

# Searchable Abstracts Document

# SIAM Conference on Analysis of Partial Differential Equations (PD25)

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#### IP1

### Overhanging Water Waves in Incompressible, Constant Vorticity Fluids

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#### IP2

### The Study of Wave Turbulence: A Meeting Ground for Mathematical Techniques

Wave turbulence theory provides a unifying framework for analyzing the nonlinear interactions of dispersive waves across diverse physical regimes. These interactions manifest over an extensive range of scales and media, encompassing phenomena such as gravitational waves in astrophysical contexts, surface waves in fluid dynamics, coherent structures in quantum systems, and even pattern formation in everyday fluid flows. Despite their widespread occurrence, the mathematical analysis of such systems remains exceptionally challenging due to the interplay between nonlinearity, dispersion, and randomness. These difficulties have catalyzed the development of sophisticated analytical and probabilistic tools, drawing from partial differential equations, statistical mechanics, and dynamical systems theory. In this lecture, I will present a short survey of recent advances in the rigorous treatment of wave turbulence, emphasizing key techniques, foundational results, and persistent open problems. Some attention will be given to the derivation and analysis of kinetic equations, scaling limits, and the role of resonant interactions in the long-time dynamics of weakly nonlinear wave systems.

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#### IP3

#### On the Vanishing Viscosity Limit and Boundary Layers in Incompressible Fluids

I will review recent advances in our understanding of incompressible fluid flow at small viscosity near rigid walls. Formally, as viscosity vanishes, solutions to the Navier-Stokes equations should converge to solutions to the Euler equations at least in the energy norm, the so-called vanishing viscosity limit. However, the validity of this limit is still poorly understood mathematically. As a matter of fact, near walls viscosity cannot be neglected, no matter how small. The effect of viscosity is captured by the behavior of the fluid in a small layer near the boundary, called a viscous layer. Such behavior is neither described by solutions to the Navier-Stokes equations nor by solutions of the Euler equations. The boundary layer can detach from the wall due to instabilities and leads to the creation of large vorticity in the flow, resulting in increased drag and potential loss of lift.

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#### IP4

### Vortex Reconnection in 3d Critical Abelian Higgs Models

The critical Abelian Higgs model (AHM) is a system of nonlinear wave equations arising in particle physics. We construct solutions of this system in 3+1 dimensions that exhibit a number of slowly-moving nearly parallel vortex filaments. The leading-order dynamics of this ensemble of filaments are described by a wave map into the modulo space, a manifold carrying a natural Riemannian structure that parametrizes stationary 2D solutions of the AHM. These results allow for the study of the poorly-understood phenomenon of vortex reconnection in this setting. In particular, it is shown that in the regime studied, reconnection is the generic outcome of collisions of pairs of vortex filaments. Extremely similar results are also proved for the critical Abelian Higgs heat flow, modeling certain superconductors, and in higher dimensions. This work is joint with Masoud Geevechi.

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#### IP5

#### SIAG/APDE Early Career Prize Lecture Regularity in Nonuniformly Elliptic Problems

I will review recent advances on the regularity of solutions to nonuniformly elliptic problems, with emphasis on the achievement of Schauder theory within the sharp nonuniformity range.

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#### IP6

### Monotonicity of the Fisher Information in Kinetic Equations.

We discuss recent results showing that the standard Fisher information is monotone in time for the space homogeneous Boltzmann and Landau equations. This new Lyapunov functional allows us to establish the existence of global smooth solutions in all cases that remained open. To prove this monotonicity, we introduce a novel doubling-variables technique and reduce the problem to an inequality in the family of the log-Sobolev inequalities.

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#### IP7

#### Stability and Growth for 2D Euler Equation

In this talk, I will discuss some recent results on the 2D incompressible Euler equation in the whole plane, where we show the orbital stability of some traveling wave solutions, and also use the stability to establish vorticity gradient growth. The results include the nonlinear stability of vortex quadrupoles with odd-odd symmetry (joint with Kyudong Choi and In-Jee Jeong), nonlinear stability

of multiple Lamb dipoles (joint with Ken Abe and In-Jee Jeong), and using stability to obtain growth of vorticity gradient (joint with In-Jee Jeong and Tao Zhou).

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#### SP1

#### SIAG/APDE Best Paper Prize Lecture-Curvature-Driven Wrinkling of Thin Elastic Shells

Archive for Rational Mechanics and Analysis 239 (2021) 1211-1325. It studies wrinkling patterns seen in thin elastic shells. Using tools from the calculus of variations, the paper develops an elegant and deep theory with amazing predictive power. The selection committee members were Maria-Carme Calderer (Chair), University of Minnesota; Nestor Guillen, Texas State University; Robert Marangell, Sydney University; Anna Mazzucato, Pennsylvania State University; and Jens Rademacher, University of Hamburg.

<u>Ian Tobasco</u> University of Michigan Department of Mathematics itobasco@umich.edu

#### CP1

### Title: Lower Bounds on the Radius of Analyticity for Parabolic Equations

In 2011, Z. Lei and F. Lin presented a proof of existence of a global mild solution of the three-dimensional Navier-Stokes equations in scale-critical function spaces based on the Wiener algebra, for sufficiently small initial data in full space. In 2015, H. Bae modified the technique used by Lei and Lin to prove lower bounds on the radius of analyticity of the solution obtained. Recently, Ambrose, Lopes Filho and Nussenzveig Lopes adapted Bae's argument to periodic flows, which changes the radius estimate a little. The same authors have adapted these ideas to revisiting the 2004 existence result by M. Cannone and G. Karch in spaces of pseudo-measures, extending it to a larger space of initial data. In addition, they applied this method to the scalar Kuramoto-Sivashinsky equation, where we improve known existence results and obtain new estimates on the radius of analyticity of the solution, both in spaces based on pseudomeasures and on the Wiener algebra and to the viscous Constantin-Lax-Majda equation. The purpose of this talk is to present the latest results obtained.

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#### CP1

#### A Quasi-Incompressible Cahn-Hilliard-Darcy Model for Two Immiscible Fluids in Porous Media.

We introduce a quasi-incompressible CahnHilliardDarcy (qCHD) model that, for the first time, integrates diffuse-interface thermodynamics with Darcy flow within a fully consistent energetic framework, permitting a non-zero velocity divergence. Derived through Onsagers variational principle, the system combines a singular FloryHuggins free energy with Darcy-type dissipation, resulting in mass-conservative, energy-dissipative, and pressure-driven dy-

namics, while maintaining adjustable compressibility. Utilizing the derived energy law, we establish the global existence of weak solutions. A matched-asymptotic expansion further demonstrates that as the interface thickness approaches zero, the model rigorously converges to the classical Muskat problem, thereby connecting the phase-field and sharp interface theories. These findings provide the first comprehensive analytical foundation for the dynamics of the interface of two immiscible fluids with unmatched densities in porous media and establish a basis for structure-preserving numerical schemes.

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#### CP1

#### Hydrodynamic Limit of a Boltzmann Equation : General Inflow Boundary Data and Poisson Coupling

Input your abstract, including TeX commands, here. The abstract should be no longer than 1500 characters, including spaces. Only input the abstract text. Don't include title or author information here. An approximation by diffusion of a Boltzmann equation is studied. A linear time relaxation model and an inflow boundary data with a general profile are considered. A corrected Hilbert expansion and the contraction property of the collision operator are used to establish a uniform L<sup>1</sup>-estimate. A correction of the boundary layer at the first order is introduced in order to prove a strong convergence and to exhibit a rate of convergence. The limit fluid model is a Drift-Diffusion model associated with effective boundary data obtained as a decay at infinity of a Half-Space problem. The analysis is performed, in the first step, for the linear case (prescribed potential). In the second step, the analysis is extended to the case of a self-consistent potential (Poisson coupling) in one dimension by carefully combining the relative entropy method and a perturbation of the Hilbert expansion; giving the convergence and a rate of convergence. This is a joint work with Samia Ben Ali

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#### CP1

# High-Resolution Finite Volume Simulation of Atmospheric Lamb Waves Based on a Modified Shallow Water Model

Atmospheric Lamb waves are low-frequency acoustic gravity waves triggered by large scale geophysical events such as major volcanic eruptions. The January 15, 2022 eruption of the Hunga TongaHunga Haapai undersea volcano produced one of the most extensively recorded Lamb waves in modern history, propagating multiple times around the Earth. Motivated by this event, we investigate a mathematical model of the Lamb wave propagation based on a modified shallow water system, following the formulation of Amores et al. (Geophysical Research Letters, 2022), in which spatial variation in tropospheric temperature is incorporated as an effective bathymetry term. We numer-

ically solve this system using high-resolution finite volume methods, specifically the Wave-Propagation Algorithm (R. J. Leveque, 2002) implemented on adaptively refined grids. Particular focus is given to the spatially varying flux terms and the accurate capture of wave structure across adaptively refined grids. The use of adaptive mesh refinement via Forestclaw (D. Calhoun, C. Burstedde 2017) allows efficient simulation of the long range propagation of Lamb waves while maintaining high-resolution near localized features. Our results demonstrate the effectiveness of this approach in capturing the observed wave behavior on a global scale.

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#### CP1

#### Breakdown of Higher-Order Epdiff Equations

A certain family of PDEs, called EPDiff equations, can be realized as the geodesic equation of the right-invariant Sobolev metric of order k on the diffeomorphism group of  $\mathbb{R}^n$ . The EPDiff equation generalizes the classic onedimensional models in fluid dynamics, such as Burgers, the Camassa-Holm, and the Hunter-Saxton equation. By exploiting the geometric framework, we obtain a breakdown criteria for solutions to EPDiff equations. Our approach relies on considering radial solutions and using Lagrangian coordinates to convert the EPDiff equation to an ODE on a Banach space, thereby obtaining  $C^1$  blow-up of the velocity field solution by direct comparison with the Liouville equation. In addition to reproducing known breakdown results for the one-dimensional fluid models, we demonstrate the novelty of our criteria by applying it to EPDiff with the Sobolev inertia operator of order k = 2, where the breakdown only occurs in higher dimensions. Moreover, we can show breakdown of solutions to EPDiff with the homogeneous Sobolev inertia operator for every  $1 \le k \le n/2 + 1$ . This is joint work with Martin Bauer and Stephen Preston.

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#### CP1

### On the Global Regularity of the Navier-Stokes Equations

We establish a sufficient condition on the initial data that ensures the existence of global smooth solutions to the incompressible Navier-Stokes equations on  $\mathbb{R}^d$ , for dimensions  $d \geq 3$ . The solutions constructed exhibit rapid decay at spatial infinity, satisfying the physical energy bounds expected of physically reasonable flows. Furthermore, we demonstrate that, for positive times, the velocity and pressure fields admit an analytic extension to the complex domain, forming a smooth curve of entire vector fields of order two. Our results provide new insights into the structure and regularity of Navier-Stokes flows under explicit and

verifiable conditions on the initial data.

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#### CP2

### Bi-Continuous Semigroups for Flows on Infinite Networks

We study transport processes on infinite metric graphs with non-constant velocities and matrix boundary conditions in the  $L^{\infty}$ -setting. In this framework, the standard translation semigroup fails to be strongly continuous, which poses significant analytical challenges. To overcome this, we develop an approach based on bi-continuous operator semigroups, well-adapted to the weak\*-topology, and establish the well-posedness of the associated abstract Cauchy problem. Our analysis covers three principal cases: a basic model with unit velocities and no absorption, a scenario with rationally dependent velocities where a transformation via graph augmentation is employed, and the general case for finite networks with arbitrary velocities. These results extend classical semigroup theory to infinitedimensional settings and offer a robust framework for future investigations of asymptotic behavior and stability in network flows. Contributions by the authors and their collaborators highlight the innovative application of bicontinuous semigroup techniques to address limitations of conventional methods.

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#### CP2

### A Variational Perspective on Auxetic Metamaterials of Checkerboard-Type

In this talk, we present a homogenization theorem via  $\Gamma$ convergence for elastic materials with stiff checkerboardtype heterogeneities under the assumptions of physical growth and non-self-interpenetration. While the obtained energy estimates are rather standard, determining the effective deformation behavior, or in other words, characterizing the weak Sobolev limits of deformation maps whose gradients are locally close to rotations on the stiff components, is the challenging part. To this end, we establish an asymptotic rigidity result, showing that, under suitable scaling assumptions, the attainable macroscopic deformations are affine conformal contractions. This identifies the composite as a mechanical metamaterial with a negative Poisson's ratio. Our proof strategy is to tackle first an idealized model with full rigidity on the stiff tiles to acquire insight into the mechanics of the model and then transfer the findings and methodology to the model with diverging elastic constants. The latter requires, in particular, a new quantitative geometric rigidity estimate for non-connected squares touching each other at their vertices and a tailored Poincar type inequality for checkerboard structures.

#### Wolf-Patrick Düll

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#### CP2

#### Global Compactness Result for a Brezis-Nirenberg-Type Problem Involving Mixed Local Nonlocal Operator

We have investigated the profile decomposition of Palais-Smale sequences associated with a Brezis-Nirenberg type problem involving a combination of mixed local nonlocal operators, given by

$$\begin{cases} -\Delta u + (-\Delta)^s u - \lambda u = |u|^{2^* - 2} u & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega. \end{cases}$$

where  $\Omega \subseteq \mathbb{R}^N$  is a smooth bounded domain with N > 3,  $s \in (0,1)$ ,  $\lambda \in \mathbb{R}$  is a real parameter and  $2^* = \frac{2N^-}{N-2}$  denotes the critical Sobolev exponent. As an application of the derived global compactness result, we further study the existence of a positive solution of the corresponding Coron-type problem [J.M. Coeon, Topologie et cas limite des injections de Sobolev (1984)] when  $\lambda = 0$ .

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#### CP2

### Normalized Solutions to a Quasilinear Equation Involving Critical Sobolev Exponent

We study the existence and regularity results of normalized solutions to the following quasilinear elliptic Choquard equation with critical Sobolev exponent and mixed diffusion type operators:

$$-\Delta_p u + (-\Delta_p)^s u = \lambda |u|^{p-2} u + |u|^{p^*-2} u + \mu (I_\alpha * |u|^q) |u|^{q-2}$$

where  $N \geq 3$ ,  $\tau > 0$ ,  $\frac{p}{2} \left( \frac{N+\alpha}{N} \right) < q < \frac{p}{2} \left( \frac{N+\alpha}{N-p} \right)$ ,  $I_{\alpha}$  is the Riesz potential of order  $\alpha \in (0,N)$ ,  $\mu > 0$  is a parameter,  $(-\Delta_p)^s$  is the fractional p-Laplacian operator,  $p^* = \frac{Np}{N-p}$  is the critical Sobolev exponent, and  $\lambda$  appears as a Lagrange multiplier. \*\*Keywords\*\*: Normalized solutions, Choquard equation, critical growth, local and nonlocal operator, existence results, Hlder regularity.

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#### CP3

#### Physics Informed Signal Reconstruction

In this paper, we develop a theoretical framework for understanding the limits of function reconstruction in cases where the function satisfies a first-order partial differential equation (PDE). We derive minimax optimality rates for interpolation and approximation algorithms for three function classes: 1-Lipschitz functions satisfying first-order linear  $(\mathcal{F}_l)$ , quasilinear  $(\mathcal{F}_{ql})$ , and nonlinear  $(\mathcal{F}_{nl})$  PDEs by leveraging the property induced by their characteristic curves. For all three classes, we establish an improved minimax rate, which scales asymptotically with the sample size n as  $\left(\frac{\log n}{n}\right)^{1/(d-1)}$  where d is the dimension of the domain. This is an improvement over the classical minimax rate of  $\left(\frac{\log n}{n}\right)^{1/d}$  known for the class of 1-Lipschitz function. Our results guarantee only the local existence of these minimax rates for the quasilinear  $\mathcal{F}_{ql}$  and nonlinear  $\mathcal{F}_{nl}$  classes. These findings highlight the significance of prior knowledge of the underlying PDE structure in enhancing function reconstruction from the sample data. We present numerical results to validate our theoretical find-

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#### CP3

### Inverse Problems for Bathymetry Estimates in a River Network

Reconstructing river bathymetry from altimetry measurements, for gauged or ungauged rivers, is a crucial issue in water surface monitoring and Earth observation (EO). Following [Thibault Malou, Jerome Monnier. Two-scale diffusive wave equation dedicated to spatial rivers observations. 2023. ?hal-03386480v2?], we derive a semi-linear steadystate equation characterizing river flow in one single single river section delimited by a domain  $\Omega$ . We then study theoretically and numerically the associated inverse problem consisting in estimating the unobserved river bathymetry b from water surface (WS) measurements H. We prove the problem is ill-posed [J.-L. Lions, Optimal Control of Systems Governed by Partial Differential Equations. Springer Berlin, Heidelberg, Jul. 1971, vol. 4, pp. 236237, and has an infinite number of solutions uniquely determined by an unitial condition of the form  $b(x_0)$ , for  $x_0 \in \Omega$ . Moreover, we identify Ithe minimal inferable scale in our model and provide both deterministic and stochastic estimates of b, through Variational Data Assimilation methods [M. Asch, M. Bocquet, and M. Nodet, Data assimilation: methods, algorithms, and applications. SIAM, 2016, vol. 11.]. Subsequently, we introduce a model for river networks in the form of watersheds, i.e. where all branches which are all directly or indirectly connected to a main one, and extend our theoretical and numerical results to this system.

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#### CP3

### Rigidity from Infinity for Ideal Alfvn Waves in 3D Thin Domains

This talk concerns the rigidity from infinity for Alfvén waves governed by ideal incompressible magnetohydrodynamic equations subjected to strong background magnetic fields along the  $x_1$ -axis in 3D thin domains  $\Omega_{\delta} = \mathbb{R}^2 \times (-\delta, \delta)$  with  $\delta \in (0, 1]$  and slip boundary conditions. We show that in any thin domain  $\Omega_{\delta}$ , Alfvén waves must vanish identically if their scattering fields vanish at infinities. As an application, the rigidity of Alfvén waves in  $\Omega_{\delta}$ , propagating along the horizontal direction, can be approximated by the rigidity of Alfvén waves in  $\mathbb{R}^2$  when  $\delta$  is sufficiently small.

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#### CP3

### Extending Qualitative Reconstruction to Biharmonic Scattering with Limited Data

In this talk, we discuss an extension of the relatively new Extended Sampling Method (ESM) to inverse shape problems in biharmonic wave scattering with limited-aperture far-field data. This computationally simple and mathematically rigorous point-sampling approach defines an indicator function that recovers the approximate location of the scatterer. Using far-field measurements, we analyze the stability of the ESM and demonstrate its effectiveness as a reconstruction tool. Our results will include scattering from clamped regions under various data regimes: a single incident wave at fixed frequency, multiple incident waves at fixed frequency, and a single incident wave across multiple frequencies demonstrating versatility that makes the method well-suited for providing reliable initial guesses or target localization in iterative reconstruction schemes. This is based on joint work with Peijun Li and Isaac Harris.

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#### CP3

### Inverse Scattering for Sources and Media with Corners

In this presentation, we consider scattering and inverse scattering for sources and penetrable media. In particular, we consider two-dimensional scattering sources and media with corners on their supports. We show that sources and media with corners non-trivially scatter all incident waves under certain admissible conditions. In addition, we discuss the inverse problem of uniquely determining the shape of a polygonal source region from its corresponding far-field data.

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#### CP4

Viscosity Solutions of Hamilton-Jacobi-Bellman

#### Equation for the Optimal Control of the Stochastic Convective Brinkman-Forchheimer Equations

In this work, we consider the following two- and three-dimensional stochastic convective Brinkman-Forchheimer (SCBF) equations in torus  $\mathbb{T}^d$ ,  $d \in \{2,3\}$ :

$$d\mathbf{u} + \left[-\mu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \alpha \mathbf{u} + \beta |\mathbf{u}|^{r-1}\mathbf{u} + \nabla p\right] dt = dW, \ \nabla \cdot \mathbf{u} = 0,$$

where  $\mu, \alpha, \beta > 0, r \in [1, \infty)$  and W is a Hilbert space valued Q-Wiener process. The above system can be considered as damped stochastic Navier-Stokes equations. Using the dynamic programming approach, we study the infinitedimensional second-order Hamilton-Jacobi equation associated with an optimal control problem for SCBF equations. For the supercritical case, that is,  $r \in (3, \infty)$  for d = 2 and  $r \in (3,5)$  for d = 3  $(2\beta\mu > 1$  for r = 3 in  $d \in \{2,3\}$ ), we first prove the existence of a viscosity solution for the infinite-dimensional HJB equation, which we identify with the value function of the associated control problem. By establishing a comparison principle for  $r \in (3, \infty)$  and r = 3with  $2\beta\mu \geq 1$  in  $d \in \{2,3\}$ , we prove that the value function is the unique viscosity solution and hence we resolve the global unique solvability of the HJB equation in both two and three dimensions.

#### Sagar Gautam

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#### CP4

# Quasilinear Schrödinger Equation Involving Critical Hardy Potential and Choquard Type Exponential Nonlinearity

In this article, we study the following quasilinear Schrödinger equation involving Hardy potential and Choquard type exponential nonlinearity with a parameter

$$\left\{ -\Delta_N w - \Delta_N(|w|^{2\alpha})|w|^{2\alpha-2}w - \lambda \frac{|w|^{2\alpha N - 2}w}{\left(|x|\log\left(\frac{R}{|x|}\right)\right)^N} = \left(\int_{\Omega} \frac{H(y,w(y))}{|x-y|^{\mu}}dy\right) \right\}$$

where  $N\geq 2$ ,  $\alpha>\frac{1}{2}$ ,  $0\leq \lambda<\left(\frac{N-1}{N}\right)^N$ ,  $0<\mu< N$ ,  $h:\mathbb{R}^N\times\mathbb{R}\to\mathbb{R}$  is a continuous function with critical exponential growth in the sense of the Trudinger-Moser inequality and  $H(x,t)=\int_0^t h(x,s)ds$  is the primitive of h. With the help of the Mountain Pass Theorem and critical level, which is obtained by the sequence of Moser functions, we establish the existence of a positive solution for a small range of  $\lambda$ . Moreover, we also investigate the existence of a positive solution for a non-homogeneous problem for every  $0\leq \lambda<\left(\frac{N-1}{N}\right)^N$ . To the best of our knowledge, the results obtained here are new even in case of N-Laplace equation with Hardy potential.

#### Shammi Malhotra

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#### CP4

# A Posteriori Error Estimates for the Parabolic Partial Differential Equations with Small Random Input Data - Elliptic Reconstruction Approach

We propose and analyze residual-based a posteriori error estimates for parabolic partial differential equations (PDEs) with small random input data in the

 $L_P^2(\Omega; L^\infty(0,T;L^2(D)))$ -norm, where  $(\Omega,\mathcal{F},P)$  is a complete probability space, D is the physical domain, T>0 is the final time. We apply the perturbation technique to deal with uncertainty, wherein solving a PDE with small random input data is equivalent to solving deterministic PDEs. To approximate the solution for these problems, we employ finite element method for the physical space approximation and backward Euler scheme for time discretization. To obtain optimality in space, we employ the elliptic reconstruction operator in more general settings. The results could be seen as a generalization of the work on deterministic parabolic PDEs with the Dirichlet boundary condition to the parabolic PDEs with small uncertainties in a Robin boundary condition. Numerical investigations confirm the theoretical findings.

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#### CP4

### Surjectivity of Elliptic Operators on Non-Compact Manifolds

Existence of solutions to elliptic equations is usually studied on bounded domains and compact manifolds. In this talk, I will discuss the  $C^{\infty}$  existence problem for elliptic equations on non-compact manifolds. In particular, I will describe a proof that a second-order elliptic differential operator P, on any manifold M, has closed range in  $C^{\infty}(M)$ . If M has no compact components, then P is surjective on  $C^{\infty}(M)$ . I will discuss applications of this theorem to a generalized Helmholtz decomposition. This talk relates to the preprint at arXiv:2203.07534.

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#### CP4

### Variational Approximation of Heat Flow of Harmonic Maps into Non-Positively Curved Manifold

We introduce a new approach to construct (weak) heat flow of harmonic maps between manifolds based on the Weighted-Energy-Dissipation (WED) approach, which involves a variational functional with a small parameter. For smooth target manifolds, we recover the well-known theorems by Eells-Sampson (on NPC target manifolds) through Dynamical Variational Principle (DCP) as well as PDE convergence approach.

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#### CP5

## Existence and Uniqueness of Weak Solutions to a Structural Acoustic Model with $C^1$ Source Term on the Plate

This talk considers a structural acoustic model consisting of a semilinear wave equation defined on a smooth bounded domain  $\Omega \subset \mathbb{R}^3$  coupled strongly with a Berger plate equation acting on the flat portion of the boundary of  $\Omega$ , where the system is influenced by arbitrary  $C^1$  source term on the plate equation. Nonlinear semigroups and monotone operator theory will be used to establish the existence of local weak solutions, said solutions will be shown to be unique, and the global existence of potential well solutions will be established. Sufficient criteria for the continuous dependence of the solutions on the initial data will also be presented.

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#### CP5

#### Shock Waves in Magnetosphere: Earth and Saturn

Solar winds from the Sun interact with planetary magnetospheres, causing shock waves. We have studied the dynamics of these shocks using Lie group theoretic methods, obtaining similarity solutions for both power and exponential law shock paths. It is interesting to note how the wave dynamics evolve with time. The models considered are one-dimensional, unsteady, and adiabatic magnetogasdynamic and classical ion fluid equations. The medium around these shocks is taken as a perfect dissipationless gas with dust particles. The dust is assumed to consist of small solid particles distributed continuously. The dusts inertia and its solid-phase behavior strongly influence the wave propagation. The analysis has been done on Earth and Saturn. It appears that raising the mass fraction of the dust does not alleviate the shock front. We have done an extensive parametric analysis that considers physical parameters. Their roles are explored both qualitatively and quantitatively. In the power-law regime, pressure and density trends sometimes reverse compared to the exponential law under variations in dust loading parameter, dust-togas density ratio, symmetry, adiabatic index, and Mach number, illustrating the dependence of shock strength on energy deposition laws. Theoretical results are supported with detailed graphical interpretation.

