

Synthetic Biology, Real Mathematics

By Dana Mackenzie

Imagine cellular factories churning out biofuels. Nanomachines hunting out cancer cells. Machines growing from living tissues. These were some of the visions proposed at the start of the new millennium by the founders of a subject they called *synthetic biology*. It was like genetic engineering but more ambitious: Instead of merely splicing a single gene from one organism into another, synthetic biologists would design entire new biological circuits from scratch.

Fourteen years later the revolution is still going on, but with tempered expectations. Living cells have proved to be more unpredictable than electric circuits. In waves of hype and anti-hype, synthetic biology has been portrayed as either a visionary technology or a bust.

One thing, though, has remained constant. Since its inception, synthetic biology has embraced mathematics in an attempt to fast-forward past the tinkering stage to the

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Life Sciences 2014

Modelling Plant Cell and Tissue Growth

By Oliver E. Jensen

Until a few years ago I knew very little about plants, despite the fact that—as harvesters of solar energy, atmospheric regulators, and a primary food source—they underpin our existence on this planet. My turning point came in 2007, when I had the good fortune to join a systems biology project at the University of Nottingham, UK (www.cpib.ac.uk), created to develop a “virtual root.” This opened my eyes to the remarkable ways in which even the most humdrum weed is able to exploit and adapt to its environment. The project also turned out to be an excellent testbed for the development of multiscale mathematical models connecting mechanics to biology.

Plant growth is tempered by numerous signals. Underground, a root can respond to gravity, water gradients, distribution of nutrients such as phosphate and nitrate, and soil stiffness. The hope for the virtual root project is that an understanding of how a plant senses, integrates, and responds to these competing signals will contribute to the development of improved crop species that can thrive in challenging climatic conditions and harsh agricultural environments.

Mathematicians, engineers, and computer scientists have an important role to play in this endeavour. Quantitative descriptions of growth and biological development are essential if we are to get to grips with the full complexity of living organisms. At the same time, to avoid problems of computational intractability, we need efficient techniques to handle interactions spanning widely disparate space- and time-scales. And alongside the traditional components of systems biology—intricate networks describing interactions between genes and proteins within individual cells—it is essential to account for the spatial organisation of multiple cells in a tissue. This requires consideration of the transport of signals between cells, and of the physical forces that provide mechanical integrity to an organ and ultimately drive its growth.

A natural focus for our project was the species *Arabidopsis thaliana*, a type of cress that—like the fruit fly in animal biology—is intensively studied because it grows easily and reproduces quickly. Its primary root provides a beautifully organised system for study (Figure 1, left). Stem cells near

the root tip divide and differentiate into a small number of cell types, which form tightly adherent concentric layers. Each cell draws in water by osmosis; a stiff cell wall resists inflation, allowing the cell’s internal turgor pressure to rise to a level comparable to that of a car tyre. The cell grows by softening the cell wall, allowing turgor-driven expansion. To stop a cell from inflating like a party balloon, arrays of stiff cellulose fibres embedded in the cell wall ensure that growth is anisotropic: Fibres, wrapped like hoops around a barrel, inhibit radial expansion of a cylindrical cell but allow axial elongation. In this way, a primary root, formed of aligned cylindrical cells (or osmotic micropumps), can expand rapidly along its length in an essentially one-dimensional manner.

To model growth, we addressed processes

occurring at four distinct spatial levels: an individual cell wall, a single cell, multiple cells in a single cross-section of a root, and multiple cross-sections along the whole axis of the root. Motivated by a simple model dating back to the mid-1960s, we found that a growing cell or tissue can be well described in mechanical terms as viscoplastic: Provided that a yield criterion is satisfied, growth can be modelled as an irreversible stretching deformation that proceeds at a rate characterised by a set of “extensibility” parameters. We adopted a variety of multiscale modelling techniques to derive yield and extensibility parameters appropriate to the different scales. Naturally, geometric factors play an increasingly important role (alongside mechanical properties) as the spatial scales increase.

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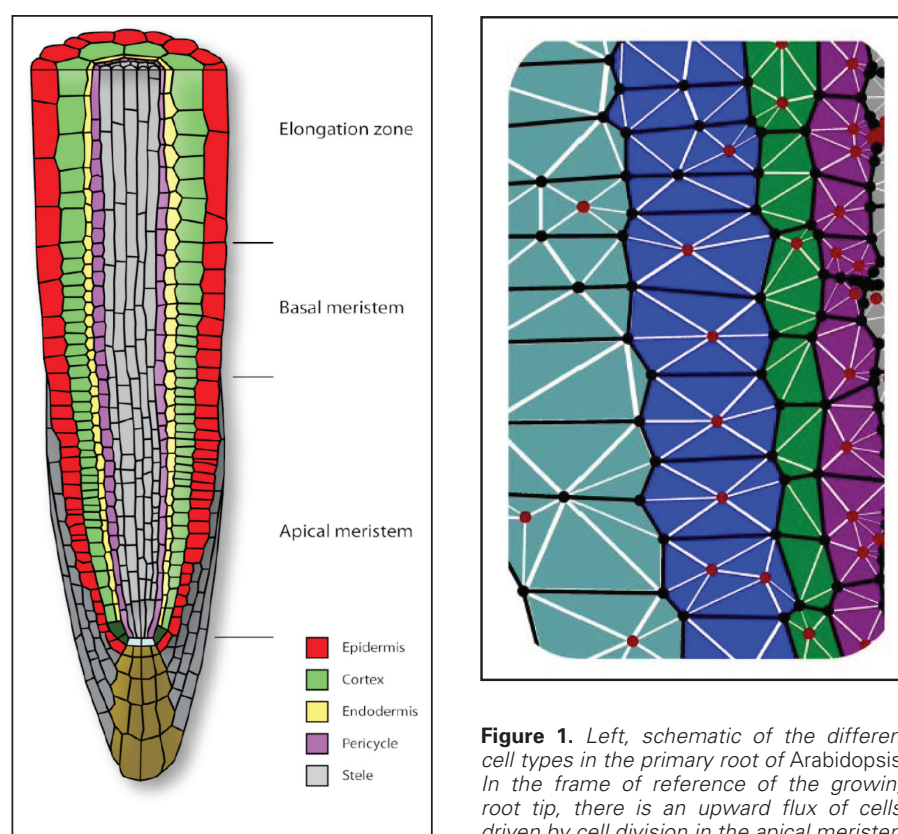


Figure 1. Left, schematic of the different cell types in the primary root of *Arabidopsis*. In the frame of reference of the growing root tip, there is an upward flux of cells, driven by cell division in the apical meristem and by cell expansion in the elongation zone.

The spatial organisation of cells along the root axis also reveals the cells’ life-histories, with short young cells near the root tip and longer older cells further behind. Gravity-sensing cells are among the brown cells at the root tip. Reprinted from *Trends in Plant Science*, 14/7, Benjamin Péret et al., *Arabidopsis lateral root development: An emerging story*, 399–408, 2009, with permission from Elsevier.

Right, in computational simulations of cell expansion, individual cells are represented as polygons (in a two-dimensional projection), and mechanical properties are ascribed to the cell walls; walls in the plane of the simulation are divided into elements that account for the reorientation of embedded cellulose microfibrils (from Fozard et al., 2013). Use of realistic cell geometries is important for accurate simulation of the transport of regulators like auxin.

The Complexity of Entanglements

By Jean-Luc Thiffeault

Complex entanglements occur everywhere in nature: in human hair, in the threads forming hagfish slime, in carbon nanotubes, in DNA, and in countless other settings. These entanglements involve physical strands, fibers, or polymers. The character of the entanglements is important, as it often affects the macroscopic integrity of the materials.

In certain other cases, the strands being entangled are more ethereal. For instance, researchers have for many years viewed magnetic field lines in solar flux tubes in the spirit of entanglements. They compute the mathematical braid formed by the field lines, and attempt to relate the complexity of the entanglement to the occurrence of solar flares [3, 15]; see Figure 1.

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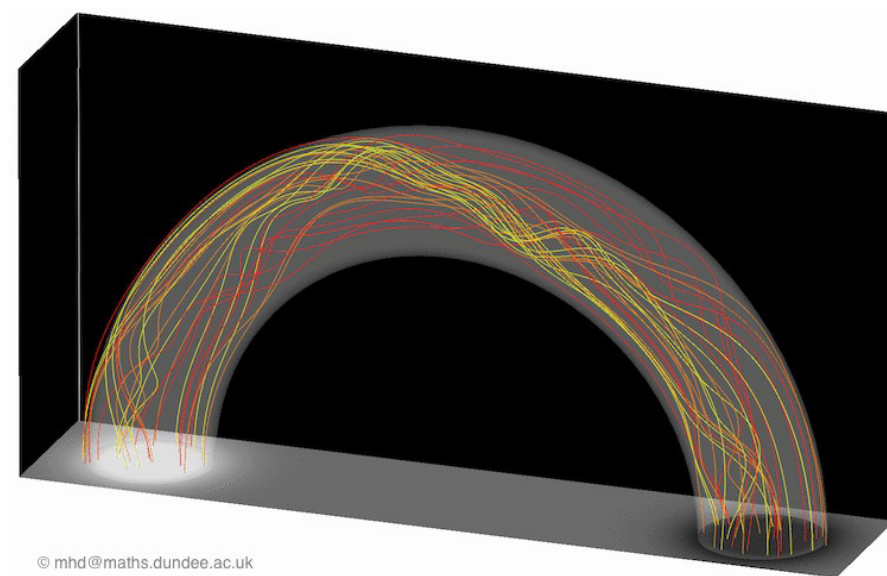
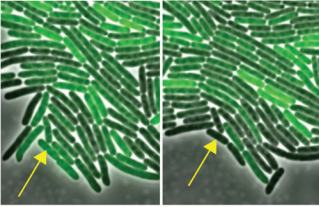


Figure 1. Model of tangled magnetic field lines representing a solar coronal loop. Courtesy of the Dundee MHD group; <http://www.maths.dundee.ac.uk/mhd/>.

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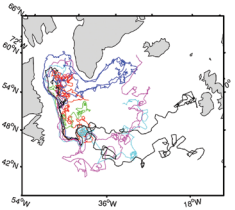


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Modelling Plant Cell and Tissue Growth

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The Complexity of Entanglements



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Mathematics of Electricity Markets Under Uncertainty

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A Role for Modeling, Simulation, and Optimization in an Agricultural Water Crisis

For a 2011 workshop on sustainability, AIM deputy director Estelle Basor solicited a problem from a berry-farming cooperative in drought-stricken California. Lea Jenkins and Kathleen Fowler addressed the problem by developing a computational farming tool, combined with optimization algorithms for selecting multiple-year planting and harvesting strategies.

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Volcanic Activity as Trigger of Climate Change

It's well known that major volcanic eruptions are often followed by decreases in global temperatures. "The arrow of causality points both ways," James Case writes, reviewing a book premised on the idea. Mathematical models now being developed explain the mechanisms by which climate change can lead not only to volcanic activity, but also to tsunamis and earthquakes.

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Optimization 2014

Mathematics of Electricity Markets Under Uncertainty

By Andy Philpott

The last twenty years have witnessed a transformation in the development of market mechanisms for supplying electric power. Applied mathematics continues to make major contributions in both understanding and implementing these market mechanisms. This article presents a personal perspective on the role of applied mathematics in this area, as well as a selected set of contributions and promising directions.

Studying market mechanisms for electricity is simplified by its being a single commodity, but complicated by features that make standard economic models difficult to apply. To begin, electricity is not easily storable, and the flow of power over a transmission network must satisfy certain laws of physics. The optimal power flow problem of meeting active and reactive power requirements at least cost is formulated with complex variables and is highly nonlinear. Scheduling power dispatch of large generation units must also satisfy unit-commitment constraints that involve costs and delays for starting turbines or ramping them up or down. All these features lead engineers toward system optimization problems of some complexity that, in their most general form, are not convex. Nevertheless, powerful optimization techniques (e.g., mixed integer programming for capacity expansion and unit commitment [8], or semi-definite programming relaxations for optimal power flow [10]) have been developed to attack these problems.

Electricity market mechanisms attempt to replace such system optimizations by individual optimizations. The goal of the market designer is to recover the system optimal solution, but to do so by providing incentives rather than control. The incentives are intended to produce an optimal supply of power in the short run (from the cheapest plants), as well as to provide sufficient profits to cover the long-run costs of generation, including the construction of new generating plants when and where needed. Ideally, incentives would give a system optimal solution in both the short and the long run, but this is not typically possible. If the underlying short-run system optimization problem is not convex, for example, some approximation is needed to make it so and thus allow its decomposition into agent problems.

Even when the system optimization problem is convex, if generators can act as price setters in these markets, the exercise of their market power often leads to Nash equilibrium solutions that incur losses of overall welfare. In other words, acting in their own self-interest, generators seek to withhold generation so as to increase prices above competitive levels; this leads to an equilibrium at which no generators are

willing to unilaterally alter their generation levels. Although the result is increased profits for generators, the losses to consumers often exceed this gain. Many papers in the applied mathematics literature deal with this phenomenon not only in electricity, but also in telecommunications, road traffic, supply chains, and other applications.

Most practical questions about market power in electricity markets are asked *ex ante*. Market designers prefer to model a market mechanism before testing it in the real world, when real money is at stake. Different mechanisms produce different overall efficiencies, but also different transfers of wealth between agents.

It has been known for some years that imperfectly competitive generation and distribution of electricity in constrained transmission networks are difficult to model *ex ante*. In a classic paper, Borenstein, Bushnell, and Stoft [3] showed that for a simple two-node example with a constrained transmission line, there is no pure-strategy Nash equilibrium for two identical generators located at each node. When the line capacity is small, each generator has an incentive to reduce its output less than the competitor. Each generator's node imports power and constrains the line to yield monopoly profits. Because both agents have this incentive, no symmetric Nash equilibrium exists unless they adopt mixed strategies. In certain cases uncertain demand serves to smooth out the discontinuities that lead to failure in the Borenstein example, and it is possible to obtain pure-strategy Nash equilibria in supply-function equilibrium [9], but these results seem to be difficult to scale to real transmission networks.

A different pathology might occur when a pair of agents are at the same location but share a transmission line to consumers. If the line has a capacity constraint, the problem becomes one of *generalized Nash equilibrium* [6]—that is, an equilibrium problem in which the feasible set of actions (as well as the payoff) of an agent depends on the other agents' strategies. An equilibrium might no longer be unique in this setting, as for some range of λ , an action by one agent that uses a proportion λ of the line capacity can be matched by a best response by the other that uses the proportion $(1 - \lambda)$, defining a continuum of equilibria. Much progress has been made in understanding the mathematics of problems of this type, led by Jong-Shi Pang [12] and Francisco Facchinei [5].

