will encounter — a world driven by data” [1]. Inspired by this conviction, Levitt and a group of advocates came together to officially form DS4E in the summer of 2021. From the beginning, the coalition has sought to transform K-12 education in data science and mathematics by advising states on standards, guiding districts in implementation, and providing professional development and collaborative programs for both teachers and students.

Mathematics teachers often seek to contextualize their curricula and demonstrate the existing connections between math and other fields. While past efforts to reimagine mathematics education primarily focused on content, today’s challenges and opportunities present a fundamentally different landscape. The rise of data and artificial intelligence (AI) demands a transformative approach that emphasizes *what* students learn and *how* they learn it. The integration of data and computing tools into the learning process extends instruction beyond traditional math concepts and helps students navigate and leverage the technologies that shape the modern world.

For example, students now have access to free, web-based tools (such as data analysis software) that redefine their exploration of the connections between data and mathematics. These resources should comprise a foundational component of mathematics

² <https://freakonomics.com/podcast/americas-math-curriculum-doesnt-add-up-ep-391>

instruction to strengthen students’ data intuition and expose them to the many applications of math. The inclusion of *data literacy* as an integral component of K-12 education is a pillar of DS4E’s mission. The coalition believes that “to be successful in a world already defined by data, students must begin learning and developing the fundamentals of data science—data intuition and data literacy—in primary and secondary school, regardless of whether they go on to college or major in data science” [5].

Recent research from the Burning Glass Institute³ and ExcelinEd⁴ revealed that roughly 22 percent of U.S. job postings

³ <https://www.burningglassinstitute.org>

⁴ <https://excelined.org>

now require at least one skill that pertains to “getting, exploring, and analyzing data” [4]. While traditional job titles like *computer scientist*, *engineer*, or *business analyst* may immediately come to mind, a broader trend is actually at play; at least 30 percent of job listings in industries such as utilities, manufacturing, agriculture, and forestry now call for one or more data science skill. This major shift in the job market means that data science expertise is no longer exclusive to data scientists and instead extends to a wide array of professions.

To address this growing need, DS4E is working to ensure that all K-12 classrooms have multiple opportunities to engage with

See **Data Science 4 Everyone** on page 6



Author Mahmoud Harding of Data Science 4 Everyone (DS4E) speaks about the importance of data science education at the 2024 iteration of SXSW EDU: an annual event in Austin, Texas, that is hosted by South by Southwest as part of its collection of conferences and film festivals that are dedicated to different industries (including education). Photo courtesy of DS4E.

Extreme Weather

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with various sinks and dissipation mechanisms. As a result, the effective randomness in atmospheric conditions follows a non-Gaussian distribution, where extreme events occur as fluctuations at the far end of the probability tail.

One relevant approach models the velocity field in a fluid as $u(t) = \langle u \rangle + \sum_l \delta u_l(t)$, where the field consists of a mean value with time-dependent fluctuations $\delta u_l(t)$ on each scale l . Though this field is written in one dimension, it is easily generalizable to higher dimensions [2]. The specific manifestation of the fluctuations can include various forms of turbulence, such as the intermittent effects that are present in real-world phenomena. Multifractal analysis can help to describe this behavior. Rather than a fixed number, the fractal dimension is a continuous function called the *singularity spectrum*: $\langle \delta u_l^q \rangle \propto l^{\zeta(q)}$ for $q \in \mathbb{R}$. Here, $\zeta(q)$ is a function that characterizes the possible scaling factors from the Navier-Stokes equations in fluid dynamics.

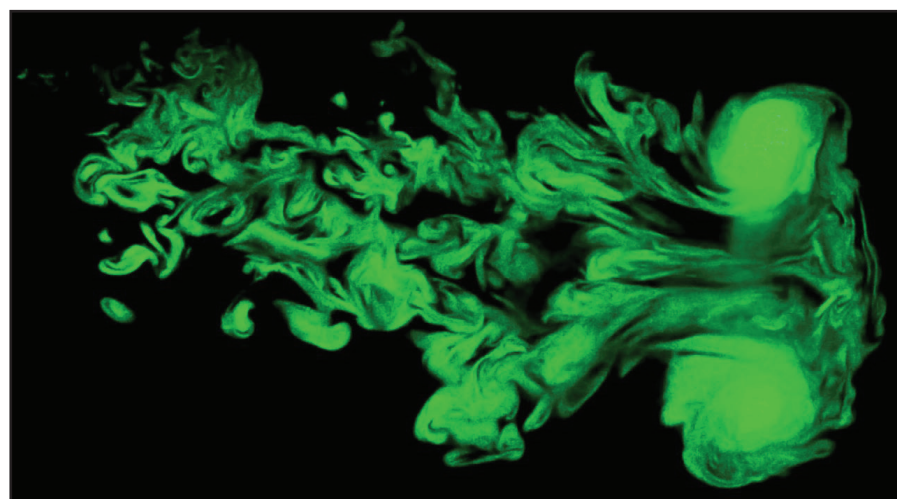


Figure 2. A laser illuminates turbulence in water vapor from a fog machine, showing the self-similarity in vortices from large scales to microscopic distances. Models must account for atmospheric turbulence to characterize the drivers of extreme weather events. Figure courtesy of Jeffrey Pilkington and Alex Meyer.

Making Scientific Analogies

A lack of data frequently complicates the attribution of specific weather events to climate change, as such extremes have either never occurred in recorded history or only began occurring recently. Messori and his colleagues cited the massive deadly heat waves in Europe in 2003 and western North America in 2021 as examples of unprecedented historical events that enter the record, providing information about the conditions that allow them to gradually become part of recent history.

“Would the same weather conditions still give as strong a heat wave in a 1.6° colder climate?” Messori asked, referencing a new Copernicus Climate Change Service report that identified a 1.6° Celsius average increase in global temperature in 2024 compared to pre-industrial conditions [1]. “If not, then we can say that based on our analog analysis, this heat wave was X degrees warmer than it would otherwise have been without global warming.”

The particular example of heat waves is especially important given their deadliness, association with wildfire risk, and heightened stress on infrastructure like roads and railroad tracks. However, French meteo-

rologists Pascal Yiou of the Laboratoire des Sciences du Climat et de l’Environnement and Aglaé Jézéquel of the Laboratoire de Météorologie Dynamique previously noted that many standard climate models only include summers that align with global trends, rather than exceptionally hot seasons [4]. Yiou and Jézéquel thus simulated heat waves that might occur once every 1,000 years under current conditions, aiming to understand the specific atmospheric physics that drive such incidents.

The *analog method* allows scientists to investigate the sufficiency of existing data or models—based on known extreme episodes—to predict events that have not yet occurred. Researchers can identify the conditions that give rise to extreme weather within a climate model, then rerun the model with perturbations around those conditions — thereby producing phenomena that the model might not otherwise predict.

The heart of Yiou and Jézéquel’s analog model is a *stochastic weather generator*, which the duo tested by simulating heat waves in France. The measurable quantity was the average temperature over a 15-day period, which was the output of a Markov chain generator; atmospheric circulation—such as anticyclonic action—provided the transition probabilities. Since this circulation is not directly observable, one can use analogs of unavailable data that do or do not incorporate knowledge of specific heat waves to tweak the probabilities. “Are there many ways in which the atmosphere can get into the particular state that created this extreme weather?” Messori asked. “Or are there very few ways to reach that state? Did the persistence of the system play a role in engendering the extreme event?”

Don’t Know Why There’s No Sun up in the Sky

Many existing methods that evaluate extreme events were initially developed for other scientific fields, including astrophysics and theoretical fluid dynamics [2]. In fact, the overall methodology encompasses

other rare geophysical incidents — such as geomagnetic storms due to solar activity, which are obviously not under human control but can still dramatically impact society. Messori and his collaborators even reference the Lorenz “butterfly” attractor, which was originally formulated as an idealized weather model but nonetheless serves as a toy framework for the physical underpinnings of rare extreme events.

However, the underlying complexity of the climate and other geophysical systems means that even sophisticated, cutting-edge methods require continued refinement. “A number of the approaches that we present assume a stationary or quasi-stationary system, so several of them are not designed to say something about rapid changes,” Messori said. “As long as you have a slow drift, they’ll still do their job.”

Nevertheless, Messori was quick to note that these approaches are neither toys nor abstract theory. “We’re not developing tools for the sake of developing tools,” he said. “The idea is to probe a complex, chaotic system, such as the Earth.”

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Energy Harvesting

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of these systems' global dynamics and bifurcations remains a complex challenge. Classical global stability methods for smooth dynamical systems often have limited applicability to nonsmooth systems [2-4]. More broadly, global stability methods for nonautonomous, nonsmooth systems are still relatively rare.

Here, we present a novel computer-assisted approach [1] that permits us to study the global dynamics of a vibro-impact pair. Our system consists of an externally forced capsule and an internal freely moving ball; neglecting the friction between the ball and capsule, the ball's motion is driven by both gravity and its impacts with the membranes at the capsule's ends (see Figure 1b, on page 1). The capsule itself is driven by an external harmonic excitation $\hat{F}(\omega\tau + \psi)$ with period $2\pi/\omega$. Using Newton's second law of motion, the dynamics between impacts are

$$\frac{d^2 X}{d\tau^2} = \frac{\hat{F}(\omega\tau + \psi)}{M},$$

$$\frac{d^2 x}{d\tau^2} = -g \sin \beta,$$

where $X(\tau)$, M and $x(\tau)$, m are the absolute displacement and mass for the capsule and ball, respectively. If $M \gg m$ and the impact time is negligible compared to the other time scales in the model, we can conveniently introduce the following two elements: (i) a dimensionless relative displacement $Z(t) = X(t) - x(t)$ that is scaled by a normalizing factor, and (ii) a relative velocity $\dot{Z}(t)$ between the capsule and ball. Doing so allows us to focus on the system dynamics as a whole, rather than the separate motions of the ball and capsule.

The instantaneous impact condition is then

$$Z_j = \pm \frac{d}{2}, \quad \dot{Z}_j^+ = -r\dot{Z}_j^-,$$

where the superscripts $-$ and $+$ respectively signify the state of the ball before and after impact [5]. The subscript j indicates the impact's index and the parameter r is the restitution coefficient: a quantitative measure of the membrane's elasticity. The parameter d is the normalized dimensionless capsule length in terms of the amplitude and frequency of forcing, dimensional capsule length, and capsule mass; it thus

contains several important features of the energy harvesting system.

By combining the impact conditions with the equations of motion between impacts, we can utilize exact nonsmooth maps for impact velocity \dot{Z}_k and impact phase ψ_k to capture the dynamics [5]. These maps are coupled transcendental equations that prevent explicit global stability of the system dynamics. Nevertheless, the nonsmooth dynamics suggest that short sequences of returns to the bottom of the capsule ∂B define the building blocks for return maps. The return maps represent distinct families of behaviors whose compositions yield any transient or attracting behavior, providing important characterizations of the states that translate into different levels of energy harvesting.

Our return maps do not align with standard map choices, such as Poincaré, stroboscopic, all impacts, or all returns to a particular state. Instead, they are based on the varied sequences of impacts that do or do not occur before the system returns to a specific impact state. This innovative perspective efficiently partitions the state space into a small number of regions, from which we can easily identify attracting and transient behavior and subsequently determine the anticipated energy harvesting sequences. For example, Figure 1c (on page 1) illustrates the division between sequences $\partial B \rightarrow \partial B$ and $\partial B \rightarrow \partial T \rightarrow \partial B$ that yields a small number of surfaces for \dot{Z}_{k+1} and ψ_{k+1} in terms of \dot{Z}_k and ψ_k (see Figure 1d, on page 1). These surfaces divide the state space into a small number of regions with distinguishing features that are useful for global stability results.

