Math Matters

Apply It.

The math behind... Compact Discs

Technical terms used:

Burst error, error-correcting code, Reed–Solomon code

Uses and applications:

Compact discs are convenient, portable ways to store music. However, frequent use can lead to problems such as fingerprints, dust particles, and scratches on the surface of the disc. Such physical disc problems cause burst errors, where several consecutive pieces of encoded information are in error. By implementing a particular type of error-correcting code, we can recover the music, despite any burst errors.

How it works:

A CD drive works by rotating the disc and scanning it with a laser. By regulating the rotational velocity based on the laser's position, we can ensure that data is read at a constant speed. The laser can detect pits (low points) and lands (high points) in the data track; this is a finely tuned system, and pits are only 0.12 nanometers deep. Physically, these encode binary bits over sections of track of length 0.3 nanometers. If a section changes from pit to land or vice versa, it represents a 1. If there is no change on that segment, it represents a 0.

Audio data is digitized by sampling the amplitude of the sound wave at a rate of 44,100 sample pairs per second. (Why pairs? One is for the left audio channel, and the other is for the right audio channel.) Each sample gets a binary representation of length 16, or two bytes. Thus, for each second of sound recording, we have 176,400 bytes of data to encode.

To combat burst errors, though, we don't encode the data in chronological order. Instead, we use a cross-interleaved Reed–Solomon code, which is a type of error-correcting code. Essentially, it separates samples on the data track that are chronologically consecutive. In the event of a burst error, data for temporally nearby sound could still be unharmed because it has been strategically moved away. If nearby data is available, the CD player can approximate the corrupt data by interpolating, using data from the reliable samples. Otherwise, the player can mute the offending data. Since each sample represents such a small fraction of a second, such a muting is quite difficult to notice.

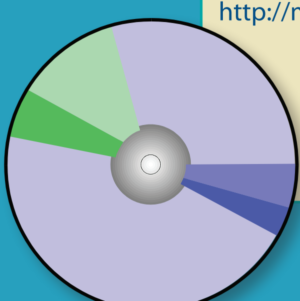
Interesting fact:

The length of the spiral track on a CD that stores the data is approximately 5 kilometers long. How does it all fit onto a disc that has a diameter of 12 centimeters? Well, the track is only 0.6 nanometers wide.

References:

W. C. Huffman, V. Pless, Fundamentals of Error-Correcting Codes (Cambridge University Press, New York, 2003)

http://micro.magnet.fsu.edu/electromag/java/cd/



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