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#### CP5

### The Hyperbolic Navier-Stokes Equations in Critical Besov Spaces

The hyperbolic Navier-Stokes (HNS) equations are a relaxation of the usual Navier-Stokes equations. In the HNS equations, the standard Fourier-type stress law is replaced by a Cattaneo-type law. The HNS equations were initially proposed as an attempt to impose finite speed of propagation, but have since gained independent mathematical interest. Because the equation is no longer parabolic, the regularity requirements on the initial data are notably more stringent and bear more resemblance to the Euler equations than the Navier-Stokes equations. We prove that the HNS equations are locally well-posed in the inhomogeneous Besov spaces  $B_{2,q}^s(\mathbb{R}^d)$  where  $s \geq 1 + \frac{d}{2}$  if q = 1 and  $s > 1 + \frac{d}{2}$  if q > 1, which includes the critical space

 $B_{2,1}^{1+\frac{d}{2}}(\mathbb{R}^d)$ . To our knowledge, this is the largest class of initial data known to produce strong solutions to the HNS equations. The main technical tool is an improved commutator-type estimate in Besov spaces.

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#### CP5

#### Multiscale Modeling, Simulation, and Analysis of Grain Growth in Polycrystals

Many technologically useful materials are polycrystalline microstructures composed of small monocrystalline grains that are separated by grain boundaries of crystallites with different lattice orientations. The changes in the grain and grain boundary structure of polycrystals highly influence the materials properties, including, but not limited to, electrical, mechanical, optical, and thermal. Thus, one of the central problems in materials science is to design technologies capable of delivering an arrangement of grains that produces a desired set of material properties. A method by which the grain structure can be engineered in polycrystalline materials is through grain growth (coarsening) of a starting structure. Grain growth in polycrystals is a very complex multiscale multiphysics process. It can be regarded as the anisotropic evolution of a large cellular network. In this talk, we will discuss recent progress in modeling, simulation, and mathematical analysis of the grain growth in polycrystalline materials. Relevant experiments will be presented as well.

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#### CP5

## An Asymptotic Preserving Scheme for the Euler-Poisson-Boltzmann System in the Quasineutral Limit

In this work, we study an asymptotic preserving (AP), energy stable, and positivity preserving semi-implicit finite volume scheme for the EulerPoissonBoltzmann (EPB) system in the quasineutral limit. The key to energy stability is the addition of appropriate stabilisation terms into the convective fluxes of mass and momenta, and the source term.

The space-time fully-discrete scheme admits the positivity of the mass density, and is consistent with the weak formulation of the EPB system upon mesh refinement. In the quasineutral limit, the numerical scheme yields a consistent, semi-implicit discretisation of the isothermal compressible Euler system, thus leading to the AP property. Several benchmark numerical case studies are performed to confirm the robustness and efficacy of the proposed scheme in the dispersive as well as the quasineutral regimes. The numerical results also corroborate schemes ability to very well resolve plasma sheaths and the related dynamics, which indicates its potential to applications involving low-temperature plasma problems.

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#### CP6

#### Well-Posedness of a Boundary Hemivariational Inequality for Stationary and Non-Stationary 2D and 3D Convective Brinkman-Forchheimer Equations

This paper investigates boundary hemivariational inequality problems associated with both stationary and non-stationary two and three-dimensional convective Brinkman-Forchheimer (CBF) equations which model the flow of viscous incompressible fluids through saturated porous media. The governing equations are nonlinear in both velocity and pressure and are subject to nonstandard boundary conditions. Specifically, we impose the no-slip condition along with a Clarke subdifferential relation between pressure and the normal velocity components. For the stationary case, we establish the existence and uniqueness of weak solutions using a surjectivity theorem. The existence of weak solutions to the non-stationary hemivariational inequality is established via a limiting process applied to a temporally semi-discrete scheme, where the time derivative is approximated using the backward Euler method-commonly referred to as the Rothe method. A novel outcome of this paper is that the existence results obtained in this work is applicable to 3D Navier-Stokes equations also.

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#### CP6

### A Hybrid Finite-Difference-Particle Method for Chemotaxis Models

We study chemotaxis systems, which are able to simulate concentration phenomena, where cell density undergoes rapid growth. Such growth can result in singular, spiky structures and lead to finite-time blowups. We focus on the dynamics of the Patlak-Keller-Segel chemotaxis system and its two-species extensions. In the latter case, different species may exhibit distinct chemotactic sensitivities, giving rise to very different rates of cell density growth. Such a situation may be extremely challenging for numer-

ical methods as they may fail to accurately capture the blowup of the slower-growing species mainly due to excessive numerical dissipation. We propose a hybrid finitedifference-particle (FDP) method, in which a sticky particle method is used to solve the chemotaxis equation(s). while finite-difference schemes are employed to solve the chemoattractant equation. Thanks to the low-dissipation nature of the particle method, the proposed hybrid scheme is particularly adept at capturing the blowup behaviors in both one- and two-species cases. The proposed hybrid FDP methods are tested on a series of challenging examples, and the obtained numerical results demonstrate that our hybrid method can provide sharp resolution of the singular structures even with a relatively small number of particles. Moreover, in the two-species case, our method adeptly captures the blowing-up solution for the component with lower chemotactic sensitivity, a feature not observed in other works.

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#### CP6

### Polynomial Decay Rates of Neutral, Collisionless Plasmas

Collisionless plasmas arise within a variety of settings, from magnetically confined plasmas in laboratories to space plasmas in planetary magnetospheres and solar winds. The fundamental electrostatic model that describes such phenomena is a system of nonlinear PDEs known as the Vlasov-Poisson (VP) system. After providing some background information concerning VP, recent results regarding the large-time behavior of solutions will be introduced. These results establish a variety of limiting self-similar behaviors for the associated charge density and electric field with time decay rates up to any integer order. This behavior is physically attributed to the degree of charge cancellation amongst moments of different particle species within neutral plasmas.

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#### CP6

#### Rarefaction Wave Interaction and Existence of a Global Smooth Solution in the Blood Flow Model with Time-Dependent Body Force

In this talk, we introduce a simplified 1-dimensional (1D) inhomogeneous system of conservation laws governing blood flow in the cardiovascular system, and discuss how a reduced 1D model can be derived using conservation principles. We then consider the interaction of two centered rarefaction waves. First, we analyze the Riemann solutions, demonstrating that the solutions lose self-similarity due to the source term. By transforming the system into non-reducible diagonal form in Riemann invariant coordi-

nates, we show how the interaction gives rise to a Goursat boundary value problem (GBVP). Consequently, discuss the existence and uniqueness of global  $C^1$  solution to the GBVP using a priori  $C^1$  bounds. Finally, we present the results of the wave interaction, establishing that either the rarefaction waves completely penetrate each other in a finite time or form a vacuum in the solution at a sufficiently large time during the process of penetration. This work has been published in "Studies in Applied Mathematics"

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#### CP7

#### Asymptotic Stability of Stationary Solutions of Quasi-One-Dimensional Gas Flows in Nozzles of Finite Length

This study addresses the asymptotic stability of steadystate solutions to the quasi-one-dimensional compressible Euler equations in a nozzle of finite length. Two classes of steady flows are considered: entirely subsonic flows, and transonic flows featuring a shock transition from supersonic to subsonic states. It is shown that all such steady solutions are nonlinearly asymptotically stable. The analysis imposes no restrictions on the mass flux, the crosssectional area derivative A'(x), or the outlet pressure. The only condition required for the transonic case is the positivity of A'(x) at the shock location. The stability proof combines energy estimates with the method of characteristics, allowing for the treatment of initial values with much lower regularity than previous works.

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#### CP7

#### Shock Wave Dynamics Via Symmetry-Driven Analysis of a Two-Phase Flow with the Chaplygin Pressure Law

This article investigates wave propagation in a two-phase flow with Chaplygin pressure law, an equation where pressure inversely depends on density. The study employs Lie symmetries and symmetry-driven analysis to derive onedimensional optimal subalgebras using the adjoint transformation and the invariant functions. Symmetry reductions yield several new exact solutions, and their physical behavior is examined graphically. Further, solutions such as peak-on waves, kinks, and parabolic solitons are identified using traveling wave transformation. Next, a framework of non-locally related PDE, including potential systems and inverse potential systems (IPS), is designed to classify nonlocal symmetries and discover more non-trivial exact solutions for the model. Next, novel conservation laws are constructed using the non-linear self-adjointness property of the model. Finally, the research explores the dynamic evolution of characteristic shock, weak discontinuity, and their interactions using one of the developed solutions. It contributes to understanding two-phase flow, offering practical implications for astrophysics, high-speed aerodynamics, and energy systems with unconventional pressure laws.

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#### CP8

# Semi-Analytical Solutions of (1+1)-Dimensional Kaup System and (2+1)-Dimensional Dispersive Long Wave Equations Using Homotopy Analysis Method

The present work employs the Homotopy analysis method (HAM) to get approximate solutions to two nonlinear partial differential equations: the (1+1)-dimensional Kaup system and the (2+1)-dimensional Long Wave Dispersive Equation. These equations represent wave propagation. The HAM is used to obtain the semi-analytical solutions while overcoming the limits of existing methods. Rigorous analysis ensures that the solutions converge. The numerical results illustrate the HAM approach's accuracy and efficiency. Convergence of the HAM-based solution is demonstrated via the squared residual error approach. It can be seen that the HAM-based method closely matches the exact solution to the problems.

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#### CP8

### Dispersive Evolution Pdes on Semi-Bounded Domains Revisited Via the Unified Transform Method

We shall discuss some of our recent findings concerning the rigorous analysis of fully non-homogeneous initial-boundary-value problems (IBVP) for several well-known dispersive evolution partial differential equations (PDE) formulated in a quarter-plane. Such PDE emerge in mathematical physics and the applied sciences as models of processes pertaining to water waves, continuum mechanics, heat-mass transfer, solid-fluid dynamics, electron physics, petroleum engineering, nanotechnology, etc. Our work is

based on the synergy between: (i) the complex-analytic unified transform method (UTM) introduced by A.S. Fokas in 1997, based on the theory of Riemann-Hilbert problems and Lax pairs, as the natural analog for IBVP of the classical Fourier and inverse scattering transforms (for wholespace initial value problems), and (ii) a recent approach developed by the speaker and numerous collaborators aiming for the rigorous refinement and extension of the UTM as well as for analytical investigation of a miscellary of qualitative properties of solutions, e.g. constructive existence (via a posteriori verification of UTM-derived integral representations of solutions in Ehrenpreis-Palamodov form, including reconstruction of prescribed initial and boundary data), well-posedness, spatiotemporal asymptotics and controllability. Various unexpected phenomena which have been uncovered (e.g., break-down, blow-up and instability effects) in the course of these investigations will be highlighted.

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#### CP8

#### Phase-Shifted Nanopterons in a Model of KdV Coupled to An Oscillatory Field

We develop nanopteron solutions for a coupled system of singularly perturbed ordinary differential equations that is a local model of various nonlocal systems that govern different problems in Fermi-Pasta-Ulam-Tsingou (FPUT) lattices. To leading order, one equation governs the traveling wave profile for the Korteweg-de Vries (KdV) equation, while the other models a simple harmonic oscillator whose small mass is the problems natural small parameter. A nanopteron solution consists of the superposition of an exponentially localized term and a small-amplitude periodic term. We construct two families of nanopterons. In the first, the periodic amplitude is fixed to be exponentially small but nonzero, and an auxiliary phase shift is introduced in the periodic term to meet a hidden solvability condition lurking within the problem. In the second, the phase shift is fixed as a (more or less) arbitrary value, and now the periodic amplitude is selected to satisfy the solvability condition. These constructions adapt different techniques due to Beale and Lombardi for related systems. As an immediate application, we construct nanopterons in a system of coupled KdV-KdV equations.

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#### CP8

# The Benjamin-Feir Instability in KdV-Like Equations with General Dispersion and Monomial Non-linearity

Nonlinear waves in dispersive media can exhibit modulation instabilities. We examine a category of scalar equations, with general dispersion and monomial nonlinearity, including a large variety of KdV-like equations. For small-amplitude traveling wave solutions, we provide a complete characterization of the spectrum near the origin of the linear operator obtained from linearizing about periodic traveling waves. We prove rigorously that, when the modulational instability is present, the spectrum connected to the origin consists of curves that invariably form a closed figure eight pattern. This is joint work with Bernard Deconinck

and Ashish Kumar Pandey.

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#### CP8

#### Two-Dimensional Solitary Gravity Wave Interactions with Infinite Depth

Solitary wave interactions in a (2+1)-dimensional dispersive model are studied. The multi-solitary wave solutions for the two-dimensional Benjamin-Ono (2DBOII) equation modeling internal waves with infinite depth are modulated and modeled as shock solutions to the conservation laws. In particular, two families of traveling wave solutions formed by four line solitary waves are suggested by numerical simulations generated using a pseudospectrum scheme. These multi-solitary wave solutions are studied using the modified Rankine-Hugoniot jump conditions defined for a multivalued discontinuity. They are also applied to modeling the Mach reflection problem, where a soliton traveling along a solid wedge encounters an inward oblique corner. In the Mach reflection for the 2DBO equation, a reflected wave, a Mach stem, and another wave with a small amplitude are generated. This is in contrast to the resonant Y-soliton, which occurs in Mach reflection for the KadomtsevPetviashvili (KP) equation. We numerically solve the jump conditions for the modulation variables and aim to find the critical angle for the corner that results in a bifurcation from one family of multi-solitary wave solutions to another.

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#### CP9

#### An Hp-Discontinuous Galerkin Time-Stepping Method for Nonlinear Sobolev-Type Equations

We investigate the hp-discontinuous Galerkin (DG) timestepping method for a class of nonlinear Sobolev-type equations. We first analyze the hp-DG semi-discretization in time. Subsequently, we consider fully discrete schemes by coupling continuous Galerkin (CG) methods in space with hp-DG in time, and establish the existence of a discrete solution. Optimal convergence results are derived for both the semi-discrete and fully discrete hp-DGCG schemes, under suitable regularity assumptions on the exact solution. A series of numerical experiments is provided to demonstrate the accuracy and effectiveness of the proposed method.

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#### CP9

#### A Higher Order in Time and Space Discretization for Maxwells Equations

We propose an implicit, arbitrarily higher-order in time discretization for a system of Maxwell's equations. The semi discretization is a generalization of the classical explicit leapfrog scheme. We use this method in conjunction with higher-order polynomial finite element spaces which form a de Rham sequence to discretize Maxwell's equations. These finite element spaces not only provide for a stable discretization of the system of partial differential equations that is Maxwell's equations but also exactly discretely preserves de Rham cohomology. Our system of Maxwell's equations uses a three-field formulation that is relatively recent and lends itself naturally to locating the fields on the smooth and exact de Rham complex. In addition to the electric field density E and magnetic flux density H which are vector fields, we also use an electric scalar pressure p in our Maxwell's system. The inclusion of p enables a spatial discretization in which the electric field Esatisfies a discrete divergence-free condition. Our proposed higher-order discretization method is stable, provably conserves energy discretely and we show that the fully discrete error converges at the appropriate rate determined by the temporal discretization order and degree of the polynomial finite element spaces. Finally, we provide some empirical validation through illustrative computations in two and three spatial dimensions.

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#### CP9

#### Adaptive Space-Time Finite Elements for Maxwells **Equations**

We present an adaptive space-time finite element framework for the solution of Maxwells equations. Traditionally, solution to the system of partial differential equations (PDEs) that is Maxwell's equations is separated by solving using the method of characteristics. This allows for a semi discretization of the space and time variables. A modern spatial discretization is usually a finite element method with, for example, a de Rham sequence of piecewise polynomial spaces or by using discontinuous Galerkin basis. The temporal partial derivatives are discretized in a finite difference scheme, implicitly or explicitly. In contrast, in a space-time framework, there is no product of solutions. We realize the space-time finite element spaces through a straightforward extension of the de Rham sequence of finite element spaces from d to (d+1) dimensions. This is facilitated by simply defining the polynomial spaces on a (d+1)-dimensional simplicial mesh. The PDE solution is then appropriately locally implicit and some recent works have adopted this approach. A motivation for doing so is to utilize the obvious parallelism in the solution scheme. In this work, we propose an adaptive extension of the tent pitching framework by generalizing the spatial a posteriori error estimators. We provide some preliminary analysis of our spatiotemporal estimators optimality, and demonstrate results for (1+1)-dimensional wave equation, and (2+1)dimensional Maxwells equations.

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#### CP9

#### Optimal L<sup>2</sup>-Norm Error Estimates for Steady and Unsteady Maxwell Equations Using Skeletal Discontinuous Galerkin Methods

This article proposes an optimal convergence analysis of two model problems: the stationary Maxwell equations, which represent the  $\mathbf{H}(\text{curl})$ -elliptic problem, and the timedependent Maxwell equations in cold plasma. We employ skeletal discontinuous Galerkin (DG) methods for spatial discretization. First, we introduce a skeletal DG method for the H(curl)-elliptic problem with variable coefficients and discuss the optimal convergence analysis in the energy and  $L^2$  norms. Next, we propose a continuous in time skeletal DG algorithm for the Maxwell equations in cold plasma. The proof for optimal convergence of error for the cold plasma equations in  $\mathbf{L}^2$  and discrete energy norms hinges on a suitably defined Ritz projection derived from the previously discussed stationary Maxwell problem. We also present numerical computations in two and three dimensions for the stationary and time-dependent Maxwell equations, including implicit and explicit time integration techniques for the time-dependent case. These computations verify the theoretical rates we have presented.

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#### CP9

### On the Role of Fractal Surfaces in Some Physical Phenomena

In this talk I will present an overview on the role of fractal surfaces in some physical phenomena modeled by local or nonlocal evolution equations. I will also discuss some recent results on some trasnmission problems for wave equations across fractal interfaces. These results are in collaboration with Simone Creo, Mirko Gallo, Massimo Cefalo and Maria di Domenico.

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#### CP9

### Legendrian Knots and Rack-Theoretic Coloring Invariants

A knot is called Legendrian if it solves the differential equation z'(t) = y(t)x'(t). The classification of Legendrian knots is an important but notoriously difficult problem in contact geometry, leading topologists to study discrete invariants of Legendrian knots. To that end, we develop the theory of generalized Legendrian racks, which are nonassociative algebraic structures based on the Legendrian Reidemeister moves. In particular, we use GL-rack coloring invariants to solve several open problems in Legendrian knot theory, and we give the first examples of GL-racks that distinguish Legendrian knots not distinguishable by

their classical or homological invariants.

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#### CP10

# Local Well-Posedness of the Electron Magnetohydrodynamics (mhd) with Partial Resistivity in $2\frac{1}{2}$ Dimensions

The electron magnetohydrodynamics (MHD) system without resistivity is generally not known to be locally well-posed. This is due to the singular and nonlinear nature of the Hall term in MHD equations. In this work, we focus on the electron MHD system in  $2\frac{1}{2}$  dimensions that includes either horizontal or vertical resistivity, and establish local well-posedness in Sobolev spaces.

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#### CP10

#### Singular Vortical Flows

Vortex spirals and cusps are special cases of self-similar singular solutions of the incompressible Euler equations Existence of algebraic spirals has been shown in some cases, but additional cases remain open. New techniques for progress on these problems will be discussed. Some old and new numerical results show various obstacles to progress, including nonexistence and nonuniqueness.

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#### CP10

### Numerical Study for the Generalized Surface Quasigeostrophic Equations

In this talk, we explore the extension of classical turbulence theory from the Navier-Stokes equations (NSE) to the generalized surface quasi-geostrophic (gSQG) model. We begin by examining the energy spectrum of gSQG and compare it with the NSE case through high resolution numerical simulations. We then investigate how enstrophy flux and the Kolmogorov-type dissipation law can be adapted to the gSQG setting and validated numerically. A key focus is on establishing the relations between characteristic wavenumbers and the "Grashof" number, understanding how these relationships behave across a range of gSQG parameters. We will also highlight the role of the critical line  $\beta = 1 + \alpha$  in the  $(\alpha, \beta)$ -plane, where our theoretical predictions begin to break down. Numerical simulations demonstrate this transition and provide insights into the limitations of current theoretical works in the supercritical region.

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#### CP10

### Stability Analysis of Mean-Field Games for Pedestrian Flow

In this presentation, we explore both the existence and the stability of minor deviations from the uniform state within the generalized Hughes model, as applied to pedestrian movement in infinitely extended corridors. We demonstrate that uniform pedestrian flows maintain their stability, provided the density meets a specific criterion. Specifically, we find that stability is achievable when the density remains below half the maximum capacity, in line with the Lasry Lions monotonicity condition. This allows us to manage deviations effectively and secure positive stability outcomes for the nonlinear Generalized Hughes model. Nevertheless, the phenomenon of wave propagation limits our ability to guarantee stability when densities exceed this threshold. Our methodology includes devising an explicit solution for the linear aspect using Fourier analysis, and employing a fixed-point technique to establish the solution for the comprehensive nonlinear mean-field games framework. This research was conducted in collaboration with Nader Masmoudi and Eliot Pacherie.

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#### CP10

### Absence of Anomalous Dissipation for Vortex Sheets

A family of solutions of the incompressible Navier- Stokes equations is said to present anomalous dissipation if energy dissipation due to viscosity does not vanish in the limit of small viscosity. In this talk we will discuss a proof of absence of anomalous dissipation for 2D flows on the torus, with an arbitrary non-negative measure plus an integrable function as initial vorticity and square-integrable initial velocity. Our result applies to flows with forcing and provides an explicit estimate for the dissipation at small viscosity. The proof relies on a new refinement of a classical inequality due to J. Nash.

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#### **CP11**

### Semi-Classical Commutator Estimates in Hartree Theory

In this talk we present new results regarding the validity of particular semi-classical bounds for minimizers of the Hartree functional. These bounds were introduced as key conditions on the initial datum by Benedikter, Porta and Schlein in order to analyze the emergence of the Hartree-Fock equation from the dynamics of large Fermi gases, in a

combined mean-field and semi-classical regime. Physically, these bounds record the underlying semi-classical structure of the system. Mathematically, they correspond to regularity estimates which are uniform in the semi-classical parameter. Proving these bounds in practice is, however, highly non-trivial. Our results is the first of its kind for non-linear systems in which particles are allowed to interact. In particular, singular pair potentials up to the Coulomb potential are included. Time permitting, we will also present new results on the application of these bounds to the study of the quantitative convergence of states for N-body Fermi gases. This talk is based on joint work with Laurent Lafleche.

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#### CP11

### Micro-Filament Modeling and Convergence of An N-Link Model

Simulating and modeling flexible fibers is an crucial issue in many microbiological problems. We focus on a recent result on convergence and well-posedness of the equations governing the dynamics of a discretized version of a continuous, flexible, inextensible filament immersed in a fluid at low Reynolds number. The elastohydrodynamic equation governing the motion of such a filament is a nonlinear 4th-order PDE system. Complexity in analytical and numerical study of the system has led to the use of simplified models, e.g. the "N-link" [F. Alouges, A. DeSimone, L. Giraldi, M. Zoppello, Self-propulsion of slender microswimmers by curvature control: N-link swimmers, Int. J. Non-Linear Mech., 2013.], a mechanical discretization into N rigid segments with elastic joints. While numerical evidence shows convergence of this model to the continuous case [C. Moreau, L. Giraldi, H. Gadlha, The asymptotic coarse-graining formulation of slender-rods, bio-filaments and flagella, J. R. Soc. Interface, 2018, rigorous convergence is nontrivial: the equations of the N-link are not a classical approximation of the underlying PDE. We prove existence and uniqueness of solutions for the N-link system and demonstrate their convergence to solutions of the classical PDE, leading also to an alternative proof of existence for the continuous PDE, complementing [Y. Mori, L. Ohm, Well-posedness and applications of classical elastohydrodynamics for a swimming filament, Nonlinearity, 2023].

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#### CP11

### Analysis of a Diffusive SIR Model on Metric Graphs and Subdomains ${\bf SIR}$

Mathematical modeling of disease dynamics requires addressing spatial interactions that occur across different topological structures. These processes simultaneously evolve over continuous subdomains, edges, and vertices, creating analytical challenges. In this work, we analyze an SIR system governed by partial differential equations and coupled across a hybrid structure. The model is formulated as a nonlinear parabolic system with junction conditions, capturing diffusion in subdomains, along edges, and at vertices. First, we establish well-posedness: using appropriate function spaces, we prove global existence and uniqueness, supported by energy estimates that bound solutions throughout the spatial domain. Second, we formulate an optimal control problem for intervention design. Vaccination and treatment rates serve as controls, minimizing a quadratic functional that balances disease prevalence and intervention costs. We establish differentiability, derive the adjoint system, and obtain first-order necessary optimality conditions, demonstrating the existence of optimal policies and enabling numerical implementation. This work combines PDEs on a hybrid structure with control theory. The framework could extend beyond epidemiology to other reaction-diffusion processes coupling continuous and discrete elements, such as ecological dispersal or chemical transport in fractured media.

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#### CP11

#### Dirichlet-Neumann and Neumann-Neumann Waveform Relaxation Methods for PDEs with Time Delay

We present a comprehensive theoretical and numerical convergence analysis of two DD-based waveform relaxation algorithms: the Dirichlet-Neumann Waveform Relaxation (DNWR) and Neumann-Neumann Waveform Relaxation (NNWR) methods, applied to a wide class of time-delayed PDEs, including parabolic, hyperbolic, and neutral equations. Time-delayed partial differential equations (PDEs) arise naturally in numerous scientific and engineering domains, including wave propagation, control systems, and biological modeling, where the system's current behavior is influenced by its past states. These methods are examined for both symmetric and asymmetric subdomains, with an emphasis on hyperbolic systems. To support the theoretical conclusions and compare the results with existing Schwarz Waveform Relaxation techniques, numerical examples are provided.

