overview of quantum error correction

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*opinions my own

goals

- introduce key concepts in quantum error correction (QEC)
- identify primary objectives and challenges
- current research topics and recent breakthroughs
- future directions
- disclaimer: focus on “standard” QEC with qubits
  - no bosonic QEC, measurement-based QC, measurement-free QEC

motivation: quantum computing

- obstacles: hardware capabilities and **error correction overheads**

- **fundamental** problem for quantum computing
  - **perfect** control and isolation of quantum information
  - **exponential** decay of "fidelity" (e.g., with # operations)
    - ~10^{-3} error rates → ~10^{3} physical operations

- **algorithmic** implications
  - exponential speedups: **rare**
  - quadratic speedups: all over the place

problem size n

runtime $= f(n) \rightarrow O(\sqrt{f(n)})$

arXiv:2306.08585 (quant-ph)

[Submitted on 14 Jun 2023]

How to compute a 256-bit elliptic curve private key with only 50 million Toffoli gates

Daniel Litinski

Focus beyond Quadratic Speedups for Error-Corrected Quantum Advantage

Ryan Babbush,[*] Jarrod R. McClean,[†] Michael Newman, Craig Gidney, Sergio Boixo, and Hartmut Neven

Google Quantum AI, Venice, California 90291, USA

(Received 10 November 2020; published 29 March 2021)

lightening crash course: repetition code

- repeat every bit: $\{0,1\} \rightarrow \{000,111\} =$ "code" C
- generally: $C =$ null space of **parity check** matrix H

$$H \cdot w = 0 \pmod 2 \qquad H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

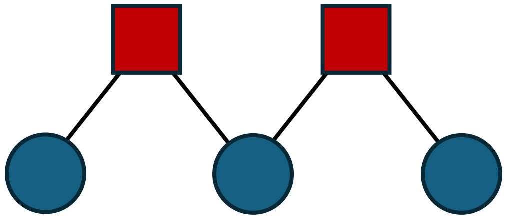
- error e induces a **syndrome** s

$$H \cdot (w + e) = H \cdot e = s \qquad e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow H \cdot e = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- **decoding**: given syndrome s, find most likely error e

NP hard in general!

- Tanner graph



code parameters: $[n, k, d]$

- $n = [\#\ \text{data bits}]$ $\qquad$ $k = [\#\ \text{message bits}]$
  - code rate: k/n
- distance d: minimum Hamming distance between code words
  - minimum Hamming weight of undetectable error

$d \sim$ “robustness”

- [# correctable errors] $= \left\lfloor \frac{d-1}{2} \right\rfloor \approx \frac{d}{2}$
- repetition code: $[n, 1, n]$
- “good codes”: $k, d \propto n$

quantum codes: $[\![n, k, d]\!]$

- n = [# physical qubits] k = [# logical qubits]
  - code rate: k/n
- distance d: ~~minimum Hamming distance between code words~~
  - minimum ~~Hamming~~ weight of undetectable error

$d \sim$ "robustness"

- [# correctable errors] $= \left\lfloor \frac{d-1}{2} \right\rfloor \approx \frac{d}{2}$
- ~~repetition code: $[n, 1, n]$~~
- "good codes": $k, d \propto n$

stabilizer codes

- code space: $\{|\psi\rangle : S|\psi\rangle = |\psi\rangle \text{ for all checks } S\}$

$Z_1Z_2 \rightarrow$ parity of qubits 1, 2

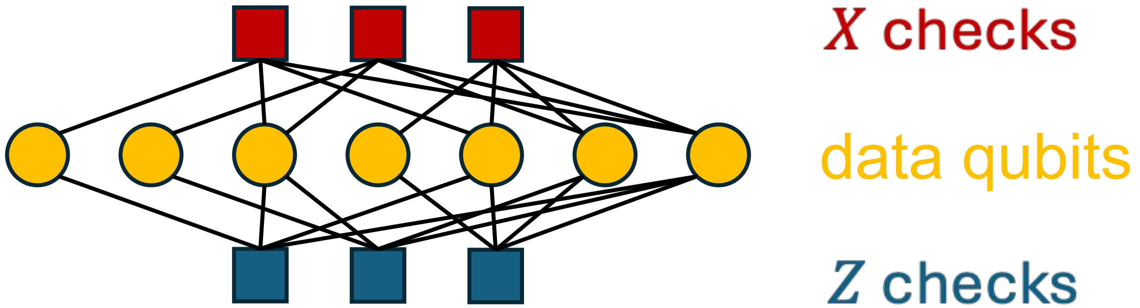
- errors:

bit flips: $|0\rangle \leftrightarrow |1\rangle$

phase flips: $|0\rangle + |1\rangle \leftrightarrow |0\rangle - |1\rangle$

- CSS codes:

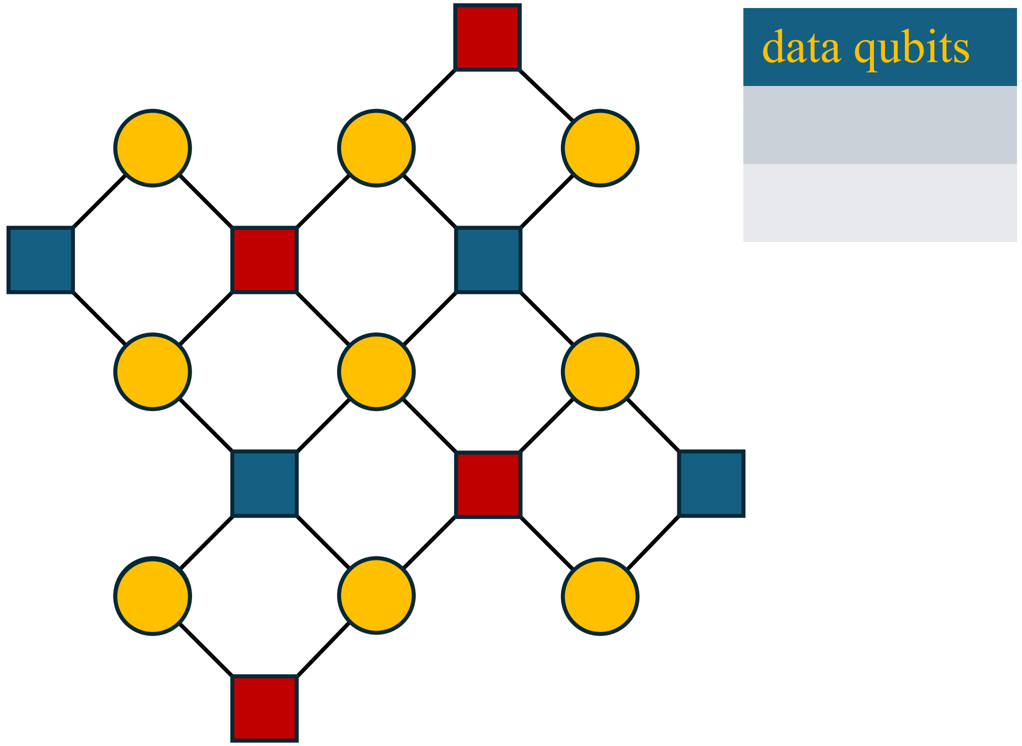
Calderbank-Steane-Shor



- stabilizer measurements $\leftrightarrow$ syndrome extraction
  - syndrome = measurement outcomes: $(+1, -1, +1, \ldots)$

surface code

- Tanner graph ↔ qubit layout
- nearest-neighbor connectivity
- $[\![n = L^2, k = 1, d = L]\!] \;\rightarrow\; [\![kL^2, k, L]\!]$
- code distance (robustness): $d \sim \sqrt{n}$
- code rate (efficiency): $\frac{k}{n} = \frac{1}{d^2}$



locality and overheads

- surface code: $kd^2 = n$
  - "good code": $k, d \propto n$

Tradeoffs for Reliable Quantum Information Storage in 2D Systems
Sergey Bravyi, David Poulin, and Barbara Terhal
Phys. Rev. Lett. **104**, 050503 – Published 5 February 2010

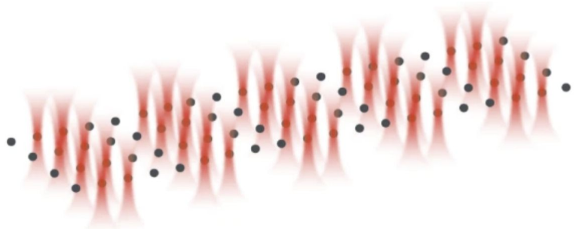
- 2D codes w/ local stabilizers: $kd^2 = O(n)$ "BPT bound"

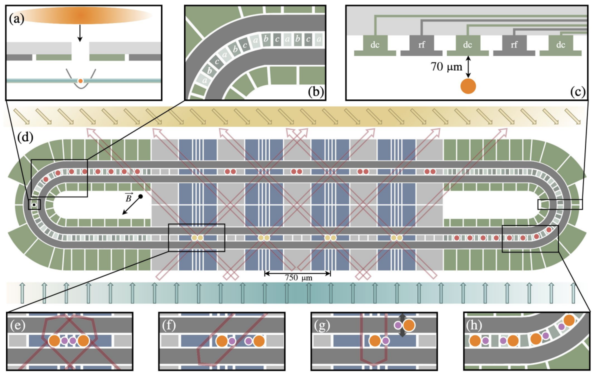
- $k = 1,\ d = 10 \implies n = 100$
  - Google: $d{\sim}30 \implies n{\sim}1000$

Article | Open access | Published: 06 December 2023
Logical quantum processor based on reconfigurable atom arrays
Nature **626**, 58–65 (2024)

A Race-Track Trapped-Ion Quantum Processor
S. A. Moses *et al.*
Phys. Rev. X **13**, 041052 – Published 18 December 2023