Several approaches can be taken to guarantee unique pure-strategy Nash equilibria in problems with transmission networks. One is to ignore the transmission network and treat all agents as being at a single location. Such models, well studied in the literature, lead to useful insights about the features of different market mechanisms but

suffer from some lack of realism. Another approach is to include a transmission network in the model, but impose conditions on the rationality of the agents that restrict them in anticipating congestion; an example can be found in [20]. A weakness in this approach is that it is hard to justify a method for choosing the limitation on rationality, and different choices can yield very different equilibrium outcomes.

A third possibility, and our focus henceforth, is to assume that agents are perfectly competitive. At first sight, this might appear to be a backward step. Because perfect competition is an ideal that won't occur in practice, models that assume it are unlikely to give good predictions when agents behave strategically—unless the system implemented computes marginal prices from a model that computes a socially optimal plan, and then requires agents to trade at these prices. This approach, called *market socialism* by economists, is essentially the electricity spot market design used in a number of regions with abundant hydro energy (such as Brazil and Chile).

Other regions with lots of hydro energy, like New Zealand and the Nordpool countries, have deregulated electricity pools. A reason to study perfectly competitive markets in these settings is to provide some *ex post benchmark* for the performance of a real market. Market data, even if not made public, should be available to the regulator. If observed prices are significantly above perfectly competitive benchmarks, a regulator has some evidence to support investigations of possible market manipulation. Social planning optimization models and the system marginal prices they yield (as Lagrange multipliers) are obvious candidates for competitive benchmarks.

Use of optimization models to benchmark electricity market performance needs an important caveat: The socially optimal outcomes delivered in theory by market socialism do not always coincide with a perfectly competitive equilibrium. If enough market instruments and agents are risk-neutral, then an equilibrium under perfect competition will correspond to a socially optimal solution. Having enough market instruments is a *market-completeness* assumption. A well-known example of market incompleteness was identified in Brazil when different agents owned generating stations at different points on the same river. As shown by Lino et al. [11], such situations can lead to a loss of welfare in competitive equilibrium unless the market is completed with contracts that enable the agents to trade in the water they use for generating electricity.

A more subtle form of incompleteness can result from risk aversion. In this case, agents might accept the same probability distribution for uncertain events, but have

See **Electricity Markets** on page 6

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A Role for Modeling, Simulation, and Optimization in an Agricultural Water Crisis

By Eleanor Jenkins and Kathleen Fowler

The authors have been working together since 2011 on a problem to do with agricultural sustainability. What brought them together was a workshop at the American Institute of Mathematics (AIM). For last summer's SIAM Annual Meeting, they organized a minisymposium, The Mathematics of Sustainability, in which they and others discussed particular aspects of the problem. Here, on the invitation of SIAM News, they tell the story of their ongoing collaboration.

Water crises in the western U.S. have been making headlines, with recent stories on California and Kansas appearing on PBS, NPR, CNBC, and CNN, as well as in *The New York Times*, *USA Today*, and nearly all of the major news outlets. Many of the stories have focused on the impact of the droughts on the local agricultural industry.

The problem is one that's all too familiar to Estelle Basor. Having grown up in the Pajaro Valley region of California, she knows the strong connections between the region and farming. In "Raspberry Fields Forever," her contribution to the 2013 Mathematics of Planet Earth blog, Basor gives a synopsis of her family's farming history and her efforts to help ensure the long-term viability of the agricultural industry in the valley.

Basor also happens to be the current deputy director of AIM (one of the National Science Foundation-supported mathematics institutes). As chief organizer of AIM's 2011 Workshop on Sustainability Problems, she solicited a problem from Driscoll's, a berry-farming cooperative in California; the problem was to assess the impact of several proposed changes in farming practices on the profitability of the berry farms and the use of groundwater resources.

The Pajaro Valley region is responsible for nearly 60% of the strawberries produced in the United States in a given year, and Driscoll's is one of the main farming cooperatives in the valley. Berries are water-intensive crops, and farmers in California rely heavily on groundwater resources to meet their irrigation needs—meaning

that the health of the underlying aquifer is directly tied to their livelihood. Studies have shown that agriculture, along with urban water usage and climate change, has damaged the aquifer in recent decades, causing seawater intrusion along the coast and removing near-shore water supply wells from use. In turn, the cost of water has risen, putting further stress on farmers, who rely heavily on groundwater pumping for irrigation in the face of three consecutive years of below-average rainfall coupled with above-average temperatures.

The Pajaro Valley Water Management Agency, which monitors and regulates the water resources in the region, has worked tirelessly to preserve and manage the available water; part of the agency's long-term management plan is a proposed irrigation limit for farmers. Operating under this limit will allow for recovery of the depleted aquifer, which currently has significant saltwater intrusion into near-coast drinking water supply wells. The irrigation limit, however, requires farmers to reassess their crop selections and irrigation strategies.

The farmers and Driscoll representatives came to the workshop with several ideas for coping with the reduced water limits, but they needed help in determining which idea, or combination of ideas, would provide the best results for their long-term profitability. With other applied mathematicians and engineers, we collaborated with the representatives to evaluate the possible effects of the proposed changes on water usage and the local farming economy. Our team, consisting of researchers from academia and industry, brought a truly multidisciplinary approach to the problem.

The practices suggested by the farmers included fallowing land, varying crop-rotation strategies, building aquifer recharge networks to capture rainfall for infiltration into the basin, and modifying irrigation



Photo by M'Liss Hinshaw

Berries are water-intensive crops, and farmers in California rely heavily on groundwater resources to meet their irrigation needs—meaning that the health of the underlying aquifer is directly tied to their livelihood.

techniques. In the course of the 2011 workshop, and over the ensuing months and years, we have used a multi-level modeling approach, combining simulation and optimization, to provide insights and offer strategies to farmers in the community.

To better understand the trade-offs between the various stakeholders in the region, we began by developing a computational farming tool, combining it with optimization algorithms for selecting planting and harvesting strategies over a multiple-year window. We used data provided by the local farmers to formulate the problem, allowing for multiple crops and incorporating constraints to model the planting rules (e.g., the planting or harvesting of crops only in certain months). The objectives for the optimization were to maximize profitability while minimizing water usage, which, based on the crop specifications, can be in opposition. We recently coupled a large-scale, multi-physics farm and groundwater simulator to an optimization suite. This allows us to study more realistic scenarios and accurately model a fully integrated environment. We presented our results to several farmers in the region; they

have used our analysis to adjust their crop selection, incorporating crops that are less water intensive, for the near future (http://www.nsf.gov/discoveries/disc_summ.jsp?cntn_id=131827).

It is clear that mathematics plays a key role in an understanding of critical environmental issues, and in particular of sustainability problems. At the 2011 AIM workshop, the other problems focused on the modeling of systems that provide their own energy, the storage and deployment of energy produced from renewable sources, and the modeling of turbines capable of producing energy from waste heat. Our session at the 2014 SIAM Annual Meeting, the Mathematics of Sustainability, included talks on farming practices, on the design of aquifer recharge networks to replenish depleted water resources, on renewable energy, and on modeling efforts under way for water-quality assessments in the Saint Lawrence Seaway. A common theme is the critical need for interdisciplinary teams from science, engineering, and computational applied mathematics to address such problems, in partnerships with industry, policy-makers, and practitioners.

Acknowledgments

We have been fortunate to work on a problem that has sustainability as its goal. We are grateful for support from AIM, and Estelle Basor in particular, for giving us the means to gather our team for our annual sessions in Palo Alto. We are also grateful to our main team members, Stacy Howington and Matthew Farthing from the U.S. Army Corps of Engineers, Engineering Research and Development Center, and John Chrispell, from Indiana University of Pennsylvania, for our progress thus far. Tsvetanka Sendova of the Michigan State University Department of Mathematics and numerous students—Mark Minick, Matt Parno, Stephen Carter, and Corey Ostrove—have also contributed to this work. We hope to continue our work and apply this analysis to other regions facing similar stresses.

Eleanor Jenkins is an associate professor in the Department of Mathematical Sciences at Clemson University. Kathleen Fowler is an associate professor of mathematics at Clarkson University.

Volcanic Activity as Trigger of Climate Change

Waking the Giant: How a Changing Climate Triggers Earthquakes, Tsunamis, and Volcanos. By Bill McGuire, New York, Oxford University Press, 2012, xiv+303 pages.

Bill McGuire is a professor of geophysical and climate hazards at University College London. He was a member of the UK Natural Hazard Working Group, established in January 2005 following the Indian Ocean tsunami, and was appointed in 2010 to the UK Scientific Advisory Group for Emergencies to address the volcanic ash problem caused by the Eyjafjallajökull eruption in Iceland that spring.

As a volcanologist, McGuire is keenly aware of the ways in which volcanic activity affects the global climate. Indeed, after a major eruption, global temperatures typically decline by several degrees, due to absorption and the increased reflectivity of the resulting cloud cover, and sometimes require years to recover. In this, his fifth book, McGuire argues that the arrow of causality points both ways. Volcanic activity causes climate change, and climate change can trigger lithospheric (including volcanic) activity. Mathematical models purporting to explain the mechanisms involved are under development.

In 1977 McGuire first visited the Valle

del Bove (Valley of the Oxen), a gigantic amphitheater carved out of the eastern flank of Mount Etna, in Sicily, to gather data for his doctoral thesis. Knowing that rainfall in the Mediterranean had increased significantly as ice age glaciers receded, he guessed that the eastern flank of Mount Etna had become saturated with water, causing (as in the Los Angeles area during an El Niño spring) a mudslide of epic proportions and—in all probability—a catastrophic tsunami. Confirmation of such a hypothesis might begin with a search for evidence of the “lost tsunami.”

Recently, a team of volcanologists employed a nonlinear PDE model of surface wave propagation to simulate such an event [2]. Assuming that about 8000 years ago a 25-cubic-kilometer landslide moving at roughly 360 km/hour entered the Ionian Sea in a matter of minutes, the team estimated the effects of the resulting waves on the soft sediments carpeting much of the Ionian seafloor, along with shoreline effects in the eastern Mediterranean. Although sea levels have risen about 10 meters in the last 8000 years, substantial quantities of evidence are still to be found.

Joseph Boussinesq derived the first relevant partial differential equation in 1872, in an effort to explain the experimental results

obtained by John Scott Russell regarding solitary traveling waves (solitons) in canals. Because a number of similar equations have since been derived, students of the subject now refer to equations of “Boussinesq type.” It was realized around 1980 that anticipated increases in computing power would soon make it possible to analyze and predict, affordably and with considerable accuracy, many instances of nearshore hydrodynamic behavior, including wave propagation and shoaling, wave/current interaction, wave breaking, and the

generation of nearshore circulation.

Derivation of an equation of Boussinesq type begins with expansion of the vertical and horizontal components of flow velocity (or potential) in Taylor series around a reference elevation, truncated after a few terms; the equations expressing the irrotational and incompressible nature of the flow are exploited to eliminate vertical partial derivatives in favor of horizontal ones. The result is a single equation involving a single

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BOOK REVIEW

By James Case

SIAM Elects Seven Board and Council Members for 2015–2017

As this issue of *SIAM News* goes to press, members' votes in the 2014 election have been tallied.

With three available seats on the SIAM Board of Trustees, the membership re-elected Mary Ann Horn of Vanderbilt University and the National Science Foundation and Tamara Kolda of Sandia National Laboratories. Newly elected to the board was Russel Caflisch of UCLA, where he has been a professor of mathematics since 1989 and, beginning in 2008, director of IPAM (the Institute for Pure and Applied Mathematics). He was named a SIAM fellow in 2009.

The four members elected to the council include one incumbent—Rachel Kuske of the

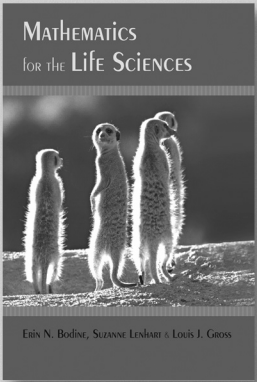
University of British Columbia—and three new people:

Raymond Chan, a SIAM fellow (2013), chairs the mathematics department at the Chinese University of Hong Kong.

Geoffrey McFadden, a fellow of the National Institute of Standards and Technology and of SIAM (2012), works mainly in materials science.

Padma Raghavan, associate vice president for research and director of strategic initiatives at Penn State, was named Distinguished Professor of Computer Science and Engineering in 2012.

A complete look at the SIAM leadership as of January 1, 2015, will appear next year.