We approximate the surfaces that define the exact return maps for \dot{Z}_k and ψ_k in each region of the state space to construct a piecewise smooth composite map for approximate velocity v_k and approximate relative impact phase ϕ_k (see Figure 2a). This framework is highly computationally efficient because it reduces the primary task to the construction of return maps for short-time realizations of impact pairs across the space of initial conditions, which is divided into a small set of dynamical building blocks. The method seeks to calculate a single return that consists of only a few impacts, which significantly reduces computational demand when compared to approaches like basin of attraction calculations or cell mapping. Our framework hence provides an efficient alternative to traditional longtime simulations for the derivation of flow-defined Poincaré

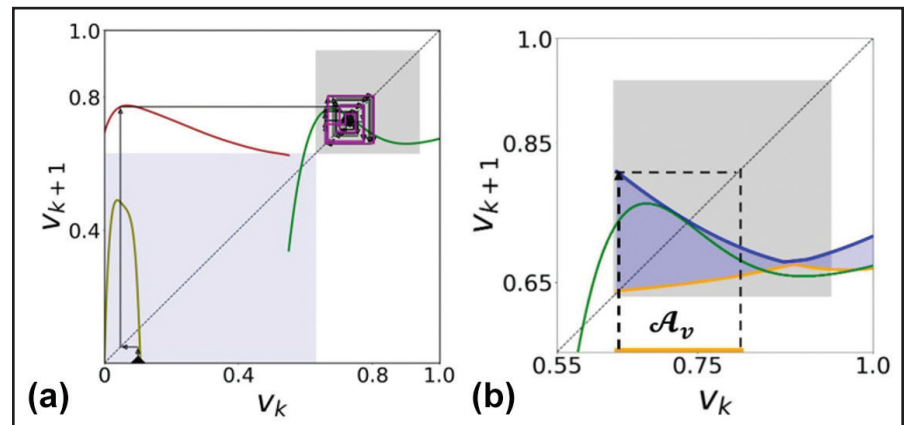


Figure 2. Approximation of the exact two-dimensional (2D) return map and its reduction to a one-dimensional (1D) auxiliary map for the analysis of vibro-impact chaotic dynamics. **2a.** The 2D map is projected onto the phase plane of the approximate velocity v_k . The curves represent maps for distinct state space regions in Figure 1d (on page 1), and the dark solid line indicates a typical trajectory that reaches an attracting domain of the chaotic attractor. **2b.** The 1D auxiliary map is derived from the 2D return maps using bounds for a potentially attracting region (in gray). Figure adapted from [1].

maps and analysis of global dynamics in limit cycle or chaotic systems.

The form of these composite maps facilitates cobweb analysis, which characterizes the nonsmooth vibro-impact dynamics via simple return maps for impacts on one end of the capsule (rather than via the full compositions of map sequences, which are not return maps). Cobweb analysis identifies attracting regions in the phase planes to yield the range of values of the relative impact velocity \dot{Z}_k , on which the net energy output relies. Figure 2a demonstrates the ability of cobweb analysis to capture this range; the shaded regions depict two-dimensional (2D) return maps that cannot be visualized in the projection. But for some strongly transient regions, the 2D return map dynamics are captured by a “separable” approximation: a pair of one-dimensional (1D) return maps that separate the dynamics of v_k and ϕ_k . Solid curves in the figure represent the 1D return maps, which facilitate transparency for the overall dynamics.

Cobweb analysis motivates a valuable definition of auxiliary maps on the regions for different building block behaviors. For regions with attracting dynamics, the auxiliary map is conservatively based on extreme bounds (e.g., the gray shaded region in Figure 2a) and can thus bound the attracting domain (see Figure 2b) that contains all of the system's nontrivial attractors.

The auxiliary maps simplify the 2D return maps into a set of 1D equations that separately track the dynamics of approximate velocity v_k and phase ϕ_k . Auxiliary maps also capture the worst-case scenario and provide conservative bounds on the maximum and minimum range of v_k and ϕ_k at each iteration. Figure 2b illustrates the worst-case scenario of cobwebbing (dashed line) and shows the maximum and minimum values of v_k after one iteration of the 1D map. This action defines the bounds for the size of the attracting domain in v_k (yellow interval \mathcal{A}_v), which directly relates to the energy output of the chaotic regime in Figure 2a.

Our computer-assisted method reduces nonsmooth, impacting systems into a composite piecewise smooth map that offers a robust framework for the study of global dynamics. Here, we focus on parameter regions that are associated with energetically favorable states in vibro-impact energy harvesting systems. Since our approach employs generic return maps that comprise short sequences of impacts that decompose the full dynamics, it is adaptable to a wide range of nonsmooth systems.

The versatility of our technique enables its application to realistic external environments and makes it a valuable tool for future exploration. Ongoing research will refine these theoretical frameworks and methodologies to deepen our collective understanding of vibro-impact dynamics and ultimately develop energy harvesting solutions that are resilient to realistic and stochastic external conditions.

This article is based on [1] and Lanjing Bao's minisymposium presentation¹ at the 2023 SIAM Conference on Applications of

¹ https://meetings.siam.org/sess/dsp_talk.cfm?p=126649

Dynamical Systems² (DS23), which took place in Portland, Ore., in May 2023. Bao received funding to attend DS23 through a SIAM Student Travel Award.³

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Lanjing Bao is a Ph.D. student in applied mathematics and bioinformatics at Georgia State University. Rachel Kuske is a professor of mathematics at the Georgia Institute of Technology and a SIAM Fellow. Daniil Yurchenko is an associate professor in the Institute of Sound and Vibration Research at the University of Southampton in the U.K. Igor Belykh is a Distinguished University Professor of Applied Mathematics at Georgia State University.

² <https://www.siam.org/conferences-events/past-event-archive/ds23>

³ <https://www.siam.org/conferences-events/conference-support/travel-and-registration-support>



Call for Nominations for the Ostrowski Prize 2025

The aim of the Ostrowski Foundation is to promote mathematical sciences. Every second year it provides a prize for recent outstanding achievements in pure mathematics and in the foundations of numerical mathematics. The value of the prize for 2025 is 100,000 Swiss francs.

The prize has been awarded every two years since 1989. The most recent winners are Peter Scholze in 2015, Akshay Venkatesh in 2017, Assaf Naor in 2019, Tim Austin in 2021, and Jacob Tsimmerman in 2023.

See www.ostrowski.ch/index_e.php for the complete list and further details.

The jury invites nominations for candidates for the 2025 Ostrowski Prize. Nominations should include a CV of the candidate, a letter of nomination and 2-3 letters of reference.

The Chair of the jury for 2025 is Mikael Rørdam, University of Copenhagen, Denmark. Nominations should be sent to ostrowski@math.ku.dk by May 31st, 2025.

Want to Place a Professional Opportunity Ad or Announcement in SIAM News?

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Introducing SIAM's New "Industry Initiatives" Webpage

By Sharon Arroyo and Dirk Hartmann

The SIAM Industry Committee¹ was established in 2012 to increase the participation of industry professionals in SIAM activities, improve SIAM services for industry members, and provide the community with information about careers in industry.² SIAM staff routinely work with the Industry Committee to facilitate interactions between industry and academia. The Committee thereby embraces an inclusive view of the industry space, fostering valuable relationships between for-profit and non-profit institutions, government organizations, and research centers. The Industry Committee is now excited to share the new "Industry Initiatives" webpage³ on the SIAM website, which emphasizes the "I" in SIAM.

¹ <https://www.siam.org/get-involved/connect-with-a-community/committees/industry-committee>

² See page 10 for an article about a virtual data science career panel that the SIAM Industry Committee organized in 2024.

³ <https://www.siam.org/programs-initiatives/industry>

The industry webpage features organizational spotlights that overview the work of SIAM members at prominent companies like The Boeing Company⁴ and Merck;⁵ describe sample projects that utilize applied mathematics, computational science, and/or data science; outline skills that contribute to successful careers in the mathematical sciences; and articulate SIAM's ongoing efforts to provide value to these organizations. These brief articles are especially valuable for students, early-career researchers, job seekers, and anyone who is considering a career change and wants to familiarize themselves with the role of applied mathematics in industry.

The page also includes personal success stories—in both video and written formats—from SIAM members who work outside of academia. In these testimonies, members discuss their career paths, daily schedules, and expectations; share useful knowledge

⁴ <https://www.siam.org/publications/siam-news/articles/spotlight-on-the-boeing-math-group>

⁵ <https://www.siam.org/publications/siam-news/articles/mathematics-and-merck-an-impactful-collaboration>



During the 2024 SIAM Annual Meeting, which took place last July in Spokane, Wash., the SIAM Industry Committee hosted a panel that explored careers outside of academia. From left to right: panelists John-Paul Sabino (The Boeing Company), Amanda Howard (Pacific Northwest National Laboratory), Gwen Spencer (Netflix), Nicole Jackson (Sandia National Laboratories), Wotao Yin (Alibaba Group/Academy for Discovery, Adventure, Momentum and Outlook), and moderator Sharon Arroyo (The Boeing Company). SIAM photo.

and skills; offer advice for other applied mathematicians; and detail applied and industrial mathematics' continued contributions to society. Other personal stories are available in the SIAM Careers Brochure.⁶

Want to learn more about the work of applied mathematicians in industry, as well as industrial innovations that are made possible by applied mathematics? We actively post a curated collection of industry-related publications, conference presentations, and software that showcases industrial applications in a range of contexts. Items in this collection include a synopsis of the work and its impact, as well as information about the authors and their organizations. **Please feel free to submit ideas⁷ for consideration.**

Finally, the "Industry Initiatives" webpage also provides an up-to-date summary of additional Industry Committee activities,

⁶ <https://go.siam.org/aa344e>

⁷ <https://www.surveymonkey.com/r/BVHSLZW>

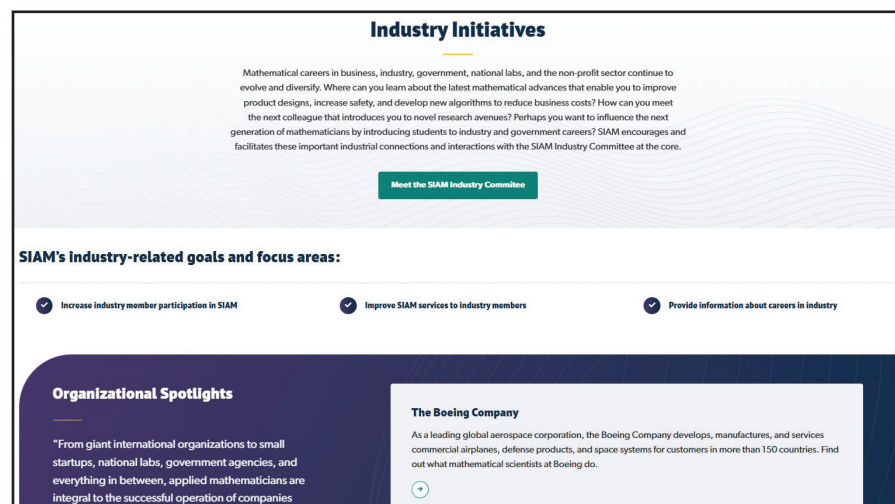
such as the new SIAM Industry Prize,⁸ impactful contributions within the mathematical sciences, SIAM's ongoing collaboration with the BIG Math Network,⁹ and regular career panels to support students who are pursuing employment in industry. This site is a great way for SIAM members to read about industry happenings and identify opportunities for involvement.

We are excited that the SIAM community now has a place to go to learn more about SIAM's industry initiatives and the important work of the Industry Committee. Let's drive the "I" in SIAM together.

Sharon Arroyo is a Senior Technical Fellow at The Boeing Company. Dirk Hartmann is a Siemens Technical Fellow at Siemens Digital Industries Software.

⁸ <https://www.siam.org/programs-initiatives/prizes-awards/major-prizes-lectures/siam-industry-prize>

⁹ <https://bigmathnetwork.org>



The new "Industry Initiatives" webpage on the SIAM website features testimonies from SIAM members who work outside of academia, as well as article spotlights that highlight the research of individuals at various companies, detail sample industry projects, outline skills that contribute to successful industry-based careers in the mathematical sciences, and describe how organizations can benefit from SIAM's programming and resources.

Elections

Continued from page 2

Court cases *Rucho v. Common Cause*⁵ in 2019 and *Allen v. Milligan*⁶ in 2022.

Although this work was inspired by specific real-world needs, Duchin noted that the field of mathematical political science continues to evoke many new questions about Markov chain sampling and graph partitioning. It also highlights broad, important challenges in network theory and data science that pertain to topics like clustering and community detection.

Social Choice Theory and Voting

The second part of Duchin's MDS24 presentation focused on social choice theory and voting theory, namely the ways in which a collection of people can arrive at a decision from a list of possible choices.