- alternatives:
  - qubit movement
  - non-local classical communication

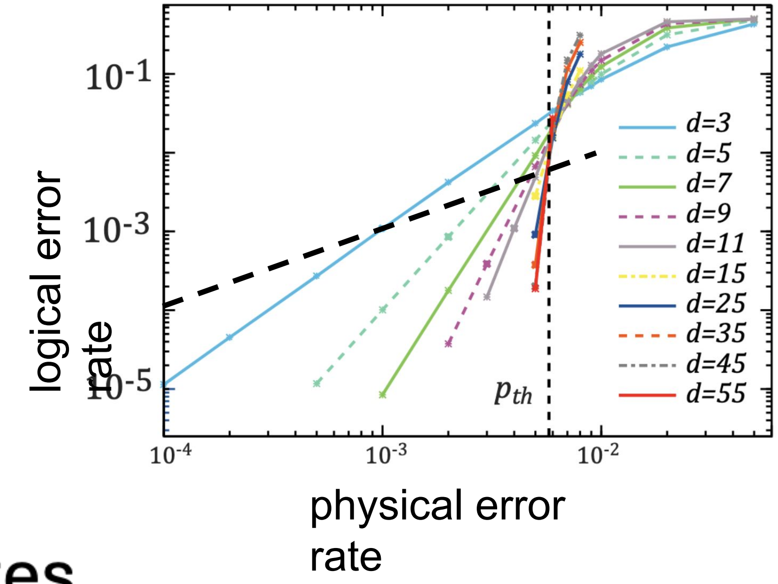




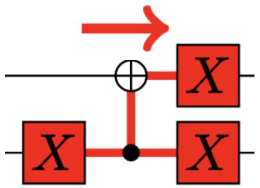
Hierarchical memories: Simulating quantum LDPC codes with local gates
Christopher A. Pattison, Anirudh Krishna, John Preskill arXiv:2303.04798

fault tolerance and universal computation

Fowler, Mariantoni, Martinis, Cleland
Phys. Rev. A 86, 032324



- correcting errors with faulty operations
  - remove more errors than you add
  - threshold thrm: need physical error rate $p < p_{\text{th}}$
- faulty measurements: repeat QEC d times
  - wanted: "single-shot" QEC
- error propagation and "transversal" logical gates
  - transversal gate sets are incomplete (Eastin-Knill theorem)
  - the transversal gates are "not quantum" (Gottesman-Knill theorem)
  - "magic" states



Restrictions on Transversal Encoded Quantum Gate Sets

Bryan Eastin and Emanuel Knill
Phys. Rev. Lett. **102**, 110502 – Published 18 March 2009

bird's eye view

codes

- local codes
  - topological codes ~ surface, color
- non-local codes
  - "qLDPC" codes
- subsystem codes
- dynamical codes
  - "Floquet" codes

challenges

- better codes
- better decoders
- fault tolerance
- universal quantum computation

recent(ish) breakthroughs!

- “good” quantum codes exist (2021+) *
  - $k, d \propto n$ LDPC
  - local architectures?!? (arXiv:2303.04798)
  - practical codes? (notable: IBM, Nature 627, p778–782 (2024))

- hardware demonstrations of QEC (2023+)
  - scalable surface code architecture
  - beyond-breakeven logical operations

- asymptotic QEC overhead reductions
  - magic state distillation
  - magic state cultivation

Asymptotically good Quantum and locally testable classical LDPC codes
Authors: Pavel Panteleev, Gleb Kalachev | Authors Info & Claims — *Proceedings* > *STOC 2022*

Good Quantum LDPC Codes with Linear Time Decoders
Authors: Irit Dinur, Min-Hsiu Hsieh, Ting-Chun Lin, Thomas Vidick | Authors Info & Claims — *Proceedings* > *STOC 2023*

Suppressing quantum errors by scaling a surface code logical qubit
Google Quantum AI — *Nature* **614**, 676–681 (2023)

Quantum error correction below the surface code threshold
Google Quantum AI and Collaborators — arXiv:2408.13687

Demonstration of logical qubits and repeated error correction with better-than-physical error rates — arXiv:2404.02280
[1] *Microsoft Azure Quantum*
[2] *Quantinuum*

Time-Efficient Constant-Space-Overhead Fault-Tolerant Quantum Computation
Hayata Yamasaki & Masato Koashi — *Nature Physics* **20**, 247–253 (2024)

How to fault-tolerantly realize any quantum circuit with local operations
Shin Ho Choe[1,2] and Robert König[1,2] — arXiv:2402.13863

Constant-Overhead Magic State Distillation
Adam Wills, Min-Hsiu Hsieh, Hayata Yamasaki — arXiv:2408.07764

Magic state cultivation: growing T states as cheap as CNOT gates
Craig Gidney, Noah Shutty, Cody Jones — arXiv:2409.17595

future of QEC research

- local codes
  - faster (large-scale, real-time) decoding algorithms
  - magic state factories
- non-local, subsystem, and dynamical codes
  - codes + code families at **small-to-moderate size**
  - universal computation
  - better decoders
- QC + QEC **architectures**
  - logical operations
  - compatible quantum algorithms

thank you