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#### CP11

# Finite -Approximate Controllability of Non-Autonomous Neutral Impulsive Evolution Equations with State-Dependent Delay

In this paper, we investigate the finite-approximate controllability of non-autonomous semilinear neutral impulsive differential equations with state-dependent delay. The results concerning the finite-approximate controllability of the first-order system are established in a separable Hilbert space framework. To derive sufficient conditions for finite-approximate controllability, it is shown that if the linear part of the non-autonomous system is approximately controllable, then, under certain natural conditions, the non-linear part of the semilinear neutral system also achieves finite-approximate controllability. Finally, a concrete example is presented to illustrate and validate the theoretical findings.

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#### CP11

#### Multiple Operator Learning with Expressivity Guarantees and Scaling Insights

Many modern learning tasks involve approximating entire families of operators that vary with underlying parameters or conditions, such as the solution maps of parametric partial differential equations with varying coefficients or boundary data. In this work, we introduce a framework for multiple operator learning, where a single model is trained to approximate a range of function-to-function maps  $\{G[\alpha]\}\$ . We propose a neural architecture that performs well in practice and, crucially, comes with theoretical guarantees. In particular, we show that our model satisfies a universal approximation property for a broad class of operator families. For Lipschitz continuous operators, we further provide explicit scaling laws: we derive how the models width, depth, and number of non-zero parameters must grow to achieve a given approximation accuracy. These results shed light on how architectural choices influence learning performance, and how scaling laws can guide both the design and analysis of operator learning systems. Our findings offer a step toward more principled and scalable approaches to learning in complex, parameter-varying environments.

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#### CP12

#### Fractional Regularity, Global Persistence, and Asymptotic Properties of the Boussinesq Equations on Bounded Domains

We address the long-time behavior of the 2D Boussinesq system, which consists of the incompressible NavierStokes equations driven by a non-diffusive density. We construct globally persistent solutions on a smooth bounded domain, when the initial data belongs to  $(H^k \cap V) \times H^k$  for  $k \in \mathbb{N}$ , and to  $H^s \times H^s$  for 0 < s < 2. The proofs use parabolic maximal regularity and specific compatibility conditions at the initial time. Additionally, we also deduce various asymptotic properties of the velocity and density in the long-time limit and present a necessary and sufficient condition for the convergence to a steady state.

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### $\begin{array}{c} \text{CP12} \\ \text{TBD} \end{array}$

TBD

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#### CP12

### Exact Boundary Controllability for the Ideal Magneto-hydrodynamic Equations

We consider the three-dimensional ideal MHD system on a domain in with a controllable part of the boundary where we prescribe the boundary data. The basic question of boundary controllability is whether, given two states, one can by means of the control on the boundary drive one state to another. We will review the existing literature on this problem and provide a positive result for domains with only Sobolev regularity. The results are based on works with Matthew Novack, Wojciech Ozanski, and Vlad Vicol.

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#### CP12

#### A Logarithmically Bounded Number of Small Rigid Bodies in a Viscous Incompressible Inhomogeneous Fluid Is Negligible

We consider a large number of inhomogeneous rigid bodies immersed in an inhomogeneous incompressible fluid contained in a bounded domain of dimension bigger or equal to two. We address the question about the asymptotic behaviour of the corresponding system of partial differential equations as the number of rigid bodies tends to infinity with a logarithmic bound and the diameter of the bodies tends to zero. We show that the rigid bodies are neglected in the limit in the sense that the limit system is given only by the inhomogeneous incompressible Navier-Stokes System. We do not require any regularity for the boundaries of the rigid bodies and the domain. Our result extends earlier work by Feireisl, Roy and Zarnescu (2023) to more general

assumptions which are of physical relevance. This includes assumptions on the mass densities of the fluid and the rigid bodies, which are allowed to be inhomogeneous, can attain the value zero, and only need to be bounded in some suitable  $L^p$ -space instead of being uniformly bounded.

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#### CP12

### Fast-Slow Dynamics of the Quasi-Geostrophic Approximation

This work is devoted to investigating the rotating Boussinesq equations of inviscid, incompressible flows with both fast Rossby waves and fast internal gravity waves. The main objective is to establish a rigorous derivation and justification of a new generalized quasi-geostrophic approximation in a channel domain with no normal flow at the upper and lower solid boundaries, taking into account the resonance terms due to the fast and slow waves interactions. Under these circumstances, We are able to obtain uniform estimates and compactness without the requirement of either well-prepared initial data or domain with no boundary. In particular, the nonlinear resonances and the new limit system, which takes into account the fast waves correction to the slow waves dynamics, are also identified without introducing Fourier series expansion. The key ingredient includes the introduction of (full) generalized potential vorticity.

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#### CP12

### Case Studies of Single and Two Phase Flow Using Finite Volume Methods

Numerical solutions of two phase flow are always being a difficult task to achieve. A systematic approach is adopted to reach to some solutions of two phase flow in typical case studies. To have confidence on the obtained solutions for two phase flow in difficult geometries the incompressible flow was first solved for regular geometry in two dimensions using conventional cell-centered staggered grid finite difference method. The same was extended to two phase flow taking different densities but with the same viscosity. Results obtained are compared with already exiting solutions including mesh less methods. A thorough study was carried out to develop a code in MATLAB in object oriented environment. Meshless solutions which have already obtained for Rayleigh Taylor Instability problem and then have good results for a typical jet problem in two dimen-

sions. It is interesting to know these problems are solved in microns (micro meter units). In the end an effort is made to solve two phase problem in highly complex nozzle problem. Results are in the process.

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#### CP13

#### Riccati Diagonal Stability in the Complex Domain

The notion of Riccati diagonal stability for a pair of real matrices (A,B) has recently been introduced. This paper extends these results to pairs of complex matrices A and B. Formally, we define a pair (A,B) as Riccati diagonally stable if there exist positive definite matrices P and Q such that the matrix  $A^TP + PA + Q + PBA^{-1}B^TP$  is negative definite. It has been shown that this condition ensures the asymptotic stability of the time-delayed system  $\dot{x} = Ax(t) + Bx(t-\tau)$ . Our characterizations provide new insights into stability analysis for complex-valued systems with delays.

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#### CP13

# A New Tempered Alikhnov Formula with Nonuniform Time-Stepping Maximum Principle Preserving Scheme for Tempered Fractional AllenCahn Equations

To address initial singularities in solutions of the tempered fractional Allen-Cahn equation, we construct a new secondorder tempered Alikhnov formula with tempering parameter  $\lambda$  on nonuniform time grids. The method uses the sum-of-exponentials (SOE) technique to efficiently approximate the Caputo tempered derivative. Spatial discretization employs a fourth-order compact finite difference operator. The resulting scheme preserves a discrete maximum principle and achieves sharp maximum-norm error bounds by capturing temporal regularity through the convolution structure of the consistency error. An adaptive time-stepping strategy improves long-time simulation efficiency, yielding computational cost  $O(MN \log M \log N)$ and memory usage  $O(M \log N)$ , where M and N are spatial and temporal grid sizes. For multidimensional problems, we develop an efficient compact ADI formulation. Numerical results confirm accuracy and efficiency, establishing this as a first-of-its-kind scheme for tempered Allen-Cahn equations.

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#### CP13

#### Robust Stability and Error Estimate of Adaptive Third-Order BDF Methods with Improved Stepsize Ratio for Nonlinear Parabolic Equations

We establish an original framework for the stability and convergence analysis of implicit and implicit-explicit variable-step BDF3 schemes for linear and nonlinear parabolic equations. By constructing a positive recursion functional, we derive a novel gradient decomposition for the variable-step BDF3 method, and prove its positive definiteness when  $0.5 \le \rho_n \le 1.7319$ , where  $\rho_n$  is the adjacent stepsize ratio. This significantly improves on existing results. Under this ratio restriction, we obtain robust stability and optimal error estimates utilizing a concise energy method in an abstract setting. The main ingredients in the proofs include the reformulation of the schemes via a class of discrete orthogonal convolution kernels, global properties of the discrete kernels, and novel convolution type inequality tools. The theoretical barrier, originating from the lack of an analytic expression and uniform positivity of the orthogonal kernels and their scaling counterparts, is addressed by using the elliptic matrix norm. This essentially extends the orthogonal kernel based framework used for the analysis of the second-order BDF method. Numerical results are included to support the analysis. The proposed framework is applicable to a wide range of nonlinear parabolic equations and demonstrates a promising approach for higher-order linear multistep methods with variable stepsizes.

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#### CP13

### Weak Galerkin Finite Element Method for Singularly Perturbed Turning Point Problems

We introduce a weak Galerkin finite element method (WG-FEM) for a class of parabolic singularly perturbed boundary turning point problem (SPBTPP). We develop our numerical scheme by discretizing time using implicit  $\theta$ -scheme over a uniform mesh, and space using weak Galerkin finite element method (WG-FEM) on a layer adapted Shishkin mesh. We analyze the stability and established the uniform convergence of the proposed scheme in the energy norm. Several numerical examples are presented validating our theoretical estimates.

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#### CP13

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#### A Hybrid Approach for Singularly Perturbed Parabolic Problems with Discontinuous Data

In this article, we study a two-dimensional singularly perturbed parabolic equation of the convection-diffusion type, characterized by discontinuities in the source term and convection coefficient at a specific point in the domain. These discontinuities lead to the development of interior layers. To address these layers and ensure uniform convergence, we propose a hybrid monotone difference scheme that combines the central difference and midpoint upwind schemes

for spatial discretization, applied on a piecewise-uniform Shishkin mesh. For temporal discretization, we employ the Crank-Nicolson method on a uniform mesh. The resulting scheme is proven to be uniformly convergent, order achieving almost two in space and two in time. Numerical experiments validate the theoretical error estimates, demonstrating superior accuracy and convergence when compared to existing methods.

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#### CP13

### Transport Sweep Based Solutions of the Multidimensional BGK Equation on Unstructured Grids

For rarefied gases and in other extreme situations, the continuum assumption underlying traditional fluid models fails. In these situations, one must resort to solving the underlying kinetic system. The most accurate kinetic model, the Boltzmann equation, is too expensive for many important problems. The famous Bhatnagar-Gross-Krook (BGK) model captures much of the behavior at a more practical computational cost. Unfortunately, in the most general setting, the BGK equation must still be solved in a seven dimensional phase space (time-1, physical space-3, velocity space—3). We will discuss a full phase space implicit BGK solver. The discretization uses digonally implicit Runge-Kutta (DIRK) time integration, nodal discontinuous finite elements in physical space, and a collocation method in velocity space. We prove that our discretization is unconditionally stable and energy dissipative for a simplified BGK equation. We demonstrate that sweep-based solution strategies enable us to solve the full phase BGK equation using a massively parallel strategy on both CPU and GPU architectures. The solver is tested on a wide range of benchmark problems.

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#### MS1

#### Wiener Algebra Methods for Linear Dispersive Bounds: Some Recent Results

In the Schrödinger operator  $H = (-\Delta)^m + V(x)$  on  $\mathbb{R}^n$ , one says that  $\widecheck{V}$  is scaling-critical if belongs to a space with the same homogeneous scaling as  $|x|^{-2m}$ . This is the endpoint case for V(x) being a compact perturbation relative to the free Laplacian. Pointwise dispersive bounds for Schrödinger operators with a large scaling-critical perturbation almost invariably use some adaptation of Wiener's inversion theorem for the Fourier transform of integrable functions. The case m = 1, n = 1 (i.e. the Schrödinger equation on the line) can even be handled using the classical statement of Wiener's theorem. More often, the functions in question are operator-valued, and there are various notions of what it means for them to be integrable. The talk will survey results obtained by these methods over the past few years, highlighting in each case the way that some fundamental obstruction was overcome. The author's results are joint work with Burak Erdogan and William Green. Work by other authors will also be presented.

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#### MS1

# The Spectrum of Schrdinger Operators Interacting at Different Scales (both Scalar and Non-Self-Adjoint)

We discuss work with Emmanuel Fleurantin and Chris Jones looking at the spectrum of scalar and matrix Schrdinger operators with potentials interacting at different spatial scales but scaled in a critical fashion. The motivation is to understand the interaction of solitons with adiabatic potentials. We discuss two results: 1) an exact spectral count in the scalar case and 2) a precise formulation of the spectrum on the imaginary axis in the matrix case

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#### MS1

### A New Approach to the Fourier Extension Problem for the Paraboloid

The plan of the talk is to describe a new approach to the socalled Restriction Conjectures in Harmonic Analysis, that Itamar Oliveira and I have developed recently. Without entering into details, this new point of view allows one to prove that (essentially) all the relevant conjectures (linear or multi-linear) are true, provided that one of the functions involved has a tensor structure.

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#### MS1

Pointwise Decay for Solutions to Nonlinear Wave

#### **Equations**

We present a robust method to establish optimal pointwise decay rates for a variety of (linear and nonlinear) wave equations. We focus on the case of the nonlinearity satisfying a generalized version of the classical null condition. This is based on previous work with Jason Metcalfe, Daniel Tataru, and Shi-Zhuo Looi.

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#### MS2

#### Plate Theory for Metric-Constrained Actuation of Liquid Crystal Elastomer Sheets

Liquid crystal elastomers (LCEs) combine the large deformation response of a cross-linked polymer network with the nematic order of liquid crystals pendent to the network. The focus of this talk is the actuation of LCE sheets where the nematic order, modeled by a director field, is specified heterogeneously in the plane of the sheet. Heating such a sheet leads to a large spontaneous deformation, coupled to the director design through a well-known metric constraint. We derive a plate theory for the pure bending deformations of patterned LCE sheets in the limit that the sheet thickness tends to zero using the framework of  $\Gamma$ -convergence. After dividing the bulk energy by the cube of the thickness to set a bending scale, we show that all limiting midplane deformations with bounded energy at this scale satisfy the aforementioned metric constraint. We then identify the energy of our plate theory as an ansatz-free lower bound of the limit of the scaled bulk energy, and construct a recovery sequence that achieves this plate energy for all smooth enough midplane deformations. We then apply the plate theory to some known examples. This is joint work with David Padilla-Garza (Hebrew University of Jerusalem) and Paul Plucinsky (University of Southern California).

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#### MS2

#### Median Filters for Multiphase Interfacial Motions: Discoveries and Insights from the Connection to the MBO Scheme

At the core of this presentation is the establishment of a precise connection between median filter (sorting-based) level set schemes and threshold dynamics (MBO scheme). This connection facilitates the development of level set methods informed by recent advances in threshold dynamics, enabling simulation of the mean curvature motion of interface networks under varying surface tension and mobility conditions. We then extend these median filters (level set methods) to an anisotropic wetting/dewetting scenario where one of the three phases remains stationary while triple junctions form at their intersections. This scenario serves as a testbed for examining complex triple junction

conditions due to anisotropy. Numerical evidence supports the correct angle condition at these junctions, encouraging further exploration into fully anisotropic multiphase flow dynamics.

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#### MS2

### A Local Discontinuous Galerkin Method to Compute Parallelogram Origami Metamaterials

Origami metamaterials made of repeating unit cells of parallelogram panels joined at folds dramatically change their shape through a collective motion of their cells. Here we develop an effective elastic model and numerical method to study the large deformation response of these metamaterials under a broad class of loads. The model builds on an effective plate theory derived in [Xu et al, 2024]. The resulting model for parallelogram origami is a generalized elastic continuum that is nonlinear in the effective deformation gradient and angle field and regularized by high-order gradients thereof. A finite element formulation of this model using the  $C^0$  interior penalty method to handle second gradients of the deformation was proposed in [Xu et al, 2025]. However, no convergence proof was provided. In this talk, we propose to use a Local Discontinuous Galerkin formulation with  $C^0$  elements. The interest is two fold: the method is stable for any value of the penalty parameter and a discrete energy is available. We then prove that the discrete energy  $\Gamma$ -converges to the continuous one. The method has been implemented using the open source computing platform Firedrake. Finally, we show the effectiveness of the method by doing computations on two canonical parallelogram origami patterns, in Miura and Eggbox origami, under a variety of loading conditions.

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#### MS2

### On the Wrinkle-to-Fold Transition in Confined Elastic Plates

Experiments on confined elastic sheets have shown two phenomena – a wrinkling response where energy is diffusely distributed, and a folding response, where energy is highly concentrated, often along lines or curves. Both are easy to observe and informally describe. However, in a hyperelastic modeling approach, there are few rigorous results on the precise differences between wrinkles and folds or on the onset of this dichotomy, called the "wrinkle-to-fold transition." Moreover, the emergence of stress concentration starting from hyperelasticity without damage or interfacial energy remains mysterious. In this work, we present a scaling laws result that distinguishes wrinkles from folds, and identifies the critical parameter dependence for the transition. In one case, fold length is a proxy for the stored energy; in the other, it is the "area lost to the wrinkles." We comment on the prefactor in these scaling laws as a function of the confinement imposed on the sheet.

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#### MS3

#### On the Lack of External Response of a Nonlinear Medium in the Second-Harmonic Generation Process

Can a nonlinear medium appear linear to an outside observer for a particular probing? In this talk we discuss the scattering problem for a nonlinear medium of compact support with second-harmonic generation. Such a medium, when probed with monochromatic light beams at given frequency generates additional waves at double frequency The response of the medium is governed by a system of two coupled semi-linear PDEs for the electric fields. We investigate whether there are situations in which the generated double frequency wave is localized inside the inhomogeneity, that is, whether the nonlinear interaction of the medium with the probing wave is invisible to an outside observer. This leads to the analysis of a semi-linear elliptic system formulated in the support of the medium with nonstandard boundary conditions. The presented analysis sets up a mathematical framework needed to investigate a multitude of questions related to nonlinear scattering with second-harmonic generation. This is presentation is based on a joint work with Narek Hovsepyan (Rutgers U), Matti Lassas (U Helsinki), and Michael Vogelius (Rutgers U).

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#### MS3

#### Forward and Inverse Problems for the Wave Equation with a Distributed Fractional Derivative

Fractional time derivative operators are successfully used to model anomalous diffusion processes and the power-law attenuation of compressional and shear waves propagating in viscoelastic microstructured media and materials with memory. The order of the fractional derivative is related to the characteristic time scale; for multiscale media, it can take several different values or change continuously, which can be formulated using the distributed fractional derivative. The talk discusses the attenuated wave equation and the inverse problem of identification of the distributed Caputo fractional time derivative. An efficient numerical method for its solution is presented.

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#### MS3

#### Quantum Diffusion and Random Matrix Theory

The random Schrdinger equation is a toy model used to describe the behavior of waves in random media. In the weak-coupling limit, it is expected that the transport of these waves is well described by an incoherent mixture of

waves traveling along random walk paths. Prior attempts to describe this limit have involved difficult combinatorial analyses of perturbative expansions for the solution. In this talk I will describe a new approach to understanding the diffusion of wave solutions which uses ideas from random matrix theory and dispersive PDE in lieu of diagrammatic expansions. This is based on joint work with Adam Black and Reuben Drogin.

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#### MS3

#### Kinetic Equations for One and Two-Photon Light in Random Media

In this talk we present a fully quantized real space model of a scalar electromagnetic field coupled to a continuum of two level atoms which are randomly distributed throughout space. After discussing some of the relevant physical background, we will restrict ourselves to states which have at most one or two photons present. The probability amplitudes satisfy a system of pseudodifferential equations with random coefficients. By applying formal multiple scale asymptotic analysis we derive kinetic equations for the associated probability density functions. In a suitable limit, these probability densities also satisfy constant coefficient diffusion equations.

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#### MS4

### Entropy Analysis and Singularities of Entropy Solutions for Nonlinear Conservation Laws

In this talk, we present some recent developments in solving several longstanding open problems involving the singularities of entropy solutions for nonlinear conservation laws and related nonlinear partial differential equations through entropy analysis and associated methods. These problems, in particular, include the minimal entropy conditions for well-posedness, the understanding of the underlying phenomena of cavitation and concentration, and the rigorous analysis of entropy solutions via the theory of divergence-measure fields, among others. Further related topics, perspectives, and open problems will also be addressed.

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#### MS4

Ill-posedness and Singularity Formation Issues for

#### MHD

We will discuss several constructions for MHD systems and models which produce either norm inflation type ill-posedness or finite time singularity formation.

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#### MS4

### Self-Similar Solutions to Two-Dimensional Riemann Problems with Transonic Shocks

Multidimensional conservation laws is an active research area with open questions about existence, uniqueness, and stability of properly defined weak solutions, even for fundamental models such as the compressible Euler system. Understanding particular classes of weak solutions, such as Riemann problems, is crucial in this context. This talk focuses on self-similar solutions to two-dimensional Riemann problems involving transonic shocks for compressible Euler systems. Examples include regular shock reflection, Prandtl reflection, and four-shocks Riemann problem. We first review the results on existence, regularity, geometric properties and uniqueness of global self-similar solutions of regular reflection structure in the framework of potential flow equation. A significant open problem is to extend these results to compressible Euler system, i.e. to understand the effects of vorticity. We show that for the isentropic Euler system, solutions of regular reflection structure have low regularity. We further discuss existence, uniqueness and stability of renormalized solutions to the transport equation for vorticity in this low regularity setting.

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#### MS4

### Concentration Phenomenon of Weak Solutions for Compressible Flows

We will discuss the concentration phenomenon of the kinetic energy  $\rho |\mathbf{u}|^2$ , associated to isentropic compressible Navier-Stokes equations, in  $\mathbb{R}^n$  with n=2,3 and the adiabatic constant  $\gamma \in [1,n/2]$ . Except a space-time set with Hausdorff dimension less than or equal to  $\Gamma(n)+1$ , no concentration phenomenon occurs. Some recent development will be also discussed.

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#### MS5

#### Nonseparable, Nonlocal Mean Field Games with Initial Distributions in Negative-index Sobolev Spaces

A fundamental question in mean field games is the convergence problem, i.e. to prove that the limit of solutions of N-player games converge as N goes to infinity. A strategy which has been used to solve the convergence problem for certain separable Hamiltonians is to prove existence of solutions for the mean field games PDE system, to use those solutions to construct solutions of the master equation, and to use the solution of the master equation to

give control over the solutions of the N-player game. The first step in this program, then, is to be able to solve the mean field games PDE system. This must be done with the appropriate data for the N-player games system, which is a sum of Dirac masses. Many previous existence theories for nonseparable mean field games (including those of the speaker) have used smooth classes of data, thus excluding Dirac masses as initial data. Now, we identify a class of nonlocal, nonseparable Hamiltonians for which we can prove existence of solutions with rough data. The speaker and collaborators have previously shown existence of solutions with data in pseudomeasure spaces, and now the speaker will give details of an existence theory with data in negative-index Sobolev spaces.

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#### MS5

### Markov Perfect Equilibria in Discrete Finite-Player and Mean-Field Games

We study dynamic finite-player and mean-field stochastic games within the framework of Markov perfect equilibria (MPE). Unlike their continuous-time analogues, discrete-time finite-player games generally do not admit unique MPE. However, we show that uniqueness is remarkably recovered when the time steps are sufficiently small, and we provide examples demonstrating the necessity of this assumption. This result, established without relying on any monotonicity conditions, underscores the importance of inertia in dynamic games. Furthermore, we discuss different learning algorithms and prove their convergence to the unique MPE.

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#### MS5

### Mean Field Games of Controls with Fractional Laplacian

We analyze a fractional mean field game of controls system, showing existence of solutions when the order of the fractional Laplacian is  $s \in (\frac{1}{2},1)$ . Here the running cost depends on the distribution  $\mu$  of not only the states but also optimal strategies. The coupling it is assumed to satisfy the Lasry-Lions monotonicity condition. We derive three types of a priori estimates on solutions. First, we use the monotonicity condition to derive moment estimates on  $\mu$ . Second, we derive abstract estimates on fractional parabolic equations and apply them to the mean field game. Third, we derive new estimates on the time regularity of the distribution  $\mu$  by analyzing the associated Lévy process. We apply these estimates and the Leray-Schauder fixed point theorem to establish existence of solutions.

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#### MS5

Self-Similar Intermediate Asymptotics for First-

#### Order Mean Field Games

We study a mean field planning problem in which the initial density is a Dirac mass. We show that there exists a unique solution which converges to a self-similar profile as time tends to 0. We proceed by studying a continuous rescaling of the solution, and characterizing its behavior near the initial time through an appropriate Lyapunov functional.

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#### MS6

#### A New Locally Divergence-Free Path-Conservative Central-Upwind Scheme for Ideal and Shallow Water Magnetohydrodynamics

I will present semi-discrete path-conservative centralupwind (PCCU) schemes for ideal and shallow water magnetohydrodynamics (MHD) equations. These schemes offer several key advantages: they locally preserve the divergence-free condition, do not require Riemann solvers, and produce high-resolution, non-oscillatory results. The design is based on the Godunov-Powell nonconservative form of the MHD equations, augmented with evolution equations for the spatial derivatives of the magnetic field. These derivatives are used in a special piecewise linear reconstruction to ensure non-oscillatory behavior and enforce the local divergence-free property. The scheme also accounts for jumps in the nonconservative product terms across cell interfaces, enhancing its stability. I will further discuss the extension to magnetic rotating shallow water equations. The extended scheme is well-balanced and exactly preserves the divergence-free magnetic field. The well-balanced property is achieved using a flux globalization approach, enabling exact preservation of both stilland moving-water steady states. Numerical results for various test problems demonstrate the accuracy, robustness, and positivity-preserving nature of the proposed PCCU schemes.