Anyone who teaches an undergraduate math course with a module on voting theory is likely familiar with Arrow's impossibility theorem, which rules out all possible voting systems that satisfy a particular set of axioms. When asking for unanimity and independence properties, Arrow's theorem suggests that the only valid system is a dictatorship. Disallowing dictatorships hence leads to the "impossibility" in social choice theory of requiring a voting system to preserve all axioms.

Beyond plurality vote, experts have proposed many systems that may provably satisfy various properties. One of these alternatives—and perhaps the most famous—is *ranked choice voting*. Duchin explored this framework in the context of

the 2021 New York City mayoral campaign, during which ranked choice voting led to the election of Eric Adams (though Adams was also the plurality leader in all rounds of voting). New York City's ranked choice voting methodology is an iterative process wherein voters rank the available choices and submit their full listings. Based on the aggregate tally of voters' first choices, the lowest ranked choice is eliminated and votes for that least popular choice are automatically reallocated to the remaining choices according to voters' rankings. The process repeats until only one winning candidate is left.

Duchin also overviewed a handful of alternate schemes with surprisingly nice properties, including *randomly selected dictator*. Another interesting scheme is *plurality veto*: a subtractive system that tallies "positive" votes in a first round, then iteratively subtracts points from candidates based on negative preferences until a single option with a nonzero tally remains.

Finally, Duchin shared her perspective on the notion of axioms in social choice theory. A recent axiom of note is *proportionality*, which asserts that the selection of preferences should somehow reflect voters' intersectional makeup. Additionally, Duchin mentioned that she would like to see the field of mathematical political science move away from the rigid demands of binary properties—as in Arrow's axioms—and respond more in terms of *tendencies*, i.e., the *typical* behaviors of these axioms (such as those in Arrow's framework, or more modern axioms like proportionality) rather than worst-case behaviors.

Applied mathematicians clearly have important roles to play as experts, modelers, and evaluators of existing political systems. Duchin repeatedly emphasized the bidi-

rectional nature of this work; fundamental research contributes to practical algorithms and impacts society while simultaneously revealing open questions in fields such as graph theory, numerical analysis, and mathematical data science. This type of two-way relationship comprises the heart of applied mathematics as a discipline. We look forward to future contributions from Duchin and other researchers as they continue to pursue fair political representation and effect positive societal change.

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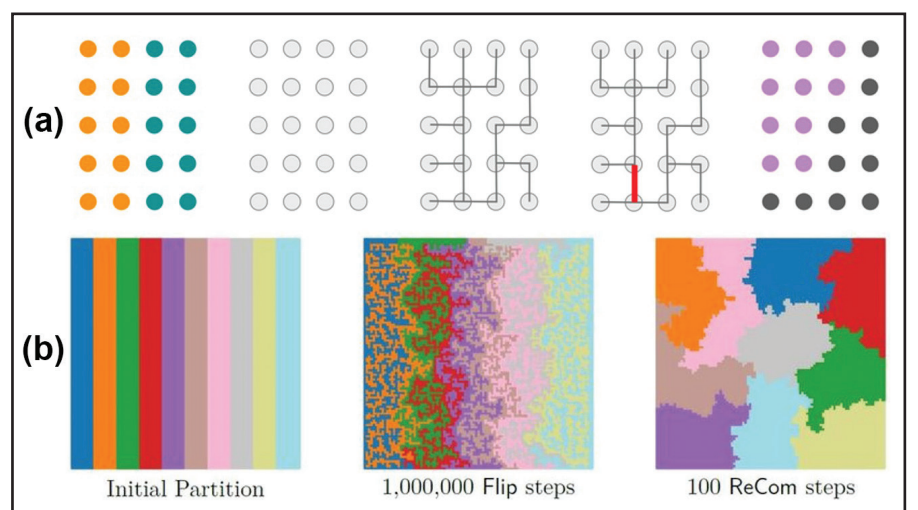


Figure 2. The recombination (ReCom) algorithm greatly accelerates sampling from an ordered initial state. **2a.** One step of the ReCom algorithm based on the balanced splitting of a spanning tree. Starting from the initial groups (orange and teal), the final groups (purple and gray) are the result of merging the two, constructing a spanning tree of the subgraph, and splitting on a balanced edge. **2b.** Acceleration of the ReCom algorithm over traditional "flip" steps for a graph with 10 subgroups. Figure adapted from [1].

⁵ https://www.supremecourt.gov/opinions/18pdf/18-422_9o11.pdf

⁶ https://www.supremecourt.gov/opinions/22pdf/21-1086_1co6.pdf

The Outreach Component of My NSF CAREER Award

By Arvind K. Saibaba

In 2019, I was fortunate to receive the Faculty Early Career Development Program¹ (CAREER) award from the U.S. National Science Foundation (NSF) after applying for the very first time. According to the NSF, this award “supports early-career faculty who have the potential to serve as academic role models in research and education.” It is my understanding that the integration of research and education is essential. For the educational component of my CAREER project, I thus proposed a collaboration with high school teachers to develop educational materials for their classrooms.² The goal of this article is twofold: In the first part, I will talk about how we created the teaching materials, and in the second part, I will share insights about the CAREER award application process.

¹ <https://new.nsf.gov/funding/opportunities/career-faculty-early-career-development-program>

² https://www.nsf.gov/awardsearch/showAward?AWD_ID=1845406&HistoricalAwards=false

Data Science 4 Everyone

Continued from page 3

data across various subjects. In mathematics education, this goal includes finding a better balance between teaching procedural processes and preparing students to use data, math, and computing to tackle complex problems. We must leverage data to connect mathematics with science and technology and ensure that students gain practical, applicable knowledge.

One of DS4E’s most impactful initiatives is the After the AP Data Science Challenge,⁵ which we launched in partnership with Skew The Script,⁶ CourseKata,⁷ and North Carolina State University’s Data Science and AI Academy.⁸ This program introduces Advanced Placement (AP) Statistics and AP Computer Science students to the fundamentals of data science with a project that analyzes data from the U.S. Department of Education’s College Scorecard.⁹ It begins with a professional development opportunity for teachers, who learn to program in R and gain a foundational understanding of exploratory data analysis and machine learning. Once the instructors are prepared, their students participate in the challenge after completing their AP exams. In its 2023 pilot year, 2,500 students took part in the competition. Based on participant feedback, we refined and expanded the program to attract more than 9,000 students in 2024.

⁵ <https://www.datascience4everyone.org/post/second-year-surge-after-the-ap-data-science-challenge-captivates-9000-students>

⁶ <https://skewthescript.org>

⁷ <https://coursekata.org>

⁸ <https://datascienceacademy.ncsu.edu>

⁹ <https://collegescorecard.ed.gov>

Developing the Teaching Materials

Every mathematics instructor encounters students who ask, “When will I use this in real life?” Although I was relatively good at mathematics while growing up in Bangalore (now Bengaluru), India, I wasn’t particularly drawn to the subject until college. I discovered applied math as an engineering major, which changed my career trajectory. Specifically, it led me to graduate school and eventually to an academic career in mathematics. Given my own path, I wanted to provide that same kind of exposure to other students at an even earlier stage of their schooling.³

I chose to work with high school teachers in my CAREER project for several reasons.⁴ First, the materials that we developed would ultimately reach more people because teachers interact with many

³ To the younger version of me, “Hang in there and good luck!”

⁴ The real reason is that after watching teen flicks like *The Karate Kid* and *Mean Girls*, I am terrified of high school students.

Until recently, data science in K-12 education has mostly been overlooked. A National Academies report estimated that only six percent of U.S. high schools offered data science programs in 2023, in which just 0.1 percent of teachers and 3.6 percent of students participated [2]. But due in part to ongoing efforts by DS4E, the number of states that offer some form of data science education has grown from only two states in 2019 to 29 in 2024. During the 2023-2024 academic year, 277 schools and 104 districts in the U.S. included data science in their curricula [5].

Despite this progress, the current state of data science education still fails to meet the requirements of most employers. Recognizing the need to close this gap, the National Council of Teachers of Mathematics¹⁰ (NCTM) recently published a formal position that “data science content should be available to all students in order to complete their high school mathematics graduation requirement” [3]. K-12 mathematics instruction should no longer teach concepts solely through memorization and procedural steps; instead, it must integrate computer software technology and data science principles to highlight mathematics’ important role in problem solving. To support state- and district-level entities as they navigate the shifting educational landscape, DS4E has partnered with groups in Utah, New York, and North Carolina. For instance, the coalition collaborated with the aforementioned Data Science and AI Academy to host the NC Data Science Education Summit¹¹ in the fall of 2023 and

¹⁰ <https://www.nctm.org>

¹¹ <https://datascienceacademy.ncsu.edu/programs/k-12-outreach/ds-edu-summit>

students throughout their careers. Second, the teachers could use their pedagogical experience to help me explain the mathematical concepts at an appropriate level and enhance the overall delivery.

I developed the materials over the course of four summers between 2020 and 2024 by working intensely with a small group

CAREERS IN MATHEMATICAL SCIENCES

of high school teachers during two-week workshops each year. For the first two years, I advertised the opportunity widely throughout my network and conducted phone interviews to identify

possible candidates. I specifically chose teachers with experience in both curriculum development and classroom instruction, and I couldn’t have asked for a better group of educators in the first year. They were full of enthusiasm and ideas, and we ultimately developed three modules that focus on image processing, X-ray imaging (see Figure 1, on page 7), and the mathematics of ranking. Each self-contained module includes an introduction to linear algebra, comprises lesson plans and ideas for imple-

mentation, and identifies the North Carolina (NC) educational standards that it addresses.

In the span of five years, I worked with eight teachers from a variety of schools in NC. The schools ranged geographically (both urban and rural) over six counties and catered to students with different backgrounds. We built on the linear algebra foundations that we developed in the first year and created modules on social networks, the spread of diseases, and cryptography. We invited several high school teachers from local districts to NC State University for a two-day workshop in 2022 to share our materials with them. In 2024, we recorded videos that offer a brief introduction to the materials, and we plan to present this work at teacher conferences in the near future.

As a numerical analyst, I tried to de-emphasize the role of inverses in the content that we developed. One can always solve a linear system—when a solution exists—with Gaussian elimination rather than by computing an inverse, since an inverse may not always exist. And even if the matrix is invertible, it may be twice

See NSF CAREER Award on page 7

2024. This event emphasized the importance of data science in early education and aimed to position North Carolina as a leader in the transformation of data science education.

As of fall 2023, the NC Department of Public Instruction¹² did not offer any data science courses. In response to this dearth, outreach organizations at NC State—with support from DS4E—led the development of course outcomes for a revised Introduction to Data Science course that falls within the Department’s Career and Technical Education¹³ sector. This course can now fulfill the computer science credit requirement for high school graduation in North Carolina, thanks to continued support from NC State and DS4E to train teachers, plan curricula, and secure access to technology. Partnerships such as these accelerate progress and generate sustained and impactful change.

Mathematics education should foster data intuition in the early grades, build data literacy in middle school, and integrate data science competencies across all high school subject areas. To achieve this goal, DS4E is working with students, educators, academics, and professionals to create a national learning progression for K-12 data science and data literacy.¹⁴ In June 2024, more than 100 education leaders gathered at the University of Chicago to further this work, building on five previous months of collaboration among 11 focus groups and organizations like the NCTM, National Science Teaching Association,¹⁵ National Council for the Social Studies,¹⁶ and Computer Science Teachers Association.¹⁷ The discussions at this convening prioritized key data education outcomes that students should meet by high school graduation.

One critical aspect of this event was the K-12 Data Science and Literacy Learning Progressions Writing Sprint, during which a group of educators drafted an outline that constitutes the foundation for the first version of the learning progression (set to be released later this year). We hope that it will serve as a resource for states and districts as they develop or revise the standards and assessments of mathematics and data science courses. Additionally, we encourage other disciplines to use the progression to incorporate data into their own curricula. In

¹² <https://www.dpi.nc.gov>

¹³ <https://www.dpi.nc.gov/districts-schools/classroom-resources/career-and-technical-education>

¹⁴ <https://www.datascience4everyone.org/post/building-a-national-learning-progression-for-k-12-data-science-education>

¹⁵ <https://www.nsta.org>

¹⁶ <https://www.socialstudies.org>

¹⁷ <https://csteachers.org>

fact, the progression could even facilitate the design of training programs in pre-service teacher procedures and in-service professional development initiatives.