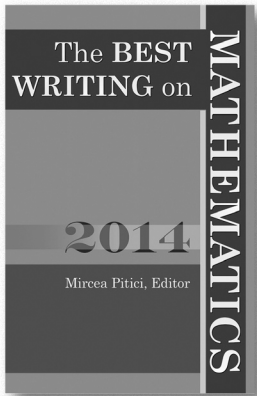


Mathematics for the Life Sciences

Erin N. Bodine, Suzanne Lenhart & Louis J. Gross

“This is the book I always wanted to write, a masterful and thorough introduction to the basic mathematical, statistical, and computational tools one needs to address biological problems, punctuated with solid and motivational applications to biology. The book is a seamless and authoritative treatment, with broad scope, that makes an ideal text for an introductory course.”
—Simon A. Levin, editor of *The Princeton Guide to Ecology*

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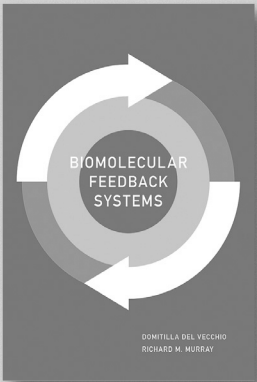


The Best Writing on Mathematics 2014

Edited by Mircea Pitici

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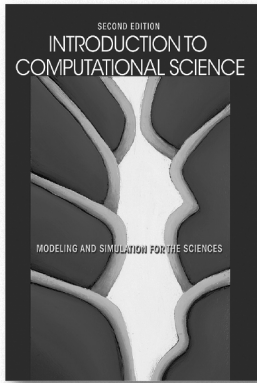


Biomolecular Feedback Systems

Domitilla Del Vecchio & Richard M. Murray

“This is an excellent compendium of the most important techniques and results in the application of feedback and control to biomolecular systems. *Biomolecular Feedback Systems* is very timely, and a must-read for students and researchers.”
—Ernesto Estrada, University of Strathclyde

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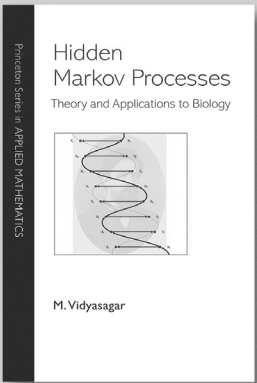
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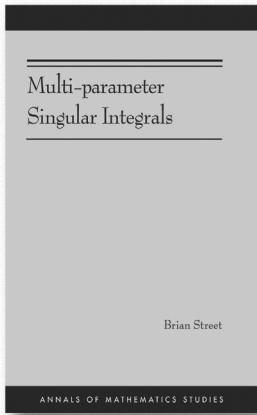


Hidden Markov Processes

Theory and Applications to Biology
M. Vidyasagar

“This book provides a terrific introduction to an important and widely studied field—Markov processes (including hidden Markov processes)—with a particular view toward applications to problems in biology. With a wonderful balance of rigor, intuition, and choice of topics, the book gives a unique treatment of the subject for those interested in both fundamental theory and important applications.”
—Sanjeev Kulkarni, Princeton University

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Multi-parameter Singular Integrals

Brian Street

This book develops a new theory of multi-parameter singular integrals associated with Carnot-Carathéodory balls. Brian Street first details the classical theory of Calderón-Zygmund singular integrals and applications to linear partial differential equations. He then outlines the theory of multi-parameter Carnot-Carathéodory geometry, where the main tool is a quantitative version of the classical theorem of Frobenius. Street then gives several examples of multi-parameter singular integrals arising naturally in various problems. The final chapter of the book develops a general theory of singular integrals that generalizes and unifies these examples.

Annals of Mathematics Studies, 189
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Climate Change

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unknown function of (one or two) horizontal space variables representing the surface elevation above the reference.

With additional mild assumptions, Boussinesq’s original equation can be reduced to:

■ the Kortweg–de Vries (KdV) equation for wave propagation in one horizontal space dimension,

■ the Kadomtsev–Petviashvili equation for wave propagation in two horizontal space dimensions, and

■ the nonlinear Schrödinger equation.

The main predictions of this “lost tsunami” model are supported by a wealth of submarine evidence, including core samples from Holocene turbidite deposits at the bottom of the Ionian Sea, and now-submerged (yet well-preserved) remains of the neolithic settlement known today as Atlit-Yan, located some hundred meters offshore from the current coast of Israel. The site shows evidence of hasty abandonment, including—beneath a protective coating of clay—a quantity of fish already gutted and sorted according to size, for anticipated consumption or trade. McGuire describes a number of other prehistoric undersea landslides, the largest being the so-called Storegga landslide and tsunami, which occurred off the coast of Norway near the end of the last ice age.

Needless to say, the effects of another such tsunami—were it to occur today—would be devastating. The most imposing current threat appears to come from the Cumbra Vieja volcano, on the island of La Palma, in the Canary Islands. The seaward flank of that still active volcano appears to be unstable, and a computer model of the tsunami that can be expected from a future collapse has been constructed [3]. The worst-case scenario predicts that a huge initial dome of water, about three-quarters of a kilometer high, would form within two minutes of failure. Within ten minutes, the shores of nearby islands would be struck

events—it gets the timing right but fails to predict the magnitude of the recorded shocks. Subsequent research by the U.S. Geological Survey suggests, however, that the New Madrid quakes may have been less powerful than previously believed, possibly falling within the range predicted by the model.

Discussing volcanic eruptions as yet another possible consequence of climate change, McGuire points out that twenty thousand years ago, Iceland was completely covered by an ice sheet more than 800 meters thick. As the ice melted, the depressed lithosphere rebounded by as much as 450 meters, dramatically decreasing the pressure on the underlying strata and thereby unleashing a 30-fold increase in magma production. The resulting burst of volcanic activity lasted more than a millennium. Much the same reaction might be expected from the Grimsvötn volcano, currently resting beneath the Vatnajökull ice cap, should the latter continue to melt at anything like the present rate.

Most of the world’s volcanos are located near the Pacific rim, close to the water’s edge. Rising sea levels, as they press down on the base of a volcano, seem to be able at times to force magma out of them, much as toothpaste can be forced out of a tube. McGuire cites evidence of such forcing on the Pavlof volcano, in the Aleutian peninsula. Rising sea levels there each autumn apparently irritate Pavlof. During the period from 1973 to 1998, 13 of the volcano’s 16 eruptions occurred between September and December, when sea levels are at their highest.

Rising sea levels threaten the stability of faults separating adjoining tectonic plates, as, for example, the San Andreas fault separates the Pacific and the North American plates. A recent study [1] from the Scripps Institution of Oceanography concluded that the bending of the lithosphere resulting from a large rise in sea level could “unclamp” the San Andreas fault, possibly unleashing “the big one” so often evoked by Californians. Similar conclusions apply to the Anatolian

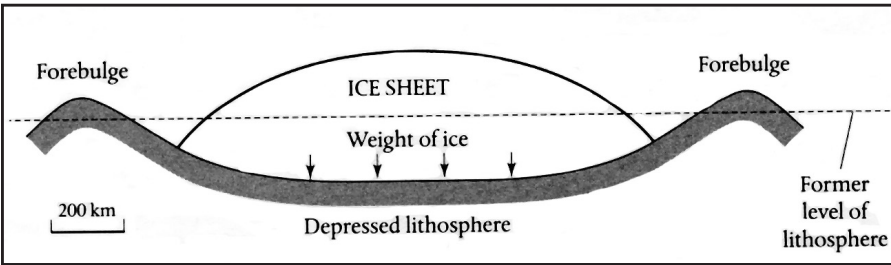


Figure 1. Bouncing back. As an ice sheet melts, the depressed underlying lithosphere rebounds, forcing the surrounding ridge—the “forebulge”—to migrate outward. The phenomenon could explain otherwise mysterious earthquakes in North America—including those near New Madrid, Missouri (1811), and Charleston, South Carolina (1886). Figure from Waking the Giant.

by a series of immense waves; perhaps an hour later, waves several stories high would reach the northwestern coast of Africa. After 12 hours, the eastern Caribbean would be deluged, and waves as high as 8 meters would inundate the coast of Florida.

Earthquakes can also be triggered by climate change. As thick ice sheets form, the underlying lithosphere is depressed. The resulting stiffness causes the ground immediately surrounding the sheet to flex upward, forming a boundary ridge, or “forebulge,” as shown in Figure 1. As the ice melts, the depressed portion of the lithosphere rebounds, forcing the surrounding ridge to migrate outward—albeit at glacial speeds—away from the former margins of the ice sheet. As applied to North America, this effect could explain not only the otherwise mysterious earthquakes near New Madrid, Missouri, in December 1811—and continuing into February 1812—but also the large and damaging quake that struck Charleston, South Carolina, in 1886.

A recently developed finite element model [4] seeks to predict the arrival of such post-glacial pulses of seismic activity at various places in North America. Although it predicts the timing and magnitude of the prehistoric quakes known to have occurred in Quebec and Indiana, the model has difficulty with the New Madrid

fault, which transects northern Turkey and was the site of the Izmit earthquake that killed 17,000 people in 1999.

It is too soon to estimate the extent of the damage to be expected from human-produced global warming. But McGuire has begun to catalog the more obvious dangers, and suggests that mathematical modeling—together with the fieldwork performed by earth scientists—seems to be capable of assessing the associated risks.

References

[1] K. Luttrell and D. Sandwell, *Ocean loading effects on stress at near shore plate boundary fault systems*, J. Geophys. Res., 115 (2010), B08411; doi:10.1029/2009JB006541.
[2] M.T. Pareschi, E. Boschi, and M. Favalli, *Lost tsunami*, Geophys. Res. Lett., 33:22 (2006), L22608; doi:10.1029/2006GL027790.
[3] S.N. Ward and S.J. Day, *Cumbre Vieja volcano—potential collapse and tsunami at La Palma, Canary Islands*, Geophys. Res. Lett., 28 (2001), 397–400.
[4] P. Wu, and P. Johnson, *Can deglaciation trigger earthquakes in North America?*, Geophys. Res. Lett., 27 (2000), 1323–1326.

James Case writes from Baltimore, Maryland.



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Synthetic Biology

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engineering stage. “You need something that goes beyond intuitive understanding, something that is quantitative. Only mathematical models can really provide that,” says Krešimir Josić, a mathematician at the University of Houston. While the equations might look familiar, biology has a way of putting a new twist on some classical problems of circuit design and electrical engineering.

Mathematical Beginnings

In the late 1990s, James Collins of Boston University, along with then-graduate student Tim Gardner, set out to “forward engineer” genetic circuits. At the time, the Human Genome Project was turning up many new gene networks inside human cells. Biologists were trying to reverse engineer some of those networks to figure out what they did and how they did it.

Collins, who was trained as a physicist, thought it made more sense to go in the other direction. He proposed to forward engineer simple gene networks whose function is known and whose behavior is predictable. For anyone with a background in computer science or electrical engineering, the benefits of such an approach are obvious. Every computer is designed from innumerable small parts—switches, logic gates, timers. Why not build biological systems the same way? The information gleaned in the engineering of synthetic parts could also help biologists understand natural networks.

At a key moment in his and Gardner’s work, Collins says, former SIAM president Martin Golubitsky, current director of the Mathematical Biosciences Institute at Ohio State University, visited their lab for a yearlong sabbatical. “He encouraged us to think about the simplest mathematical models possible,” Collins says. Before you can run, you have to walk. And before you can walk, you have to be able to switch a cell on and off.

In the end, Collins and Gardner focused on one of the simplest systems possible—a genetic toggle switch, comprising two genes. “Each one has a promoter, a strip of DNA that’s basically an on switch for the gene,” Collins explains. “You engineer it so that the protein produced by gene A wants to bind to the on switch for gene B and turn it off, and vice versa.”

The system can be modeled with two differential equations, analogous to the equations describing two competing species. It’s a well-known theorem that under many conditions, one species (or in this case, one gene) will win. If it is to behave as a switch, the system needs to have two stable equilibria, one with gene A on and gene B off, the other with gene B on and gene A off. It also needs a chemical trigger that will move it from one equilibrium state to the other.

“What surprised us was how hard it was

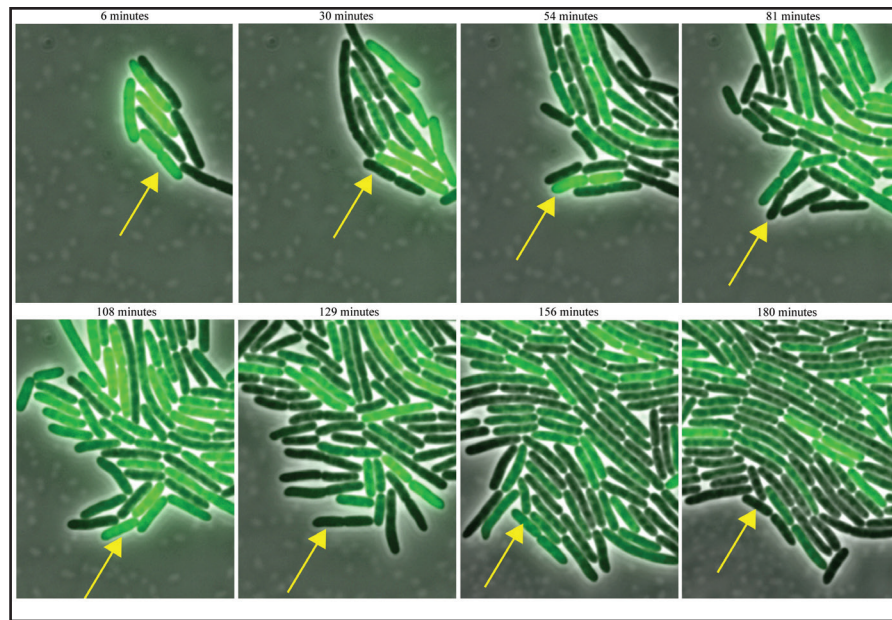


Figure 1. An *E. coli* bacterium (arrow) equipped with a synthetic genetic circuit blinks on and off, as the circuit activates and deactivates a green fluorescent protein. A mathematical model helped to predict a mutation that keeps the genetic clock running at a steady tempo, independent of temperature. Image courtesy of the Bennett Lab, Rice University.

experimentally to find regions of bistability,” Collins says. “Our mathematical models and our biological understanding are not as good as we thought, and not as good as they should be to do predictive design.”

Collins’s team introduced a gene into the human intestinal bacterium *E. coli* that would enable it to produce a green fluorescent light; they also put in a DNA switch that would turn the fluorescent protein on and off. After nine months and plenty of false starts, the system finally worked. Their paper was published in *Nature* in 2000, right next to a paper by a Princeton University team, Michael Elowitz and Stanislas Leibler, who had engineered a simple oscillating circuit in the same bacterium. The two papers are still universally cited as the starting point for synthetic biology.

But several not-so-obvious problems would slow progress in the field. Electrical circuits can be wired up from off-the-shelf parts that work together in a predictable way. This is still far from the case for gene circuits. “Pretty much every sophisticated circuit that was published in *Nature* or *Science* probably took four or five years to assemble, fine tune, and test,” says Domitilla Del Vecchio, a dynamical systems theorist at MIT. The Princeton group’s oscillator is not robust—it often fails to work, depending on conditions in the cell. As for Collins’s switch, it works qualitatively the way the equations predict, but “in a quantitative way it’s far away,” Josić says. By now, hundreds of simple gene circuits have been produced, but many of them have the same problems—they are poorly documented and finicky, and the models describing them are inaccurate.

Another problem that has become increasingly evident is what synthetic biologists call “context-dependent behavior.” Gene networks may work differently, or not at all, in different cells. And different genetic circuits can interfere with each other.

Engineers are used to the fact that electrical circuits are modular: Different components either do not interfere with one another, or they interact in predictable ways. So far, the jury is out on whether biological systems can be coaxed into working the same way. “Synthetic biology is a test of how modular biology really is,” says Pamela Silver, a systems biologist at Harvard University.

Mathematical Cures for Biological Ills?

Much as mathematics helped synthetic biology get started, it also may help the field overcome some of its growing pains. Two recent examples suggest how synthetic biology can live up to its name and become a *synthesis* of biology and math.