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#### MS6

### Analysis of Data Assimilation Algorithms for the Navier-Stokes Equations

This talk is based on joint work with Aytekin Cibik and Rui Fang. Nudging based data assimilation has a well-developed mathematical theory (comprising hundreds of papers) but is little used in practice. Kalman filter variants dominate practical simulations but have limited supporting theory, often with spherical cow type assumptions. This talk will present a recent development (from these approaches) of assimilation algorithms with lower computational complexity than KF variants and a supporting mathematical theory without SC assumptions. It is (provably) insensitive to model error, in all cases increases predictability horizons and for dense enough data and large enough parameters induces an infinite predictability horizon.

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#### MS6

### Existence of Weak Solutions for a Model of the Geodynamo

In this talk, I will discuss a model of the earth's magnetic field that has previously been simulated numerically, but has not been shown to be well-posed. This model couples solid physics for the electrically conducting inner core with magnetohydrodynamic (MHD) equations in the liquid outer core, as well as the magnetic field outside of the core, which is taken to be non-conducting. I will define and prove existence of Leray-Hopf-type weak solutions for this system. Particular challenges include the transmission conditions of the magnetic field coming from non-constant physical parameters and extending the magnetic field to a non-conducting exterior. To address these problems, we must carefully define a function space and prove appropriate embeddings. I will also describe progress toward short-time strong solutions for this model.

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#### MS6

#### **Euler Equations and Transonic Flows**

In this talk, we will consider the Euler equations of gas dynamics and applications in transonic flows. In particular, we will present the results on the transonic flows past obstacles, transonic flows in the fluid dynamic formulation of isometric embeddings, and the transonic flows in nozzles. We will discuss global solutions and stability obtained through various techniques and approaches.

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#### MS7

#### A Global Well-posedness Result for the Threedimensional Quasi-geostrophic Equations

A three-dimensional geophysical flow model over a cylindrical domain with a multiply connected cross-section is considered. The model is based on the quasi-geostrophic dynamics, and is supplied with homogeneous Neumann boundary conditions on the top and bottom, and no-flux boundary conditions on the lateral, in addition to various other dynamical constraints chosen to enforce a unique solvability. The existence and uniqueness of a strong solution is then obtained using the rather classical strategies that were previously employed by Yudovich and Kato for the two-dimensional Euler equations.

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#### MS7

### Semi-Analytic Physics-Informed Neural Networks for Singular Layer Problems in Fluid Mechanics

Singular perturbation problems in fluid mechanics, characterized by a small (viscosity) parameter multiplying the highest-order derivatives, give rise to multiscale behavior

with sharp transitions such as boundary and interior layers. These features pose significant challenges for both analysis and computation. In this talk, I will present recent developments in semi-analytic Physics-Informed Neural Network (PINN) methods designed to address these difficulties in slightly viscous fluid flows. By incorporating viscous layer correctors into the learning framework, the proposed approach significantly improves both accuracy and computational efficiency. This work highlights the synergy between asymptotic analysis and machine learning in advancing the simulation of singularly perturbed fluid equations.

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#### MS7

### Navier-Stokes: What Happens in the Boundary Layer?

What happens in the boundary layer for solutions to the incompressible Navier-Stokes equations, asymptotically for small viscosity (high Reynolds number), is not generally known. Heuristic and numerical methods have been proposed and, in special circumstances, verified. I will discuss aspects of the history of this problem, and demonstrate a connection between the work of Chorin (with Marsden and others) in the 1970s, Kato in the 1980s, and Maekawa and others over the past decade or so.

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#### MS8

### Gevrey Regularity for a Fluid-Structure Interaction Model

We discuss a result of Gevrey regularity for a semigroup which models a particular fluid-structure interaction model. This PDE system was derived and studied by the late Igor Chueshov to mathematically model the phenomenon of "sloshing". In this model, the fluid evolves in a piecewise smooth or convex geometry. On a portion of the boundary, a fourth order plate equation is coupled with the fluid through pressure and matching velocities. In order to obtain the conclusion of Gevrey regularity, we obtain an appropriate norm estimate of the resolvent of the associated strongly continuous semigroup generator. These resolvent estimates also serve to quantify the precise extent of smoothness. Moreover, we provide a numerical example - conducted through the finite element method - which suggests that the index of Gevrey regularity we obtain is sharp. Such regularity means that there is some measure of "infinite propagation of signals", and so solutions are (locally) smooth in character. Consequently, when one wishes to investigate controllability properties of the given fluid-structure model, one is necessarily compelled to consider null controllability; i.e., the problem of finding control functions which will "steer" the dynamics to the zero state. This is joint work with Dylan McKnight and Sara McKnight.

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#### MS8

### Nonlinear Spectral Stability for Parabolic Evolution Equations

I will present a new linearization principle for the nonlinear stability of solutions to semilinear evolution equations of parabolic-type. We assume that the set of equilibria forms a finite dimensional manifold, and  $\lambda = 0$  is a semisimple eigenvalue of the relevant linear operator L, arising after linearizing the equation about an equilibrium. A "linearization principle" usually provides conditions under which the nonlinear stability (or instability) of the equilibria can be determined only by the spectral properties of L. Unfortunately, classical linearization principles do not apply in our setting. More recent "generalized principles of linearized stability" either require strong regularity assumptions on the linearized equations (for example, L must satisfy the property of maximal  $L^p$ -regularity) or assume that L is the generator of a strongly stable semigroup. Our new linearization principle assumes that L is the generator of an analytic semigroup (not necessarily stable) plus mild assumptions on the structure of the evolution equation. Specifically, we show that if a (mild) solution to our evolution equation exists globally in time and remains "close" to the manifold of equilibria at all times, then the solution must eventually converge to an equilibrium point at an exponential rate. These abstract results will be then applied to the equations governing the motion of a heavy fluid-filled solid.

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#### MS9

#### To be determined

Discrete maximum principles (DMPs) for elliptic equations are typically established under the assumption that all offdiagonal entries of the stiffness matrix are non-positive. For the 2D Poisson equation discretized using  $P_1$  elements, this leads to the so-called "angle condition", which requires that the sum of the angles opposite each edge does not exceed  $\pi$ . This is essentially a local condition, which is propagated to the entire domain using matrix analytic tools. In this work, we introduce a novel macro-element-based technique for establishing global DMPs in finite element discretizations of both linear and semilinear elliptic equations, without relying on the traditional angle condition. The key idea is to adapt a connectivity-based argumentcommonly used in the continuous setting to demonstrate how strong DMPs can propagate from individual macroelements to the entire computational domain. We apply our technique to prove DMPs for the Poisson equation on meshes with minor defects (i.e., where the angle condition is violated in isolated areas) and on nearly degenerate meshes. Additionally, we provide an independent proof of a strong DMP for semilinear elliptic equations.

Andrei Draganescu

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#### MS9

### PARAMETRIC HYPERBOLIC CONSERVATION LAWS

We propose a parametric hyperbolic conservation law (SymCLaw) for learning hyperbolic systems directly from data while ensuring conservation, entropy stability, and hyperbolicity by design. Unlike existing approaches that typically enforce only conservation or rely on prior knowledge of the governing equations, our method parameterizes the flux functions in a form that guarantees real eigenvalues and complete eigenvectors of the flux Jacobian, thereby preserving hyperbolicity. At the same time, we embed entropy-stable design principles by jointly learning a convex entropy function and its associated flux potential, ensuring entropy dissipation and the selection of physically admissible weak solutions. A corresponding entropystable numerical flux scheme provides compatibility with standard discretizations, allowing seamless integration into classical solvers. Numerical experiments on benchmark problems, including Burgers, shallow water, Euler, and KPP equations, demonstrate that SymCLaw generalizes to unseen initial conditions, maintains stability under noisy training data, and achieves accurate long-time predictions, highlighting its potential as a principled foundation for data-driven modeling of hyperbolic conservation laws.

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#### MS9

### A New Sparsity Promoting Residual Transform Operator for Lasso Regression

Lasso regression is a widely employed approach within compressive sensing algorithms to promote sparsity and recover piecewise smooth signals when the given observations are obtained from noisy, blurred, and/or incomplete data environments. The underlying assumption regarding the sparse-promoting operator is typically pre-determined. This assumption of a particular type of variability, such as piecewise constant or piecewise linear behavior across the entire domain, is problematic in more general cases. To address the limitations of assuming a fixed (and typically low order) variability for the sparsity-promoting operator, this investigation proposes a novel residual transform operator. In a nutshell, the idea is that for a general piecewise smooth signal, it is possible to design two operators  $\mathcal{L}_1$  and  $\mathcal{L}_2$  such that  $\mathcal{L}_1 f \approx \mathcal{L}_2 f$  but  $\mathcal{L}_1 \not\approx \mathcal{L}_2$ . The corresponding residual transform operator,  $\mathcal{L} = \mathcal{L}_1 - \mathcal{L}_2$ , yields a result that (1) effectively reduces the variability dependent error that occurs when applying either  $\mathcal{L}_1$  or  $\mathcal{L}_2$  to f, a property that holds even when  $\mathcal{L}_1 f \approx \mathcal{L}_2 f$  is not a good approximation to the true sparse domain vector of f, and (2) does

not require  $\mathcal{L}_1$  or  $\mathcal{L}_2$  to have prior information regarding the variability of the underlying signal.

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#### **MS10**

### Radiative Decay of Edge States in a Time-Forced SSH Model

We study the effect of time-periodic forcing on the edge state of the semi-infinite SuSchriefferHeeger (SSH) model, a 1D tight-binding model. Numerical simulations and an asymptotic expansion demonstrate that if the frequency of forcing is in resonance with the continuous spectrum of the unforced Hamiltonian, then on a time scale proportional to the inverse square of the forcing amplitude, the edge state decays in amplitude due to the radiation of its energy into the bulk. A proof is work in progress, and makes use of a new dispersive decay estimate for the time-evolution induced by the Hamiltonian.

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#### **MS10**

### Pseudo-magnetism in the Slowly Strained Discrete Honeycomb Lattice

In photonic crystals, slowly varying nonuniform strains in honeycomb lattices generate effective gauge fields that couple the Dirac operator to a magnetic vector potential, producing flat-band structures (Landau levels) for certain strain patterns. Although such systems are not generally governed by discrete tight-binding models, our discrete honeycomb framework reproduces the same effective magnetic Dirac operator observed in the continuum regime. The derivation of the effective magnetic Dirac operator is obtained via a formal discrete multiscale expansion. We also establish a rigorous proof of convergence for this multiscale expansion for deformations with bounded gradients. This is a joint work with Michael I. Weinstein.

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#### MS10

### Schrdinger Operators with Lattice Invariant Potentials

We develop a systematic framework to study the dispersion surfaces of Schrdinger operators  $H = -\Delta + V$ , where the potential  $V \in C^{\infty}(\mathbb{R}^n, \mathbb{R})$  is both periodic with respect to a lattice  $\Lambda \subset \mathbb{R}^n$  and respects its symmetries. Using techniques from Floquet-Bloch theory and representation theory, we prove a series of results that can be

used to analyze the operator H where the lattice  $\Lambda$  is arbitrary. As an application of this framework, we describe the generic structure of some singularities in the band spectrum of Schrdinger operators invariant under various two- and three-dimensional lattices. Specifically, we study the square, hexagonal, rectangular, simple cubic, body-centered cubic, face-centered cubic, and stacked hexagonal lattices, in the process reproducing results due to [R. T. Keller et al. Spectral Band Degeneracies of  $\pi/2$ -Rotationally Invariant Periodic Schrdinger Operators] and [Charles L. Fefferman and Michael I. Weinstein. Honeycomb Lattice Potentials and Dirac Points], and also proving a conjecture of [Haimo Guo, Meirong Zhang, and Yi Zhu. Three-Fold Weyl Points for the Schrodinger Operator with Periodic Potentials].

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#### **MS10**

#### Flat Bands in 2D Structures

Magic angles are a hot topic in condensed matter physics: when two sheets of graphene are twisted by those angles, the resulting material is superconducting and the so-called energy bands are flat. In 2011, Bistritzer and MacDonald proposed a model that is experimentally very accurate in predicting magic angles. In this talk, I will introduce some recent mathematical progress on the twisted bilayer graphene, including the mathematical characterization of magic angles and flat bands, and generic existence of Dirac cones. I will also discuss some new mathematical discoveries in twisted multilayer graphene. This talk includes joint works with Simon Becker, Bryan Li, Solomon Quinn, Zhongkai Tao and Alex Watson.

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#### **MS11**

Measure-valued Solutions and Selection Criteria: Do we Want to Solve the Euler System or Deterministic Turbulence?

We show that the measure-valued solutions of the Euler system of gas dynamics generated by oscillatory sequences of consistent approximations violate the principle of maximal entropy production formulated by Dafermos. Actually solutions satisfying Dafermos' criterion must be weak solutions. Numerical results illustrate that solutions obtained by standard numerical methods may be oscillatory and thus do not comply with the Dafermos criterion. A natural question arsises: Do we want to compute low viscosity limits obtained by "standard' methods or to see exact solutions of the Euler system?

#### Eduard Feireisl

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#### MS11

### On Maximal Dissipation Criteria for the Compressible Euler Equations

In the past years, results based on a technique called convex integration have drawn lots of interest within the community of mathematical fluid mechanics. Among other fascinating results, this technique allows to prove existence of infinitely many solutions for the multi-dimensional compressible Euler equations. All these solutions satisfy the energy inequality which is commonly used in the literature to identify physically relevant solutions. On the other hand, intuitively at least some of the infinitely many solutions still seem to be non-physical. For this reason one has studied additional admissibility criteria regarding maximal energy dissipation, to no avail: such criteria do not select the solution which is expected to be the physical one. In this talk we give an overview on the aformentioned non-uniqueness results and we explain why maximal dissipation fails to single out the solution which is presumably the physical solution.

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#### **MS11**

### Failure of the Least Action Admissibility Principle in the Context of the Compressible Euler Equations

Over the past decade, convex integration has reshaped our understanding of weak solutions to the multi-dimensional barotropic compressible Euler equations, revealing severe non-uniqueness, even among solutions satisfying classical entropy conditions. Infinitely many admissible weak solutions for certain Riemann-type initial data were constructed using convex integration, showing that genuinely multi-dimensional solutions can emerge from onedimensional data. This spontaneous symmetry breaking challenges physical intuition and motivates the search for new selection criteria. One such proposal is the least action admissibility principle, which selects solutions minimizing an action functional inspired by variational mechanics. In this talk, we evaluate this principle, showing that it excludes a solution that may be regarded as the intuitively correct one. This suggests that either our understanding of a single physically relevant weak solution needs to be reconsidered, or the principle itself must be discarded as a viable selection criterion.

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#### MS11

### Non-uniqueness for Continuous Solutions to 1D Conservation Laws

In this talk, we show that a geometrical condition on  $2 \times 2$  systems of conservation laws leads to non-uniqueness in the class of 1D continuous functions. This demonstrates that the Liu Entropy Condition alone is insufficient to guarantee uniqueness, even within the mono-dimensional setting. We provide examples of systems where this pathology holds, even if they verify stability and uniqueness for small BV solutions. Our proof is based on the convex integration

process. Notably, this result represents the first application of convex integration to construct non-unique continuous solutions in one dimension. This is a joint work with Robin Ming Chen, and Cheng Yu.

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#### **MS12**

### Convolution Estimates for the Gain Boltzmann Operator with Hard-spheres

We discuss new moment-preserving polynomially weighted convolution estimates for the gain operator of the Boltzmann equation with hard potentials, including the critical case of hard-spheres. Our approach relies crucially on a novel cancellation mechanism dealing with the pathological case of energy-absorbing collisions (that is, collisions that accumulate energy to only one of the outgoing particles). This difficulty is specific to hard potentials, and is not present for Maxwell molecules. Our method quantifies the heuristic that, while energy-absorbing collisions occur with non-trivial probability, they are statistically rare, and therefore do not affect the overall averaging behavior of the gain operator. At the technical level, our proof relies solely on tools from kinetic theory, such as geometric identities and angular averaging. This is joint work with Tristan Lger.

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#### **MS12**

#### Global Well-posedness and Scattering Results for Radial Solutions of Nonlinear Wave Equations

This talk will focus on the long-time behavior of large solutions to nonlinear wave equations in the energy-supercritical setting. First, we will briefly review the earlier applications of concentration compactness and rigidity methods through the seminal works of Kenig-Merle and Duyckaerts-Kenig-Merle for the energy-critical and energy-supercritical problems in three dimensions. We will then discuss the generalization of rigidity arguments to higher dimensions while pointing out the challenges in even dimensions.

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#### MS12

### Global Dynamics for Solutions to Nls with Non-Radial Symmetry

In this talk consider the nonlinear Schrödinger equation in the intercritical setting, and questions of global existence versus finite time blow-up. We introduce classes of initial data with non-radial symmetries. For each of these classes, we establish new dichotomy results. By restricting initial data to suitable classes, this results in dichotomy results beneath an arbitrarily large mass-energy threshold.

#### Ian Miller

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#### **MS12**

### On the NLS with Combined Nonlinear Terms, Well-posedness and Blow-up Results

We first discuss local and global well-posedness for any positive power of nonlinearity in a weighted subset of a Sobolev space with the global results requiring extra constraints. We prove scattering for initial data with a quadratic phase and discuss a possible threshold for the global vs finite time existence, then discuss how the same quadratic phase can lead to blow up solutions. We proceed by showing numerical examples for exponential type nonlinearities, which are then backed up by analytical results.

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#### **MS13**

### Phase-field Fracture 2.0: "New Formulation!", "Extra Strength!".

Crack propagation in brittle materials can be described in terms of trade-off between a bulk and surface energies, characterized by a material's fracture toughness (Griffith, 1921; Francfort and Marigo, 1998). Crack nucleation, however, is a much more complex process involving fracture toughness, material strength, i.e. the range of stresses a material can sustain while deforming elastically, and complex scale effects. Phase-field models of fracture (Bourdin et al., 2000; Bourdin et al., 2008) have become ubiquitous due to their ability to account for complex fracture patterns in a wide range of materials and multi-physics settings. When seen as gradient damage models (Pham et al., 2011; Marigo et al., 2016), they properly account for tensile crack nucleation only (Tann et al., 2018; Kumar et al., 2020). A case can be made that crack nucleation cannot be accounted for in variational models based on Griffith-like surface energy. It has been suggested that one needs to renounce to the variational nature of the models (Lopez-Pamies et al., 2025), I will propose a different approach using a cohesive energy depending on the crack opening. Unlike existing models, this approach is capable of handling arbitrary strength surfaces and cohesive energies. In this talk, I will present the proposed model and its properties, including a postulated "sharp interface" limit, its properties, and its numerical implementation.

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#### **MS13**

#### Adaptive Finite Element Approximation for Quasi-Static Crack Growth

We present an adaptive finite element approximation for a model of quasi-static crack growth in dimension two. The discrete setting consists of integral functionals that are defined on continuous, piecewise affine functions, where the triangulation is a part of the unknown of the problem and adaptive in each minimization step. The limit passage is conducted simultaneously in the vanishing mesh size and discretized time step, and results in an evolution for the continuum Griffith model of brittle fracture with isotropic surface energy [FriedrichSolombrino18] which is characterized by an irreversibility condition, a global stability, and an energy balance.

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#### **MS13**

# Ambrosio-Tortorelli Approach to Topological Singularities and Connections with Jump Minimizing Liftings

We study the Gamma-convergence of Ambrosio-Tortorellitype functionals, for maps u defined on an open bounded set  $\Omega \subset \mathbb{R}^n$  and taking values in the unit circle  $\mathbb{S}^1 \subset \mathbb{R}^2$ . Depending on the domain of the functional, two different Gamma-limits are possible, one of which is nonlocal, and related to the notion of jump minimizing lifting, i.e., a lifting of a map u whose measure of the jump set is minimal. The latter requires ad hoc compactness results for sequences of liftings which, besides being interesting by themselves, also allow to deduce existence of a jump minimizing lifting. This is based on a joint work with Giovanni Bellettini and Riccardo Scala.

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Riccardo Scala Dip. Ing. Inf. Sc. Mat. Univ. Siena riccardo.scala@unisi.it

#### MS13

### Concentration-compactness for Fracture Mechanics

Motivated by variational models for fracture, we discuss a new proof of compactness for GSBVp functions without a priori bounds on the function itself. Our proof is based on the classical idea of concentration-compactness, making it quite intuitive. Further, so far as we are aware, this is the first time the connection to concentration-compactness has been made explicit for problems in fracture mechanics.

Based on joint work with W. M. Feldman.

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#### MS14

#### Suppressing Instability in Vlasov-Possion Systems

Plasma instabilities are a major concern in plasma science, for applications ranging from particle accelerators to nuclear fusion reactors. In this work, we consider the possibility of controlling such instabilities by adding an external electric field to the VlasovPoisson equations. Our approach to determining the external electric field is derived from a linear analysis that examines the revised dispersion relation. Allowing the external electric field to depend on time and space, we show that it is possible to completely suppress the plasma instabilities when the equilibrium distribution and the perturbation are known, with one particular choice of external field turning the system back to free-streaming. Numerical simulations of the nonlinear two-stream and bump-on-tail instabilities verify our theory and demonstrate the effectiveness of the few choices of external electric field that we derive.

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#### MS14

### Generative Diffusion Models from a PDE Perspective

Generating new images has become more common, anyone can easily generate new images using a simple prompt. Although there are various techniques such as autoencoders or GANs (Generative Adversarial Networks), the most popular approach nowadays is to use diffusion models. The goal of this presentation is to introduce the core idea of this method, which consists of reversing a diffusion process. However, reversing diffusion is an ill-posed problem. We will show how to bypass this restriction by transforming a diffusion equation into a transport equation. We then analyze how the generated density relates to the original samples, proving that diffusion models (if done perfectly) do not generalize.

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#### **MS14**

#### Super-resolution Imaging in Disordered Media

We propose a methodology that exploits large and diverse data sets to accurately estimate the ambient mediums Greens functions in strongly scattering media. Given these estimates excellent imaging results are achieved, with a resolution that is better than that of a homogeneous medium. This phenomenon, also known as super-resolution, occurs because the ambient scattering medium effectively enhances the physical imaging aperture.

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#### MS14

#### Effective Dynamics of Translationally Invariant Magnetic Schrdinger Equations in the High Field Limit

We study the large field limit in Schrdinger equations with magnetic vector potentials describing translationally invariant B-fields with respect to the z-axis. In a first step, using regular perturbation theory, we derive an approximate description of the solution, provided the initial data is compactly supported in the Fourier-variable dual to  $z \in \mathbb{R}$ . The effective dynamics is thereby seen to produce high-frequency oscillations and large magnetic drifts. In a second step we show, by using the theory of almost invariant subspaces, that this asymptotic description is stable under polynomially bounded perturbations that vanish in the vicinity of the origin.

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#### **MS15**

#### Weak Solutions of the Compressible Poisson-Nernst-Planck-Navier-Stokes Equations

We consider the compressible Poisson-Nernst-Planck-Navier-Stokes (PNPNS) system of equations, which models the transport of charged particles, under the influence of the self-consistent electrostatic potential, in a compressible fluid. We study the equations posed on a smooth bounded spatial domain of  $\mathbb{R}^3$  and consider a set of boundary conditions motivated by physical considerations. Then we prove the existence of global weak solutions for the initial/boundary value problem, without restrictions on the size of the initial data. This is a joint work with Dehua Wang.

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#### MS15

#### Rayleigh-Taylor Instability and Beyond

It is known in physics that steady state of fluids under the influence of uniform gravity is stable if and only if the convection is absent. In the context of incompressible fluids, convection happens when heavier fluids are on top of lighter fluids, known as Rayleigh-Taylor instability. However, in real world, heat transfer plays an important role in convection of fluids, such as the weather changes, and or cooking a meal. In this context, the compressibility of the fluids becomes important. Indeed, using the more realistic model of compressible flow with heat transfer, the behavior of solutions is much closer to the real world and more complicated. We will discuss these topics in this lecture, including some on-going research projects. The lecture will be accessible to audiences with basic knowledge on multivariable calculus, and little of differential equations.

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#### MS15

### Oscillations in Hyperbolic-parabolic Systems and the Issue of Deriving Homogenized Equations

The objective of the talk is to provide examples of sustained oscillations for hyperbolic-parabolic systems. This problem was motivated by work on the existence theory for viscoelasticity of Kelvin-Voigt type with non-convex stored energies which shows propagation of  $H^1$ -regularity for the deformation gradient of weak solutions for semiconvex stored energies. While weak solutions still exist for initial data in  $L^2$ , oscillations on the deformation gradient can now persist and propagate in time. The existence of sustained oscillations in hyperbolic-parabolic system is studied systematically via examples, for paradigm systems from viscoelasticity and for the compressible Navier-Stokes system with non-monotone pressures. In several space dimensions oscillatory examples are associated with lack of rank-one convexity of the stored energy. The subject naturally leads to the problem of deriving effective equations for the associated homogenization problems. This is in general a hard problem that can in simpler models be addressed by ideas from the kinetic formulation for conservation laws.

#### Athanasios Tzavaras

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#### MS15

## On the Vanishing Viscosity Limit for Incompressible Flows With Inflow/Outflow Boundary Conditions

We study the vanishing viscosity limit for the incompressible Navier-Stokes equations (NSE) in a general bounded domain with inflow-outflow boundary conditions. Extending the work of Gie, Hamouda, and Temam (Netw. Heterog. Media 7, 2012) and also of Lombardo and Sammartino (SIAM J. Math. Anal. 33 2001), we allow for a general injection and suction angle, as long as it is bounded away from zero. We rigorously establish the convergence of NSE solutions to those of the Euler equations (EE) as viscosity vanishes in the energy norm. We prove interior convergence in both the  $L^2$  and the Sobolev  $H^1$  norms at the same rates as in the case of injection/suction normal to the boundary. The proof relies on the construction of boundary layer correctors via Prandtl-type equations and a higher-order asymptotic expansion that improves the convergence rate.