As the world grows increasingly reliant on data, all students must develop data literacy and data science skills. Beyond success in higher education, such skills will help students access diverse career opportunities and become informed, contributing members of society.

Mahmoud Harding delivered an invited presentation¹⁸ on this topic at the 2024 SIAM Conference on Applied Mathematics Education,¹⁹ which took place in Spokane, Wash., last July in conjunction with the 2024 SIAM Annual Meeting.²⁰

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Mahmoud Harding is the Instructional Design Director of Data Science 4 Everyone and a former math and data science teacher at the North Carolina School of Science and Mathematics.

¹⁸ https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=80057

¹⁹ <https://www.siam.org/conferences-events/past-event-archive/ed24>

²⁰ <https://www.siam.org/conferences-events/past-event-archive/an24>



College students engage in discussion during a 2024 Data Science 4 Everyone (DS4E) focus group event that aimed to develop the first-ever K-12 data science and data literacy learning progression. The first version of this progression will be released in 2025. Photo courtesy of DS4E.

openCARP: Personalized Computational Model of the Heart Examines Cardiac Rhythm

By Axel Loewe, Aurel Neic,
Gunnar Seemann, Edward
Vigmond, and Gernot Plank

What makes your heart beat? The answer may lie in advanced computational models that provide a deeper understanding of the fundamental mechanisms that drive the heartbeat or cause it to go awry — ultimately facilitating the development of personalized therapies to treat heart diseases. The openCARP software package,¹ which uses cardiac bidomain equations to simulate the electrophysiology of the heart, marks a significant step forward in this important research area [6].

Atrial fibrillation is a heart rhythm disorder that originates in the atria (the heart's upper chambers) and causes the heart to beat rapidly and irregularly. According to the Heart Rhythm Society,² nearly 40 million people around the world experience

¹ <https://opencarp.org>

² <https://www.hrsonline.org/afib-awareness-month-2023>

this common condition, which is linked to poor sleep and daytime fatigue. Treatment is crucial to prevent stroke, heart failure, and other detrimental consequences, and tailored computer simulations can help identify optimal treatment pathways. For instance, individuals with atrial fibrillation can undergo a magnetic resonance imaging scan that enables the creation of a three-dimensional model of their heart. Specialists are then able to incorporate additional electrocardiogram (ECG) measures and adjust this computational model to reflect a patient's unique heart function — eventually yielding a

digital twin that allows cardiologists to identify precise targets for treatment procedures like ablation, which creates scar tissue in the heart that blocks the defective electrical signals that cause atrial fibrillation. Medical practitioners can also computationally simulate ablation to visualize a patient's risk of developing new heart rhythm disorders [2]. If such risks are indeed detected, they adjust

the ablation targets until an optimal therapy is reached. This innovative approach promises to enhance the precision and effectiveness of treatments for heart rhythm disorders, potentially improving outcomes and reducing risks for patients worldwide.

The combination of numerical simulations and mathematical models of biophysical systems can improve our overall understanding of cardiac function. Because the

human body is inherently a multiscale system, associated models must encompass spatial dimensions that range from nanometers to meters. The smallest components of many cardiac

electrophysiology models are ion channels, which are only a few nanometers wide. These channels are integrated into cell models that are then combined into tissue models to study phenomena at the organ and body levels — including the optimization of atrial fibrillation therapy, assessment of the impact of specific gene mutations on the likelihood of heart rhythm

disorders and their effects on ECG measurements, and optimal design of new pacemakers or cardiac catheters. This type of comprehensive numerical approach bridges the gap from the nanometer to meter scale (see Figure 1, on page 8).

The openCARP software package [6] has emerged as a pivotal tool in the rapidly evolving field of cardiac electrophysiology simulations, setting new standards for the computational modeling and simulation of cardiac function. This open platform aims to facilitate cutting-edge research and enhance reproducibility and productivity in the scientific community. For added context, openCARP is a public-source cardiac electrophysiology simulation environment that enables *in silico* experiments and supports a wide range of users, including biomedical engineers, computational scientists, and specifically trained biologists and medical doctors. The software simulates cardiac electrophysiological processes at various scales, from the single-cell level to the entire heart. Its robust, flexible

See openCARP on page 8

SOFTWARE AND PROGRAMMING

NSF CAREER Award

Continued from page 6

(or more) as expensive to compute⁵ when compared to solving the linear system. Furthermore, computing the inverse is not numerically stable under some circumstances, which is an important consideration in floating-point arithmetic [1].

While creating these materials, I encountered some new applications. Of these, my favorite is the friendship paradox [2], which says that one is not as popular as one's friends.⁶ This paradox is especially

⁵ This fact takes on increased importance in the scenario of a timed exam.

⁶ I had always suspected as much, but I finally found a mathematical proof (Cauchy-Schwartz for the win).

relevant to students since it may help to explain their levels of dissatisfaction with what they see on social media.

I learned several things from my outreach experiences. To start, any new modules that teachers wish to implement in their classrooms must be tied to specific teaching standards. This requirement ensures that teachers can incorporate these materials as part of their work, rather than as extra curricula. While designing the materials, the teachers always began with the end in mind by first planning the assessments and exams, then developing the actual instruction and activities to adhere to the learning goals. Interestingly, many teachers in NC seemed to like mathematics but were sometimes intimidated by the subject, either because they hadn't encoun-

tered certain topics before or because they had encountered them, but with negative memories. It seems to me that it doesn't bode well for students if their instructors feel this way, or for society if the current attitudes towards mathematics persist.

On the other hand, the teachers that I worked with were all eager to learn new concepts, explore novel instructional tools, and share best teaching practices. In that sense, their enthusiasm was so infectious that I felt like I was working with my university colleagues. Finally, my outreach efforts benefited my own teaching as well. I was able to adapt some of the materials for my undergraduate classes in linear algebra through lectures and homework problems. I also find that I can now better calibrate the level at which I teach, at least in the initial lectures.

The materials that we developed are all freely available online.⁷ Please let me know if you use them in any way, and don't hesitate to share any comments or feedback.

Applying for the NSF CAREER Award

Some readers may be interested in applying for the CAREER award themselves, and I hope that my knowledge can guide future applicants. NSF grants typically require both scientific intellectual merit and broader impact. Most faculty members can comfortably write the research portion of the proposal based on prior experience with standard NSF proposals and/or their involvement on review panels. However, the education component of the CAREER award is an extremely important part of the application that distinguishes it from standard NSF proposals and certain other early career awards; as such, it is often more difficult to compose. Assuming that a standard CAREER award is worth \$400,000 USD, an acquaintance of mine somewhat jokingly valued each page of the CAREER proposal at \$25,000 USD and the first page at \$50,000 USD. With this perspective in mind, I spent nearly three pages of the main narrative (out of a maximum of 15) on the education component and tried to weave it throughout the narrative of the proposal.

For additional credibility, I provided an appropriate budget plan for the educational component, an assessment plan, and supporting letters. It is important to obtain a strong letter from the department head/chair that commits the necessary resources for success. As part of some exploratory legwork before submitting the proposal,

I organized smaller-scale outreach workshops for teachers that helped me refine my narrative and braced me for some of the challenges ahead. In addition, I had other educational efforts at the undergraduate and graduate levels that supported the education portion of my proposal and bolstered the research portion. Feedback from reviewers was positive, but I did receive a comment that recommended an even more detailed education plan.

While the award is in my name, I certainly didn't do it all alone. Jason Painter, director of The Science House⁸ (an outreach unit within the College of Sciences at NC State), helped me craft the educational component of the CAREER proposal. He shared some valuable insights about the language, helped me put together an assessment plan, and wrote me a letter of support. I also received great feedback from mentors and colleagues who generously provided their time, as well as my wife, Ivy Huang, who offered constant encouragement and support. Tamar Avineri, Chair of Mathematics at the North Carolina School of Science and Mathematics, worked with me for all five years and was instrumental in bringing my hazy vision to life. It may take a proverbial village to raise a child, but it takes a whole community to educate and inspire the next generation of students.

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⁷ asaibab.math.ncsu.edu/outreach

⁸ <https://sciencehouse.ncsu.edu>

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Figure 1. Flyer for one of the six high school teaching modules that resulted from the author's Faculty Early Career Development Program award from the U.S. National Science Foundation. This self-contained module uses puzzles such as nonograms to introduce X-ray imaging and the concept of ill-posedness. Flyer courtesy of Matthew Purser.

openCARP

Continued from page 7

platform permits the creation and sharing of simulation pipelines, thereby automating *in silico* experiments and ultimately increasing reproducibility.

The package has already had a profound impact on scientific research, particularly in the realms of drug safety, diagnosis, risk stratification, therapy planning, and intra-procedural support. The software's instrumental role in more than 150 peer-reviewed publications³ highlights its versatility and effectiveness in advancing our understanding of cardiac electrophysiology.

The following cardiac bidomain equations constitute the backbone of the openCARP simulator:

$$\nabla \cdot \sigma_i \nabla \phi_i = \beta I_m$$

$$\nabla \cdot \sigma_e \nabla \phi_e = -\beta I_m - I_e$$

$$I_m = C_m \frac{\partial V_m}{\partial t} + I_{\text{ion}} - I_{\text{trans}}.$$

These equations relate the intracellular electrical potential ϕ_i to the extracellular electrical potential ϕ_e through the transmembrane current density I_m . Here, σ_i and σ_e are the intracellular and extracellular conductivity tensors, β is the ratio of cell membrane surface to tissue volume, C_m is the cell membrane capacitance per unit area, V_m is the transmembrane voltage ($\phi_i - \phi_e$), and I_{ion} is the current density that flows through the ion channels in the cell membrane. Finally, I_{trans} is a transmembrane current density stimulus and I_e is an extracellular current density stimulus.

To solve these equations (or the simplified monodomain model), openCARP employs either PETSc⁴ (the Portable, Extensible Toolkit for Scientific Computation) or Ginkgo.⁵ Ginkgo offers high-performance linear algebra operations that are specifically optimized for graphics processing unit architectures and thus enhance computational efficiency [1]. Additionally, CVODE⁶ (part of Lawrence Livermore National Laboratory's SUNDIALS⁷ suite) can expedite the solution of ordinary differential equations that arise in simulations of ionic currents and membrane voltages. All of these numerical

backends collectively allow openCARP to perform detailed, accurate cardiac simulations that facilitate research and development in cardiac health. MICROCARD,⁸ a Centre of Excellence funded by EuroHPC,⁹ extends openCARP towards microstructural models with even higher resolution for deployment on large supercomputers.

Since its inaugural release in 2019, openCARP has undergone significant growth and development. The openCARP community has put together a comprehensive set of documentation and training materials,¹⁰ including 30 didactic hands-on examples and a suite of video tutorials. These resources are meant to help new users get started and empower experienced users to maximize the software's potential. Regular user meetings,¹¹ which take place both in person and online, foster community bonding and encourage knowledge sharing among users and developers. To date, the software's codebase has seen over 5,000 commits from more than 30 contributors. Tailored infrastructure—including a searchable question-and-answer system, as well as tools and processes that ensure the long-term availability of citable software versions—promotes reproducibility and open science.

Sustainability is a central tenet of openCARP. The development team is committed to minimizing the required level of long-term maintenance and support through a comprehensive software management plan and extensive continuous integration, delivery, testing, and benchmarking pipelines. The openCARP package adheres to the FAIR Principles¹² (findability, accessibility, interoperability, and reuse) for scientific data management and stewardship, which are essential for the promotion of open science. The software development team actively advocates for these principles and contributes to initiatives like the FAIR Principles for Research Software [3]. The integration of these ideas into the openCARP framework ensures the software's continued value within the scientific community.