The Biological Amp. In her research with biologist Alex Ninfa of the University of Michigan, and more recently with biological engineer Ron Weiss of MIT, Del Vecchio has studied a context-dependent behavior called *retroactivity*. This occurs when a “downstream” system affects an “upstream” system. For example, you might introduce a gene network into the cell that produces an oscillating signal. Then you might add a counter and a switch to tell the cell to release a certain protein after a certain number of oscillations. Alas, as soon as you connect up these components, the oscillator stops working. Because the counter and the switch do not receive the oscillating signal, the protein is not released. What happened?

The problem is well known in engineering. The downstream components, often called the “load,” might use a chemical that the upstream circuit needs to produce its oscillations. Electrical engineers solve the problem by introducing a third device, such as an isolation amplifier, that insulates the oscillator from the load while still allowing it to transmit the signal to the load. The isolation amplifier uses negative feedback. When it detects a deviation from the desired signal, it adds a compensating signal to counteract the error. Often, it needs to amplify the compensating signal.

“When I saw that this standard negative feedback design would help with the loading effect, I asked Alex if he knew any biological system that would implement the idea,” Del Vecchio recalls. “He immediately said, ‘Phosphorylation.’” This is a chain reaction that is the cell’s natural method for amplifying signals. By adding a phosphorylation cascade to her circuit, Del Vecchio isolated the upstream oscillating module from the load, making the whole system work again.

“That circuit is now a submodule of a bigger system I’m working on with Ron Weiss, to control the level of beta cells in the pancreas,” Del Vecchio says. The dream application would be to engineer pancreas cells to self-medicate, in order to treat diabetes. But, she emphasizes, that’s a long way off. “Honestly, we’re just starting to build all the parts. If you ask me in five

years, I hope to tell you we’re close to getting the circuit to work.”

A Matter of Timing. As Del Vecchio’s example suggests, both organisms and machines need reliable ways to keep time. In the ordinary world, clocks require a steady oscillator, such as a pendulum or a quartz crystal. The same is true at the cellular level. Unfortunately, genetic oscillators depend on chemical reactions, and chemical reactions speed up when the temperature rises. The molecular clock therefore speeds up as well.

This year, in the *Proceedings of the National Academy of Sciences*, Josić, along with mathematician William Ott of the University of Houston and physicist-turned-biologist Matthew Bennett of Rice University, described the first synthetic genetic clock that compensates for temperature changes. The discovery began with an improved mathematical model of the oscillator. (See Figure 1.) Ordinary differential equations ignore the time that it takes for a bacterium like *E. coli* to transcribe its DNA into RNA and to translate the RNA blueprint into proteins. “It’s not like electric circuitry, where signals go at the speed of light,” Ott says. “There are always lags.” A more accurate model would involve *delay* differential equations.

Bennett had a general theoretical idea of how temperature compensation could work, and the model gave them more specific ideas of the kind of intervention they needed. We decided to make a point mutation,” Ott says. In particular, they wanted a mutation that would “repress the ability of the repressor to repress.”

The back-and-forth between biology and mathematics continued. Bennett suggested several known mutations that were believed to make an inhibitor bind less effectively to its target as the temperature goes up. According to the mathematical model, this could neatly compensate for the increased reaction rates. And, in fact, when they introduced one of the mutations, the oscillator kept steady time over a 10-degree (Celsius) range of temperatures.

Josić emphasizes that they still don’t have experimental proof that the mutation affects the inhibitor in the way that the model suggests. The model can only propose a mechanism. The other plausible mutations didn’t work, moreover, which means that true predictive engineering is still a work in progress. “We have shown that temperature compensation can be achieved, but that’s not the same as saying you can exactly control it and exactly understand what is happening,” Ott says.

■■■

In both of these cases, mathematicians and biologists complemented each others’ strengths. “We as engineers can write equations and do the mathematics, but when it comes to implementing these things, the collaborations have been instrumental,” says Del Vecchio.

Synthetic biology will certainly continue to be a good source of math problems. Control theory might improve biological circuits that are too finicky to work autonomously. There is some evidence that engineered cell communities behave more dependably than engineered networks of genes. These “cellular consortia” display new behaviors, such as pattern formation, and involve partial differential equations to track how chemicals diffuse and how cells move through space. Some may even require agent-based models to track individual cells.

Finally, cells do one thing that no electrical engineer ever had to worry about: They divide. What happens to a green blinking bacterium when it splits into two bacteria? As Ott says, “The number of interesting problems is large, and they are different from the types of problems that come from within math.”

Dana Mackenzie writes from Santa Cruz, California.

The Right Way To Describe Neural Activity?

To the Editor:

Eric Shea-Brown’s exciting article “Exploring Connectivity in the Brain’s Network of Neurons” (*SIAM News*, October 2014) makes me wonder if a slightly different target should be pursued. He asks, “Just what is the right way to describe . . . neural activity?” I propose one right way.

It is hard to believe that the nervous system can process information without a fundamental unit corresponding to the word in a computer. Of course, different systems in the brain may use different fundamental units. Of course, the word is not indivisible—it can be divided into fields used separately in microcode. Nonetheless, it seems to me that the wonderful methods Shea-Brown describes might be focused on the questions: What is the word? How do impulses code the word?

Clues might be found by taking an exist-

ing logic system, in which impulses are not all the same size, because of noise and speeds approaching the physical limits of the device, and focusing Shea-Brown’s methods on that system. There we know the answer, and so can refine methods to be sure that they can detect what we know. It is hard to believe

that methods that fail in known digital systems will work in the brain, so this approach may help with the choice of what will work best for biology.—

Bob Eisenberg, Department of Molecular Biophysics and Physiology, Rush Medical Center.

In agreement that the neural code is an important, still unresolved question in the field, Eric Shea-Brown suggests a resource for interested readers: Spikes: Exploring the Neural Code (Rieke, Bialek, Warland, and de Ruyter van Steveninck).

LETTERS TO THE EDITOR

Electricity Markets

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different attitudes to risk. (We do not consider here the real possibility that their probability distributions might differ, which leads to a different model.) Each agent's attitude to risk gives a higher weight to bad outcomes in a way that has been formalized by the theory of coherent risk measures (see [1, 17, 19]). Here, the risk-adjusted cost of a random cost Z can be expressed as

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z],$$

that is, the worst-case expectation of the costs when taken with respect to some probability measure that lies in a convex set \mathcal{D} . For the simplest setting, \mathcal{D} can be taken to be polyhedral, as illustrated in Figure 1.

When all agents maximize their risk-adjusted profits, the competitive equilibrium obtained is often different from an optimal social plan. To see why, it is helpful to imagine what risk measure a social planner would use to account for the different preferences of all the different agents. When a hydro producer dislikes low reservoir inflows that yield low revenues, and a thermal producer dislikes high hydro reservoir inflows that yield low energy prices, any coherent risk measure used by the social planner will fail to cover both as worst cases.

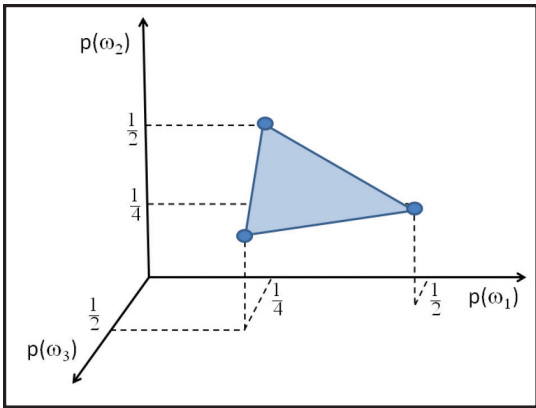


Figure 1. Given random cost outcomes $Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$ with equal probability, the coherent risk measure $\rho(Z) = \frac{3}{4} \mathbb{E}[Z] + \frac{1}{4} \max Z$ has risk set $\mathcal{D} = \text{conv} \{(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})\}$, as depicted by the shaded triangle, yielding $\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z] = \frac{1}{4} Z(\omega_1) + \frac{1}{4} Z(\omega_2) + \frac{1}{2} Z(\omega_3)$.

To get closer to an optimal social plan, someone has to make tradeoffs. The word is well chosen, as the tradeoffs can be shown to come from specific contracts traded between the players. In the example of the hydro reservoir, the hydro agent arranges a contingent claim (a two-way option) with the thermal plant that pays the hydro player an amount from the thermal plant when reservoir inflows are low and a (possibly) different amount in reverse when reservoir inflows are high. The actual amounts paid are based on the probability distribution of inflows, and

reduce the risk of both parties. Such a contract was recently arranged in New Zealand between a thermal and a hydro generator (see [2]).

In the last few years a very elegant theory of market equilibrium and coherent risk measures has emerged ([7, 16]). The key result, established by Danny Ralph and Yves Smeers [16], states that if each agent i has a coherent risk measure with risk set \mathcal{D}_i , and $\cap_i \mathcal{D}_i$ is polyhedral or has nonempty relative interior, and the market for trading risk is complete, then a risk-averse social planning solution with risk set $\cap_i \mathcal{D}_i$ corresponds to a competitive equilibrium in which all agents optimally trade their risk. In this equilibrium, all agents and the social planner view the random

future through the same lens, placing higher weight on exactly the same scenarios. The risk trading, that is, alters the players' payoffs so that all agree on the worst-case future scenarios.

This correspondence with a socially optimal solution enables regulators and market designers to go some way toward establishing good market mechanisms under perfect competition. The theorem indicates that sufficient traded instruments are needed if a social optimum is to be achieved. Numerical

experiments with different mixtures of contracts show that some existing instruments are better than others, and can get most of the way to a socially optimal solution. These models are being used to understand how capital investment decisions are affected by risk in competitive markets [4], and how prices in markets with hydroelectricity increase as water shortages approach [15].

Many mathematical challenges remain. Hydrothermal scheduling problems with many reservoirs are high-dimensional stochastic control problems. Approximate solutions can be obtained with variations of the stochastic dual dynamic programming (SDDP) algorithm [13], in both risk-neutral and risk-averse settings [14, 18]. This method constructs an outer approximation of the Bellman function of dynamic programming through Monte-Carlo sampling of the random variables and cutting planes. A version of the Ralph–Smeers theorem is also available for multistage problems when formulated in scenario trees. Regulators can use this method to postulate a system dynamic risk measure that enables them to benchmark actual market prices against those obtained with a competitive equilibrium and complete markets for risk.

The main challenge with such an approach arises in the computation. As mentioned above, computing approximate solutions to multistage social planning problems uses a

dynamic programming approach that approximates the Bellman functions with cutting planes. This approach relies on the random processes having stagewise-independent noise terms, a serious restriction. For most problems with no more than about ten reservoirs, the approximate Bellman functions converge to a policy that, when simulated, is provably close to optimal (with high probability). The subgradients of the Bellman function, however, show poorer convergence behavior. This matters, as these marginal water values are estimates of equilibrium energy prices.

Finally, the numerical solution of multistage competitive equilibrium in the incomplete case remains a challenge that mathematicians are actively pursuing. The goal is to compute a competitive equilibrium at large scale, which makes decomposition essential. Policy makers and regulators are in desperate need of such computational tools, which will enable them to oversee market behavior in the hope of making it more competitive and therefore more efficient.

References

[1] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, *Coherent measures of risk*, Math. Finance, 9 (1999), 203–228.
[2] K. Barker, *Meridian signs new hedge contract with Genesis*, NZ Energy News, July 8, 2014.
[3] S. Borenstein, J. Bushnell, and S. Stoft, *The competitive effects of transmission capacity in a deregulated electricity industry*, RAND J. Econom., 31:2 (2000), 294–325.
[4] A. Ehrenmann and Y. Smeers, *Generation capacity expansion in a risky environment: A stochastic equilibrium analysis*, Oper. Res., 59:6 (2011), 1332–1346.
[5] F. Facchinei and C. Kanzow, *Generalized Nash equilibrium problems*, Ann. Oper. Res., 175:1 (2010), 177–211.
[6] P.T. Harker, *Generalized Nash games and quasi-variational inequalities*, Eur. J. Oper. Res., 54:1 (1991), 81–94.
[7] D. Heath and H. Ku, *Pareto equilibria with coherent measures of risk*, Math. Finance, 14:2 (2004), 163–172.
[8] B.F. Hobbs, *The next generation of electric power unit commitment models*, International Series in Operations Research & Management Science, Vol. 36, Springer, New York, 2001.
[9] P. Holmberg and A.B. Philpott, *Supply function equilibria in transportation networks*, Tech. Rep., Electric Power Optimization Centre, 2013; www.epoc.org.nz.
[10] J. Lavaei and S.H. Low, *Zero*

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Graduate Summer School: Games and Contracts for Cyber-Physical Security

July 7–23, 2015

Organizing Committee

Saurabh Amin (MIT) and Galina Schwartz (UC Berkeley)

Scientific Overview

This summer school will provide an advanced introduction on how the mathematical tools of game theory can be applied to improve the resilience (security and reliability) of cyber physical systems (CPS) that control critical national infrastructures, such as our electricity, water, and transportation networks. The operations of such CPS are driven by actions of many human decision makers who need to make decisions based on limited information. In addition, these humans frequently have conflicting objectives, which make them reluctant to share even partial information with others. Game-theoretic tools allow analyzing strategic behavior of the entities upon whose choices the CPS operations depend.

The first two weeks will provide an overview of the relevant mathematical tools of game and contract theory, and an outline of the incentive theory framework. The third week will give applications of game theory tools to improving CPS security in various infrastructure domains.

Participation

The summer school will provide a rare opportunity for researchers in mathematics, computer science, engineering, and related sciences to learn about recent research directions and future challenges in this area. Funding is available to support graduate students and postdoctoral researchers in the early stages of their career, as well as more senior researchers interested in undertaking new research in this area. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission, and we welcome their applications. The application is available online, and is due March 31, 2015.

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SIAM's CSC Workshop Series Marks 10th Year

By Albert-Jan N. Yzelman
and Bora Uçar

The 2014 SIAM Workshop on Combinatorial Scientific Computing, held at the École Normale Supérieure in Lyon, France, July 21–23, was the sixth in a series that began ten years ago in San Francisco. True to CSC tradition, the 2014 workshop program comprised a wide range of combinatorial topics arising from many corners of scientific computing. Presenting recent results were a diverse set of speakers—PhD students, postdocs, early-career researchers, and well-established researchers, from academia, national laboratories, and industry; speakers from industry accounted for more than 10% of the talks.