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#### **MS16**

#### Machine-Precision Operator Learning

Neural operator methods have shown promise in approximating infinite-dimensional nonlinear operators, but increasing their complexity often yields minimal improvement. Moreover, their relatively low accuracy is unacceptable for many physical applications. To address this, we introduce CHONKNORIS Cholesky-Newton-Kantorovich Neural Operator for Residual Iterative Systems a novel operator learning framework that iteratively refines an ini-

tial guess of an operators output using a learned update mechanism. This mechanism guarantees convergence to the correct solution under mild conditions. Remarkably, CHONKNORIS can achieve machine-level accuracy even when initialized with a coarse approximation. The method is general and applies to both invertible operators and those defined implicitly via constraints, such as in physics-informed systems. We demonstrate its accuracy and flexibility through applications to nonlinear PDEs and inverse problems.

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#### **MS16**

### Kernel Collocation Methods for Inferring Differential Equations from Data

We introduce a kernel method learning differential equations and their solution maps that is highly efficient in data requirements, both in terms of solution examples and amount of measurements from each example. This method is based on combining a least-squares kernel collocation scheme for general nonlinear PDEs with a RKHS approximation of the PDE itself in terms of the solution and its derivatives. Numerical experiments demonstrate significant improvements in computational speed and robustness compared to PINN based methods.

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#### **MS16**

#### Krom: Kernelized Reduced Order Model

Due to technological advancements, we are increasingly dealing with complex systems as exemplified in the digital twin modeling which requires efficient and accurate simulations withinmicroseconds. However, efficient and accurate simulations of these processes require significant computational resources, which are no longer feasible given physical and engineering limitations such as Landauers Principle, Moores Law, and energy constraints. For this reason, Reduced Order Models (ROMs) are employed to project these high-dimensional problems onto lower-dimensional spaces while still maintaining sufficient accuracy. In this paper, we propose an approach based on Gaussian Processes that can solve high-dimensional problems accurately and efficiently, with accompanying theoretical guarantees.

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#### MS16

#### Fast Bayesian Multilevel Quasi-Monte Carlo

Existing multilevel quasi-Monte Carlo (MLQMC) methods often rely on multiple independent randomizations of a low-discrepancy (LD) sequence to estimate statistical errors on each level. While this approach is standard, it can be less efficient than simply increasing the number of points from a single LD sequence. However, a single LD sequence does not permit statistical error estimates in the current framework. We propose to recast the MLQMC problem in a Bayesian cubature framework, which uses a single LD sequence and quantifies numerical error through the posterior

variance of a Gaussian process (GP) model. When paired with certain LD sequences, GP regression and hyperparameter optimization can be carried out at only  $\mathcal{O}(n\log n)$  cost, where n is the number of samples. Building on the adaptive sample allocation used in traditional MLQMC, where the number of samples is doubled on the level with the greatest expected benefit, we introduce a new Bayesian utility function that balances the computational cost of doubling against the anticipated reduction in posterior uncertainty. A series of numerical experiments illustrate the performance of our fast Bayesian MLQMC method and error estimates for multilevel PDE problems. The Bayesian error estimates obtained using digital nets are found to be reliable, although, in some cases, mildly conservative.

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#### **MS17**

### Competition, Cooperation, and Common Pool Resources in Mean Field Games

The tragedy of the commons (TOTC) states that the individual incentives will result in overusing common pool resources (CPRs) which in turn may have detrimental future consequences that affect everyone involved negatively. However, in many real-life situations this does not happen and researchers such as the Nobel laureate Elinor Ostrom suggested mutual restraint by individuals can be the preventing factor. In mean field games (MFGs), since individuals are insignificant and fully non-cooperative, the TOTC is inevitable. This shows that MFG models should incorporate a mixture of selfishness and altruism to capture real-life situations that include CPRs. Motivated by this, we will discuss different equilibrium notions to capture the mixture of cooperative and non-cooperative behavior in the population. First, we will introduce mixed individual MFGs and mixed population MFGs where we also include the CPRs. The former captures altruistic tendencies at the individual level and the latter models a population that is a mixture of fully cooperative and non-cooperative individuals. For both cases, we will briefly discuss definitions and characterization of equilibrium with the forward backward stochastic differential equations. Later, we will discuss a real-life inspired example of fishers where the fish stock is the CPR. We will analyze the existence and uniqueness results, and discuss the experimental results. (This is a joint work with Mathieu Lauriere.)

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#### **MS17**

### Non-Potential Mean-Field Games La Benamou-Brenier

Mean-field games (MFG) theory is a mathematical framework for studying large systems of agents who play differential games. In the PDE form, MFG reduces to a Hamilton-Jacobi equation coupled with a continuity or Kolmogorov-Fokker-Planck equation. Theoretical analysis and computational methods for these systems are challenging due to the absence of strong regularizing mechanisms and coupling between two nonlinear PDE. One approach that proved successful from both theoretical and computational perspectives is the variational approach, which interprets the PDE system as KKT conditions for suitable con-

vex energy. MFG systems that admit such representations are called potential systems and are closely related to the dynamic formulation of the optimal transportation problem due to Benamou-Brenier. Unfortunately, not all MFG systems are potential systems, limiting the scope of their applications. I will present a new approach to tackle non-potential systems by providing a suitable interpretation of the Benamou-Brenier approach in terms of monotone inclusions. In particular, I will present advances on the discrete level and numerical analysis and discuss prospects for the PDE analysis.

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#### **MS17**

### Regularization of Mean Field Game Partial Differential Inclusions

Mean field Game (MFG) Partial Differential Inclusions (PDI) are generalizations of the standard system of Partial Differential Equations (PDE) from mean field games to situations where players in the game may have possibly nonunique optimal controls, and the resulting Hamiltonian is not required to be differentiable. We study second-order MFG PDI with convex, Lipschitz continuous, but possibly nondifferentiable Hamiltonians, and their approximation by systems of classical MFG PDE with regularized Hamiltonians. Under very broad conditions on the problem data, we prove that, up to subsequences, the solutions of the regularized problems converge to solutions of the MFG PDI. In particular, we show the convergence of the value functions in the  $H^1$ -norm and of the densities in  $L^q$ -norms. Under stronger hypotheses on the problem data, we also establish rates of convergence between the solutions of the original and regularized problems, without requiring any higher regularity of the solutions. We give concrete examples that demonstrate the sharpness of several aspects of the analysis.

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#### **MS17**

### Finite Dimensional Projections of HJB Equations in the Wasserstein Space

In this talk, we consider the optimal control of particle systems with mean-field interaction and common noise, and their limit as the number of particles tends to infinity. First, we prove the convergence of the value functions  $u_n$  corresponding to control problems of n particles to the value function V corresponding to an appropriately defined infinite dimensional control problem. Then, we prove, under certain additional assumptions,  $C^{1,1}$  regularity of V in the spatial variable. In the second part of the talk, we discuss conditions under which the value function V projects precisely onto the value functions  $u_n$ . Using this

projection property, we show that optimal controls of the finite dimensional problem correspond to optimal controls of the infinite dimensional problem and vice versa. This talk is based on [A. Swiech, L. Wessels, Finite Dimensional Projections of HJB Equations in the Wasserstein Space, https://arxiv.org/abs/2404.05185 (2024)].

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#### **MS18**

#### FedCB<sup>2</sup>O: Reaching Group Consensus in Clustered Federated Learning and Robustness to Backdoor Adversarial Attacks

Adversarial attacks pose significant challenges in many machine learning applications, particularly in the setting of distributed training and federated learning, where malicious agents seek to corrupt the training process with the goal of jeopardizing and compromising the performance and reliability of the final models. In this talk, I will discuss the problem of robust federated learning in the presence of such attacks by formulating the training task as a bi-level optimization problem. I'll present some theoretical results on the resilience of consensus- based bi-level optimization (CB2O), an interacting multi-particle metaheuristic optimization method for solving bilevel optimization problems, in adversarial settings. Specifically, I'll present a global convergence analysis of CB2O in mean-field law in the presence of malicious agents, demonstrating the robustness of CB2O against a diverse range of attacks. Extensive experiments demonstrate the robustness of the FedCB2O algorithm against label-flipping attacks in decentralized clustered federated learning scenarios, showcasing its effectiveness in practical contexts.

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#### MS18

### Residual Diffusivity for the Stochastically Forced Navier Stokes Equations

Consider a diffusive passive scalar that is advected by a periodic incompressible flow. On long time scales it is known that the effect of the periodic drift averages and the behavior is the same as that of a purely diffusive scalar with an effective diffusion coefficient. Residual diffusivity is the remarkable pheonomenon where the effective diffusivity stays bounded and non-degenerate as the molecular diffusivity vanishes. It is conjectured to happen in situations where the advecting drift is chaotic. In this talk we show that the stochastically forced Navier Stokes equations, (and more generally any random velocity fields that satisfies a uniform in diffusivity mixing condition) exhibits residual diffusivity.

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#### **MS18**

#### Non-Local Perimeters from Adversarial Learning

Recent work in machine learning has recognized that many standard algorithms for classification are strongly affected by adversarial attacks. Accordingly, a growing body of research has tried to identify ways to mitigate this issue. This talk will discuss a natural non-parametric formulation of this objective, which can be transformed into a standard classification problem that utilizes a non-local perimeter as a regularizer. Connections with optimal transportation, mean curvature flow, and minimal surfaces, and related open problems will also be discussed. This represents joint work with Nicolas Garcia Trillos, Leon Bungert, and Rachel Morris.

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#### MS18

### Extended Convexity and Uniqueness of Minimizers for Interaction Energies

Linear interpolation convexity (LIC) has served as the crucial condition for the uniqueness of interaction energy minimizers. We introduce the concept of the LIC radius which extends the LIC condition. Uniqueness of minimizer up to translation can still be guaranteed if the LIC radius is larger than the possible support size of any minimizer. Using this approach, we obtain uniqueness of minimizer for power-law potentials  $W_{a,b}(\mathbf{x}) = \frac{|\mathbf{x}|^a}{a} - \frac{|\mathbf{x}|^b}{b}, -d < b < 2$  with a slightly smaller than 2 or slightly larger than 4. The estimate of LIC radius for a slightly smaller than 2 is done via a Poincaré-type inequality for signed measures. To handle the case where a slightly larger than 4, we truncate the attractive part of the potential at large radius and prove that the resulting potential has positive Fourier transform. We also propose to study the logarithmic power-law potential  $W_{b,\ln}(\mathbf{x}) = \frac{|\mathbf{x}|^b}{b} \ln |\mathbf{x}|$ . We prove its LIC property for b=2 and give the explicit formula for minimizer. We also prove the uniqueness of minimizer for b slightly less than 2 by estimating its LIC radius.

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#### **MS19**

# A Merging of Fields: Using PDE Analysis, Algebra, & Optimization to Investigate Stability of Shear Flows

Determining nonlinear stability of steady states for complex dynamical systems is a notoriously difficult problem, even for the seemingly simplest cases. For example, it is expected that the standard steady state shear profile, 2D planar Couette flow, is globally stable for all Reynolds numbers, however the state-of-the-art analysis is decades

old and only proves the stability for relatively low Reynolds numbers. In recent years, a promising computational approach uses polynomial sum-of-squares optimization to find Lyapunov functions based on low-mode projections onto an orthogonal basis of  $L^2 \cap H^1$ . Critically, physical symmetries inherent in the system can be exploited to reduce the complexity of the resulting optimization problem. We will present on rigorous and practical extensions of this work.

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#### **MS19**

### Analysis of a Compressible Flow Problem Describing Tumor Growth

In this presentation, we discuss a nonlinear model of tumor growth, described by a coupled hyperbolic-elliptic system of partial differential equations. In this model, the compressible flow of tumor cells is modeled by a continuity equation for the cell density, which takes into account transport via a background flow (given by a potential solving a Brinkman-type equation), and which has a source term modeling cell growth and death. We show that for sufficiently large viscosity, the tumor growth system admits nontrivial global strong solutions for positive initial data having a gradient with sufficiently small norm. This illustrates the regularizing effects of the source term representing tumor cell growth and death on the resulting transport dynamics of the equation. Furthermore, we characterize the long-time behavior of global strong solutions to the tumor growth system using a level-set analysis, in which we analyze how level sets evolve as they are transported by the flow, in terms of expansion/contraction and accretion/depletion of cells. While there has been past work on global existence of weak solutions for this tumor growth system, these results open the study of well-posedness in terms of more regular strong/classical solutions, which exist globally in time. This is joint work with Konstantina Trivisa (University of Maryland, College Park).

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#### **MS19**

### On the Infinite-rotation Limit of the Stochastic Primitive Equations

This talk will discuss a recent result for the stochastic primitive equation in the limit of infinite-rotation , namely, that the laws of the solutions to the stochastic primitive equation are asymptotically attracted to the unique invariant probability measure corresponding to its limit resonant system. This result is obtained by establishing an averaging principle that allows one to properly identify the covariance structure of the noise in the infinite-rotation regime and an asymptotic coupling technique to establish the existence of a spectral gap for the limit resonant system. This is joint work with Quyuan Lin (Clemson) and Rongchang Liu (U

Arizona).

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#### MS20

#### Complex Scattering Makes for Simple Numerics: A Method for Simulating Junctions of Several Semiinfinite Domains

Scattering problems involving unbounded interfaces occur frequently in physics and engineering settings. Due to this prevalence, there exist many numerical methods for solving such problems. Unfortunately, the complicated behavior of solutions in the vicinity of infinite interfaces can make it challenging to deriving explicit error bounds for these methods. Many of these methods also require a large computational domain, and so require a large number of discretization points to accurately solve the problem. In this talk, we present a class of decomposable scattering problems. For this class of problems, the PDE domain can be decomposed into a collection of simple subdomains. The fundamental solutions for these simple regions can then be used to reduce the scattering problem into an integral equation on the interfaces between these subdomains. These integrals equations can be analytically continued into the complex plane, where they can be safely truncated with controllable accuracy. We demonstrate this procedure for the example of two dielectric waveguides meeting at an interface. For this problem, we show that the fundamental solutions and densities decay exponentially in the complex plane, and so the analytically continued integral equation can be truncated with exponential accuracy. We will also demonstrate how this method can be applied to a collection of other scattering problems, including interfaces between periodic gratings.

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#### MS20

### Momentum Space Method for Double-Walled Carbon Nanotubes

Moir physics has yielded exotic many-body effects including correlated insulators and superconductivity. To understand many-body models, a thorough understanding of the single-particle picture is critical as the single-particle basis is used to construct two-body models. Here we introduce an algorithm using momentum space, the analogue of momentum space for moir 2D materials, to construct quasi-band structure for multi-walled carbon nanotubes. Particularly, we focus on the case where the chirality of

the nanotubes results in moir patterns.

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#### **MS20**

# From Schroedinger to Diffusion: Speckle Formation of Light in Random Media and the Gaussian Conjecture

Laser light passing through strongly scattering media form random interference patterns called speckle. Understanding the formation and propagation of speckle finds important applications in imaging and optimization of optical links. A well-known conjecture in physical literature states that high frequency waves propagating over long distances through turbulence eventually become complex Gaussian distributed. The intensity of such wave fields then follows an exponential law, consistent with speckle formation observed in physical experiments. Though fairly well-accepted and intuitive, this conjecture is not entirely supported by any detailed mathematical derivation. In this talk, I will discuss some recent results demonstrating the Gaussian conjecture in a weak-coupling regime of the paraxial approximation. I will describe two scaling regimes, one is a kinetic scaling where the second moment is given by a transport equation and a second diffusive scaling, where the second moment follows an anomalous diffusion. In both cases, the limiting complex Gaussian distribution is fully characterized by its first and second moments. This is joint work with Guillaume Bal.

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#### **MS20**

### Bounds and Limitations on Broadband Quasistatic Passive Cloaking in the Near-Field

In this talk, we discuss our recent results on the following challenging question: is it possible to use a passive cloak to make invisible a dielectric inclusion on a finite frequency interval in the quasistatic regime of Maxwells equations for an observer close to the object? First, we will present our theorem that gives the answer to this question: No one cannot due to the passivity of the cloaking device. Second, we discuss how the passivity assumption allows us to derive sum rules for the Dirichlet-to-Neumann map in the nearfield cloaking problem, based on its properties related to two important classes of analytic functions, namely, Herglotz and Stieltjes functions. Third, we present our theorem on bounds that impose fundamental limits on passive cloaking devices over a finite frequency interval. These results are achieved by combining two techniques for producing bounds: i) variational bounds (using the Dirichlet and Thomson variational principles) from abstract theory of composite; ii) the analytic approach to bounds using sum rules for passive systems. This is joint work with Maxence Cassier and Graeme W. Milton.

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#### **MS21**

### Maximal Turbulence as a Selection Criterion for Measure-valued Solutions

The quest for a good solution concept for the partial differential equations (PDEs) arising in mathematical fluid dynamics is an outstanding open problem. An important notion of solutions are the measure-valued solutions. It is well known that for many PDEs there exists a multitude of measure-valued solutions even if admissibility criteria like an energy inequality are imposed. Hence in recent years, people have tried to select the relevant solutions among all admissible measure-valued solutions or at least to rule out some solutions which are not relevant. In this paper another such criterion is studied. In particular, we aim to select generalized Young measures which are "maximally turbulent'. To this end, we look for maximizers of a certain functional, namely the variance, or more precisely, the Jensen defect of the energy. We prove existence of such a maximizer and we show that its mean value and total energy is uniquely determined. Our theory is carried out in a very general setting which may be applied in many situations where maximally turbulent measures shall be selected among a set of generalized Young measures. Finally, we apply this general framework to the incompressible and the isentropic compressible Euler equation. Our criterion of maximal turbulence is plausible and leads to existence and uniqueness in a certain sense (in particular, the mean value and the total energy of different maximally turbulent solutions coincide).

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#### **MS21**

### Are $L^{\infty}$ Solutions to Hyperbolic Systems of Conservation Laws Unique?

For hyperbolic systems of conservation laws in 1-D, fundamental questions about uniqueness and blow up of weak solutions still remain even for the apparently simple systems of two conserved quantities such as isentropic Euler and the p-system. Similarly, in the multi-dimensional case, a longstanding open question has been the uniqueness of weak solutions with initial data corresponding to the compressible vortex sheet. We address all of these questions by using the lens of convex integration, a general method of constructing highly irregular and non-unique solutions to PDEs. Our proofs involve computer-assistance. This talk is based on joint work with Lszl Szkelyhidi, Jr.

#### Sam Krupa

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#### **MS21**

### Flexibility for an Isotropic Landau Equation and the 2D Stationary Navier-Stokes Equations

In this talk, I will describe flexibility results of h-principle type (non-uniqueness, existence of non-trivial stationary solutions) for an isotropic Landau equation, which are the first of their kind in the kinetic theory setting. The main new tool is an interpretation of the nonlinear term using

paraproducts. This tool is applicable in the Navier-Stokes equations as well and gives sharp non-uniqueness results in the 2-dimensional case.

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#### MS22

### Recovering Optimal Spectral Measure from Noisy Data

We study the problem of noisy data fitting and spectral measure recovery in the classes Stieltjes and Completely monotone functions. The analytical setting is non-negative least squares over the conical hull of a curve in  $\mathbb{R}^n$ . While the non-negative least squares problem receives much attention, no method exists to analyze convergence to the true solution, and popular numerical algorithms come with very few guarantees. We remedy this by developing a simple theory of minimization over convex cones, by which we can convert the non-negative constraints to convex ones. In this context, convergence analysis is well understood. Algorithms for the convex problem provide means to compute a function along with its spectral measure which provide a good fit for the noisy data and approximately satisfy the optimality conditions. Our theory then gives us tools to analyze the support of the optimal spectral measure.

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#### MS22

#### On Crack Nucleation in Phase-Field Fracture

Variational formulations of Griffith fracture based on global minimization, and even local minimization, intertwine propagation and (unphysical) nucleation. This is true also of phase-field approximations of sharp Griffith fracture. Recently, there have been attempts to combine experiment-based nucleation criteria with Griffith propagation in phase-field models. We describe difficulties in doing this, together with recent progress.

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#### MS22

### A New Lower Bound for the Kirchhoff Bending Energy

The bending energy of a surface in  $\mathbb{R}^3$  is the square of the  $L^2$ -norm of its second fundamental form. In elasticity, this energy arises as the leading-order term in the elastic energy of thin plates, where the admissible configurations are isometric immersions of the surface. When the intrinsic geometry of the surface is non-trivial, a natural question is to obtain bounds on this bending energy in terms of appropriate norms of the Gaussian curvature. In this talk, I will present a new lower bound for the bending energy in terms of the  $H^{-1}$ -norm of the curvature. Unlike previously known bounds, this one captures curvature concentration. I will illustrate its use by deriving optimal energy scalings for cones and E-cones.

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#### MS22

### Scaling Law Analysis of Crystals on Curved Surfaces

Curved crystals are different from their flat counterparts. Experiments where colloidal particles are put onto liquid droplets have shown that, with enough particles, the resulting crystals are highly anisotropic and fractal-like, even though they form slowly enough that energy minimization should apply. Motivated by the surrounding literature, we study a continuum model of curved crystals involving minimization of a bulk, non-Euclidean elastic energy plus a surface energy (perimeter) term. We prove upper and lower bounds on the scaling law of the minimum energy with respect to its parameters, and find two laws distinguishing anisotropic and isotropic crystal growth. Mathematically, the key ingredient is a new "thin isoperimetric inequality" whose optimizers are not spheres. This interpolation-style inequality yields lower bounds on the energy of a curved crystal, which are optimal in the expected anisotropic regime.

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#### MS23

### Continuum Model for Relaxed Moire Bilayer Graphene

Twisted bilayer graphene (TBG) has drawn significant interest due to recent experiments which show that TBG can exhibit strongly correlated behavior such as the superconducting phases. Much of the theoretical work on TBG has been based on analysis of the Bistritzer-MacDonald model which includes a phenomenological parameter to account for lattice relaxation. In this work, we propose a newly developed model which systematically accounts for the effects of structural relaxation, in which atom positions are altered to minimize total mechanical energy. The tightbinding model contains a natural separation of scales between the atomistic scale and a much larger moire scale. Via a systematic multiple-scales expansion, we obtain an effective continuum PDE model with accurate structural relaxation corrections. We show that in the low energy limit, the continuum model accurately approximates the dynamics of tight-binding wave functions spectrally localized to the monolayer Dirac points. This work can naturally be extended to bilayer graphene with uniform strain.

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#### **MS23**

#### A Topological Index for 1D Interacting Quantum Systems

A central aim in the mathematics of topological phases of matter is to identify macroscopic observables with global topological indices. Analogous to the non-interacting setting, we would like to classify various symmetry classes by formulating infinite volume topological indices also in the interacting setting. When possible we would also like to show that these indices are complete, i.e., given two states with the same index, there is a continuous, symmetry preserving path which connects them. We discuss a  $\mathbb{Z}_2$ -valued index on the space of 1D parity invariant "local" pure states which is stable under appropriate perturbations. This is then applied to Class D interacting systems in one-dimension to yield an infinite-volume many-body Majorana number. Based on joint with Jacob Shapiro.

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#### MS23

#### Dispersion in Floquet Materials

Floquet media, materials subject to time-periodic forcing, naturally give risevia homogenization to Dirac Hamiltonians that differ from those traditionally studied in quantum mechanics. A key question is whether wavepackets disperse in such media. This, in turns, leads to the study of dispersive dynamics of nonautonomous Hamiltonians, a largely open problem.

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#### MS23

#### Electronic and Mechanical Continuum Models of Twisted Bilayer Graphene

Moiré materials, made by stacking 2D materials with slightly mismatched Bravais lattices, have attracted huge attention in recent years because of their remarkable ability to realize diverse quantum phenomena such as "unconventional" superconductivity. These phenomena are generally predicted using continuum PDE models which extract the essential physics at the scale of the material's moiré pattern. I will show how these models can be derived using

systematic multiple-scales analysis with rigorous error estimates. I will first present the derivation for models of single-particle electronic properties, such as the Bistritzer-MacDonald model of twisted bilayer graphene. I will then present the case of mechanical models describing structural relaxation and vibration (phonons). Our derivations and proofs clarify the parameter regime of validity of such continuum approximations and allow for systematic derivations of higher-order corrections.

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#### **MS24**

#### Resolvent Expansion and Edge State Dynamics for Honeycomb Media with Irrational Edges

In this work, we study wave propagation in twodimensional continuous honeycomb structures with a straight line defect. We focus on a Schrdinger operator  $H := -\Delta + Q(x)$  in  $\mathbb{R}^2$  with a perturbed honeycomb potential, where the perturbation consists of a transition, along an edge, between two distinct symmetry-breaking bulk periodic structures. Our goal is to understand the nature of the states that fill the bulk spectral gaps for edges with irrational slopes. Our approach relies on the observation that the perturbed potential can be viewed as a slice of a 3D potential with translation-invariance in the direction of the edge. This property allows us to reframe our problem in a higher-dimensional framework, involving an augmented 3D degenerate Schrdinger operator. Using this framework, we provide an asymptotic expansion for the resolvent of the augmented Schrdinger operator in terms of effective 1D Dirac operators. We show that the asymptotic expansion for irrational edges involves infinitely many effective Dirac operators, unlike rational edges. We use the resolvent estimates to study wavepackets dynamics along the edge.

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#### **MS24**

#### On Optimality and Bounds for Internal Solutions Generated from Impedance Data Driven Gramians

We consider the computation of internal solutions for a time domain plasma wave equation with unknown coefficients from the data obtained by sampling its transfer function at the boundary. The computation is performed by transforming the background snapshots for a known background coefficient using the Cholesky decomposition of the data-driven Gramian. We show that this approximation is asymptotically close to the projection of the internal solution onto the subspace of background snapshots. This allows us to derive a generally applicable bound for the error in the approximation of internal fields from boundary data only for a time domain plasma wave equation with an unknown potential q. We use this to show convergence for general unknown q in one dimension. We show numerical experiments and applications to SAR imaging in higher

dimensions.