In recognition of its essential role in cardiac electrophysiology simulation, openCARP was honored with the Newcomer Prize of the first Helmholtz Incubator Software Award¹³ in 2024. Amir Jadidi,

⁸ <https://www.microcard.eu>

⁹ <https://eurohpc-ju.europa.eu>

¹⁰ <https://opencarp.org/getting-started>

¹¹ <https://opencarp.org/community/user-meetings>

¹² <https://www.go-fair.org/fair-principles>

¹³ <https://helmholtz.software/news/2024-08-28/awardees-of-the-first-helmholtz-software-award>

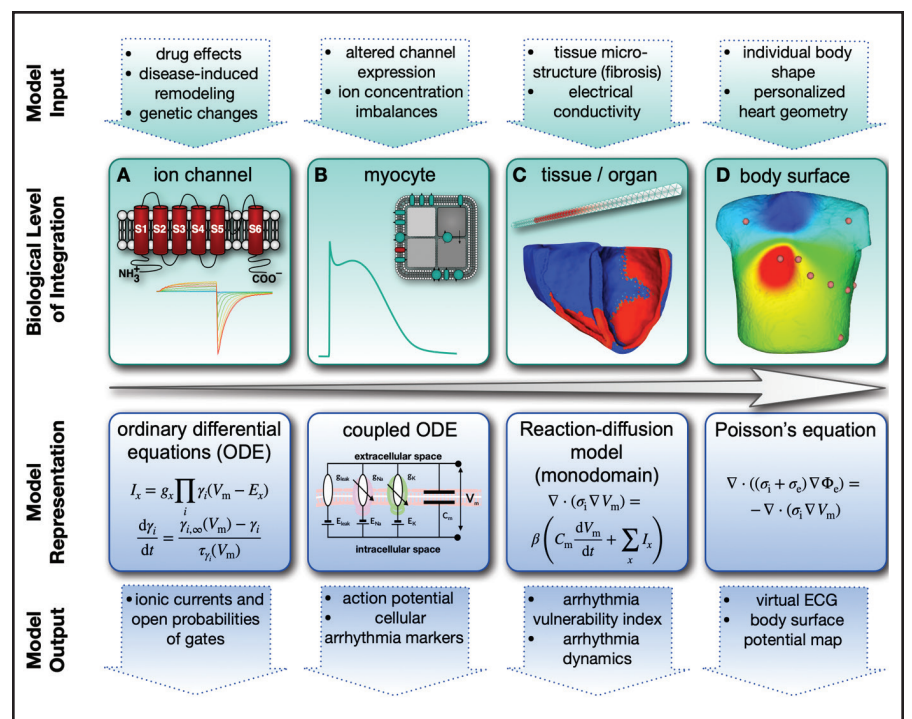


Figure 1. Hierarchy of multiscale cardiac electrophysiology models, ranging from ion channels (1a) via integrated cell-level (1b) and tissue-level models (1c) to body surface and electrocardiogram measurements (1d). The simulation system enables the investigation of “what-if” scenarios by changing the input parameters of the model (top row) and analyzing the subsequent effect on simulation outputs at numerous scales (bottom row) in comparison with experimental and clinical data. Figure courtesy of [4].

a clinical cardiologist at the Heart Center in Lucerne, Switzerland, is involved with several open-CARP-based research projects and praised its contributions to the broader cardiac community. “Computer models enhance our understanding of clinical observations and enable the transfer of anecdotal knowledge to other patients,” he said. “They are invaluable for systematically evaluating new algorithms that analyze clinical signals, as they can synthetically generate virtually any number of signals with known origins. This capability will become increasingly crucial as machine learning methods continue to proliferate.”

To conclude, we eagerly anticipate a future where simulations and prototyping accelerate the development of medical devices and personalized therapies. *In silico* methods provide quantitative, biophysical, and highly controlled approaches that have become integral to physiological and medical research. At the same time, computer simulations can complement biological experiments, animal testing, and clinical trials — thereby propelling the understanding of fundamental disease-causing processes and the creation of personalized therapies [5]. Mechanistic and data-driven modeling must synergistically work together to advance preventive, diagnostic, and therapeutic approaches. This integration of computer simulation and medical research

promises to enhance the precision and effectiveness of new medical technologies, ultimately improving patient outcomes and furthering the field as a whole.

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Geometry of the Phase Lag

Standard sophomore discussions of forced linear oscillations tend to rely on formulas, often without intuitive answers to some of the following basic questions:

1. Why does the phase lag φ lie between 0 and π for damped oscillators?
2. Why is $\varphi=0$ or π in the absence of damping?
3. Why is $\varphi=\pi/2$ when $\omega=\omega_0$, the natural frequency (i.e., the frequency of the same oscillator without damping)?

All of these questions refer to the harmonic oscillator with viscous drag and subject to sinusoidal forcing:

$$\ddot{x} + c\dot{x} + \omega_0^2 x = F \cos \omega t, \quad (1)$$

where $\omega_0^2=k$ is Hooke's constant. We chose mass $m=1$ without a loss of generality. As a standard step, one complexifies (1), i.e., considers x as the coordinate of point z in the plane satisfying

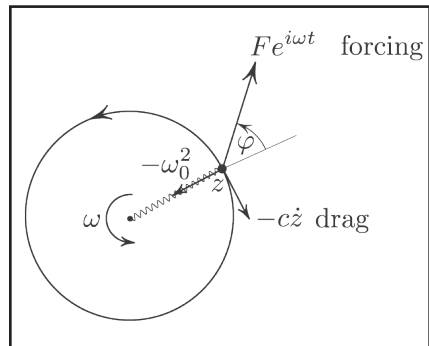


Figure 1. Point mass that is subject to the Hookean force $-\omega_0^2 z$, the drag, and the external force $F e^{i\omega t}$. The x -coordinate satisfies (1), and the steady state solution of (1) corresponds to the circular motion of z with angular velocity ω of the forcing.

$$\ddot{z} + c\dot{z} + \omega_0^2 z = F e^{i\omega t}. \quad (2)$$

Since complex numbers may spook some students, we can equivalently say that x in (1) is the x -coordinate of the point mass that is pulled by the Hookean spring with force $-\omega_0^2 z$ towards the origin, also subject to both the drag force $-c\dot{z}$ that is linear in velocity and the external force of constant magnitude F that spins with angular velocity ω , like a spinning jet engine attached to the mass.

The steady state solution of (1) corresponds to the circular motion of the point mass, and the phase lag φ in Figure 1 is the angle by which the direction of response z lags behind the direction of the forcing.

Figure 1 answers the aforementioned questions 1-3 as follows:

1. In the presence of damping, the tangential component $F \sin \varphi$ must be positive to compensate the drag, hence $\varphi \in (0, \pi)$.

2. In the undamped case $c=0$, the tangential component of forcing must vanish so that $\varphi=0$ or π ; otherwise, the particle will have nonzero tangential acceleration and be unable to maintain constant speed.

3. The case $\varphi=\pi/2$ amounts to purely tangential forcing. Since the drag is also tangential and the speed of circular motion is constant, the forcing exactly cancels the drag. We can then pretend that neither drag nor forcing is present; what's left is a circular motion of the undamped unforced oscillator with Hooke's constant ω_0^2 and hence with frequency ω_0 . But the frequency of our circular motion is also ω , so that $\omega=\omega_0$. Thus, $\omega=\omega_0$ is equivalent to $\varphi=\pi/2$.

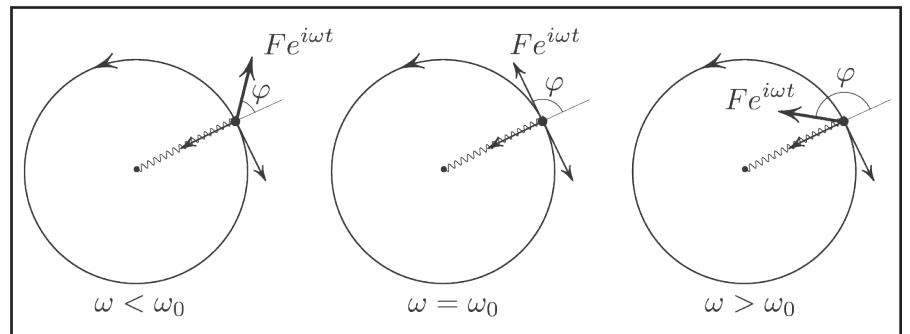


Figure 2. The damped case. As the driving frequency ω —i.e., the angular velocity—increases, more inward force is needed to help the spring.

Figures 2 and 3 illustrate several additional properties that are stated and explained in the respective captions.

I deliberately avoided calculations, which are typically done via complex numbers. These calculations are actually made redundant through the use of Newton's second law: from Figure 1,

$$F \sin \varphi = c\omega R$$

$$F \cos \varphi = \omega^2 R - \omega_0^2 R.$$

The first equation expresses the balance between the tangential component of forcing and the drag, thus assuring constant speed along the circle. The second equation states that the centripetal force $\omega^2 R$ is the resultant of the spring force and the radial component of the driving force.

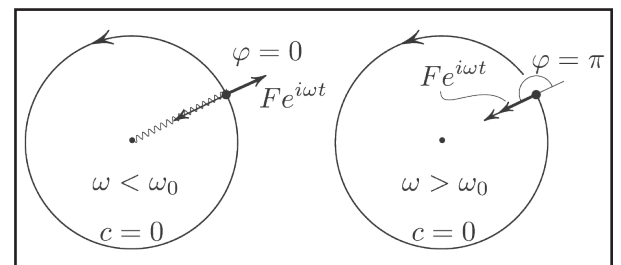


Figure 3. The undamped case. If the forcing frequency is smaller than the natural frequency, more inward force is needed to weaken the spring to maintain a frequency that is lower than ω_0 . Conversely, if we insist on a high-frequency circular motion, we must help the spring by pulling the mass inwards, thus explaining $\varphi=\pi$. The borderline case $\omega=\omega_0$ is possible only if $F=0$ in the undamped case.

MATHEMATICAL CURIOSITIES

By Mark Levi

A New Biography of a Great Ancient Scientist

Archimedes: Fulcrum of Science.
By Nicholas Nicastro. The University of Chicago Press, Chicago, IL, November 2024. 240 pages, \$22.50.

By common consensus, Archimedes (c. 287-212 BCE) was the greatest inventor of classical antiquity and the greatest mathematician prior to Isaac Newton. He introduced the idea of limiting sums that, 1,700 years later, formed the foundation of integral calculus. He computed the area of a parabola, the volume of a sphere, and several other such quantities. He calculated rational bounds on the value of π —fixing its value to within a range of 0.002—and rational bounds on the value of $\sqrt{3}$, within a range of 0.00003. He offered mathematical formulations for the law of the lever and the law of buoyancy. His war machines and defensive devices allowed the defenders of Syracuse to hold off the besieging Romans for a year and a half—much longer than the Romans expected.

Largely due to Archimedes' important role in the defense of Syracuse, there are many more ancient biographical accounts about him than any other contemporaneous mathematician. Unfortunately, these records are of uneven reliability and—barring a few facts that Archimedes himself mentioned in the introductions to his treatises—all date from decades or even centuries after his death. The first surviving account of Archimedes appears in Polybius' *The Histories*, which was written in approximately 140 BCE. Marcus Tullius Cicero mentions Archimedes in several of his writings. Most famously, Cicero's *Tusculan Disputations* (c. 45 BCE) recounts his own successful search for Archimedes' lost gravesite in Syracuse. In *Life of Marcellus* (c. 100 CE), Plutarch details Archimedes' life and his death at the hands of a Roman soldier [3]. Other ancient references exist in the works of Vitruvius (c. 20 BCE), Livy (c. 10 BCE), Pliny the

Elder (c. 60 CE), Quintilian (c. 70 CE), and Silius Italicus (c. 70 CE).¹

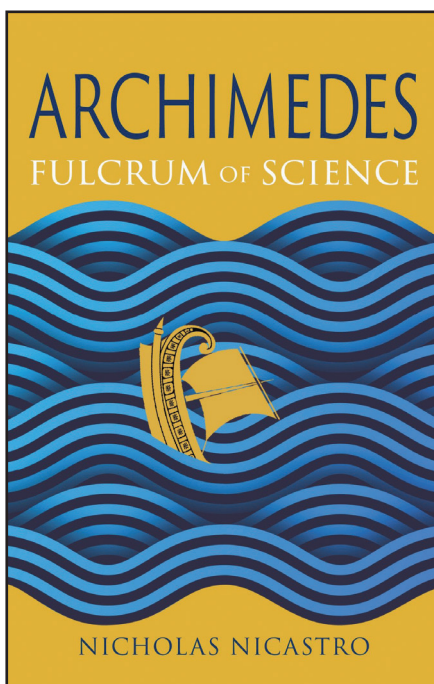
Nicholas Nicastro's newest book, titled *Archimedes: Fulcrum of Science*, is a comparatively short text that is meant for lay readers. After a brief prologue that sets the stage with a description of Cicero finding Archimedes' grave, the book breaks into three sections: "Engineer" (72 pages), "Mathematician" (48 pages), and "Legacy" (39 pages).