The origins of CSC, which go back to well before 2004, are found in data analysis, sparse matrix computations, meshing, automatic differentiation, partitioning, parallel computing, and related areas. The 2014 workshop sessions covered many intriguing problems from these domains. The three invited speakers focused on two interesting research areas: randomized algorithms for numerical linear algebra and network analysis.

Network Analysis

Tamara Kolda (Sandia National Laboratories) opened the workshop with an invited talk on large-scale networks. Much research has gone into the development of theoretical models that generate graphs with properties similar to those of social networks, the web, and other applications. Different synthetic graphs do well on different similarity metrics; Kolda focused on reproducing degree distributions and clustering coefficients from empirical observations.

One newer model, the block two-level Erdős–Rényi (BTER) graph, hypothesizes that high clustering coefficients observed in real-world networks can be captured well by a collection of dense Erdős–Rényi graphs of various sizes and connectivity. The ER subgraphs form the fine level of the BTER graphs; on the coarse level, the nodes in the ER subgraphs are connected according to the Chung and Lu (CL) graph model, which captures the heavy-tailed degree distribution. CL connects the fine components without adversely affecting the community structure of any of the ER subgraphs [4].

In closing, Kolda considered the matching of BTER graphs to real-world data sets, as well as the analysis of time-dependent graphs. The subject of network analysis was by no means exhausted with Kolda's talk. Subsequent speakers elaborated on the theme of network analysis, with discussions of scalable graph spectrum estimation, anomaly detection, b -matching, centrality computation, coarsening and partitioning methods, as well as software implementing many of these algorithms.

Randomized Algorithms

In the second invited talk Petros Drineas (Rensselaer Polytechnic Institute) discussed randomization in numerical linear algebra. Especially useful in the handling of large-scale matrices, random sampling can greatly reduce the size of the input problem. Drineas began by discussing matrix sampling methods that randomly select rows from a large input matrix to build a matrix of smaller dimension that retains the important information of the original. Early methods select rows uniformly at random in independent identically distributed trials; an example is the Kaczmarz method, as sketched out below.

More effective methods, however, do not select rows according to a uniform distribution. Instead, a row is selected with a probability proportional to its Euclidean norm or, even better, with respect to its so-called leverage score. One application, from a data analysis perspective, provides an alternative to the well-known singular value decomposition $A = U\Sigma V^T$, where the singular vectors forming U , V are hard to interpret in terms of the original data represented by the $m \times n$ input matrix A . Instead, columns from A can be sampled to form an $m \times c$ matrix C , $c \ll n$, for which an approximation $A \approx CX$ can be constructed that strives to keep the error $\|A - CX\|_F$ as close as possible to the error of an optimal low-rank representation, as given by a truncated SVD. The big advantage is that the matrix C , formed from the original columns of A , has a more natural interpretation than the SVD [1, 3].

The sampling of matrix columns (or rows) with preference for influential ones can also be applied to the solution of linear systems $Ax = b$. The randomized Kaczmarz method iteratively refines an approximation of x , using only information from a randomly chosen row a_i of A . In line with the above, the original algorithm selects rows i.i.d. uniformly; sampling the i th row with probabilities proportional to $\|a_i\|$, however, leads to faster convergence.

Bridging algorithms like the randomized combinatorial Kaczmarz algorithm and scientific computing, Sivan Toledo (Tel Aviv University) gave the final invited talk. The Kaczmarz algorithm solves symmetric and diagonally dominant linear systems $Sx = b$ in near-linear time. In [2] Kelner et al. proposed a variant of the Kaczmarz method, describing their method from a physical point of view. The initial linear system S is rewritten in a graph formulation, such that the solution vector x can be retrieved from the solution of $Lu = v$ for u , where L is the Laplacian of the graph formulation of S . With the graph formulation interpreted as an electrical flow problem, the system of equations $Lu = v$ can be solved efficiently.

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duality gap in optimal power flow problem, IEEE Trans. Power Systems, 27:1 (2012), 92–107.

[11] P. Lino, L.A.N. Barroso, M.V.F. Pereira, R. Kelman, and M.H.C. Fampa, *Bid-based dispatch of hydrothermal systems in competitive markets*, Ann. Oper. Res., 120:1 (2003), 81–97.

[12] J.-S. Pang and M. Fukushima, *Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games*, Comput. Manag. Sci., 2:1 (2005), 21–56.

[13] M.V.F. Pereira and L.M.V.G. Pinto, *Multi-stage stochastic optimization applied to energy planning*, Math. Program., 52 (1991), 359–375.

[14] A.B. Philpott, V.L. de Matos, and E.C. Finardi, *On solving multistage stochastic programs with coherent risk measures*, Oper. Res., 2013.

[15] A.B. Philpott, M.C. Ferris, and R.J.-B. Wets, *Equilibrium, uncertainty and risk in*

hydrothermal electricity systems, Tech. Rep., Electric Power Optimization Centre, 2014; www.epoc.org.nz.

[16] D. Ralph and Y. Smeers, *Pricing risk under risk measures: An introduction to stochastic-endogenous equilibria*, 2011; http://ssrn.com/abstract=1903897.

[17] R.T. Rockafellar and S. Uryasev, *Optimization of conditional value at risk*, J. Risk, 2 (2000), 21–41.

[18] A. Shapiro, *Analysis of stochastic dual dynamic programming method*, Eur. J. Oper. Res., 209 (2011), 63–72.

[19] A. Shapiro, D. Dentcheva, and A. Ruszczyński, *Lectures on Stochastic Programming*, SIAM, Philadelphia, 2009.

[20] J. Yao, I. Adler, and S.S. Oren, *Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network*, Oper. Res., 56:1 (2008), 34–47.

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Institute for Computational and Experimental Research in Mathematics

SUMMER TOPICAL WORKSHOPS

Computational and Analytical Aspects of Image Reconstruction

July 13-17, 2015

Organizers: Gaik Ambartsoumian (University of Texas), Vladimir Druskin (Schlumberger-Doll), Esther Klann (Johannes Kepler University), Venkateswaran P. Krishnan (TIFR Centre for Applicable Mathematics), Alfred Louis (Universität des Saarlandes), and Eric Todd Quinto (Tufts University)

Description: The mathematical study of image reconstruction problems can have a huge impact on human life. More efficient mathematical algorithms for X-ray tomography and more accurate mathematical models in seismic or hybrid imaging can lead to better imaging devices in fields such as medicine and remote sensing. This workshop will bring together experts working in computational and analytical aspects of image reconstruction (including but not limited to electron-microscope tomography, hybrid imaging, radar and sonar, full waveform inversion of seismic imaging and X-ray CT) as well as postdoctoral fellows and graduate students.

Mathematics in Data Science

July 28-30, 2015

Organizers: Philip Kegelmeyer (Sandia National Lab), Tamara Kolda (Sandia National Lab), Randall LeVeque (University of Washington), Aleksandra Mojsilovic (IBM T. J. Watson Research Center), Linda Ness (Applied Communication Sciences), and Alyson Wilson (NC State University)

Description: The goal of this workshop is to bring together mathematicians and data scientists to participate in a discussion of current methods and outstanding problems in data science. The workshop is particularly aimed at mathematicians interested in pursuing research or a career in data science who wish to gain an understanding of this rapidly evolving field and the ways in which mathematics can contribute. Researchers currently working in data science are also encouraged to attend, to share ideas about mathematical methodologies and challenges. A number of experienced data scientists with a variety of backgrounds from academics, national laboratories, and industry (including startups) will be invited. The program will include overview and technical talks, several panels consisting of practitioners with different experience levels, and a poster session.

Numerical Methods for Large-Scale Nonlinear Problems and Their Applications

August 31 – September 4, 2015

Organizers: Tim Kelley (NC State University), Homer Walker (Worcester Polytechnic Institute/ICERM), and Carol Woodward (Lawrence Livermore National Lab)

Description: Over the last 20 years, Newton-Krylov methods have developed to maturity, allowing effective fully-coupled treatment of a broad range of large-scale nonlinear problems. This has set the stage for addressing more difficult problems with more challenging features. Additionally, applications for which state-of-the-art Newton-Krylov approaches are inapplicable have recently exposed several basic research questions. This workshop will include mathematicians and computer scientists who work on algorithm design, implementation, and analysis, together with disciplinary scientists and engineers who use the algorithms in applications and have a working knowledge of their capabilities and limitations.



To learn more about ICERM programs, organizers, program participants, to submit a proposal, or to submit an application, please visit our website: **icerm.brown.edu**.

Ways to participate:

Propose a:

- semester program
- topical workshop
- summer undergrad or early career researcher program

Apply for a:

- semester program or workshop
- postdoctoral fellowship

Become an:

- academic or corporate sponsor

About ICERM: The Institute for Computational and Experimental Research in Mathematics is a National Science Foundation Mathematics Institute at Brown University in Providence, Rhode Island. Its mission is to broaden the relationship between mathematics and computation.



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Got a Problem?

Problem ideas sought for high school math modeling competition

Challenging the next generation of mathematicians is critical. You can help, and impact thousands of students, by submitting a problem statement to Moody's Mega Math (M³) Challenge

In 2014, more than 5,000 high school students in the U.S. participated in and submitted solutions to an open-ended, realistic, math-modeling problem presented to them in the M³ Challenge, an Internet-based applied math contest that occurs annually in February/March. The contest, which is organized by SIAM, poses a problem that students, working independently in teams of 3–5, must solve in just 14 hours.

In previous years, students tackled timely issues such as making school lunches healthy, implementing recycling guidelines, persistent drought, the census, the stimulus package and job creation, energy independence, Social Security solvency, and choosing stocks for maximum gain (see below right). Coming up with great problem ideas year after year is not easy, and that's where we're hoping you can help.

SIAM is looking for ideas for problems to be used in upcoming M³ Challenges

Problems should:

- Be accessible to 11th and 12th graders
- Be suitable for solution in 14 hours
- Provide the possibility for significant mathematical modeling
- Be of current interest and involve interdisciplinary problem solving and critical thinking skills
- Have enough data available for a variety of approaches and depth of solutions
- Be broken down into parts with some parts easier than others so that all teams can make some progress
- Identify references to help get students started

Submit your problem statement idea in the format of previous Challenge problems, which can be found at m3challenge.siam.org/problem.

Problem structure

Within the problem statement, there should be three questions for teams to answer:

- Question One: The Warm Up — every serious team can answer.
- Question Two: The Guts — framed so that every team can have some success and many teams can cover it well.
- Question Three: The Discriminator — many teams can do something, while only a few will have exceptional results.



Moody's Mega Math Challenge®

\$125,000 in SCHOLARSHIP PRIZES!



**Challenge weekend:
February 28–March 1, 2015**

High school* juniors and seniors:

- Form a team of 3-5 students with one teacher-coach.
- Choose your work day.
- Submit a solution to the open-ended modeling problem.

Participation is FREE and entirely Internet-based!

Register by February 20, 2015

M3Challenge.siam.org

Organized by **siam**
Society for Industrial and Applied Mathematics

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Previous problem titles

- 2006: Solving the Social Security Stalemate
- 2007: Beat the Street!
- 2008: Energy Independence Meets the Law of Unintended Consequences
- 2009: \$787 Billion: Will the Stimulus Act Stimulate the Economy?
- 2010: Making Sense of the 2010 Census
- 2011: Colorado River Water: Good to the Last Acre-Foot
- 2012: All Aboard: Can High Speed Rail Get Back on Track?
- 2013: Waste Not, Want Not: Putting Recyclables in Their Place
- 2014: Lunch Crunch: Can Nutritious be Affordable and Delicious?

Future topics

We are open to any topic!

Of particular interest are problems based on timely, relevant, hot-button issues facing the U.S. and the rest of the world.

Honoraria

- Problems selected to be used as "the" Challenge problem receive a \$1,000 honoraria.
- Problems found suitable to add to the M³ problem reserve bank receive \$150; an additional \$150 will be paid if it is used as a sample or resource on the Challenge website

**Submit problem statements by uploading your file at
m3challenge.siam.org/problem/submitproblems.php
or via email to montgomery@siam.org.**

High schools in the following states are eligible to participate in M³ Challenge 2015: Alabama, Arizona, Arkansas, Colorado, Connecticut, Delaware, Florida, Georgia, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, North Dakota, Ohio, Oklahoma, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Virginia, Washington, D.C., West Virginia, Wisconsin, and Wyoming.



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Idea submissions or additional information needed?

Contact:



Michelle Montgomery, Project Director
Moody's Mega Math Challenge
montgomery@siam.org
m3challenge.siam.org

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Society for Industrial and Applied Mathematics
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The National Association of Secondary School Principals placed this program on the NASSP National Advisory List of Student Contests and Activities since 2010.



CSC14

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Kelner et al. recognized that the vector v in the above formulation has a very specific structure, which can be exploited for efficient solution by the randomized Kaczmarz algorithm through the use of a specifically constructed spanning tree of the derived graph. Toledo gave an alternative interpretation of the method, while relying solely on the graph formulation, column and null spaces, and orthogonal projections. In this way, he related the new Kaczmarz algorithm to earlier combinatorial preconditioning techniques.

Parallel Computing

Randomized algorithms like the Kaczmarz method often lend themselves well to parallelization, and parallel computing has always been one of the focus areas of combinatorial scientific computing. For the randomized combinatorial Kaczmarz algorithm, a parallel approach was presented after Toledo’s talk; indeed, about a third of the presentations at the workshop were related to parallel computing. Today, with the increasing core counts of processor architectures, including those in our hand-held devices, such a focus is hardly a surprise.