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#### **MS24**

### A Reduced-Order Model for Radiative Transport and Its Applications in Imaging

One of the main challenges in inverse radiative transport problems is their high computational cost due to the high-dimensionality nature of the radiative transport equation. In this talk, I will discuss some computational strategies based on reduced-order models to accelerate inverse transport computations. While the methods are mainly motivated by physical intuitions, we will provide some justifications in simplified settings.

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#### MS24

#### Inverse Problems in Quantitative PAT

Photoacoustic tomography (PAT) is a hybrid modality that combines optics and acoustics to obtain high-resolution and high-contrast imaging of heterogeneous media. In this talk, I will focus on the inverse problems in the quantitative step of PAT, which aim to reconstruct the optical coefficients of the governing radiative transport equation from the ultrasound measurements. I will discuss the uniqueness and stability from several perspectives and models. Some numerical experiments are performed to validate the results.

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#### MS25

### Stability of Reflecting Shock Waves for 1D Isentropic Flows

In this talk, we show the weak/BV principle (in the spirit of the Di Perna and Dafermos weak/strong principle) for reflecting shock waves in 1D. We consider the isentropic Euler equation with in-flow and out-flow boundary conditions, focusing on the subsonic case. We prove that reflecting shocks are stable among all weak entropic solutions verifying a mild condition known as the strong trace property. The proof uses the method of a-contraction with shifts.

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#### **MS25**

#### Vanishing Viscosity Limit of Two Interacting Shocks from Same Family for Compressible Isentropic Navier-Stokes Equations

We study the vanishing viscosity limit for one-dimensional (1D) compressible and isentropic Navier-Stokes equations when the corresponding inviscid Euler equations admit two interacting shocks from the same characteristic family. Unlike the case two interacting shocks from two different characteristic families form two outgoing shocks for compressible isentropic Euler equations, two interacting shocks with the overtaking of a shock by another from the same family will produce a stronger shock in the interacting field and a weak rarefaction wave in the transverse field. Then two new difficulties are encountered. One is that two interacting shocks from the same family is more singular than the head-on shock collisions from different families in energy norm. Another one is the incompatibilities of the stability proof frameworks of the shock and rarefaction waves after the interaction. Both difficulties are overcome by ultimately using the a-contraction method with time-dependent shifts for the shocks to justify the vanishing physical viscosity limit process. Moreover, the decay rate with respect to the viscosity is obtained. The talk is based on the joint work with Dr. Lin-an Li from Beijing Normal University and Prof. Dehua Wang from University of Pittsburgh.

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#### MS25

### Location of Free Boundaries in a Finitely Long Nozzle

In this talk, we introduce our recent work on the existence and unique location of free boundaries in a finitely long nozzle with appropriate boundary conditions at the entrance of the nozzle and the receiver pressure at the exit of the nozzle. It includes the three-dimensional transonic shock with large swirl velocity, two-dimensional transonic shock with large combustion process, and two-dimensional subsonic contact discontinuity in nozzles. They are the first mathematical results on transonic shocks with large swirl velocity or large combustion processes with non-trivial background solutions being constructed. It is also the first mathematical result to determine the location of the contact discontinuity without giving a starting point. The key ideas about the higher-order approximation, compatibility conditions to determine the free boundaries' location, and careful modification of the equations and boundary conditions will be introduced.

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#### MS25

### Incompressible Limits of Weak Solutions for the Compressible Euler Equations

In this talk, I will discuss the non-uniqueness of global weak solutions to the compressible Euler equations using convex integration. Additionally, I will explore how any  $L^2$ -

bounded weak solution of the incompressible Euler equations can be obtained as a limit of solutions to the compressible Euler equations in the vanishing Mach number, also within the convex integration framework.

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#### **MS26**

#### Scattering vs. Non-scattering

I shall discuss obstacle regularity and geometry implications of non-scattering (in the context of the Helmholtz equation) and relate these results to well known results for the Pompeiu/Schiffer conjecture. This represents joint work with Fioralba Cakoni, Narek Hovsepyan and Jingni Xiao.

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#### **MS26**

### Non-Scattering, Always-Scattering, and Inverse Scattering

In this talk, I will present some recent progress on alwaysscattering, non-scattering, and their connections to inverse scattering. We consider scattering problems when a medium is probed by incident waves and as a result scattered waves are induced. The aim of inverse scattering is to deduce information about an unknown medium by measuring the corresponding scattered waves outside the medium. Inverse scattering has applications in many fields of science and technology, of which radar is one of the most prevalent. Non-scattering is a particular phenomenon that arises when a medium is probed but no scattered waves can be measured externally. Non-scattering impacts inverse scattering, and it has applications in invisibility where one tries to avoid detection or damage of an object. Moreover, the mathematical formulation of non-scattering is of similar nature to free boundary problems, as well as the Schiffer conjecture, or equivalently, the Pompeiu problem. There can be situations when non-scattering never occurs for a given medium; this phenomenon is called alwaysscattering. The always-scattering feature has applications in inverse problems for uniquely determining the shape or other properties of a medium from scattering measurements.

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#### MS27

### Uncovering Jupiters Radiation Belt Through PDE Learning

Jupiters magnetosphere is the largest structure in the solar system, capable of trapping high-energy electrons with energies reaching hundreds of MeVposing significant risks to spacecraft systems and instruments. Recent advances in data-driven partial differential equation (PDE) modeling offer promising avenues for improving our understanding of Jupiters radiation environment beyond current models. This work leverages kernel-based PDE learning methods, which are particularly well-suited for the extremely

sparse data settings inherent to space missions, where insitu measurements are limited. These methods address key challenges faced by traditional machine learning approaches in such regimes. The objective of this study is to develop a physics-informed model of the electron environment around Jupiter by learning a diffusion PDE governing the electron distribution, using all available data from past missions such as Galileo and forthcoming observations from the Juno spacecraft.

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#### **MS27**

### Fluids you can Trust: Operator Learning for Incompressible Flows

This paper introduces the Kernel Neural Operator (KNO), a novel operator learning technique that uses deep kernelbased integral operators in conjunction with quadrature for function-space approximation of operators (maps from functions to functions). KNOs use parameterized, closedform, finitely-smooth, and compactly-supported kernels with trainable sparsity parameters within the integral operators to significantly reduce the number of parameters that must be learned relative to existing neural operators. Moreover, the use of quadrature for numerical integration endows the KNO with geometric flexibility that enables operator learning on irregular geometries. Numerical results demonstrate that on existing benchmarks the training and test accuracy of KNOs is higher than popular operator learning techniques while using at least an order of magnitude fewer trainable parameters. KNOs thus represent a new paradigm of low-memory, geometrically-flexible, deep operator learning, while retaining the implementation simplicity and transparency of traditional kernel methods from both scientific computing and machine learning.

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#### MS28

# Homogenization and Stieltjes Analytic Representation of the Effective Properties for Domains with Rough Interfaces

Stieltjes analytic representation has been successfully used in homogenization of the transport properties of two phase composite materials as well as advection diffusion processes with different types of fluid velocity fields. Here we extend the Stieltjes representation to the case of quasilinear problem formulated in a domain filled by two materials occupying subregoins with rough boundaries.

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#### **MS28**

### Band Structure Calculation of Dispersive Photonic Crystals

We propose a new method for band structure calculation of photonic crystals. It can treat arbitrarily frequencydependent, lossy or lossless materials. The band structure problem is first formulated as the eigenvalue problem of an operator function. Finite elements are then used for discretization. Finally, the spectral indicator method is employed to compute the eigenvalues. Numerical examples in both TE and TM cases are presented to show the effectiveness. There exist very few examples in literature for the TM case, and the examples in this paper can serve as benchmarks.

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#### MS29

#### Semicontinuous Comparison of Viscosity Solutions for a Class of Second Order PDEs on the Wasserstein Space

We prove a comparison result for viscosity solutions of second order parabolic partial differential equations in the Wasserstein space. The comparison is valid for semisolutions that are semicontinuous in the measure. The class of equations we consider is motivated by Mckean-Vlasov control problems with common noise, filtering problems, and prediction problems with expert advice. Joint work with Erhan Bayraktar, Xihao He and Xin Zhang.

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#### MS29

### Mean Field Games with Common Noise Via Malliavin Calculus

We study a new approach for McKean-Vlasov control in the probabilistic weak formulation. We characterize optimal controls by establishing a Pontryagin maximum principle. Our results relate the control problem to a generalized McKean-Vlasov backward stochastic differential equation rather than the usual forward backward system obtained in the strong formulation. The new point of view is particularly relevant when regularity is lacking in the problem. We illustrate this by considering conditioned interaction. That is, when the particles interact conditionally on remaining in a given domain similar to the problem introduced by Lions in his lectures at the Collge de France in November 2016. We further provide two applications. First, we present a version of the Schrdinger problem that allows for particles to leave a domain and show how to solve it using

a penalization scheme. Second, we shed new light on the link between McKean-Vlasov control and potential mean field games. Based on join works with Ren Carmona and Kevin Zhang.

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### **MS29**

### Convergence Rate of Particle System for Second-Order PDEs on Wasserstein Space

In this talk, we provide a convergence rate for particle approximations of a class of second-order PDEs on Wasserstein space. We show that, up to some error term, the infinite-dimensional inf(sup)-convolution of the finite-dimensional value function yields a super (sub)-viscosity solution to the PDEs on Wasserstein space. Hence, we obtain a convergence rate using a comparison principle of such PDEs on Wasserstein space. Our argument is purely analytic and relies on the regularity of value functions established in [Daudin, Jackson, Seeger, 23]. This is based on the joint work with Erhan Bayraktar and Ibrahim Ekren.

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#### **MS30**

#### On the Kinetic Vicsek Model

In this talk, I will present some results on the kinetic Vicsek model. The mixing and enhanced dissipation phenomena stabilize the dynamics within specific parameter regimes and ensure effective communication among agents. Consequently, the solution exhibits features similar to those of a spatially-homogeneous system. As a result, we confirm the phase transition observed in the agent-based Vicsek model at the kinetic level. This is a joint work with Mengyang Gu.

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#### MS30

# The Vanishing Viscosity Limit with Anisotropic Viscosity

We study the vanishing viscosity limit in bounded domains for special classes of flows with symmetry when the viscosity coefficient is anisotropic, that is, there is different viscosity in the tangential and normal directions to the boundary. We establish the zero-viscosity limit using correctors under suitable relative size conditions on the normal and tangential viscosities. This is a preliminary report.

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#### MS30

## A General Framework for Convergence of Statisti-

#### cal Solutions and Applications

A trajectory statistical solution of a given evolution equation is a probability measure on the space of trajectories whose support is contained in the corresponding set of solutions, in addition to obeying a suitable regularity condition. I will present a general result yielding a set of verifiable assumptions that yield convergence of statistical solutions of a given parametrized family of evolution equations towards a statistical solution of a limiting system. As illustration, I will apply this result to the inviscid limit of the Navier-Stokes equations towards the Euler equations, and to Galerkin approximations of the Navier-Stokes equations.

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## MS31

### Wellposedness Analysis in Multiscale Interface Couplings of Poroelasticity and Lumped Hydraulic Systems

In biomechanics, local phenomena, such as tissue or organ perfusion, are strictly related to the global features of the whole blood circulation. We propose a heterogeneous model where a local, accurate, 3D description of tissue perfusion by means of poroelastic equations is coupled with a systemic 0D lumped model of the remainder of the circulation. This represents a multiscale strategy, which couples an initial boundary value problem to be used in a specific tissue region with an initial value problem in the rest of the circulatory system. We discuss new results related to the well-posedness analysis for this multiscale model.

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### MS31

# Existence of Weak Solutions to Navier Stokes-Plate Interaction PDE Dynamics

We consider a Navier Stokes fluid-plate interaction (FSI) system which describes the evolutions of the fluid contained within a 3D cavity, as it interacts with a deformable elastic membrane on the free upper boundary of the cavity. We analyze the wellposedness of weak solutions to the stationary coupled PDE system by way of invoking the nonlinear generalization of the abstract variational formulations, wherein an inf-sup approach is followed to show existence-uniqueness of solutions under a small data assumption.

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### MS31

# On Rotational-Form Equations: Global Well-Posedness and a Geometric Numerical Algorithm

A new 3D equation is discussed, which is formed like Burgers' equation by starting with the 3D incompressible Navier-Stokes equations (NSE) and eliminating the pressure and the divergence-free constraint, but instead the Bernoulli pressure is eliminated, leaving only the rotational form of the nonlinearity. In the viscous case, we prove the

global existence, uniqueness, and higher-order regularity of solutions to this equation with no restriction on the initial data other than smoothness. The inviscid case is more challenging than the usual 3D Burgers equation as it has unavoidable derivative loss, but we prove local existence via Nash-Moser iteration and give an example of a class of solutions with smooth initial data that develop a singularity in finite time. Moreover, a new numerical algorithm is presented in the 2D case, and simulations will be shown to illustrate the dynamics. In addition, a rotational-form modification for the 2D and 3D Kuramoto-Sivashinsky equations (KSE) is proposed, and global well-posedness is also established. Global well-posedness for the original 3D NSE and 2D KSE remains a challenging open problem, but it is hoped that by focusing on the rotational term, new insight may be gained.

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#### **MS31**

# Hyperbolic Navier-Stokes and Hyperbolic MHD Equations

The hyperbolic Navier-Stokes equations include an additional second-order time derivative term in the velocity equation, while the hyperbolic magnetohydrodynamics (MHD) equations differ from the standard MHD system by a similar term in the magnetic field equation. These terms are not merely mathematical artifacts. They emerge naturally from fundamental physical laws. Mathematically, the global regularity of solutions to these hyperbolic systems remains a significant open problem. Even for the two-dimensional case, global bounds on the  $L^2$ -norm of solutions are unknown in general. This talk will present recent progress on regularity and convergence issues, as well as the construction of non-unique weak solutions.

 $\begin{array}{c} {\rm Jiahong~Wu} \\ {\rm \overline{University~of~Notre~Dame}} \\ {\rm jwu29@nd.edu} \end{array}$ 

### MS32

# Properties of Discretely Self-Similar Solutions to the Navier-Stokes Equations

This talk primarily reports asymptotic properties of discretely self-similar solutions to the 3D Navier-Stokes equations. These are solutions that are scaling invariant for at least some scaling factors but perhaps not all. This property leads to a time-periodic quality. Due to their exact scaling, they are a good test case for properties of solutions in more general classes. Following this theme, I will discuss an asymptotic expansion in the time variable near the initial time for these solutions. I will also explore an application to the quantification of the severity of hypothetical non-uniqueness and generalizations to other classes of solutions.

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### MS32

Periodic Solutions for Coupled, Partially-

#### Dissipative Systems

Time-periodicity arises in multi-physics systems as (i) responses to forcing (blood flow in human tissues) or (ii) as self-excitations (aeroelastic flutter). We consider an idealized fluid-structure interaction: a heat-wave system coupled across a lower-dimensional interface, with a timeperiodic external force. For undamped systems, resonance can preclude periodicity. For parabolic systems, the existence and uniqueness theory of periodic solutions is wellunderstood. Yet in the heat-wave case, the system is partially damped, leading to an indeterminate case. As the unforced system is not uniformly stable, periodicity is not characterized by the abstract theory of dissipative systems. We provide a construction of unique periodic solutions for this heat-wave system. Wave a priori estimates must be reconstructed through the interface from the heat dissipation via boundary control estimates, introducing geometric constraints into the problem. Thus, for certain classes of domains, unique periodic solutions are obtained from temporally smooth forcing, eliminating the possibility of resonance. We call attention to the regularity loss in the dynamics, and it remains open whether the geometric and regularity constraints are technical, or necessary to circumvent resonance.

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# MS32

# The Variational Structure and Time-periodic Solutions for Mean-field Games Systems

The Lasry-Lions monotonicity condition is crucial for establishing generic (long-time) well-posedness of mean-field games (MFG) systems, as well as stability with respect to long-time-horizon limits (turnpike property). In this work, we show that this monotonicity is, in fact, necessary by providing an example of an MFG system with non-monotone coupling that admits a time-periodic solution and thus does not possess the turnpike property. An essential property that yields the periodicity is the variational structure of the MFG system, which is interesting on its own right.

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#### MS32

# Systematic Search for Singularities in Periodic 3D Euler Flows

It remains one of the central questions in mathematical fluid mechanics whether solutions of the three-dimensional incompressible Euler equations can develop finite-time singularities from smooth initial conditions, i.e., whether certain norms of the solutions blow up in finite time. In this talk, I will present a numerical approach to this problem where we develop a PDE optimization method to systematically search for initial data that may lead to a potential singularity. The behavior of the obtained extreme flow, which features two colliding distorted vortex rings, suggests a finite-time singularity formation. This is based on a joint work with Bartosz Protas.

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#### MS33

# Guaranteeing Higher Order Convergence for Accelerated Minimizing Movement Schemes in Optimal Transport

The minimizing movement scheme provides a variational generalization of the implicit Euler method for gradient flows in metric spaces, and in particular for Wasserstein gradient flows arising in Optimal Transport. Ambrosio, Gigli, and Savar established that, under convexity assumptions, the minimizing movement scheme converges with the optimal linear rate. Several accelerated minimizing movement schemes have been proposed in the literature, but so far they have only been able to rigorously guarantee, at best, sublinear convergence. In this talk, I will present a new accelerated second-order minimizing movement scheme on the Wasserstein-2 space that leverages its differential structure. For sufficiently smooth energy functionals, we prove that this scheme achieves the optimal quadratic convergence rate. Under weaker assumptions, namely continuity and displacement convexity, we still establish rigorous linear convergence rates. Beyond rates, the scheme enjoys numerical stability properties: the energy is nearly monotone along the discrete flow, and when the energy is L-smooth and  $\lambda$ -convex with  $\lambda > 0$ , the Wasserstein gradient norm of the energy functional along the discrete solution decays exponentially in time with an asymptotically optimal rate. Finally, I will outline how these ideas extend toward the systematic design of general higher-order minimizing movement schemes. This is joint work with Matt Jacobs.

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#### MS33

# Michaelis-Menten Kinetics for Enzyme Reactions in the Presence of Diffusion

Michaelis-Menten kinetic is frequently used when modelling enzyme-, or more generally catalytic-, reactions. In the case of homogeneous medium, i.e. the (bio-)chemical concentrations depend solely on time, both formal and rigorous derivations of MM from mass action kinetic have been studied extensively and thoroughly in the last decades. For heterogeneous medium, the modelling should take into account the diffusion of substances, which leads to a system of partial differential equations. In this case, interestingly, only formal derivation of MM from mass action kinetic has been investigated. In this talk, we present the derivation of MM in the presence of diffusion. The proof utilises an improved duality technique and a modified energy method. This is based on a joint work with Bao-Ngoc Tran (University of Graz).

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#### **MS33**

### Mean Field Limit for Congestion Dynamics in 1D

In this talk, I will present recent joint work with Inwon Kim and Antoine Mellet in which we derive a model for congested transport (a PDE at a macroscopic scale) from particle dynamics (a system of ODEs at the microscopic scale). Such PDEs appear very naturally in the description of crowd motion, tumour growth, and general aggregation phenomena. We begin with a system where the particle trajectories evolve according to a gradient flow constrained to some finite distance of separation from each other. This constraint leads to a Lagrange multiplier which, in the mean field limit (infinite number of particles), generates a pressure variable to enforce the hard-congestion constraint. Our results are confined to one spatial dimension wherein we rely on both the Eulerian and Lagrangian perspectives for the continuum limit.

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#### **MS34**

# Dense Point Spectrum for Honeycomb Media with Irrational Edges

We study edge states along a straight interface between two distinct continuum honeycomb structures in  $\mathbb{R}^2$ . For interfaces with rational slopes, the perturbed medium remains translation-invariant along the interface. In this case, prior work has shown the existence of edge states. For irrational slopes, however, the lack of translation invariance along the interface makes the very definition of edge states nontriv-

ial. Our approach relies on the observation that the overall structure can be viewed as a slice of a 3D structure which is translation-invariant in the direction of the interface. This property allows us to reframe our problem in a higher-dimensional framework, defining edge states as restrictions of solutions to an augmented 3D degenerate Schrdinger eigenvalue problem. These solutions propagate along the interface and decay in the transverse direction. Using this framework, we construct exact edge states whose behavior is governed by effective 1D Dirac operators. Under appropriate spectral and Diophantine conditions, we prove that irrational interfaces lead to a dense set of eigenvalues within the bulk spectral gaps. This is in sharp contrast with rational interfaces.

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#### **MS34**

# Scattering Theory for Models of Topological Insulators

An edge transport robust to perturbations is guaranteed along interfaces separating two-dimensional topological insulators in different phases. It is characterized by a topological invariant that is often computed using bulk information via a bulk-edge correspondence (BEC). The invariant may also be computed by means of spectral flows whenever the latter is available. In such a setting, we present a scattering theory that allows for a quantitative description of the asymmetric edge transport in the presence of perturbations.

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# **MS34**

# Semiclassical Localization with Applications to TMDs

We discuss localization in random and quasi-periodic Schroedinger operators and how this is related to the small-angle limit of twisted TMDs. In the case of disorder, our study is motivated by random displacements/strain, the quasi-periodicity arises when periodic Schroedinger operators are exposed to perpendicular magnetic fields.

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### **MS34**

# Bulk-Edge Correspondence for $\mathbb{Z}_2$ Topological Insulators with Curved Interfaces

Topological insulators are insulating phases of matter distinguished by non-trivial topological invariants. A striking feature arises when two insulators with distinct invariants are joined: conducting states emerge along their interface, a phenomenon known as the bulk-edge correspondence (BEC). In this talk, we investigate qualitative

BEC for Fermionic time-reversal-invariant topological insulators, characterized by the Z2-valued Fu–Kane–Mele index, with particular attention to curved interfaces. We will begin with Chern insulators, classified by their Chern numbers, and then extend the discussion to Z2 topological insulators. In particular, we will highlight how the seemingly contradictory properties of translation invariance and local determinacy in Z2 insulators naturally arise from different formulations of the index.

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#### MS35

# On the Large Data Limit of T-Sne Visualizations

We present new work on the continuum limit of a graphbased data visualization technique called the t-Stochastic Distributed Neighbor Embedding (t-SNE), which is widely used for visualizing data in machine learning and the natural sciences. The objective function has two competing terms, which can be interpreted as attraction and repulsion between pairs of particles, and balancing these terms leads to effective visualization. In this work we present some results on a continuum limit variational problem that describes the large data behavior of t-SNE. While the variational problem is highly non-convex in the gradient, the one dimensional version of the problem admits a unique Lipschitz minimizer, and numerical experiments confirm that t-SNE adheres to the smooth solution. We will sketch the arguments in the one dimensional setting and describe a number of open problems.

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### MS35

# Minimax Rates for the Estimation of Eigenpairs of Weighted Laplace-Beltrami Operators on Manifolds

In this talk, we discuss the problem of estimating eigenpairs of elliptic differential operators from samples of a distribution supported on a manifold. The operators that we consider are relevant in unsupervised learning and in particular are obtained by taking suitable scaling limits of widely used graph Laplacians over data clouds. We study the minimax risk for this eigenpair estimation problem and explore the rates of approximation that can be achieved by commonly used graph Laplacians built from random data. Our work showcases novel connections between PDE theory and statistics and utilizes recent advances in quantitative homogenization of partial differential equations on random media, here in the setting of random geometric graphs. It also sets the stage for the analysis of other important estimation problems that we believe are of relevance to the modern literature of operator learning and inverse problems.

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#### MS35

### A Variational Approach to Nonlinear Dimension Reduction in Data Science

Dimension reduction algorithms, such as principal component analysis, multidimensional scaling, and stochastic neighbor embeddings, are a widely used tool for data exploration, visualization, and subgroup identification. These algorithms are typically posed as an optimization problem over a finite family of particles involving attraction and repulsion energies. While these algorithms see broad application across many scientific fields, our theoretical understanding of non-linear dimension reduction algorithms remains limited. This talk will describe new results that identify properties of minimizers of the multidimensional scaling problem using tools from the Calculus of Variations. This problem contains many analogies with optimal transport (OT), and can been seen as a quadratic programming variant of the linear programming formulation of OT. We'll discuss results which prove existence and piecewise smoothness of the solutions to this problem. Along the way, we will showcase situations where standard libraries give outputs that are misleading and provably suboptimal, and propose new computational algorithms to mitigate these issues and improve efficiency. Connections with the celebrated Gromov-Wasserstein distance will also be highlighted.

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#### MS35

# Higher-Order Hypergraph Learning

Hypergraphs provide a natural framework for modeling higher-order interactions, yet their theoretical underpinnings in semi-supervised learning remain limited. We provide an asymptotic consistency analysis of variational learning on random geometric hypergraphs, showing convergence to a weighted p-Laplacian equation. Motivated by this, we propose Higher-Order Hypergraph Learning (HOHL), which regularizes via powers of Laplacians from skeleton graphs for multiscale smoothness. HOHL converges to a higher-order Sobolev seminorm and can be efficiently applied to categorical hypergraph-structured data. Empirically, it outperforms standard graph and hypergraph baselines, especially in low-label and geometry-free settings.

