MATH MATTERS

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The math behind... Smoke Animation

Technical terms used:

Incompressible fluid, smoke ring flow, vortex filament, (discrete) integrable system

Uses and applications:

In fluid dynamics and computer graphics, an incompressible fluid is often modeled as a finite collection of vortex filaments. Fast and physical correct algorithms based on a structure-preserving discretization of the motion of these filaments are used to illustrate smoke in computer games and motion pictures.

How it works:

Due to the Biot--Savart law, the velocity field of an incompressible ideal fluid is uniquely determined by its vorticity, which describes the rotation around a point of the fluid. The typical dynamic behavior of smoke rings arises when the vorticity is concentrated in small tubular neighborhoods of closed curves, called vortex filaments. Conversely, realistic flows can be accurately approximated by computing the motion of the filaments.

The time evolution of infinitely thin filaments is described by a differential equation known as the smoke ring flow. Now, the key to discretize this dynamical system is to preserve its integrable structure. Whereas there is no universal notion of integrability in the modern theory of integrable systems, discrete integrable equations can be commonly characterized by their multidimensional consistency.

Discretizing the time direction yields a doubly discrete smoke ring flow. Noting that the smoke ring flow models only close range effects of the vortex filaments, an additional term is added to describe effects at the long range. Finally, moving obstacles are incorporated into the model. Actually, the obstacles can be replaced by a set of certain vortex filaments, leading to a fast and physically correct simulation driven purely by filaments.

Interesting fact:

Vehicle exhausts and smoke columns in the computer-animated film Megamind (2010) were produced using an algorithm based on recent research on a filament-based model of smoke with arbitrary moving obstacles.

References:

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S. Weißmann and U. Pinkall, Filament-based smoke with vortex shedding and variational reconnection. ACM Trans. Graph. 29(4), 115:1-115:12 (2010).

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