"Engineer" is by far the best section. Nicastro provides clear, readable accounts of the devices—both practical and military—that were invented by or attributed to Archimedes. Particularly entertaining is his depiction of the huge pleasure ship *Syracusia*: "longer and heavier than HMS *Victory*, Nelson's 104-gun flagship at Trafalgar, [and] 50 percent bigger than SS *Great Western*, the largest passenger ship in the world in 1838." *Syracusia* was equipped with a gymnasium, gardens, a mosaic that illustrated the story of *The Iliad*, a temple of Venus, 10 stalls for horses, and so on (Archimedes' primary contribution was presumably to ensure that the craft was seaworthy).

¹ Almost all of these works are by Roman authors writing in Latin rather than Hellenistic authors writing in Greek; the one exception is Plutarch.

BOOK REVIEW

By Ernest Davis



Archimedes: Fulcrum of Science. By Nicholas Nicastro. Courtesy of the University of Chicago Press.

Like all biographers of Archimedes, Nicastro had to decide which stories about the scientist to accept and which to reject. Regarding the well-known tale of the golden crown, Nicastro—contrary to Galileo—believes that Archimedes might have plausibly measured the crown's volume using water displacement, and that jumping naked out of his bath while shouting "Eureka!" would have been in character. However, he dismisses the narrative that Archimedes used mirrors to create a heat ray that set fire to Roman boats at a distance, deeming it physically impossible. He also rejects the idea that the "claw of Archimedes" could have lifted Roman boats out of the water, but he does think that it might have knocked them over—which would have been just as militarily effective.

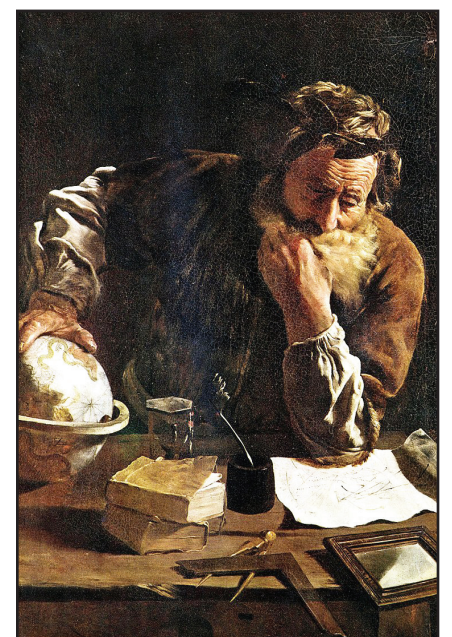
Nicastro spends several pages discussing the famous Antikythera mechanism (an ancient Greek model of the solar system) and speculating about a possible connection to Archimedes. Cicero wrote that Archimedes produced similar models of the celestial spheres that used gears to replicate the correct relative (apparent) motions of the sun, moon, and planets against the fixed stars. But since the ship that carried the Antikythera sank 150 years after Archimedes' death, Nicastro ultimately decides that Archimedes

was at most an inspiration for the device, rather than its designer.

The "Engineer" section of the book also summarizes the historical background of the Roman siege of Syracuse. Nicastro presents a detailed prior history of Syracuse and Sicily, but his account of the broader Mediterranean context is thin. He mentions in passing that Hannibal was in Italy at the time, but readers would hardly guess that a fierce, bloody war between the two greatest Mediterranean powers was raging. In fact, the Second Punic War is never acknowledged by name.

Although the "Mathematician" segment is not as much fun or as well written, it is nevertheless a useful general introduction to Archimedes' mathematical contributions. Nicastro offers a clear explanation of Archimedes' use of exponentiation to express enormous numbers in *The Sand Reckoner*, and of his proof that the

See *Ancient Scientist* on page 10



1620 oil painting called *Archimedes Thoughtful (or Portrait of a Scholar)* by Italian artist Domenico Fetti. Image courtesy of Guillermind81/Wikimedia Commons; the painting itself is in the public domain.

Virtual Panel Dives Into Data Science Careers in Industry and Government

By Jillian Kunze

As the wide-reaching relevance of data science, artificial intelligence (AI), and machine learning (ML) continues to grow, exciting new opportunities are unfolding for applied mathematicians and computational scientists across a multitude of industries. In response to the heightened demand for mathematical expertise from organizations that are actively adopting these technologies, the SIAM Industry Committee¹ organized a virtual panel—which took place last October, just before the 2024 SIAM Conference on Mathematics of Data Science²—that addressed data science careers outside of academia. Moderators William Pack of Leonardo DRS³ and Nessay Tania of Pfizer⁴ guided an hourlong conversation between panelists Jiahao Chen of the New York City Office of Technology and Innovation,⁵ Alyssa Cuyjet of Google,⁶ Marta D’Elia of Atomic Machines,⁷ Justina Ivanauskaite of MSD Czech Republic,⁸ and Raymond Perkins of Intuit,⁹ all of whom offered an inside view of their experiences as data scientists within various industry and government settings.

As the discussion commenced, several panelists acknowledged the importance of

¹ <https://www.siam.org/get-involved/connect-with-a-community/committees/industry-committee>

² <https://www.siam.org/conferences-events/siam-conferences/mds24>

³ <https://www.leonardodrs.com>

⁴ <https://www.pfizer.com>

⁵ <https://www.nyc.gov/content/oti/pages/>

⁶ <https://www.google.com/about/careers>

⁷ <https://www.atomicmachines.com>

⁸ <https://www.msd.cz>

⁹ <https://www.intuit.com>

practical research opportunities as motivating factors in their pursuit of industry or government careers. “I really appreciated that I could use mathematics to solve real problems,” Perkins, a senior data scientist at Intuit, said. “It was more a question of finding problems that I found interesting, then using that mathematical toolset.” D’Elia explored several different avenues before landing her current role as Director of AI and Modeling and Simulation at Atomic Machines. “I tried all types of jobs,” she said. “I started in academia, spent a lot of time at the labs, and then landed in industry. Throughout this path, I got closer and closer to solving actual problems; that’s when I realized that I wanted to stay in industry.”

Certain competencies from academic training become particularly relevant in the workforce. For example, a solid background in applied mathematics and statistics can help data scientists develop creative approaches to a variety of scenarios. “Strong mathematics and scientific computing skills are so important,” D’Elia said. “Students now are so lucky because they have training in ML and data science.”

A rigorous mathematical foundation also engenders a pragmatic understanding of the most effective ways to make suitable approximations according to a project’s demands and timelines. “Applied mathematics really prepares you to tread that ground,” Chen said, noting that realistic problems generally have ill-defined domains. As Director of AI and ML at the New York City Office of Technology and Innovation, Chen collaborates with his

social scientist colleagues when addressing ethical questions that cannot be solved with mathematical techniques alone.

Regardless of educational track, there are some aspects of real-world careers that researchers do not typically encounter in the classroom. Ivanauskaite, who over-

sees a 15-person team as a data science lead at MSD, found that her own training did not prepare her to solve distinctly human problems. “When you study math, it’s very individual,” she said.

“Then you come to industry and need to constantly adjust to other people.”

Early-career scientists will also find that they transition away from the clear-cut problems that are typically associated with academic coursework. “One of the big culture shocks for me was coming from a world where the problems were well defined,” Chen said. “In industry, I feel like 70 percent of data science is trying to find out what data I have and what I can do with it.” The panelists thus encouraged attendees to embrace a self-starter mindset; learn to evaluate the available data; and determine possible, practical courses of action. “When I started working, I realized that I could define the problem myself,”

Ivanauskaite said. “I can create problems that are relevant for the business to solve.”

Maintaining focus on this practical applicability to business is crucial, as fancy mathematical techniques that do not produce useful results are irrelevant in industry. “You can have the technical solution and build something very complex and nuanced,” Cuyjet, a technical program manager at Google, said. “But if it’s not dealing with the problem at hand, it will only help you so far in the job.”

Most data science job candidates are technically proficient, so what can applied mathematicians do to set themselves apart during the application process? First, they should demonstrate an ability to clearly explain an approach, articulate the importance of different metrics, and describe a model’s risks to nontechnical colleagues. “How do you think about the problems that you’re going to solve?” Perkins asked. “If you can answer that, it will really distinguish you.” Understanding the drivers of profit, potential consumer responses, and the cascading impacts of decisions can likewise differentiate an applicant, but gaining this kind of knowledge often requires some amount of industry experience. “Where I see people struggle the most is actually

See *Data Science Careers* on page 11

CAREERS IN MATHEMATICAL SCIENCES



The SIAM Industry Committee hosted a virtual panel in October 2024 that explored data science careers within industry and government settings. Top row, left to right: panelists Jiahao Chen (New York City Office of Technology and Innovation) and Alyssa Cuyjet (Google). Middle row, left to right: panelists Marta D’Elia (Atomic Machines), Justina Ivanauskaite (MSD Czech Republic), and Raymond Perkins (Intuit). Bottom row, left to right: moderators William Pack (Leonardo DRS) and Nessay Tania (Pfizer).

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Ancient Scientist

Continued from page 9

volume of a sphere is two-thirds that of the circumscribing cylinder. He sketches Archimedes’ derivation of the area of a parabola via its approximation as the union of triangles, as well as the ancient mathematician’s approximation of π based on the inscribed and circumscribed 96-sided regular polygon. Nicastro likewise details some of Archimedes’ proof techniques and describes (though not especially well) the state of contemporary mathematics within which he worked. Nicastro’s grasp of the relevant mathematics seems a little shaky, and at one point he refers to π as a “transcendental” number when he instead means “irrational.” Later on, he expresses surprise that “even modern computer-derived estimates, running now to the hundreds of trillions of digits, are still exactly within the bounds Archimedes placed it more than 22 centuries ago.”

The book’s final part, “Legacy,” is disappointing. It focuses almost entirely on a group of 11 statues in Egypt called the Philosophers Circle—one of which might represent Archimedes—and on the Pantheon in Rome (built 126 CE) and Hagia Sophia in modern-day Turkey (built 537 CE), as Nicastro thinks that their designs reflect an Archimedean influence.

Archimedes: Fulcrum of Science also includes a timeline, notes, a list of further

readings, an index, and some geometric and physical diagrams. Though the text contains 12 black and white photographs of artworks and archeological findings, the reproduced photos are of such poor quality that I don’t know why they bothered.

If you are seriously interested in Archimedes’ contributions to mathematics and his place within the history of math, then—as Nicastro himself admits in his introduction—there are much better resources on the subject. In fact, at least two previous English-language scholarly biographies of Archimedes have published since 2000 [1, 2]. But if you are in the mood for a light, readable introduction to the life and works of a great scientist and a short history of Syracuse at the height of its wealth and power, then this book is very enjoyable.

References

- [1] Geymonat, M. (2011). *The great Archimedes*. Waco, TX: Baylor University Press.
- [2] Hirshfeld, A. (2009). *Eureka man: The life and legacy of Archimedes*. New York, NY: Walker Publishing Company.
- [3] Jaeger, M. (2008). *Archimedes and the Roman imagination*. Ann Arbor, MI: University of Michigan Press.

Ernest Davis is a professor of computer science at New York University’s Courant Institute of Mathematical Sciences.

Robust Poster Sessions at MDS24 Highlight Diverse Applications of Data Science

By Manuchehr Aminian

The 2024 SIAM Conference on Mathematics of Data Science¹ (MDS24), which took place last October in Atlanta, Ga., featured a vibrant five-day sequence of evening poster sessions. To promote these sessions and encourage lively discussions about the meeting's various themes, the MDS24 co-chairs² required all minisymposium organizers to submit a series of posters that matched the number of talks in each minisymposium [1]. This stipulation—a first for the SIAM community—resulted in a rich spread of posters, attracted large crowds at the daily sessions, and encouraged broader engagement between attendees at different career stages and across employment sectors.

To provide a robust sense of the variety of featured research topics, we highlight a stratified random sample of the roughly 550 posters on display at MDS24.