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### **MS36**

# Enhancing the Inverse Born Approximation using Neural Networks

We consider time-harmonic acoustic scattering from a bounded penetrable obstacle in free space. In particular we wish to reconstruct the refractive index of the scatterer from far field measurements of the scattered field due to incident plane waves. When the scattering is weak, the Born approximation provides a linearized model for recovering the desired refractive index. We develop two neural network algorithms—Born-CNN (BCNN) and CNN-Born (CNNB)—to correct the Born approximation when the scat-

tering is not weak. BCNN applies a post-correction to the Born reconstruction, while CNNB pre-corrects the data. Both methods leverage the Born approximation's excellent fidelity in weak scattering, while extending its applicability beyond its theoretical limits. CNNB particularly exhibits a strong generalization to noisy and absorbing scatterers. This paper reports on [1]. [1] A. Desai, T. Lahivaara and P. Monk, A Neural Network Enhanced Born Approximation for Inverse Scattering, https://arxiv.org/abs/2503.01596

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#### **MS36**

# Scattering by Periodic Structures and Related Spectral Problems

We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients in the Helmholtz equation, and we describe the boundary corrections. The coefficients are assumed to be periodic functions of the fast variable, specified over the unit cell with characteristic size  $\epsilon$ . When the periodicity is in the squared index of refraction, we obtain improved convergence results that assume lower regularity than when the periodicity is in the second-order operator. In this case we describe the asymptotic behavior of boundary correctors for general domains at all orders. In particular we show that, in contrast to Dirichlet problems, the  $O(\epsilon)$  boundary corrector is nontrivial and can be observed in the far field. We further demonstrate the latter far-field effect is larger than that of the bulk corrector the so-called periodic drift, which is found to emerge only at  $O(\epsilon^2)$ . We also show how this expansion can be used to invert for the periodic structure. We illustrate the analysis by examples in one and two spatial dimensions. We describe how these expansions are related to the perturbation of the resonances and transmission eigenvalues of such a scatterer.

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### **MS37**

On the Duality Between Temporally Multi-

### Stepped Media and Spatially Multi-Layered Media

Graded metamaterials are commonly characterized as either spatially or temporally varying media, where a spatiotemporally varying medium is considered a combination of them. In recent years, the scientific community has paid much attention to spatially and temporally graded metamaterials, deriving analytical solutions and developing numerical algorithms to analyze these structures. Spatially varying metamaterials are mostly studied using the timeharmonic approach, where the wave propagation is spatially described. A spatial multi-layered metamaterial is represented by possessing material parameters that alternate between two values over space. As an analogous counterpart, an equivalent space-harmonic method has been developed to describe wave propagation in time for temporally varying metamaterials. This involves employing a temporal effective medium approach, which effectively represents a temporal multi-stepped metamaterial by alternating the material parameters of the medium between two values over time. Exact analytical solutions for the fields have been obtained using a spatially and temporally periodic metamaterial, demonstrating the duality between temporal multi-stepped media and spatially multi-layered media for different medium profiles, both with and without impedance-matching. The wave equations have been generalized for graded and periodic material functions, and the field solutions lead to special functions such as Mathieu and hypergeometric functions.

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# **MS37**

# The Bi-Anisotropic Maxwell System on Anisotropic Fractal Structures

In this talk, I will discuss a mathematical framework to study differential equations on anisotropic fractal solid structures. These are fractal structures that could have different dimensions in each direction and appear, for instance, in the modeling of composite structures with fractal type microstructures. We consider the time-dependent, bi-anisotropic Maxwell system on such fractal structures. Well-posedness of the Maxwell system is shown in this framework under appropriate conditions on the material parameters.

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### MS38

## Spot Patterns in Single and Multiple Species Chemotaxis Models with Logistic Growth

The KellerSegel models, a class of strongly coupled PDEs, were introduced by E. Keller and L. Segel in the 1970s to describe cell motility driven by chemical signals. Due to their relatively simple structures yet rich dynamical behaviors, KellerSegel systems have attracted extensive attention, with numerous studies devoted to the qualitative properties of the solutions, including global well-posedness, singularity formation, etc. This talk focuses on the localized pattern formation in single-species and multi-species Keller-Segel models with logistic growth, where the results

concerning the existence and stability of multi-spikes will be discussed. In particular, we shall introduce the application of logistic Keller-Segel models to explain economic agglomeration.

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#### MS38

# Fokker-Planck Equations for the Prediction of Disease Outbreak Severity under Stochastic Disease Transmission

In compartmental models of epidemiology, stochastic fluctuations are often considered in parameters such as the contact rate to account for uncertainties originating from environmental factors, variability in human behavior patterns, or changes in the pathogen itself. The usual choice for modeling stochastic fluctuations is white noise; however, white noise cannot incorporate the correlations arising in human social behavior. The mean reverting OrnsteinUhlenbeck (OU) process is a more adequate model for the stochastic contact rate that includes correlations in time. For the Susceptible-Infected-Susceptible (SIS) model with randomly fluctuating contact rate, we derive its stationary probability density analytically, for both white and correlated OU noise: employing the classical Fokker-Planck equation in the case of white noise, and by introducing a novel nonlinear Fokker-Planck equation in the case of OU noise. This allows us to give a complete description of the models predictions for disease outbreak severity as a function of its bifurcation parameters, i.e., the basic reproduction number, noise intensity, and correlation time. We observe that the SIS model with white noise uncertainties undergoes transitions that lead to the systematic underestimation of the spread of the disease. In contrast, modeling the contact rate with the OU process significantly hinders such unrealistic noise-induced transitions.

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# MS39

# Thermodynamically Consistent Models for Non-Isothermal Magnetoviscoelastic Fluids

In this talk, we consider the motion of a magnetoviscoelastic fluid in a nonisothermal environment. When the deformation tensor field is governed by a regularized transport equation, the motion of the fluid can be described by a quasilinear parabolic system. We will establish the local existence and uniqueness of a strong solution. Then it will be shown that a solution initially close to a constant equilibrium exists globally and converges to a (possibly different) constant equilibrium. Further, we will show that that every solution that is eventually bounded in the topology of the natural state space exists globally and converges to the set of equilibria. If time permits, we will discuss some recent advancements regarding the scenario where the deformation tensor is modeled by a transport equation. In particular, we will discuss the local existence and uniqueness of a strong solution as well as global existence for small initial data.

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Gieri Simonett Vanderbilt University gieri.simonett@vanderbilt.edu

### **MS39**

# On the Global Stability for the MHD Equations with Partial Dissipation

In this talk, I will talk about some recent well-posedness and stability results for two incompressible fluid equations. More precisely, I will first discuss a global stability result for the 3D Navier-Stokes equations. When the Navier-Stokes is coupled with the magnetic field in the magnetohydrodynamics (MHD) system, solutions near a background magnetic field are shown to be always global in time. The magnetic field stabilizes the fluid. If time permits, I will discuss some open problems. This is based on joint works with Bradshaw, Feng, and Wu.

Weinan Wang University of Oklahoma ww@ou.edu

#### **MS40**

### Analysis of a Structural Acoustics Model

In this talk, I will discuss a structural acoustics model consisting of a semilinear wave equation defined on a 3D bounded domain, coupled with a Berger plate equation acting on an elastic wall a flat portion of the boundary. The system is influenced by competing forces, including boundary and interior source and damping terms. We consider nonlinearities of polynomial, logarithmic, and exponential types. I will present our results on the well-posedness of weak solutions, the large-time behavior, and blow-up phenomena, under different assumptions on damping and source terms. The central challenge arises from the coupling between the wave and plate dynamics on the elastic wall.

Yanqiu Guo Florida International University yanguo@fiu.edu

#### **MS40**

# Euler Equations in An Elastic Domain

We address the compressible Euler equations in a domain with a free elastic boundary, evolving according to a damped fourth-order plate equation forced by the fluid pressure. We establish a priori estimates on local-in-time solutions in low regularity Sobolev spaces, namely with velocity and density initial data in  $H^3$ . The results are joint with S. Necasova and A. Tuffaha.

Igor Kukavica University of Southern California kukavica@usc.edu

### **MS40**

# Uniqueness Challenges in Poro-Elasticity

Uniqueness of weak solutions to hyperbolic-parabolic PDE systems are addressed, motivated by a linear poro-elastic

filtration system (3D system of elasticity coupled to a 3D free flow). As is typical, weak solutions do not reside in the test class, thus additional arguments are needed for well-posedness. The challenge to obtain an energy estimate for weak solutions comes comes through interface coupling, bringing about regularity loss and ill-defined interface traces. To obtain energy estimates for arbitrary weak solutions—and thus continuous dependence and uniqueness—we present two approaches, encompassing several parameter regimes for a Biot-Stokes model. For a degenerate case, we utilize "hyperbolic regularization", adapting a classical wave argument in more general spaces. In the non-degenerate (semigroup) cases, we connect finite energy solutions to a particular adjoint notion of weak solutions. The latter are unique whenever there is an underlying semigroup, so we obtain uniqueness in the former sense through identification. Time-permitting, we discuss recent results (complementing classical ones) which provide a more general tool for obtaining weak well-posedness.

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### **MS40**

# Recent Developments on Global Well-posedness and Non-uniqueness of Singular Stochastic PDEs

Singular stochastic PDEs refer to PDEs forced by random noise that are so rough so that the nonlinear terms become ill-defined as products. Global solution theory for such PDEs remain a very challenging research direction while the technique of convex integration has demonstrated nonuniqueness in some cases. We review such developments.

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#### **MS42**

# Singular Limits of Temperature Driven Flows of Compressible Fluids

We consider several singular limits for temperature driven compressible fluid motion. In contrast with the conventionally used Oberebeck-Boussinesq system we show that: 1. In the case of mild stratification, the limit system contains a nonlocal boundary term 2. In the case of strong stratification, the limit system is Majda's pancake problem instead of commonly used anelastic approximation 3. In the case of rotating fluids, the limit fluid motion is purely horizontal in contrast with the Oberbeck-Boussinesq system driven by centrifugal force used in applications

#### Eduard Feireisl

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#### MS42

Uniform Error Estimates for Mixed Finite Element Approximations of Low-Mach Large Friction Flows

#### on Networks

We discuss a fully discrete mixed finite element method for the simulation of barotropic flow of gas in long pipes and pipe networks that is based on a Hamiltonian reformulation of the governing system. Assuming the existence of a smooth subsonic solution bounded away from vacuum, we present a full convergence analysis based on relative energy estimates. Particular attention is paid to establishing error bounds that are uniform in the friction parameter. As a consequence, the method and results also cover the parabolic problem arising in the asymptotic low Mach and large friction limit. The error estimates work analogously for single pipes and pipe networks alike, provided appropriate coupling conditions (ensuring energy conservation at the junctions) are imposed.

### Jan Giesselmann

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Herbert Egger Johannes Kepler University Linz Institute for Computational Mathematics herbert.egger@jku.at

### **MS42**

# Uncertainty Quantification for Low Mach Number Flows

In this talk we review our recent results on random weakly compressible cloud models in atmospheric science [1,2]. We will derive a higher-order finite volume method that is asymptotic-preserving for low Mach number flows. To approximate uncertainty, we apply the stochastic Galerkin method. Numerical experiments demonstrate its efficiency compared to the Monte Carlo method often used in meteorological applications [3]. We will also present our results on rigorous convergence and error analysis for the Monte Carlo finite volume method applied to compressible random Euler and Navier-Stokes equations [4]. [1] Chertock, A., Kurganov, A., Lukáčová-Medvidová, M., Spichtinger, P., Wiebe, B. Stochastic Galerkin method for cloud simulation. Math. Clim. Weather Forecast. 5 (2019), no. 1, 65106. [2] Chertock, A., Kurganov, A., Lukáčová-Medvidová, M., Spichtinger, P., Wiebe, B. Stochastic Galerkin method for cloud simulation. Part II: A fully random Navier-Stokes-cloud model. J. Comput. Phys. 479 (2023), Paper No. 111987, 24 pp. [3] Janjic, T., Lukáčová-Medvidová, M., Ruckstuhl, Y., Wiebe, B. Comparison of uncertainty quantification methods for cloud simulation, Q. J. R. Meteorol. Soc. (2023) 149:28952910 [4] Feireisl, E., Lukáčová-Medvidová, M., She, B., Yuan, Y. Convergence of numerical methods for the NavierStokesFourier system driven by uncertain initial/boundary data, Found Comput Math (2024).

Maria Lukacova Johannes Gutenberg University Institute of Mathematics lukacova@uni-mainz.de

#### **MS42**

# Diffusive Equations As High-Friction Limits of Compressible Gas Dynamics

Several recent studies considered the high-friction limit for systems arising in fluid mechanics. Following this approach, we rigorously derive the nonlocal Cahn-Hilliard equation as a limit of the nonlocal Euler-Korteweg equation using the relative entropy method. Applying the recent result about relations between non-local and local Cahn-Hilliard, we also derive rigorously the large-friction nonlocal-to-local limit. The result is formulated for dissipative measure-valued solutions of the nonlocal Euler-Korteweg equation which are known to exist on arbitrary intervals of time. This approach provides a new method to derive equations not enjoying classical solutions via relative entropy method by introducing the nonlocal effect in the fluid equation. During the talk I will also discuss the high-friction limit of the Euler-Poisson system.

Agnieszka Swierczewska-Gwiazda University of Warsaw aswiercz@mimuw.edu.pl

#### **MS43**

# A Single Player and a Mass of Agents: A Pursuit Evasion-Like Game

We investigate an infinite dimensional partial differential equation of Isaacs type, which arises from a pursuit-evasion game between two populations of agents. The evolution of the two populations is described by a controlled transport/continuity equation, where the control is given by the velocity vector field. Our study is set in the framework of the viscosity solutions theory in Hilbert spaces, and we prove the uniqueness of the value functions as solutions of the Isaacs equation.

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#### MS43

# Finding Optimal Controls Using Diffusion Models

We will leverage the recent advances in diffusion models from machine learning to find optimal controls for classical landing problems. In a nutshell, we propose a Sinkhorn type algorithm to find optimal controls. We show that the algorithm satisfies a large deviation principle, with the associated cost as the rate functional.

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## MS44

# Dirac Operators and Domain Wall Networks in Topological Materials

In this talk, we will present effective Dirac models for topological modes propagating along domain wall in the presence of large gapped domains, such as the AB/BA triangular domains in small-angle twisted bilayer graphene in the presence of a strong potential difference between layers, which has attracted interest in recent years. We will discuss spectral theory aspects as well as efficient numerical discretization strategies for these nonstandard effective Dirac equations, both for the computation of edge invariants as well as the simulation of time-dependent wavepacket propagation on the network formed by the domain walls. We will illustrate numerically the robust behavior in the presence of strong disorder, curved and/or intersecting domain

walls.

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#### MS44

# Discrete Breathers in a Honeycomb Lattice Near a Semi-Dirac Point

Discrete breathers are spatially localized and time-periodic solutions that exist on nonlinear lattices. We develop an effective continuum PDE description of breathers in a 2D dimerized honeycomb lattice near a point in the dispersion landscape known as a semi- Dirac pointvarying linearly in one direction and quadratically in the other. We then analyze the existence of spectrally stable breathers near the so-called anti-continuum limit and, through numerical continuation seeded from exact solutions in this limit, obtain spatially extended breathers that approach gap solitons of the derived PDE approximation.

Andrew Hofstrand New York Institute of Technology ahofstra@nyit.edu

#### MS44

### Nonlinear Homogeneous Compact States on Flat Band Lattices

Periodic lattices exhibiting flat bands have drawn considerable attention in physics and engineering. A remarkable feature of flat band lattices is the existence of compactly supported linear eigenfunctions. In many cases, the flat bands reside at the top of the spectra, and the associated compact states possess uniform amplitudes along their supports (homogeneous). As a result, these compact states persist as solitary wave solutions to the Discrete Nonlinear Schrodinger Equations (DNLS). In this talk, we present a rigorous framework to analyze the spectral and dynamical stability of such nonlinear homogeneous compact states within DNLS models. Our approach combines Sherman-Morrison formula with functional analytic tools. We demonstrate our method through concrete examples, including 1D diamond lattices and 2D Kagome lattices, highlighting the interplay between lattice symmetries, spectral structure, and nonlinear stability.

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### MS45

# Validity of the Nonlinear Schrdinger Approximation for a General Class of Dispersive Quasilinear Systems with Quadratic Nonlinearities

In order to describe the evolution of the envelopes of small oscillating wave packet-like solutions to nonlinear dispersive systems such as the water wave system the Nonlinear Schrdinger (NLS) equation can be derived as a formal approximation equation. If the systems are quasilinear, the rigorous justification of the NLS approximation by proving error estimates over a physically relevant timespan is a highly nontrivial problem. In this talk, we present a new and systematic procedure of constructing energies for proving the error estimates of the NLS approximation for a general class of dispersive quasilinear systems with

quadratic nonlinearities by taking advantage of the Hamiltonian structure of the quasilinear terms. This procedure is very useful for the justification of the NLS approximation for the water wave system with and without surface tension in three spatial dimensions.

#### Wolf-Patrick Düll

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### MS45

# Asymmetric Capillarygravity Water Waves in the Steady Periodic Setting

We discuss ongoing work on asymmetric capillarygravity surface waves in the Euler equations. It has been known for a long time that the setting of weak surface tension allows for higher-dimensional bifurcation from still water, giving rise to multimodal waves with more than one crest in a period. These waves have, however, all been symmetric, although numerical calculations indicate the presence of truly asymmetric waves in the steady periodic setting. Recently, Mhlen and Svensson Seth extended earlier bifurcation results for the gravitycapillary Whitham equation, showing that asymmetric solutions exist as natural extensions of bimodal waves. In this work, we investigate the existence of such asymmetric solutions in the Euler equations.

### Mats Ehrnström

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### **MS45**

# Ion-acoustic Wave Dynamics in a Two-Fluid Plasma

We investigate the evolution of ion and electron populations in hot and cold plasmas using a collisionless Euler-Poisson system within the two-fluid framework. By combining analytical phase-space analysis with computational simulations in Python and Wolfram Mathematica, we investigate the generation and propagation of ion-acoustic waves. In the long-wavelength limit, we show that the system reduces to the Korteweg-de Vries (KdV) equation and supports traveling wave solutions via homoclinic orbits at specific wave speeds. Our results and visualizations offer new insights into wave dynamics in plasmas, demonstrating the versatility of the two-fluid model across various temperature regimes.

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### MS45

# The NLS-Benjamin-Ono Model Arising in the Continuum Approximation of the Calogero-Moser-Sutherland Model

A nonlocal derivative NLS (nonlinear Schrodinger) equation describes modulations of waves in a stratified fluid and a continuous limit of the Calogero–Moser–Sutherland system of particles. For the defocusing version of this equation, we prove the linear stability of the nonzero constant background for decaying and periodic perturbations and the nonlinear stability for periodic perturbations. For the focusing version of this equation, we prove the linear and nonlinear stability of the nonzero constant background under some restrictions. For both versions, we characterize the traveling periodic wave solutions by using Hirota's bilinear method, both on the nonzero and zero backgrounds. For each family of traveling periodic waves, we construct families of breathers which describe solitary waves moving across the stable background.

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#### **MS46**

# Analysis of the Volume-preserving MBO Scheme on Graphs

We investigate a graph-based variant of the Merriman-BenceOsher (MBO) scheme that incorporates a volume constraint, offering a computationally efficient approximation to mean curvature flow. This method has shown strong empirical performance in data-driven applications such as clustering and semi-supervised classification. In this work, we focus on the theoretical convergence of the scheme to continuum mean curvature flow, leveraging its interpretation as a gradient flow. Our analysis is carried out in the regime of vanishing time step and increasing data size, providing a rigorous foundation for the schemes use in large-scale graph-based learning problems. This is joint work with Tim Laux.

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### **MS46**

# Momentum-Based Acceleration and the Allen-Cahn Equation on Euclidean Spaces and in Machine Learning

Both gradient flows and momentum methods are first order methods in optimization modeled on Newton's second law: Gradient flows for massless particles under the influence of friction and a potential force, momentum methods for particles with positive inertial mass. In convex optimization, momentum methods generally converge significantly faster than gradient flows and are therefore dubbed 'accelerated' methods. We study momentum methods for the optimization of a phase-field approximation of the perimeter functional. While the gradient flow is the (parabolic) Allen-Cahn equation, the momentum flow is a less well understood damped hyperbolic equation. We study its geometric properties, obtain a singular limit by formal asymp-

totic expansions, and numerically study its solutions and applications to semi-supervised learning.

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#### **MS47**

## On an Inverse Initial Value Problem for Time-Space Fractional Parabolic Equations

Timespace fractional differential equations, which involve derivatives of non-integer order in both time and space, arise in a wide range of applicationsparticularly in modeling diffusion processes with memory and nonlocal spatial effects due to their ability to accurately capture complex transport phenomena. The corresponding inverse problem, which involves reconstructing the initial state of a system from later-time measurements, plays a crucial role in environmental monitoring, biomedical imaging, geophysics, and materials science. However, the inherent ill-posedness of such problems, combined with the complexity of fractional dynamics, presents significant analytical and computational challenges, necessitating the development of robust regularization techniques and numerical algorithms. This talk will present stability estimates, along with the Tikhonov and quasi-reversibility approaches, for a class of timespace fractional parabolic equations. Each method is supported by theoretical error estimates and numerical validation.

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#### **MS47**

## A Factorization Method Approach to the Biharmonic Transmission Problem in Absorbing Media

In this talk, I will discuss an extension of the factorization method to biharmonic transmission scattering in penetrable, absorbing media. The model comes from flexural wave propagation in a thin Kirchhoff-Love plate, described by the two-dimensional biharmonic wave equation in the frequency domain. Using far-field data, we justify the FM and show it provides a clear inside-outside criterion for identifying the scatterer based on the spectral data of the far-field operator. I'll also discuss the Born approximation for weak scatterers in this setting and compare its accuracy against exact far-field data for simple weak scatterers, illustrating both its usefulness and limitations. This is based on joint work with Rafael Ayala and Isaac Harris.

Isaac Harris, <u>General Ozochiawaeze</u> Purdue University harri814@purdue.edu, oozochia@purdue.edu

### **MS47**

From Linear to Nonlinear: Interactions of Topolog-

#### ical Light with Matter

The study of optical topological structures provides fundamental insight into the coupling of lights spatial, spectral, and polarization degrees of freedom in three-dimensional space, revealing conserved quantities and field symmetries analogous to those in condensed-matter and high-energy physics. In this talk, we will discuss how various two- and three-dimensional topological field configurations evolve through interactions with both linear and nonlinear media, uncovering the mechanisms that govern their transformation and stability. In the linear regime, we examine the propagation of optical knots and skyrmion textures through atmospheric turbulence. Experiments and simulations reveal that weak turbulence preserves their knotted topology, while stronger fluctuations induce reconnection events and topology transitions to lower order states. For skyrmion textures, we retrieve their key parameters, including polarity, vorticity, and helicity directly from the Stokes parameters, enabling quantitative identification of their initial topological state. In nonlinear media, we show that intensity-dependent refractive-index variations lead to simultaneous reconnections and untying of knotted vortex lines, while a variational technique applied to Kerr systems captures the generation of new modes due to the nonlinear dynamics. Finally, we present topology-imprinting nonlinear metasurfaces that replicate the same structured wavefront at fundamental and harmonic frequencies. Together, these results reveal the mechanisms governing the stability, transformation, and replication of optical topological fields in complex environments.

<u>Danilo G. Pires</u> Duke University danilo.gomes.pires@duke.edu

### **MS48**

# Regularity Estimates for a Class of Parabolic Equations with Degenerate Coefficients

In this talk, I will present some recent work on global gradient estimates for a class of parabolic equations with degenerate or singular coefficients. Our result is based on a perturbation method and a level set argument. This is a joint work with Tuoc Phan.

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#### **MS48**

# Optimal Control for Anti-Abeta Treatment in Alzheimers Disease Using a Reaction-Diffusion Model

Alzheimers disease (AD) is a progressive neurodegenerative disorder marked by the accumulation of amyloid-beta (A) plaques in the brain. While anti-A therapies aim to slow disease progression, their use can lead to side effects such as amyloid-related imaging abnormalities (ARIA). In this talk, I will present a spatially explicit reaction-diffusion model for A dynamics, combined with an optimal control framework that balances treatment efficacy with the risk of side effects. Using the Finite Element Method and PET scan data from real patients, we compute personalized treatment strategies that achieve lower amyloid burden compared to constant dosing regimens. This approach highlights how mathematical modeling can help refine therapeutic planning in Alzheimers disease by incorporating

both spatial distribution and treatment safety considerations.

### Sun Lee

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#### **MS48**

## Stability for Periodic Solution for a Free-Boundary Vascular Tumor Model

Within a typical tumor model, e = Tdiffusion/Tgrowth, is a small coefficient describing the relationship between two crucial processes: how quickly nutrients reach the tumor and how fast the tumor grows. It is small and therefore reasonable to set it to zero in many papers in the literature. In our work, we shall set e to be non-zero, add the consideration of vascularization and the size dependence permeability, resulting in a special Robin periodic boundary condition. In this case, we shall establish the periodic solutions well-posedness (existence and uniqueness). We shall further show that the periodic solution serves as a global attractor within the class of radially symmetric initial conditions and investigate the linear stability with respect to non-radial symmetric perturbation, where we establish a critical threshold of the tumor aggressiveness constant for which the stability changes.

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### MS48

# Optimal Control of Free Boundary Models for Tumor Growth

In this talk, we will investigate the optimal control of treatment in free boundary PDE models for tumor growth. The optimal control strategy is designed to inhibit tumor growth while minimizing side effects. In order to characterize it, the optimality system is derived, and a necessary condition is obtained. Numerical simulations will be presented to illustrate the theoretical findings and assess the impact of the optimal control strategy on tumor growth dynamics.

Xinyue Zhao

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# MS49

# L2 Stability for Shock Waves in Compressible Fluid

In this talk, I will introduce the recent progress on the L2 theory for compressible fluid. It is well known that solutions for compressible fluid often form singularities, such as shock waves. The stability of solution including shock waves is a very interesting and challenging topic. This talk will focus on the compressible Navier-Stokes equations, Euler equations, and multi-d Burgers' equation.