Social Computing

Kamesh Madduri (Pennsylvania State

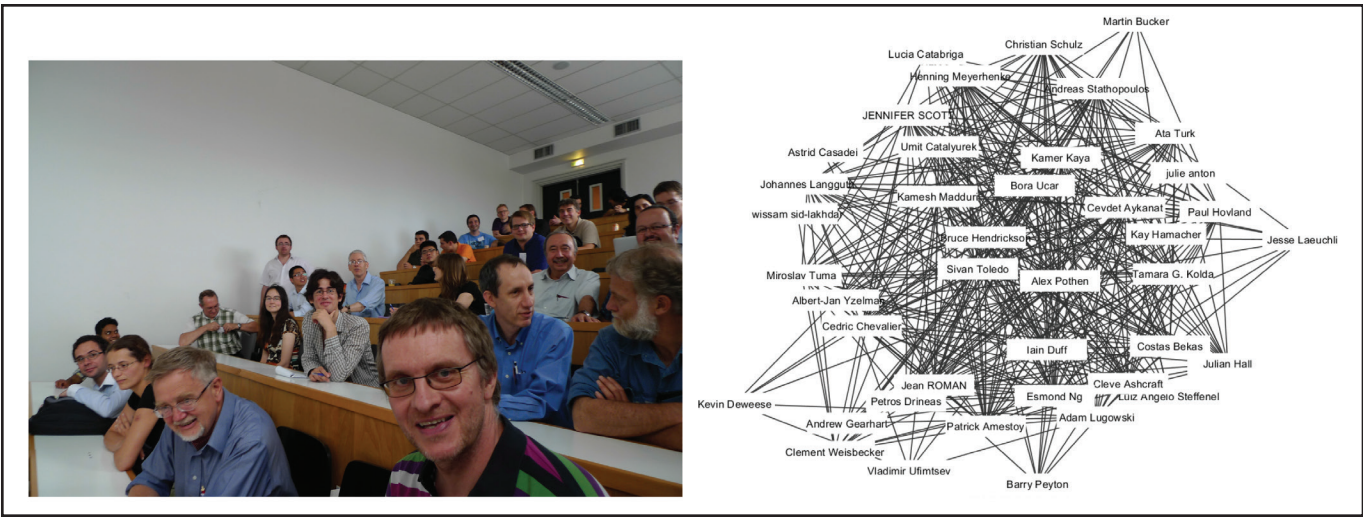


Figure 1. CSC14 participants, part of the network shown in the graph at right, by Kamesh Madduri.

University) reminded us that a conference has both scientific and social aspects, by illustrating parallel shortest-path algorithms with a graph of the CSC workshop participants (Figure 1). Outside the busy workshop schedule, the breaks and lunches were indeed lively events and provided plenty of opportunities for discussions, scientific and otherwise. These intermissions were catered well with drinks and refreshments, while the lunches provided by ENS lived up to the French reputation for excellent cuisine.

A book of abstracts of the workshop presentations is publicly available at <http://hal.inria.fr/hal-01054876>. More information

about the workshop is available at <http://www.siam.org/meetings/csc14/>.

The workshop was supported financially by the LABEX MILYON (ANR-10-LABX-0070) of Université de Lyon within the program “Investissements d’Avenir” (ANR-11-IDEX-0007), operated by the French National Research Agency, and SIAM.

References

[1] C. Boutsidis, P. Drineas, and M. Magdon-Ismail, *Near-optimal column-based matrix reconstruction*, SIAM J. Comput., 43 (2014), 687–717.
[2] J.A. Kelner, L. Orecchia, A. Sidford, and Z.A. Zhu, *A simple, combinatorial algo-*

rithm for solving SDD systems in nearly-linear time, Proc. Forty-Fifth Annual ACM Symposium on Theory of Computing, STOC ’13, 2013, 911–920.

[3] M.W. Mahoney and P. Drineas, *CUR matrix decompositions for improved data analysis*, Proc. Natl. Acad. Sci. USA, 106 (2009), 697–702.

[4] C. Seshadhri, T.G. Kolda, and A. Pinar, *Community structure and scale-free collections of Erdős-Rényi graphs*, Phys. Rev. E, 85 (2012), 056109.

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Professional Opportunities

Send copy for classified advertisements to: Advertising Coordinator, SIAM News, 3600 Market Street, 6th Floor, Philadelphia, PA 19104–2688; (215) 382–9800; fax: (215) 386–7999; marketing@siam.org. The rate is \$2.85 per word (minimum \$350.00). Display advertising rates are available on request.
Advertising copy must be received at least four weeks before publication (e.g., the deadline for the March 2015 issue is January 31, 2015).
Advertisements with application deadlines falling within the month of publication will not be accepted (e.g., an advertisement published in the March issue must show an application deadline of April 1 or later).

Dartmouth College

Department of Mathematics

New or recent PhD graduates with research interest in applied and computational mathematics may apply for instructorships in these areas for terms of two to three years. Successful candidates will teach three 10-week courses spread over three terms. Appointments are for 26 months, with a possible 12-month renewal. Positions offer a monthly salary of \$5,202, which includes a two-month research stipend for instructors in residence during two of the three summer months; if an instructor is not in residence, the salary will be adjusted accordingly.

Applicants should apply online at www.mathjobs.org (position ID: IACM #6022). The application can also be accessed at <http://www.math.dartmouth.edu/activities/recruiting/>. General inquiries can be directed to Tracy

Moloney, Administrator, Department of Mathematics, tfmoloney@math.dartmouth.edu. Applicants whose completed applications are received by January 5, 2015, will be considered first.

Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

Dartmouth College

Department of Mathematics

The Department of Mathematics anticipates the availability of a senior opening with an initial appointment in the 2015–2016 academic year. The successful candidate will have a research profile with a concentration in computational or applied mathematics, will be appointed at the level of full professor, and is expected to have an overall record of achievement and leadership consonant with such an appointment.

Students (and others) in search of information about careers in the mathematical sciences can click on “Careers and Jobs” at the SIAM website (www.siam.org) or proceed directly to

www.siam.org/careers

Applicants should apply online at www.mathjobs.org (position ID: PACM #6023). Applicants whose completed applications are received by December 15, 2014, will receive first consideration.

For more information about this position, please visit <http://www.math.dartmouth.edu/activities/recruiting/>.

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Dartmouth College

Department of Mathematics

John Wesley Young Research Instructorships are available for two to three years for new or recent PhD graduates whose research overlaps a department member’s. Successful candidates will teach three 10-week courses spread over three terms. Appointments are for 26 months, with a possible 12-month renewal; the monthly salary is \$5,202, including a two-month research

stipend for instructors in residence during two of three summer months. If an instructor is not in residence, the salary will be adjusted accordingly.


To initiate an application, applicants should go to <http://www.mathjobs.org> (position ID: JWY #6021). The application can also be accessed through a link at <http://www.math.dartmouth.edu/activities/recruiting/>. General inquiries can be directed to Tracy Moloney, Administrator, Department of Mathematics, tfmoloney@math.dartmouth.edu. Applicants whose completed applications are received by January 5, 2015, will be considered first.

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Dartmouth College

Department of Mathematics

The Dartmouth College Department of Mathematics is pleased to announce a tenure-
See Opportunities on page 10



Los Alamos
NATIONAL LABORATORY
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Los Alamos National Laboratory (LANL),
a multidisciplinary research institution engaged in strategic science on behalf of national security, has a single opening for a Center Leader position in our National Security Education Center (NSEC). It will be filled either at the Executive Advisor 4 or the R&D Scientist 5 level.

Information Science & Technology Institute

EXECUTIVE ADVISOR 4

The Institute Leader is responsible for establishing a charter, vision and implementation strategy for the institute in collaboration with the relevant Laboratory division leadership that builds on and complements the Laboratory’s Information and Science and Technology Pillar. Coordinates external visible workshops and working groups on topics in information science and technology. Manages an active seminar program, coordinates visiting scholars, and represents the Institute at scientific and administrative meetings. Management responsibilities include accountability for quality research, management of financial and human resources, proactive support of Laboratory safety, security, environment and diversity objectives, and the communications/marketing strategy for the Institute.

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Applicants may apply to both job postings at careers.lanl.gov

R&D SCIENTIST 5

EOE

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Geisinger’s IAA consists of 3 centers, 9 labs, a computational core facility, and an IT trials office. Work is under the direction of Gregory J. Moore, MD, PhD, Chief, Emerging Technology and Informatics, & Director, Institute for Advanced Application.

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Opportunities

continued from page 9

track opening for the academic year 2015–2016. There is a preference for a junior appointment, but appointment at higher rank, with tenure, is possible. The successful candidate will have a research profile with a concentration in applied or computational mathematics.

Applicants should apply online at www.mathjobs.org (position ID: APACM #6024). Applicants whose completed applications are received by December 15, 2014, will receive first consideration.

For more information about this position, please visit <http://www.math.dartmouth.edu/activities/recruiting/>.

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California Institute of Technology

Department of Computing and Mathematical Sciences

The Department of Computing and Mathematical Sciences (CMS) at Caltech invites applications for a tenure-track faculty position. The department is a unique environment in which innovative, interdisciplinary, and foundational research is conducted in a collegial atmosphere. The department seeks candidates who have demonstrated exceptional promise through novel research with strong potential connections to natural, information, and engineering sciences. Research areas of particular interest include applied mathematics and computational science as well as computing. A commitment to high-quality teaching and mentoring is expected.

The initial appointment at the assistant-professor level is for four years and is contingent upon the completion of a PhD in applied mathematics, computer science, or a related field. Exceptionally well-qualified applicants may also be considered at the full professor level.

To ensure the fullest consideration, applicants are encouraged to have all their application materials on file by December 28, 2014. For a list of documents required and full instructions on how to apply online, visit <http://www.cms.caltech.edu/search>. Questions about the application process may be directed to: search@cms.caltech.edu.

Caltech is an Equal Opportunity/Affirmative Action Employer. Women, minorities, veterans, and disabled persons are encouraged to apply.

California Institute of Technology

Department of Computing and Mathematical Sciences

The Department of Computing and Mathematical Sciences (CMS) at the California Institute of Technology seeks applications for the position of lecturer in applied and computational mathematics. This is a (non-tenure-track) career

teaching position, with full-time teaching responsibilities. The initial term of appointment can be up to three years. An advanced degree in applied mathematics or related field is necessary, as well as a track record of dedication to (and excellence in) teaching.

The successful candidate will teach courses in applied and computational mathematics and is expected to work closely with the CMS faculty on instructional matters. The lecturer may also assist in other aspects of the undergraduate program, including curriculum development and involvement in research projects with undergraduate students. Courses to be taught cover various methods of applied mathematics such as, but not limited to, complex analysis, ordinary and partial differential equations, real and functional analysis, linear algebra and applied operator theory, optimization, stochastic processes and modeling, applied statistics and data analysis, and numerical methods. The lecturer will have opportunities to be involved in ongoing research projects in the department, but such involvement is not required. More important, the successful candidate will be expected to be dedicated to, and passionate about, teaching upper-level undergraduates as well as graduate students.

Applicants should view the instructions and apply online at <https://applications.caltech.edu/job/acmlect>.

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University of California Los Angeles

Institute for Pure and Applied Mathematics

The Institute for Pure and Applied Mathematics (IPAM) at UCLA is seeking an associate director (AD) to begin a two-year appointment on August 1, 2015.

The AD is expected to be an active and established research mathematician or scientist in a related field, with experience in conference organization. The primary responsibility of the AD is running individual programs in coordination with the organizing committees. The selected candidate will be encouraged to continue his or her personal research within the context of the responsibilities to the institute.

For a detailed job description and application instructions, applicants should visit www.ipam.ucla.edu/adsearch. Applicants whose completed applications are received by February 15, 2015, will receive fullest consideration, but applications will be accepted as long as the position remains open.

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The University of Manchester

School of Mathematics

The University of Manchester has established

the highly prestigious Turing Fellowships in Mathematics, open to truly outstanding mathematicians from across the world. The fellowships are of four years and are funded by a generous external donation.

Applicants will be engaging in research at the highest international standards and show the promise of becoming world leaders in their fields. Applicants will usually be outstanding early-career postdoctoral researchers, but recent doctorands of exceptional ability and potential also will be considered.

Salary is negotiable and may include a significant enhancement to reflect the calibre of the fellows.

For further details and to submit an application, applicants should visit <https://www.jobs.manchester.ac.uk/displayjob.aspx?jobid=8660>. The deadline for applications is January 12, 2015.

The School of Mathematics is committed to promoting equality and diversity and particularly welcomes applications from women.

Clarkson University

Department of Mathematics

The Clarkson University Department of Mathematics (www.clarkson.edu/math) invites applications for a tenure-track assistant professor position in statistics or applied mathematics to start in August 2015.

The department is especially interested in applicants with expertise in statistics and computational areas of applied mathematics, but applicants from all areas of applied mathematics will be considered. Responsibilities will include teaching undergraduate- and graduate-level mathematics courses, and directing graduate students. Minimum requirements are a PhD in mathematics or statistics by the date of appointment, demonstrated excellence in both research potential and teaching ability, and fluency in English. In addition, the successful candidate should be able to interact with other faculty in the department and the university.

Applications, including a curriculum vitae and three reference letters, should be submitted to <https://clarkson.peopleadmin.com/>. Completed applications will be reviewed starting immediately.

Women and minorities are urged to apply. Clarkson University is an AA/EOE Employer.

New Jersey Institute of Technology

Department of Mathematical Sciences

The Department of Mathematical Sciences (DMS) at the New Jersey Institute of Technology (NJIT) seeks candidates to fill a tenure-track/tenured position at the assistant/associate/full professor level in the general area of applied mathematics. Applicants are sought from all fields of applied mathematics. The department is particularly interested in applicants whose research interests are consistent with the existing research strengths in scientific computation/

numerical analysis, modeling/asymptotic analysis, and PDEs and dynamical systems, with focused research groups in applications to fluid dynamics, mathematical biology, and wave propagation.

DMS has experienced tremendous growth in research over the past two decades and is now recognized as having a leading national program in applied mathematics. The department offers BS, MS, and PhD degrees, with PhD program tracks in applied mathematics as well as in applied probability and statistics. For more information about DMS faculty and programs, visit <http://math.njit.edu>.

Applicants should have a PhD in applied mathematics or a related field and postdoctoral experience with strong research and teaching potential for consideration at the assistant professor level, and an appropriate record of accomplishment in classroom teaching, mentoring of doctoral students, research publication, and funding for consideration at the associate or full professor level. At the university's discretion, the prerequisites may be excepted where the applicant can demonstrate to the satisfaction of the university an equivalent combination of education and experience that prepares the applicant for success in the position.

To apply, applicants should visit <https://njit.jobs> and search for posting number 0602414. Applicants should submit a cover letter, resume/curriculum vitae, research and teaching statements, a summary of teaching evaluations (if available), and the names and contact information of three references. Review of applications will begin on November 15, 2014 and will continue until the position is filled.

To build a diverse workforce, NJIT encourages applications from individuals with disabilities, minorities, veterans, and women. NJIT is an EEO employer.

Georgia Institute of Technology

School of Mathematics

The School of Mathematics at Georgia Tech is accepting applications for an academic professional position. Applications should include a curriculum vitae, three or more letters of reference, and evidence of teaching interest and abilities. A PhD is required for this position.

The responsibilities for the position are

- teaching two courses per semester, typically selected from the department's 1000- or 2000-level offerings;
- assisting in the evaluation of transfer credit;
- organizing or assisting with TA training;
- advising and tracking of majors; and
- other duties as assigned.