Supervised Learning Framework for Efficient Sampling of High-dimensional, Multimodal Distributions

Hoang Tran, a staff mathematician at Oak Ridge National Laboratory, presented work on the synthetic generation and sampling of high-dimensional, multimodal probability distributions.³ This mathematical setting describes most generative models for text and images, among other applications; Tran's research focuses specifically on supervised methods for the sampling of multimodal distributions.

Common frameworks such as Markov chain Monte Carlo or generative adversarial networks suffer from issues like slow convergence, vanishing gradients, poor mixing, and mode trapping. Tran's framework alleviates these complications by using a divide-and-conquer approach within the subdomains of a multimodal distribution. His technique relies on several

modular steps. First, it solves an optimization problem on the target distribution to identify potential modes, then trains a classifier whose decision boundaries partition the domains that correspond with each mode. A diffusion model generates labeled data for the unimodal distribution in every subdomain, and a neural network learns the same sampler in a supervised manner. One can then join and rescale these samplers to correct the ratios between modes.

Tran illustrated this procedure with a proof-of-concept implementation that he tested on both images and synthetic data (see Figure 1). Given the generality of the modular steps in this process, he and his colleagues hope to gradually refine the framework piece by piece.

Analysis of Reporting Trends in Russo-Ukrainian Conflicts Through Topic Modeling Algorithms

Nya Feinstein, a recent graduate of West Virginia University, presented an analysis of 15,537 news articles from the Ukrainian News Agency that she mined from LexisNexis,⁴ with a query on articles from 2011 to 2023 that contain information about Russia.⁵ She utilized latent Dirichlet allocation (LDA) and BERTopic⁶—two natural language processing tools—to evaluate the contents of these articles and quantify the relative frequency of the appearance of certain topics over time, such as *energy* (e.g., “gas” or “nuclear”) and *strategy* (e.g., “defense” or “security”). This database also allowed her to identify relative jumps or declines in different topics before and after major events—like Russia's 2014 annexation of Crimea and the 2022 Russian invasion—from a Ukrainian lens. While Feinstein observed many similarities between the content that LDA and BERTopic extracted, she also encountered some differences and challenges—especially with the use of BERTopic. She aims to refine these tools as a method of conflict forecasting and trend analysis based on news data.

⁴ <https://www.lexisnexis.com>

⁵ https://meetings.siam.org/session/dsp_talk_cfm?p=143247

⁶ <https://maartengr.github.io/BERTopic/index.html>



Nya Feinstein (center), a recent graduate of West Virginia University, shares her research poster with two attendees during one of five poster sessions at the 2024 SIAM Conference on Mathematics of Data Science, which took place in Atlanta, Ga., last October. Feinstein's work uses natural language processing tools to analyze reporting trends in Ukrainian news articles that contain information about Russia. SIAM photo.

Wasserstein Convergence Guarantees for a General Class of Score-based Generative Models

Hoang Nguyen, a graduate student at Florida State University who will soon join Meta as a research scientist, prefaced his work by formulating generative diffusion models in the context of stochastic differential equations (SDEs).⁷ A common *forward process* in SDEs is $dx_t = -f(t)x_t dt + g(t)dW_t$, where $x_0 \sim p_0$; here, p_0 is a desired but unknown data distribution, and $f(t)$ and $g(t)$ are respectively drift and diffusion terms. A *reverse process* for \tilde{x}_t runs this procedure backward in time and—under certain assumptions—obeys a more sophisticated SDE that depends on f , g , and the probability density of x_t . The formalization of this reverse process enables the recovery of p_0 for various choices of f , g , and other parameters.

Nguyen and his collaborators proved an overall bound on the 2-Wasserstein distance of a simulated density of the reverse process—that is, $\mathcal{L}(y_K)$ —to the target distribution p_0 , specifically $\mathcal{W}(\mathcal{L}(y_K), p_0)$. They used this knowledge to identify bounds on

⁷ https://meetings.siam.org/session/dsp_talk_cfm?p=140101

combinations of algebraic and/or exponential forms for $f(t)$ and $g(t)$, and ultimately prescribed an optimal step size based on the drift and diffusion terms. The group implemented this scheduler for image generation with the well-known CIFAR-10 dataset,⁸ which led to practical performance improvements. In the future, Nguyen hopes to relax some of the theoretical assumptions and apply the framework to large-scale generative diffusion models.

Classifying Imbalanced Data

Karen Medlin, a graduate student at the University of North Carolina at Chapel Hill, studies methods that handle imbalanced datasets in machine learning (ML) classification tasks.⁹ Datasets in which the number of data points of the “majority” class (e.g., class 0) greatly outnumber those of the “minority” class (e.g., class 1) pose a fundamental challenge for scientists. Because out-of-the-box techniques to train a robust classifier often fail with imbalanced data, Medlin's approach instead considers majority undersampling with bilevel optimization as a framework for any ML model. Here, the formation of an energy functional $J(S_M, w^*)$ is based on the undersampling of majority class S_M and model parameters w^* , where the constraint is dependent on solving the parameters on this subset of data. In practice, this type of optimization uses an alternating scheme wherein a proposed undersampling trains a classifier that then passes the new set of learned parameters to the functional J . Class imbalance is common in anomaly

See **Poster Sessions** on page 12

⁸ <https://www.cs.toronto.edu/~kriz/cifar.html>

⁹ https://meetings.siam.org/session/dsp_talk_cfm?p=140008

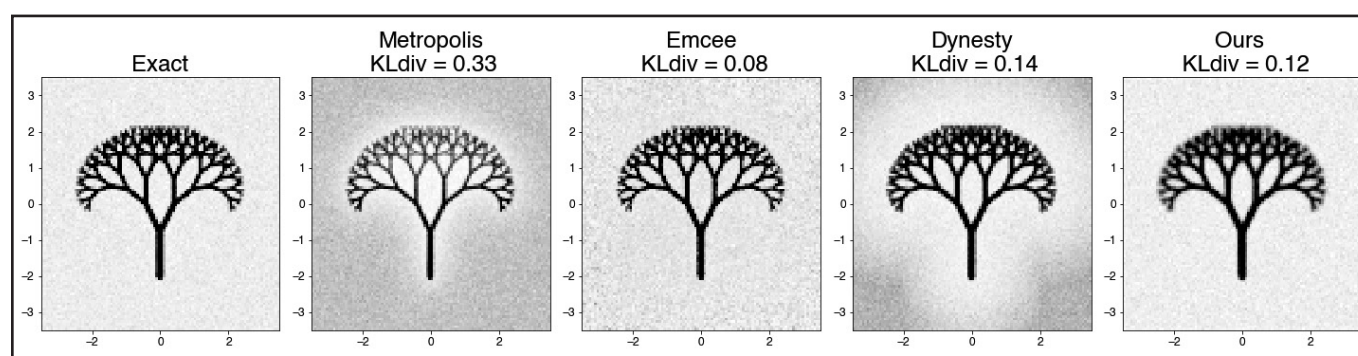


Figure 1. Performance benchmark for the sampling of a high-dimensional distribution that stems from image data. Hoang Tran's algorithm provides a close reconstruction when compared to other algorithms, as measured by the Kullback-Leibler divergence. Figure courtesy of Hoang Tran.

Data Science Careers

Continued from page 10

having the business sense, specifically around products,” Perkins said. “Yes, you can build this type of model, but the question is, why would we want you to?”

Internships provide unique opportunities for students to familiarize themselves with an industry and gain experience with real-world problems. “If you are still in undergraduate or graduate school, I would recommend reaching out to alumni from your program,” Cuyjet said. “Ask about their experience getting into industry.” When students graduate and enter the job market, this network becomes even more crucial. Because most job postings attract a high volume of applications, a referral from a

connection within the company can increase one's chances of securing an interview.

For entry-level roles that target recent graduates, academic background and demonstrated interest in the field are especially relevant. Applicants must show that they are motivated and able to deliver what the company asks; for instance, candidates can detail their contributions to a useful project that addressed a practical issue in order to demonstrate a broader perspective beyond their technical expertise. “Be able to clearly describe the value of your work in these small projects,” Ivanauskaite said.

Educational requirements for data science jobs depend heavily on the company, location, and specific position in question. In the U.S., researchers with Ph.D.s are often offered higher salaries right out of

school, though the tradeoff is that they must first spend several years in low-paying graduate programs. But this situation is not universal, and Ivanauskaite noted that a Ph.D. does not significantly accelerate one's industry career advancement in many European countries.

Some data science roles are heavily research-driven, meaning that a Ph.D. is very beneficial (though not always strictly required) to remain competitive. However, many master's programs now focus specifically on data science and ML and prepare students for tech-oriented positions. Perkins has observed that some Ph.D.-level scientists who enter the tech industry still need to attend bootcamps to gain certain technical skills, while other individuals with only master's degrees possess the

necessary competencies because of their programs' concentrations. “I would begin by asking, ‘Where do you see yourself and want to be in industry?’” Perkins said. “Then work backwards to figure out what education you need.”

Overall, applied mathematicians and computational scientists have much to be excited about in the data science space. In the coming years, many opportunities will likely continue to emerge for lucrative careers across a wide range of organizations. “It's very heartening that the field of AI and ML is a place where technically-minded people can find no shortage of interesting problems to work on,” Chen said.

Jillian Kunze is the associate editor of SIAM News.

Poster Sessions

Continued from page 11

detection and rare event forecasting, so Medlin hopes that researchers will eventually apply this methodology to such data.

Embrace Rejection: Kernel Matrix Approximation by Accelerated Randomly Pivoted Cholesky

Ethan Epperly, a graduate student at the California Institute of Technology, focuses his research on computational linear algebra at the interface of scientific ML.¹⁰ Many kernel methods in ML require the evaluation of a function $\kappa(\mathbf{x}, \mathbf{x}')$ at all pairs of a dataset's elements. The formation of this positive semidefinite kernel matrix, which necessitates $O(N^2)$ kernel evaluations with N datapoints, can act as a bottleneck due to limitations in space or compute time. To alleviate this barrier, one can replace data matrix A with a rank- k approximation $\hat{A} = FF^T$ that accelerates downstream algorithms. For any symmetric positive semidefinite matrix A , the standard randomly pivoted Cholesky (RPCholesky) algorithm produces this type of F based on outer product updates $\hat{A} \leftarrow \hat{A} + A_s A_s^T / A_{s,s}$, fol-

¹⁰ <https://meetings.siam.org/sess/dsp-talk.cfm?p=140535>

lowed by a deflation step on A . The pivot $s = s_i$ is randomly selected and weighted by the diagonal entries of the current A . This process is sequential, as the deflation at step i affects the distribution for sampling s_{i+1} .

To accelerate the algorithm, Epperly and his colleagues developed a block version of RPCholesky that samples sets of indices with rejection sampling and uses a block-based update for \hat{A} . Their method begins with a proposed set of pivots at iteration i , S_i ; after submatrix A_{S_i, S_i} is scanned, each proposed pivot s' is rejected with probability $A_{s', s'}^T A_{S_i, S_i}^{-1} A_{s', S_i} / A_{s', s'}$. Once a final set of pivots S_i is selected, one must then perform a block update of \hat{A} and a deflation of A (possibly greater than rank 1). Experiments revealed a five to 40 \times speedup over traditional RPCholesky. In practice, this method achieves state-of-the-art speed in fitting potential energy functionals when compared to alternatives like uniform sampling, greedy sampling, and ridge leverage score sampling.

A Training-free Conditional Diffusion Model for Learning Stochastic Dynamical Systems

Yanfang Liu, an assistant professor at Middle Tennessee State University, presented her work on the supervised learn-

ing of unknown SDEs through what she calls a *conditional diffusion model for flow map learning*.¹¹ Conditional diffusion models consist of the forward SDE $dZ_\tau = h(\tau)Z_\tau d\tau + g(\tau)dW_\tau$ and the associated reverse-time equation. Liu and her collaborators determined that it is possible to approximate the score function $S(Z_\tau, \tau) = \nabla_z \log p(Z_\tau)$ via a Monte-Carlo-based estimation that requires a prior dataset of the SDE's input/output pairs. Given this newfound knowledge, Liu proposed an algorithm that iteratively refines the estimated score function $S(Z_\tau, \tau)$ and utilizes it to produce a new data sample. With an expanded training dataset, one can then use a neural network to approximate the generative model. Liu ultimately observed a performance level that matched or exceeded generative adversarial networks on a collection of classical SDE models, as measured by end-time mean and standard deviation.