For more details and to submit an application, applicants should visit <https://www.mathjobs.org>. Review of applications will begin immediately and will continue until the position is filled.


Milwaukee School of Engineering

Department of Mathematics

The Milwaukee School of Engineering (MSOE) invites applications for a full-time mathematics faculty position starting in Fall 2015. The department has new and growing programs in actuarial science and operations research. The successful candidate is expected to teach the mathematics courses in the undergraduate curriculum. Those with expertise in areas such as probability, applied probability models, and game theory will be preferred. Applicants should possess an appropriate doctoral degree at the time of hiring.

MSOE offers degrees in engineering, engineering technology, construction management, business, actuarial science, operations research, technical communications and nursing. The school is located in the heart of downtown Milwaukee and is recognized in several national publications for its "applications-oriented" approach. The faculty is judged primarily on excellence in teaching. MSOE graduates are in high demand, as evidence by the school's strong job placement rate.

See Opportunities on page 11




Tenure Track Assistant Professor Position

The Department of Statistical Science at Duke University invites applications for appointments at the level of Assistant Professor to begin in Fall 2015. Preference will be given to candidates whose core statistical science research interests are complemented by collaborative research interests in areas including economics, finance or other areas of the social and policy sciences, computer science, neurosciences, and environmental science. We are particularly interested in applicants with demonstrated experience in complex stochastic modeling and computation for large-scale problems and data sets.

The Department is an internationally recognized center of excellence for research and education in contemporary statistical methodology. With leading strengths in Bayesian analysis, interdisciplinary applications and computationally intensive methods, the Department offers outstanding computational facilities and opportunities for interdisciplinary research. We currently have 18 regular rank faculty, 16 visiting, adjunct, and postdoctoral faculty, and over 75 graduate students. Beyond core research in statistical and computational sciences, we have many collaborative interactions with multiple other Duke departments, institutes and centers, including the Information Initiative at Duke (iiD), Duke Institute for Brain Sciences (DIBS), and Social Science Research Initiative (SSRI). Complementary interactions involve long-standing associations with the Statistical and Applied Mathematical Sciences Institute (SAMS) and the National Institute of Statistical Science (NISS), located nearby in the Research Triangle Park, as well as many collaborators, institutes and companies around the US and worldwide. Our internationally recognized PhD program is complemented by our MS in Statistical Science, our MS in Statistical and Economic Modeling, and our Statistical Science undergraduate degree. More information is available at the department website <http://stat.duke.edu>.

To apply, submit a letter, curriculum vitae, personal statement of research and teaching and names/letters from three references via <https://academicjobsonline.org/ajob>. Enquiries can be emailed to search@stat.duke.edu. The application pool will remain open until the position is filled; screening will begin on December 1st 2014.

Duke University, located in Durham, North Carolina, is an Affirmative Action/Equal Opportunity Employer committed to providing employment opportunity without regard to an individual's age, color, disability, genetic information, gender, gender identity, national origin, race, religion, sexual orientation, or veteran status. Applications from women and minorities are strongly encouraged. Individuals in dual career couples are encouraged to visit <http://provost.duke.edu/faculty/partner/>, the website on Duke's Advantages for Faculty, for information on opportunities for dual career couples in the area and how the university can help.



Assistant Professor of the Practice

The Department of Statistical Science invites applications for faculty appointment at the level of Assistant Professor of the Practice to begin in Fall 2015. This position is a regular rank faculty with a term renewable appointment. Preference will be given to candidates demonstrating outstanding teaching and strong interests in developing and growing our undergraduate major in Statistical Science and our MS in Statistical Science. We are also interested in applicants with complementary interests in Bayesian statistical science research and collaborative applications.

The Department is an internationally recognized center of excellence for research and education in contemporary statistical methodology. With leading strengths in Bayesian analysis, interdisciplinary applications and computationally intensive methods, the Department offers outstanding teaching support, computational facilities, and opportunities for interdisciplinary statistics teaching and collaboration. We currently have 18 regular rank faculty, 16 visiting, adjunct, and postdoctoral faculty, and over 75 graduate students.

The educational program (graduate and undergraduate) as well as the Department's research agenda benefit from strong connections with many other groups at Duke, with the Statistical and Applied Mathematical Sciences Institute (SAMS) and the National Institute of Statistical Sciences (NISS) located nearby in the Research Triangle, and with other collaborators, centers, companies and organizations nationwide. Visit <http://www.stat.duke.edu> for more information.


To apply, submit a letter, curriculum vitae, personal statement of teaching and research, and three reference letters via <https://academicjobsonline.org/ajob>. For inquiries and e-mail correspondence please write to dalene@stat.duke.edu. The application pool will remain open until the position is filled; screening will begin on 1 December 2014.

Duke University, located in Durham, North Carolina, is an Affirmative Action/Equal Opportunity Employer committed to providing employment opportunity without regard to an individual's age, color, disability, genetic information, gender, gender identity, national origin, race, religion, sexual orientation, or veteran status. Applications from women and minorities are strongly encouraged. Individuals in dual career couples are encouraged to visit <http://provost.duke.edu/faculty/partner/>, the website on Duke's Advantages for Faculty, for information on opportunities for dual career couples in the area and how the university can help.

**College of Engineering:
Open Rank Faculty**
Department of Aerospace Engineering
College of Engineering
University of Illinois at Urbana-Champaign

The Department of Aerospace Engineering at the University of Illinois at Urbana-Champaign is seeking highly qualified candidates for multiple faculty positions with emphasis on the areas of space systems/propulsion, autonomous aerospace systems, multi-disciplinary design optimization, aeroelasticity, and aerospace materials and structures. Particular emphasis will be placed on qualified candidates who work in emerging areas of aerospace engineering and whose scholarly activities have high impact.

Please visit <http://jobs.illinois.edu> to view the complete position announcement and application instructions. Full consideration will be given to applications received by **November 3, 2014**. Applications received after that date will be considered until the positions are filled.



Illinois is an EEO Employer/Vet/Disabled - www.inclusiveillinois.illinois.edu and committed to a family-friendly environment (<http://provost.illinois.edu/worklife/index.html>).

Opportunities

continued from page 10

The review of candidates will begin immediately and continue until the position is filled. To apply or for more information, applicants should visit <http://www.msos.edu> and submit a single file that includes a detailed resume, evidence of teaching excellence, and three reference letters.

MSOE is an equal opportunity employer. All qualified applicants will receive consideration for employment without regard to race, religion, color, national origin, sex, age, status as a protected veteran, among other things, or status as a qualified individual with disability.

Texas Tech University

Department of Mathematics and Statistics

The Department of Mathematics and Statistics (M&S) at Texas Tech University invites applications for four tenure-track assistant professor positions beginning in fall 2015. M&S has active research groups in both pure and applied mathematics as well as in statistics (see <http://www.math.ttu.edu/FacultyStaff/research.shtml>). The department fosters a spirit of interdisciplinary collaboration across areas of mathematics and statistics as well as with engineering and the physical and biological sciences.

M&S is seeking candidates who will be engaged in nationally visible scholarship, establish externally funded research programs, interact with the existing research groups in the department, participate in interdisciplinary collaborations and service, involve graduate students in their research, and show excellence in teaching at the graduate and undergraduate levels. A PhD at the time of appointment is required.

One position will be in statistics, with a preference for applicants in probability theory/stochastic processes. The second position will be in biostatistics, with a preference for applicants who will collaborate with researchers in environmen-

tal toxicology, biological sciences and/or public health. The third position will be in complex analysis and/or applications of complex analysis. The fourth position will be in mathematical and computational modeling, with a preference for applicants who will collaborate with researchers in biomathematics, applied mathematics, and/or computational mathematics. Applicants with very strong records who will bring externally sponsored research to Texas Tech will be considered for associate or full professor ranks.

Applicants should apply at <http://www.texastech.edu/careers/>, using the Requisition ID 1818BR. Include a completed AMS standard cover sheet and a curriculum vitae. Three letters of reference plus any material in addition to that completed online should be sent to: Alex Wang, Hiring Committee Chair, Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409-1042; alex.wang@ttu.edu. Review of applications will begin immediately.

Texas Tech University is committed to diversity among its faculty. The university strongly encourages applications from women, minorities, persons with disabilities, and veterans, and it considers the needs of dual-career couples.

Texas Tech University is an Affirmative Action/Equal Opportunity Employer.

University of Michigan

Department of Mathematics

Pending approval, the University of Michigan's Department of Mathematics is initiating a multi-year search for anticipated appointments at the tenure-track assistant, associate, or full professor levels. These would be university-year appointments for September 1, 2014 or 2015. All ranks are encouraged to apply.

Applicants should hold a PhD in mathematics or a related field (e.g., statistics); have extraordinary credentials in any area of pure, applied, computational or interdisciplinary mathematics; and show outstanding promise and/or accom-

plishments in both research and teaching. Salaries are competitive and are based on credentials. Junior candidates should furnish a placement dossier, consisting of a letter of application, curriculum vitae, and three letters of recommendation; senior candidates should send a letter of application, curriculum vitae, and names of three suggested references. In all cases, please provide a statement of teaching philosophy and experience, evidence of teaching excellence, and a statement of current and future research plans.

Application materials should preferably be submitted electronically through the AMS website www.mathjobs.org. Alternatively, applications may be sent to: Personnel Committee, University of Michigan, Department of Mathematics, 2074 East Hall, 530 Church Street, Ann Arbor, MI 48109-1043. Applications will be considered on a continuing basis, but applicants are urged to apply by December 1, 2014. Inquiries may be made by e-mail to math-fac-search@umich.edu.

More detailed information regarding the department may be found at www.math.lsa.umich.edu.

Women and minority candidates are encouraged to apply. The University of Michigan is supportive of the needs of dual-career couples and is an equal opportunity/affirmative action employer.

University of Michigan

Department of Mathematics

The University of Michigan Department of Mathematics expects to have lecturer positions available for 2015–2016. These are not tenure-track positions but may be renewed annually for up to the first four years and, thereafter, for intervals of three to five years. These are 100% university-year appointments with expected start dates of September 1, 2015. A typical full-time (100% effort) load for a lecturer III in the College of Literature, Science, and the Arts is three courses per semester or the equivalent in

other duties.

The criterion for selection and for renewal is excellence in the classroom. Interest and activity in pedagogical research is encouraged but not essential for reappointment. The successful candidate is likely to have substantial experience in teaching mathematics and a doctorate in mathematics or a closely related area.

Applicants should submit a curriculum vitae, a statement of teaching philosophy and experience, evidence of teaching excellence, and the names of at least three references. Application materials should preferably be submitted electronically through the AMS website www.mathjobs.org. Alternatively, applications may be sent to: Personnel Committee, University of Michigan, Department of Mathematics, 2074 East Hall, 530 Church Street, Ann Arbor, MI, 48109-1043. Applications will be considered on a continuing basis, but applicants are urged to apply by December 1, 2014. Inquiries may be made by e-mail to math-fac-search@umich.edu. More detailed information regarding the department may be found at www.math.lsa.umich.edu.

Women and minority candidates are encouraged to apply. The University of Michigan is an equal opportunity/affirmative action employer. This appointment is subject to the UM/LEO Agreement.

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The Institute for Computational Engineering and Sciences (ICES) at The University of Texas at Austin is searching for exceptional candidates with expertise in computational science and engineering to fill several Moncrief endowed faculty positions at the Associate Professor level and higher. These endowed positions will provide the resources and environment needed to tackle frontier problems in science and engineering via advanced modeling and simulation. This initiative builds on the world-leading programs at ICES in Computational Science, Engineering, and Mathematics (CSEM), which feature 16 research centers and groups as well as a graduate degree program in CSEM. Candidates are expected to have an exceptional record in interdisciplinary research and evidence of work involving applied mathematics and computational techniques targeting meaningful problems in engineering and science. For more information and application instructions, please visit: www.ices.utexas.edu/moncrief-endowed-positions-app/. This is a security sensitive position. The University of Texas at Austin is an Equal Employment Opportunity/Affirmative Action Employer.

THE UNIVERSITY OF TEXAS AT AUSTIN

Faculty Positions
School of Operations Research & Information Engineering (ORIE)

Cornell is a community of scholars, known for intellectual rigor and engaged in deep and broad research, teaching tomorrow's thought leaders to think otherwise, care for others, and create and disseminate knowledge with a public purpose.

Cornell University's School of Operations Research and Information Engineering (ORIE) seeks to fill multiple tenured/tenure-track faculty positions for its Ithaca campus. Applicants with research interests in all areas of operations research and information engineering will be considered, but applicants in areas aligned with the School's current strategic plan will receive primary consideration: the plan seeks to strengthen the School's leading role in advancing the analytical, methodological, and modeling tools of operations research together with the potential of "Big Data" and the information revolution.


Requisite is a strong interest in the broad mission of the School, exceptional potential for leadership in research and education, an ability and willingness to teach at all levels of the program, and a PhD in operations research, mathematics, statistics, or a related field by the start of the appointment. Salary will be appropriate to qualifications and engineering school norms.

Cornell ORIE is a diverse group of high-quality researchers and educators interested in probability, optimization, statistics, simulation, and a wide array of applications such as manufacturing, e-commerce, supply chains, scheduling, transportation systems, health care, financial engineering, service systems and network science. We value mathematical and technical depth and innovation, and experience with applications and practice. Ideal candidates will have correspondingly broad training and interests. Please apply online at <https://academicjobsonline.org/ajo/jobs/4552> with a cover letter, CV, statements of teaching and research interests, sample publications, at least three reference letters and, for junior applicants, a doctoral transcript. Applicants attending INFORMS annual meeting are strongly encouraged to submit all application materials by November 1, 2014. All applications completed by November 15, 2014 will receive full consideration, but candidates are urged to submit all required material as soon as possible. Applications will be accepted until the positions are filled.

ORIE and the College of Engineering at Cornell embrace diversity and seek candidates who can contribute to a welcoming climate for students of all races and genders. Cornell University seeks to meet the needs of dual career couples, has a Dual Career program, and is a member of the Upstate New York Higher Education Recruitment Consortium to assist with dual career searches. Visit <http://www.unyherc.org/home/> to see positions available in higher education in the upstate New York area.