Conclusions and Broader Trends

These and the numerous other poster topics at MDS24 reflected emerging trends in the mathematical data science space. The presentations spotlighted here include appli-

¹¹ <https://meetings.siam.org/sess/dsp-talk.cfm?p=140011>

cation projects, classical applied frameworks as lenses for algorithm development and computational study, and analytical inroads that reflect the state of the art in data science. No one theme overwhelmingly dominated the poster sessions; out of the approximately 550 displays, 30 addressed generative modeling; 44 focused on topological, geometrical, and/or distributional/Wasserstein-based perspectives; 66 examined neural-related subjects; and 10 or so detailed other research areas.

The applied mathematics community undoubtedly spans a diverse set of interests and entry points within the broader data science community, and we look forward to witnessing the continued evolution of these connections next year at MDS26.

References

[1] Chi, E.C., Gleich, D.F., & Ward, R. (2025, January 21). Artificial intelligence tools facilitate MDS24 conference scheduling. *SIAM News*, 58(1), p. 8.

Manuchehr Aminian is an assistant professor in the Department of Mathematics and Statistics at California State Polytechnic University, Pomona. His research interests include mathematical modeling, partial differential equations, and mathematical methods in data science.

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April 14, 2025: Posters
April 28, 2025: Travel Fund Applications

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SIAM International Conference on Data Mining

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July 7–11, 2025

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Student Chapter Funding for 2024–2025

SIAM awarded over \$70,000 to more than 170 chapters for events and activities throughout the 2024–2025 academic year. This funding supports SIAM Student Chapters in organizing meetings, hosting seminars, engaging in professional development, and fueling their passion for applied mathematics. Each year, these chapters host a wide range of exciting events. Below, are some highlights of the activities held by our chapters in 2024. For information about obtaining funding for your chapter or starting a chapter, go to www.siam.org/start-a-chapter.



California State University student chapter members participate in games.

California State University student chapter hosted a competition day where small groups of SIAM student members went head-to-head in team building exercises.

The Chinese University of Hong Kong student chapter hosted their annual student workshop, inviting students from across the globe to participate in the event. The workshop included a host of speakers who graduated from CUHK and excelled in their academic or professional careers. Attendees had the opportunity to hear engaging talks and network with peers and past graduates.

George Washington University student chapter hosted its annual spring conference on applied mathematics, focusing on emerging trends in data science and deep learning. Held in Washington, D.C., the conference featured seven speakers from both academic and industrial sectors, who presented their latest research in applied mathematics, discussed career development, and provided students with opportunities to network and establish potential collaborations with peers and professionals.

National Central University student chapter hosted a monthly *SIAM News* reading group, allowing students from various mathematics disciplines to come together and share the information they learned from *SIAM News*.

In celebration of the International Day of Women in Mathematics 2024, the Institute of Mathematics at Pontificia Universidad Católica de Valparaíso (PUCV), together with the Pontificia Universidad Católica de Valparaíso SIAM Student Chapter, organized a special screening of the French film *Le Théorème de Marguerite*. At the event, participants engaged in the "Conversations with Women in Mathematics" panel discussion featuring industry and academic professionals. The event was supported by the PUCV Gender Equity Office, InES Género PUCV, Science Up, and SIAM PUCV.



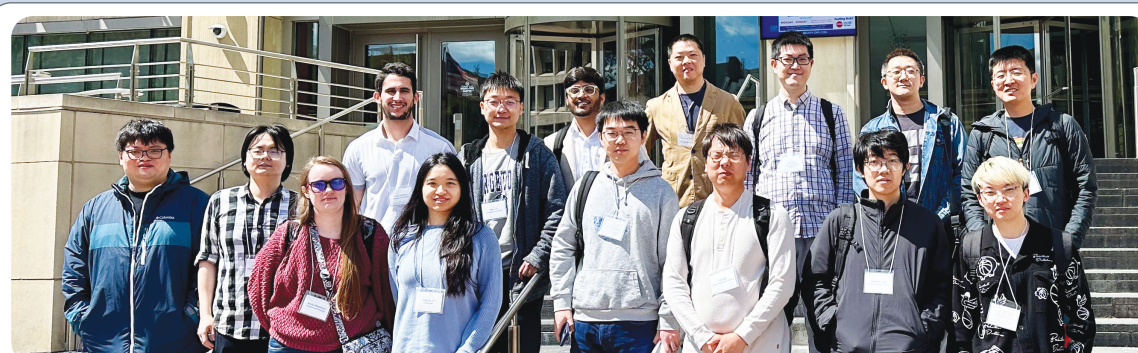
Attendees of the annual student workshop hosted by the Chinese University of Hong Kong student chapter pose for a picture during the workshop.

University of Tennessee student chapter hosted a faculty showcase where several faculty members presented a condensed version to students to assist them in picking a faculty advisor that fit their interest.

Utah State University student chapter demonstrated their creativity by volunteering at the Science Unwrapped event, themed "Randomness & Chance." Chapter members engaged participants with Roll It!, a game designed to teach players about probability.

SIAM would like to extend a warm welcome to our new student chapter members for the 2024–2025 academic year:

- Denison University
- Dublin Area
- UniPi-SNS
- University of Cincinnati
- University of Evansville
- Washington University



Attendees of the annual spring conference on applied mathematics hosted by the George Washington University student chapter pose together after the event.



The PUCV student chapter event featured panelists and moderator Paulina Sepúlveda, faculty advisor and event organizer.



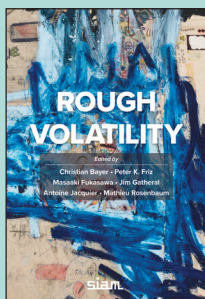
Utah State University student chapter members teach Science Unwrapped attendees how to play Roll It!

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SIAM members (excluding student members) can nominate up to two students per year for free membership. Go to siam.org/nominate-student to make your nominations.

The 2025 Class of SIAM Fellows will be announced March 26, 2025. Learn more at siam.org/Prizes-Recognition/Fellows-Program.





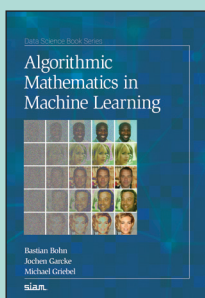
Rough Volatility

Christian Bayer, Peter K. Friz, Masaaki Fukasawa, Jim Gatheral, Antoine Jacquier, and Mathieu Rosenbaum, Editors

Volatility has traditionally been modeled as a semimartingale, with consequent scaling properties, but a new paradigm has emerged, whereby paths of volatility are rougher than those of semimartingales. According to this perspective, volatility behaves as a fractional Brownian motion with a small Hurst parameter.

Rough Volatility is the first book to offer a comprehensive exploration of the subject, organizing the material to reflect the subject's development and progression. It contributes to the understanding and application of rough volatility models by equipping readers with the tools and insights needed to delve into the topic and explores the motivation for rough volatility modeling and provides a toolbox for its computation and practical implementation.

2023 / xxviii + 263 / Soft / 978-1-61197-777-6 / List \$85.00 / SIAM Member \$59.50 / FM02



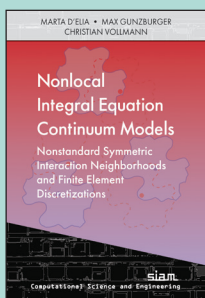
Algorithmic Mathematics in Machine Learning

Bastian Bohn, Jochen Garcke, and Michael Griebel

This unique book explores several well-known machine learning and data analysis algorithms from a mathematical and programming perspective. The authors present machine learning methods, review the underlying mathematics, and provide

programming exercises to deepen the reader's understanding. They provide new terminology and background information on mathematical concepts, as well as exercises, in "info-boxes" throughout the text. Application areas are accompanied by exercises that explore the unique characteristics of real-world data sets (e.g., image data for pedestrian detection, biological cell data).

2024 / xii + 225 pages / Soft / 978-1-61197-787-5 / List \$64.00 / SIAM Member \$44.80 / DI03



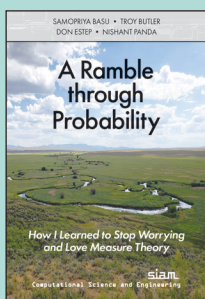
Nonlocal Integral Equation Continuum Models: Nonstandard Interaction Neighborhoods and Finite Element Discretizations

Marta D'Elia, Max Gunzburger, and Christian Vollmann

This book presents the state of the art of nonlocal modeling and discretization as well as novel analyses of a class of nonstandard nonlocal models.

These models have recently become a viable alternative to classical partial differential equations when the latter are unable to capture effects such as discontinuities and multiscale behavior in a system of interest. Because of their integral nature, nonlocal operators allow for the relaxation of regularity requirements on the solution and for capturing multiscale effects and thus have been successfully used in many scientific and engineering applications. Although the use of nonstandard models is novel, this book provides extensive background and a thorough analysis and description of their discretization methods, offering a gentle and practical introduction to nonlocal modeling for readers who are not familiar with nonlocality.

2024 / x + 176 pages / Soft / 978-1-61197-804-9 / List \$65.00 / SIAM Member \$45.50 / CS31



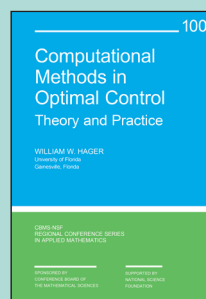
A Ramble through Probability: How I Learned to Stop Worrying and Love Measure Theory

Samopriya Basu, Troy Butler, Don Estep, and Nishant Panda

Measure theory and measure-theoretic probability are fascinating subjects. Proofs describing profound ways to reason lead to results that are frequently startling, beautiful, and useful. Measure theory

and probability also play roles in the development of pure and applied mathematics, statistics, engineering, physics, and finance. This book traces an eclectic path through the fundamentals of the topic to make the material accessible to a broad range of students. It brings together the key elements and applications in a unified presentation aimed at developing intuition; contains an extensive collection of examples that illustrate, explain, and apply the theories; and is supplemented with videos containing commentary and explanations of select proofs on an ancillary website.

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Computational Methods in Optimal Control: Theory and Practice

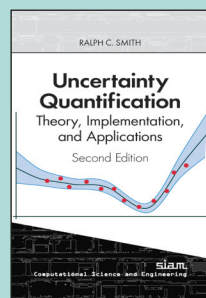
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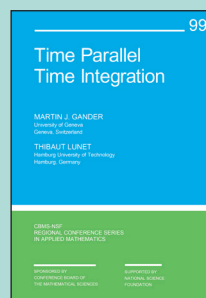
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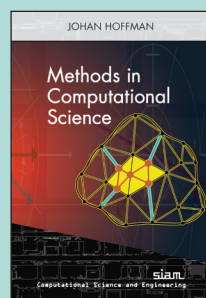
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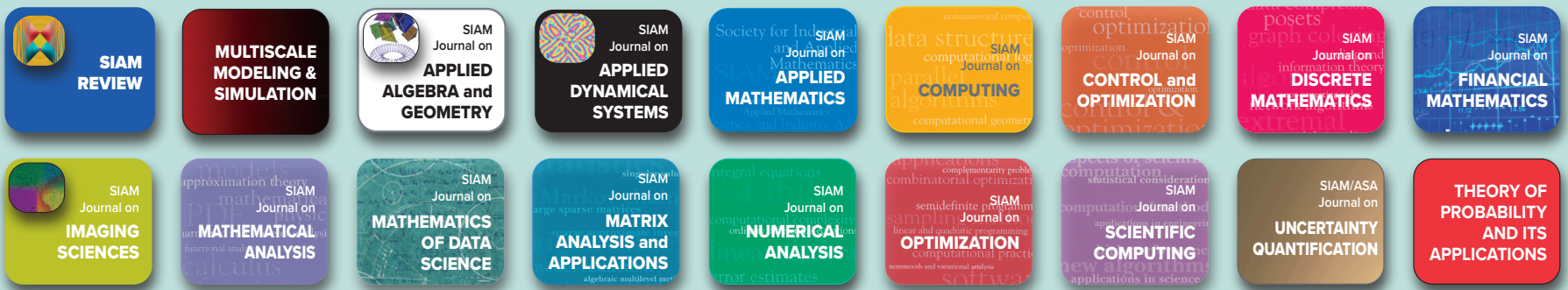
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