Find us online at <http://hr.cornell.edu/jobs> or [Facebook.com/CornellCareers](https://www.facebook.com/CornellCareers)

Cornell University is an innovative Ivy League university and a great place to work. Our inclusive community of scholars, students and staff impart an uncommon sense of larger purpose and contribute creative ideas to further the university's mission of teaching, discovery and engagement. Located in Ithaca, NY, Cornell's far-flung global presence includes the medical college's campuses on the Upper East Side of Manhattan and in Doha, Qatar, as well as the new CornellNYC Tech campus to be built on Roosevelt Island in the heart of New York City.



Diversity and inclusion have been and continue to be a part of our heritage. Cornell University is a recognized EEO/AA employer and educator.

Entanglements

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But entanglements also occur in a more abstract sense—in oceanic float trajectory data, for instance (see Figure 2). Oceanographers would like to know features of the underlying ocean currents, such as how much mixing is taking place. But the float data is a coarse tool: It samples the flow only very sparsely. The scientists often perform local analyses of the data, such as computing an effective diffusivity of individual trajectories. A method of analysis that simultaneously uses all the data would be preferable.

In [12] we analyzed oceanic trajectories by turning them into a mathematical braid. Such a braid is a symbolic sequence of generators describing how the different trajectories cross each other’s paths. We can associate to the braid of trajectories a braiding factor [13], which can be

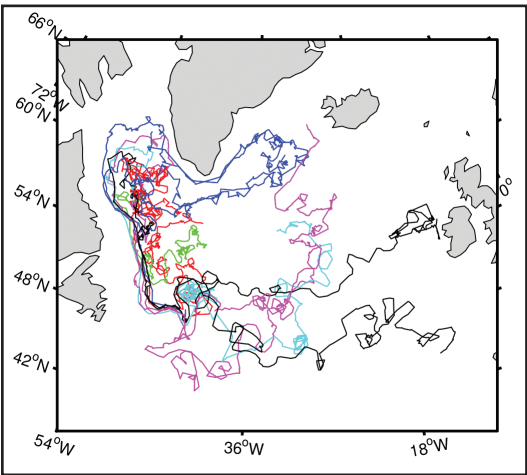


Figure 2. Float trajectories starting in the Labrador Sea. Reprinted with permission from J.-L. Thiffeault, Braids of entangled particle trajectories, *Chaos*, 20 (2010), 017516. Copyright 2010, AIP publishing LLC.

described as follows: Imagine the braid as a rigid structure; at the base hook a rubber band around some of the strands. Now force the rubber band along the rigid braid; the rubber band will typically stretch and grow. The amount by which it grows is the braiding factor. Its time-normalized logarithm, called the braiding exponent, is closely related to the geometric complexity of the braid [6]. If the braid is infinitely long or is repeated infinitely often, the exponent gives the topological entropy of the braid. The latter name has its origin in the realization of the braid in terms of pseudo-Anosov mapping classes of the punctured disk [4].

Unlike previous measures of complexity, such as the degree of the Alexander polynomial [10], the braiding factor enables us to distinguish between trivial braiding, as with strands twisted like a ribbon, and more intricate braiding, such as the classic pigtail braid (in fact, the pigtail braid is in some sense the “optimal” braid [7]).

What makes the rubber band calculation described above feasible is the fairly recent development of tools for rapidly computing the action of braids on loops [5, 9, 12]. These clever coordinates, called “Dynnikov coordinates,” efficiently encode the homotopy class of a simple closed loop by a small set of integers. They exploit the fact that a two-dimensional curve that winds without intersecting itself

quickly runs out of places to go. If a dynamical system has positive topological entropy, many loops will grow exponentially when acted upon by a braid of trajectories. If the state space of a system is partitioned into several invariant regions, however, there will exist special loops that grow very slowly or do not grow at all—corresponding to boundaries of the invariant regions. Together with Michael Allshouse [2], we developed a numerical method for detecting such slowly growing loops. We applied this method to trajectories generated by a Lagrangian chaotic mixing device; see Figure 3. This is a proposed method for identifying Lagrangian coherent structures [8]. A trajectory-based approach would have the advantage of not requiring the full velocity field to identify the coherent structures.

The method is not perfect: It requires rather long trajectories, and the data from oceanic floats is often too limited. In order to test and refine the method, Margaux Filippi, Tom Peacock, and Séverine Atis are designing a laboratory experiment that we hope to analyze using the braid approach. Marko Budišić is testing the limits of the braid approach using our freely available software package, braidlab [14].

The braid approach does not actually require an underlying flow. The trajectories generated can arise from processes of any type, not necessarily smooth. For instance, we used braids to analyze the dynamics of the grains in a two-dimensional granular medium [11]. An intriguing new application involves crowd dynamics [1]: If we track the position of individuals in a crowd, can we gauge the “complexity” of the crowd motion? S. Ali [1] has computed the braid complexity for several types of crowds, as shown in Figure 4. The Hajj appears at the bottom left (lowest complexity, because the motion of the pilgrims is so well ordered). How useful it is to classify motion this way remains to be investigated, but it is a promising new direction.

References

[1] S. Ali, *Measuring flow complexity in videos*, in 2013 IEEE International Conference on Computer Vision (ICCV), 1097–1104.
[2] M.R. Allshouse and J.-L. Thiffeault, *Detecting coherent structures using braids*, *Phys. D*, 241 (2012), 95–105.
[3] M.A. Berger, *Coronal heating by dissipation of magnetic-structure*, *Space Sci. Rev.*, 28 (1994), 3–14.
[4] P.L. Boyland, H. Aref, and M.A. Stremler, *Topological fluid mechanics of stirring*, *J. Fluid Mech.*, 403 (2000), 277–304.
[5] I.A. Dynnikov, *On a Yang–Baxter map and the Dehornoy ordering*, *Russian Math. Surveys*, 57 (2002), 592–594.
[6] I.A. Dynnikov and B. Wiest, *On the complexity of braids*, *J. Eur. Math. Soc.*, 9 (2007), 801–840.
[7] M.D. Finn and J.-L. Thiffeault, *Topological optimization of rod-stirring devices*, *SIAM Rev.*, 53 (2011),

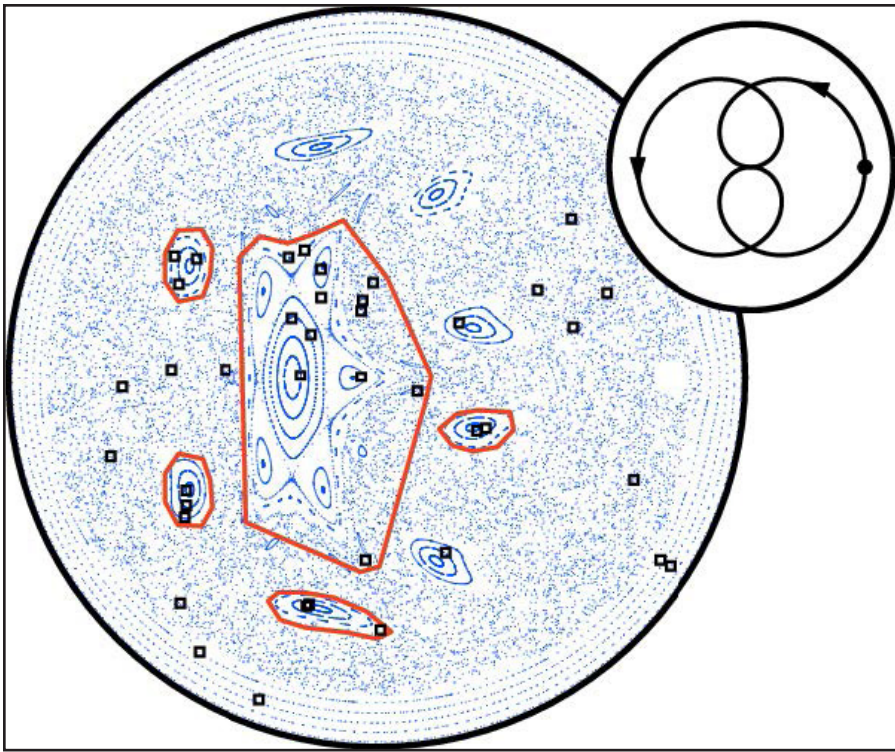


Figure 3. Poincaré section (stroboscopic map) for a vat of viscous fluid stirred by a rod (the rod’s path is shown in the inset). A chaotic sea and several islands are visible, along with regular orbits near the wall. The squares indicate the initial positions of the trajectories, with the detected non-growing loops drawn in. Reprinted from M.R. Allshouse and J.-L. Thiffeault, Detecting coherent structures using braids, *Phys. D*, 241 (2012), 95–105, with permission from Elsevier.

723–743.
[8] G. Haller and G. Yuan, *Lagrangian coherent structures and mixing in two-dimensional turbulence*, *Phys. D*, 147 (2000), 352–370.
[9] J.-O. Moussafir, *On computing the entropy of braids*, *Func. Anal. Other Math.*, 1 (2006), 37–46.
[10] S.K. Nechaev, *Statistics of Knots and Entangled Random Walks*, World Scientific, London, 1996.
[11] J.G. Puckett, F. Lechenault, K.E. Daniels, and J.-L. Thiffeault, *Trajectory entanglement in dense granular materials*, *J. Stat. Mech. Theory Exp.*, 6 (2012), P06008.
[12] J.-L. Thiffeault, *Braids of entangled particle trajectories*, *Chaos*, 20 (2010), 017516.
[13] J.-L. Thiffeault, *Measuring topological chaos*, *Phys. Rev. Lett.*, 94 (2005), 084502.
[14] J.-L. Thiffeault, M.R. Allshouse, and M. Budišić, *braidlab: A Matlab package for*

analyzing data using braids, 2014; <http://www.bitbucket.org/jeanluc/braidlab>.
[15] A.L. Wilmot-Smith, D.I. Pontin, and G. Hornig, *Dynamics of braided coronal loops. I. Onset of magnetic reconnection*, *Astron. Astrophys.*, 516 (2010), A5.

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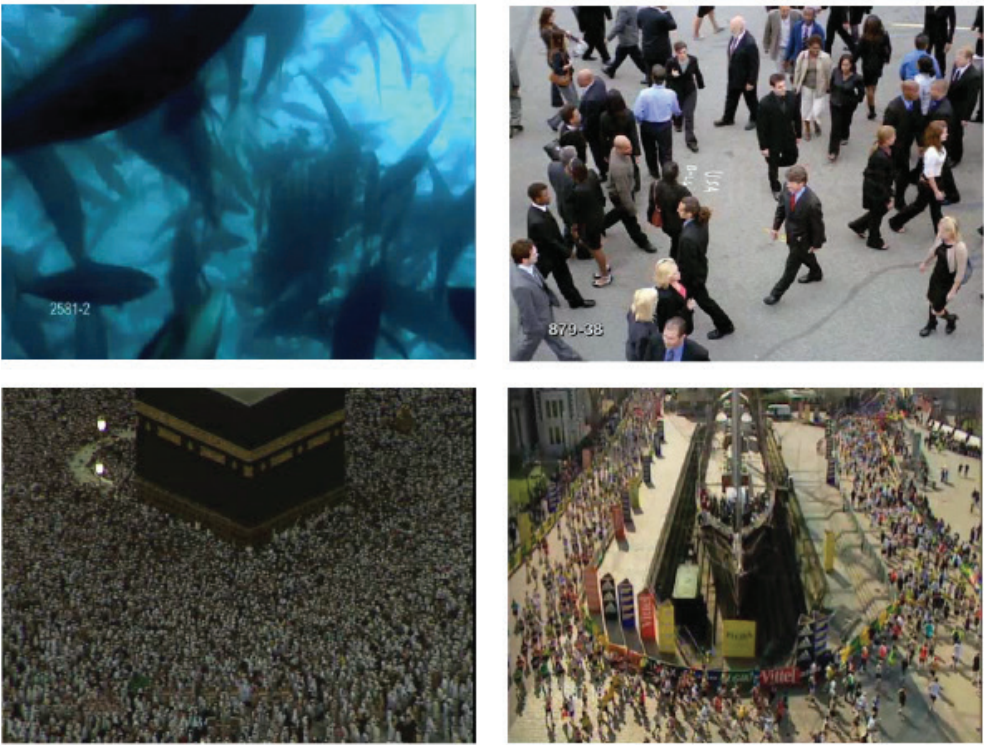


Figure 4. Various “crowds.” Clockwise from top left, in order of decreasing braid complexity: a school of fish; people walking in random directions; athletes participating in a marathon; and pilgrims performing the Hajj. © 2013 IEEE. Reprinted with permission from S. Ali, Measuring flow complexity in videos, in 2013 IEEE International Conference on Computer Vision (ICCV), 1097–1104.

Plant Growth

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Our model predicted, for example, that the outermost cell layer in a cylindrical root (the epidermis) has a predominant influence in regulating stretching and bending rates. It is therefore no surprise that the mobile hormone auxin, a key growth regulator, preferentially targets epidermal cells. Auxin also plays a central role in gravitropism, the bending response of a growing root that reorients its tip in the direction of the gravity vector. The mechanism has been known since the 1920s: Gravity-sensing cells

in the root tip bias the transport of auxin preferentially toward the underside of the root, promoting a reduction in the cell expansion rate, thereby inducing a bend. Our computational simulation of a gravitropic bend, which represents individual cells as polygons (in a 2D projection), allows the coupling of transport between individual cells and mechanical models that resolve individual cell walls (see Figure 1, right, and www.simuplant.org).

The UK funding councils that supported our project showed considerable vision in allowing the formation of a new multidisciplinary team, giving it time to gel, and allowing an ambitious research

programme to develop and mature. Importantly, the mathematical component of the programme was given the space to develop new modelling frameworks that extend beyond *Arabidopsis*, as well as the opportunity to develop models in tandem with experiments. And, of course, I now have an enduring fascination for plants, weeds included.

References

[1] R.J. Dyson, L. Band, and O.E. Jensen, *A model of crosslink kinetics in the expanding plant cell wall: Yield stress and enzyme action*, *J. Theoret. Biol.*, 307 (2012), 125–136.

[2] R.J. Dyson and O.E. Jensen, *A fibre-reinforced fluid model of anisotropic plant root cell growth*, *J. Fluid Mech.*, 655 (2010), 472–503.
[3] R.J. Dyson, G. Vizcay-Barrena, L.R. Band, et al., *Mechanical modelling quantifies the functional importance of outer tissue layers during root elongation and bending*, *New Phytologist*, 202 (2014), 1212–1222.
[4] J.A. Fozard, M. Lucas, J.R. King, and O.E. Jensen, *Vertex-element models for anisotropic growth of elongated plant organs*, *Front. Plant Sci.*, 4 (2013), 233.

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