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#### **MS49**

On the Hydrodynamics Stability of Close-to-

#### Couette Shear Flows in a Channel

In this talk, I will present two threshold theorems for the 2D Navier-Stokes equations posed on the periodic channel, supplemented with Navier/Non-slip boundary conditions. The initial datum is taken to be a suitable perturbation of a shear flow that is close to the Couette flow. For such a datum, we prove nonlinear enhanced dissipation and inviscid damping for the resulting solution. This is joint work with Jacob Bedrossian, Sameer Iyer, and Fei Wang.

Siming He University of South Carolina siming@mailbox.sc.edu

#### **MS49**

# Shear Flow of Nematic Liquid Crystals: Existence, Multiplicity, and Stability of Stationary solutions

We discuss our program toward a comprehensive study for shear flows of nematic liquid crystals via the Ericksen-Leslie model. Focusing on a critical case, we establish a one-to-one correspondence between the set of the stationary solutions with the set of solutions of an algebraic equation. The existence of multiple stationary solutions is established through countably many saddle-node bifurcations at critical shear speeds for the algebraic equation. This motivates us to establish "saddle-node bifurcations for stability of stationary solutions. The talk is based on joint works with K. Huang, J. Jiao, and M. Sofiani.

Weishi Liu University of Kansas wsliu@ku.edu

### MS50

# The Birkhoff-Rott Integral for Non-Decaying, Non-Periodic Vortex Sheets and Water Waves

A classical vortex sheet has vorticity concentrated on a the boundary between two irrotational fluids shearing past each other. The vorticity of the overall fluid, then, is a Dirac mass of the interface multiplied by an amplitude (the vortex sheet strength). To recover velocity from this vorticity the Biot-Savart law is used, and when this is specialized to the interface itself, the Birkhoff-Rott integral is found. The relevant integrals giving the velocity converge when the sheet is asymptotic to the real axis at horizontal infinity and the vortex sheet strength is decaying. The horizontally periodic case can also be treated, by carefully summing over periodic images to again yield convergent velocity integrals but with a different kernel than in the decaying case. In this talk, we give new formulas for the velocity field associated to the vortex sheet and for the Birkhoff-Rott integral which apply in both the decaying and periodic cases, and which further apply to new cases for which the previous integrals would be divergent. Examples of non-periodic, non-decaying cases of interest are non-periodic perturbations of periodic flows, and horizontally quasiperiodic flows.

<u>David Ambrose</u> Drexel University dma68@drexel.edu

### MS50

# Existence of Periodic Hollow Vortices

In this talk, we will present new results on the existence

of periodic configurations of hollow vortices. A hollow vortex is a region of constant pressure bounded by a vortex sheet and suspended inside a fluid, whose flow is governed by the 2D Euler equations. We will show that there exists a one-parameter family of steady, periodic hollow vortices that bifurcates from a periodic point vortex configurations and extends up to the onset of certain singularities, which we also classify. One notable application is the existence of von Krmn hollow vortex streets. The technique involves reformulating the problem using complex analysis tools, including conformal mappings and layer potential representations, then carrying out a global bifurcation argument.

Vasileios Oikonomou, Samuel Walsh University of Missouri oikonomou@missouri.edu, walshsa@missouri.edu

#### **MS50**

# Rigidity of Symmetric Doubly-Periodic Water Waves Near Shear Flows

While there is a vast literature on 2D steady water waves, much less is known for 3D waves. Only more recently 3D waves have been rigorously constructed, typically in the vicinity of either uniform (i.e., irrotational) or Beltrami background flows. This raises naturally the question near which background shear flows one can expect truly 3D waves at all. In this talk, we will focus on symmetric, doubly periodic, capillary-gravity water waves, but our approach is in fact also applicable to other cases like hydroelastic waves. More precisely, we will demonstrate that near unidirectional, non-uniform, non-stagnant shear flows one cannot expect 3D waves at all (which is in stark contrast to 2D). This is joint work with D.S. Seth (Trondheim), K. Varholm (Pittsburgh), and E. Wahln (Lund).

Jörg Weber University of Vienna joerg.weber@univie.ac.at

#### **MS50**

# Stable and Unstable Manifolds for Capillary Gravity Water Waves and a Class of Nonlinear PDEs

Invariant manifold theory is a fundamental tool in the study of local dynamics near invariant structures in smooth evolution systems. It ensures the existence of nonlinearly invariant structures from linear ones. The theory has been well developed for diffeomorphisms, ODEs, semilinear PDEs, and some quasilinear parabolic PDEs. However, it becomes subtle for quasilinear or more nonlinear PDEs due to regularity issues when there is no smoothing effect. In this talk, we consider a class of nonlinear PDEs whose linearizations satisfy certain energy estimates. We prove that the linear exponential dichotomy implies the existence of local stable/unstable manifolds of the equilibria. In particular the result applies to a class of nonlinear Hamiltonian PDEs including the capillary gravity water waves of one or two fluids, quasilinear wave and Schrödinger equations, KdV type equations, etc., for which the linear analysis is also discussed. Basically, for such systems under certain conditions, spectral instability implies the existence of stable and unstable manifolds, which in particular yields the nonlinear instability in rough Sobolev norms and/or the existence of solutions decaying in high Sobolev norms. This is a joint work with Jalal Shatah.

Chongchun Zeng

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#### MS51

## Regularized Fluid Models and Their Structure-Preserving Discretization

Recently, an inviscid regularization of the barotropic Euler equations using techniques from information geometry was proposed. This regularization, based on the interpretation of ideal fluid mechanics as geodesic flow, involves a modification of the flow's metric structure, which smooths shocks in a dissipation-free, non-dispersive manner. As geodesic flows generically possess Hamiltonian structure, we show that the information-geometric regularization of the barotropic Euler equations can be formulated as a Lie-Poisson Hamiltonian system. This clarifies the system's conservation laws, facilitates the design of structurepreserving numerical methods, and aids in deriving extensions of the model. Conservative extensions of the model may be obtained via extensions of the underlying Lie algebra structure, such as a semi-direct sum to add advected quantities. Concretely, regularized ideal MHD and thermal fluids are obtained by adding a magnetic field and entropy, respectively. Dissipative extensions to the model are facilitated by the metriplectic formalism, which builds on the Hamiltonian structure. This allows us to study how dissipative models such as Navier-Stokes-Fourier and resistive MHD interact with the information-geometric regularization. By contextualizing the information-geometric regularization of Euler's equations in the language of geometric mechanics, the regularization technique may be applied to a multitude of fluid and plasma models.

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### MS51

# Hamiltonian Sub-Riemannian Approach to Discretizing Ideal Fluid Simulation

This talk introduces a geometric framework for simulating ideal (incompressible, inviscid) fluids, where the discretization itself defines a non-holonomic constraint distribution on admissible velocities. Inspired by Arnolds interpretation of Euler flows as geodesics on the group of volume-preserving diffeomorphisms, we construct a sub-Riemannian system where trajectories evolve along constrained paths induced by the chosen discrete representation. From this viewpoint, we develop CO-FLIP (Coadjoint Orbit Fluid Implicit Particles), a high-order, structure-preserving method in the hybrid EulerianLagrangian framework. Using a local, divergence-free interpolation, we define a modified Hamiltonian system whose discrete flow respects the geometry of coadjoint orbitscapturing circulation and Casimirs of the continuous theory. Paired with a geometric integrator, the method preserves energy and invariants over long times. While CO-FLIP follows the FLIP algorithm in form, it avoids key pitfalls such as advection projection splitting and instability, and features exact gridparticle transfers. The result is a robust and high-fidelity solver, even at low resolutions.

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### **MS51**

# Preconditioning Transformations of Adjoint Systems for Evolution Equations

In conjunction with gradient-based methods, adjoint systems are widely used in the optimization of systems subject to differential equation constraints. In optimization, gradient-based methods are often transformed using suitable preconditioners to accelerate the convergence of the optimization algorithm. Inspired by preconditioned gradient descent methods, we introduce a framework for the preconditioning of adjoint systems associated to evolution equations to reshape the dynamics of the adjoint system. We develop two classes of adjoint preconditioning transformations: those that transform both the state dynamics and the adjoint equation and those that transform only the adjoint equation while leaving the state dynamics invariant. Both classes of transformations have the flexibility to include generally nonlinear state-dependent transformations. Using techniques from symplectic geometry, we show that these transformations preserve the property that the adjoint system backpropagates the derivative of an objective function. We apply this framework to the setting of coupled evolution equations, where we develop a notion of scale preconditioning of the adjoint equations when the state dynamics exhibit large scale-separation. We demonstrate the proposed scale preconditioning on an inverse problem of a perturbed thick Marshak wave governed by the radiation diffusion equations.

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#### MS51

# Energy-Dissipative and Conservative Discrete Gradient Particle Methods for Aggregation-Diffusion Equations and the Landau Equation

Structure-preserving particle methods have recently been proposed for a class of aggregation-diffusion equations and the Landau equation. Both of such models can be viewed as a class of nonlinear continuity equation with a velocity field depending on the variational derivative of some energy functional. While they can be semi-discretized using particle methods to satisfy energy dissipation and preservation of conserved quantities with suitable regularization, conventional time integrators applied to such semidiscretizations do not preserve these properties in general. In this talk, we introduce a notion of compatibility condition for the regularized energy functional which will enable the variational derivatives to be expressed in terms of the gradient of some particle energy functional. This approach will allow us to make use of discrete gradient integrators and to prove the resulting full-discretization preserves energy dissipation and conserved quantities simultaneously. We demonstrate the dissipative and conservative properties of our method on various numerical example. In addition, we showcase the decay of Fisher information and entropy dissipation rate in the case of the Landau equation with the Coulomb kernel. This is joint work with Jingwei Hu and Samuel Van Fleet [J. Hu, S. Van Fleet, A. T. S. Wan, SIAM J. Sci. Comput., 47(3), 2025].

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#### MS52

# Forced Oscillations in Fluid-Structure Interactions

The flow of a viscous fluid around structures is a fundamental problem that lies at the heart of the broad research area of Fluid-Structure Interactions (FSI). A fundamental aspect of this problem concerns the study of the oscillations (vibrations) induced by the fluid on the structure, when the flow is in a time-periodic regime. An important feature of this interaction is the phenomenon of resonance, which occurs when the frequency of the flow reaches a multiple of one of the natural frequencies of the structure. In fact, it may lead to destructive consequences, as damage or even collapse of the structure. Objective of this talk is to analyze this phenomenon from a rigorous mathematical viewpoint, when the structure is modeled as a spring-mounted rigid body and the fluid is governed by the Navier-Stokes equations.

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### MS52

# The Incompressible Euler- $\alpha$ Equations in the Exterior of a Vanishing Disk

We consider the Euler- $\alpha$  equations in the exterior of a small fixed disk of radius  $\varepsilon$ . We assume that the initial potential vorticity is compactly supported and independent of  $\varepsilon$ , and that the circulation of the unfiltered velocity on the boundary of the disk does not depend on  $\varepsilon$ . We prove that the solution of this problem converges, as  $\varepsilon \to 0$ , to the solution of a modified Euler- $\alpha$  equations in the full plane, where an additional Dirac located at the center of the disk is imposed on the potential vorticity.

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### MS52

# The Linearized Israel-Stewart Equations with a Physical Vacuum Boundary

The Israel-Stewart theory models relativistic viscous fluids, with important applications in astrophysics and cosmology.

In this talk, I will present recent progress on an Israel-Stewart type system with bulk viscosity in the presence of vacuum. By allowing vacuum, we introduce degeneracy near the boundary. In this case, the decay rates of fluid variables play a crucial role in solving the problem. We focus on decay rates that ensure the boundary maintains a finite, nonzero acceleration-a condition we refer to as the physical vacuum boundary condition. This allows us to model physical phenomena such as star rotation. The core of the talk is on establishing the local well-posedness of the linearized system under these vacuum conditions. Classical hyperbolic theory fail due to the degeneracy near the free boundary, so we incorporate weights in our functional framework and derive weighted energy estimates to construct solutions. Finally, we will briefly discuss how the linearized result contributes to solving the fully nonlinear problem.

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#### MS53

# A New All-Mach Asymptotic-Preserving Method Based on a Primitive Formulation of the Compressible Euler Equations

This talk presents a new numerical method for solving the compressible Euler equations in regimes with a wide range of Mach numbers, including the challenging low Mach number limit where the system becomes stiff. In such cases, explicit schemes face severe time-step restrictions, making them inefficient. We propose an asymptotic preserving (AP) scheme that maintains uniform stability and accuracy across all Mach numbers. Unlike classical approaches based on conservative hyperbolic flux splitting, our method reformulates the system in primitive (nonconservative) variables and introduces a nonconservative hyperbolic splitting. The resulting system is discretized semi-implicitly: the stiff part is treated with second-order implicit central differences, while the nonstiff part is evolved using a secondorder explicit path-conservative central-upwind scheme. A key component of the method is solving a Poisson-type equation for the pressure, which ensures the AP property. In parallel, the conservative form is evolved using a semidiscrete central-upwind scheme. At each time step, a postprocessing step selects the appropriate solution depending on the Mach number, ensuring that the nonconservative AP method is used in the low Mach limit, while the conservative scheme captures shocks and discontinuities in the compressible regime. Numerical tests demonstrate the accuracy and robustness of the method.

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#### MS53

### A New Semi-Discrete Finite-Volume Active Flux Method for Hyperbolic Conservation Laws

In this work, we present a novel Active Flux framework for hyperbolic systems of partial differential equations. Given two different formulations of the system under investigation, a conservative and a primitive (nonconservative) one, we discretize them on overlapping staggered meshes. Both conserved and primitive variables are discretized via cell averages over such staggered meshes. In particular, the primitive cell averages are used to reconstruct point values, which are suitably employed for the computation of the needed fluxes for the conservative formulation. In order to prevent convergence to incorrect solutions, typical of methods directly working on primitive formulations, we introduce a novel conservative post-processing that provides a suitable coupling of the two sets of considered variables, guaranteeing conservation and hence a correct handling of discontinuities. The approach is numerically validated on several benchmarks for the one- and two-dimensional Euler equations of gas dynamics.

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### MS54

#### The Fuzzy Landau Equation

The fuzzy Landau equation models interactions between particles that can have different spatial location. In this talk we present the latest results concerning global wellposedness and quantitative properties of the solution.

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### MS54

# Regularity and Existence Results for Some Relativistic Kinetic Equations

This talk will present some recent results on relativistic kinetic equations. Relativistic effects play an important role in models where particle speeds approach the speed of light with nontrivial probability, which happens for example in hot plasmas. The talk will focus on some novel mathematical challenges that arise from relativistic corrections to kinetic PDEs, particularly with regard to regularity and well-posedness theory.

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# **MS54**

### On the Relativistic Landau Equation

In 1936 Landau introduced a modification to the Boltzmann equation that is used to model a dilute hot plasma where charged particles interact via Coulomb interactions. This famous model, known as the Landau equation, does not include the effects of Einsteins theory of special relativity. When particle velocities are close to the speed of

light, which happens frequently in a hot plasma, relativistic effects become important. An equation that captures relativistic effects was introduced in 1956 by Budker and Beliaev and is today referred to as the relativistic Landau equation. This equation has an extremely complex kernel and lacks scaling symmetries, which present major challenges in its analysis. In this talk we will discuss recent results on the relativistic Landau equation and tools that helped us overcome structural difficulties of the relativistic setting. This is based on a joint work with Strain and a joint work with Henderson, Snelson and Tarfulea.

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#### MS55

# Asymptotic Stability of Smooth Solitons for the Camassa-Holm Equation

In this talk, we briefly present a recent result on the asymptotic stability of smooth solitons in the energy space for the CamassaHolm (CH) equation. Specifically, we show that a solution to the CH equation that is initially close to a soliton, after suitable translation, converges weakly in  $H^1$  to (possibly) a different soliton as time tends to infinity. The proof is inspired by the bi-Hamiltonian structure of the CH equation and relies on a Liouville-type theorem for CH dynamics near solitons. A key new ingredient in the proof of the Liouville theorem is the use of completeness relations for the squared eigenfunctions of the CH recursion operator. We will also discuss some applications of this approach to the classification of solutions to linear problems associated with the KdV and mKdV equations. This is a joint work with Y. Lan, Y. Liu and Z. Wang.

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#### MS55

# Sticky Particle Solutions for the Euler Alignment System

We discuss the theory of weak solutions for the Euler Alignment system with bounded or weakly singular communication protocol, in 1 spatial dimension (with remarks on ongoing work in the unidirectional case if time allows). The special structure in 1D allows one to reduce the two equations of the system to consideration of a single scalar balance law, the natural entropy conditions of which are compatible with a discretization of the system via "sticky particle" dynamics. This framework is inspired by work by Brenier and Grenier on the pressureless Euler Equations. The main work discussed is joint with Changhui Tan (Uni-

versity of South Carolina).

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#### MS55

# Existence of Mono-Kinetic Solutions for Euler Alignment Equations

The hydrodynamic description for emergent behavior of interacting agents is governed by Euler alignment equations, driven by different protocols of pairwise communication kernels. We discuss the role of a reversed entropy inequality in selecting mono-kinetic closure for emergence of strong solutions, proving the existence of such solutions subject to sub-critical threshold condition. In this work we settle the open question of existence in dimension D<sub>i</sub>2.

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#### **MS56**

# Modulations of Wave Trains in the Ostrovsky Equation

We study the orbital stability of smooth solitary wave solutions of the Novikov equation, which is a Camassa-Holm type equation with cubic nonlinearities. These solitary waves are shown to exist as a one-parameter family (up to spatial translations) parameterized by their asymptotic endstate, and are encoded as critical points of a particular action functional. As an important step in our analysis, we must study the spectrum of the Hessian of this action functional, which turns out to be a nonlocal integro-differential operator acting on  $L^2$ . We provide a combination of analytical and numerical evidence that the necessary spectral hypotheses always hold for the Novikov equation. Together with a detailed study of the associated Vakhitov-Kolokolov condition, our analy- sis indicates that all smooth solitary wave solutions of the Novikov equation are nonlinearly orbitally stable.

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### **MS56**

# Asymptotic Stability of Smooth Solutions to the Degaperis-Procesi Equation

The Camassa-Holm (CH) equation, originally derived as an asymptotic model in shallow water wave theory, is notable for admitting weak soliton solutions known as peakon-ssolitary waves with a peaked profile and a discontinuous first derivative. Since its discovery, a variety of related peakon equations have been studied, both integrable and non-integrable. Among these, the integrable Degasperis-Procesi (DP) equation stands out. It was identified by applying asymptotic integrability conditions up to third order to a family of nonlinear evolution equations, revealing that only the KdV, CH, and DP equations satisfy these constraints. In this talk, we focus on the asymptotic sta-

bility of smooth one-soliton solutions to the DP equation. The analysis presents several challenges: the associated linearized operator includes a non-local term, and the soliton solutions evolve on a nonzero background. Moreover, the Lax pair for the DP equation involves a third-order non-self-adjoint operator, in contrast to the CH equations second-order self-adjoint formulation. These differences significantly impact the techniques required to obtain the stability results.

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#### MS56

# Multi-Modal Bright and Dark Solitary Wave Solutions to a Fourth-Order Nonlinear Schrödinger Equation

In this talk, we consider the existence and spectral stability of multi-modal solitary wave solutions to a nonlinear Schrdinger equation incorporating both fourth and second-order dispersion terms. In the bright soliton regime, we show that we can construct multi-modal solitary waves by "gluing together" consecutive copies of the primary solitary wave, as long as certain geometric constraints are satisfied. Under additional assumptions, which can be verified numerically, we prove that all such multi-pulses are spectrally unstable. By contrast, numerical results suggest that, in the dark soliton regime, some multi-modal solutions are in fact stable.

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# MS56

# Renormalized Oscillation Theory for Hamiltonian Pencils

Working with a general class of linear Hamiltonian systems with nonlinear dependence on the spectral parameter, we show that renormalized oscillation results can be obtained in a natural way through consideration of the Maslov index associated with appropriately chosen paths of Lagrangian subspaces. By reduction to a generalized nonlinear eigenvalue problem, we apply our results to a class of models such as magneto-hydrodynamics systems and the Saint-Venant equations.

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### MS57

# Semiclassical Commutator Bounds in Hartree Theory

In this talk we present new results regarding the validity of

semi-classical bounds for minimizers of the Hartree functional. These bounds were introduced as key conditions on the initial datum by Benedikter, Porta and Schlein in order to analyze the emergence of the Hartree-Fock equation from the dynamics of large Fermi gases, in a combined mean-field and semi-classical regime. Physically, these bounds record the underlying semi-classical structure of the system. Mathematically, they correspond to regularity estimates which are uniform in the semi-classical parameter. Proving these bounds in practice is, however, highly non-trivial. Our results is the first of its kind for non-linear systems in which particles are allowed to interact. In particular, singular pair potentials up to the Coulomb potential are included. Time permitting, we will also present new results on the application of these bounds to the study of the quantitative convergence of states for N-body Fermi gases. This talk is based on joint work with Laurent Lafleche.

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#### **MS57**

# The Gibbs State of the Mean-Field Bose Gas

We consider the homogeneous mean-field Bose gas at temperatures proportional to the critical temperature of its Bose-Einstein condensation phase transition. We prove a trace norm approximation for the grand canonical Gibbs state in terms of a reference state, which is given by a convex combination of products of coherent states and Gibbs states associated with certain temperature-dependent Bogoliubov Hamiltonians. The convex combination is expressed as an integral over a Gibbs distribution of a onemode F4-theory describing the condensate. This result justifies an analogue of Lee and Yang's extension of Bogoliubov theory to positive temperatures, and it allows us to derive various limiting distributions for the number of particles in the condensate, as well as precise formulas for the one- and two-particle density matrices of the Gibbs state. Key ingredients of our proof, which are of independent interest, include two novel abstract correlation inequalities. The proof of one of them is based on an application of an infinite-dimensional version of Stahl's theorem.

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### **MS57**

# Diffusion of the Lorentz Mirror Walk in High Dimensions

The Lorentz mirror walk is a discrete model of transport in a random environment, in which a particle scatters off of randomly placed mirrors with density p at vertices in  $\mathbb{Z}^d$ . A folklore conjecture is that these trajectories are confined to a cycle in d=2 at any p>0, whereas in  $d\geq 3$  it is expected that most trajectories escape to infinity for

small enough p. In this talk I will overview a proof that these trajectories are diffusive in  $d \ge 4$  for time scales that are superpolynomial in  $p^{-1}$ . The proof combines diffusive estimates for the walk which are valid at large scales with an analysis of the small-scale ballistic behavior of the walk. This is joint work with Dor Elboim and Antoine Gloria.

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#### **MS57**

# On the Onset of Renormalized Quantum Boltzmann Fluctuation Dynamics Near a Bose-Einstein Condensate

Hilbert's 6th problem asks for an axiomatic treatment of physical laws. In particular, the emergence of macroscopic laws, such as fluid or heat equations, from microscopic laws, the classical or quantum Liouville equations, are longstanding open problems in mathematical physics. The classical problem has seen tremendous progress, including the recent long-term derivation by Deng and Hani. However, the quantum case remains largely elusive. I will describe how for short times, quantum Boltzmann corrections emerge when describing a Bose gas at low temperatures. Crucially, we provide rigorous error control. This is based on joint work with Thomas Chen.

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# MS58

# A Regularizing Property of the 2D Eikonal Equation in Connection with the AvilesGiga Functional

The 2D Eikonal equation is closely related to the variational analysis of a classical energy functional, called the Aviles-Giga functional in connection with smectic liquid crystals and thin film blisters. In the variational setting, significant effort has been devoted toward understanding solutions of the 2D Eikonal equation with low fractional Besov regularity. Notably, weak solutions under certain low regularity conditions exhibit automatic regularization. In this talk, I will present a new regularizing effect for weak solutions of the 2D Eikonal equation under a weak fractional Besov regularity. This regularity lies at the borderline between continuity and the presence of vortex singularities. This is joint work with Xavier Lamy and Andrew Lorent.

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### **MS58**

### Regularity of a family quasilinear wave equation

The cusp singularity, with only Hlder continuity, is a typical singularity formed in the quasilinear hyperbolic partial differential equations. We are going to talk about the global existence of Hlder continuous energy conservative weak solution for a family equations including Hunter-Saxton and Camassa-Holm and Novikov equations, where the regularity of solution varies with respect to a parameter. We will also look into generic regularity of such equations. This result can help us predict regularity of cusp singularity for many other models.

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## **MS58**

# Poiseuille Flow of Hyperbolic Ericksen-Leslie System in Dimension Two

In this talk, we study the Poiseuille laminar flow in a tube for the full Ericksen-Leslie system. It is a parabolic-hyperbolic coupled system which may develop singularity in finite time. We will prove the global existence of energy weak solution, and the partial regularity of solution to system.

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# MS59

# Steady Viscous Surface Waves on an Incline

In this talk we consider free-boundary incompressible Navier–Stokes flow down an incline in two dimensions. By reformulating the problem as an elliptic system (in the sense of Agmon–Douglis–Nirenberg), we find a local branch of steady solutions close to laminar flow. We then extend this branch to a global curve of solutions using global bifurcation theory. This is joint work with Miles Wheeler.

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# MS59

## On the Existence and Stability of Large Periodic Traveling Wave Solutions to the Free Boundary Stokes and Navier-Stokes Equations

We study the free boundary problem for a finite-depth layer of viscous incompressible fluid in arbitrary dimension, modeled by the Stokes or Navier-Stokes equations. In addition to the gravitational field acting in the bulk, the free boundary is acted upon by surface tension and an external stress tensor posited to be in traveling wave form. We prove that for any isotropic stress tensor with periodic profile, there exists a locally unique periodic traveling wave solution, which can have large amplitude. Moreover, we prove that the constructed traveling wave solutions are

asymptotically stable for the dynamic Stokes problem.

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### MS59

# Gravity Driven Traveling Bore Wave Solutions to the Free Boundary Incompressible Navier-Stokes Equations

We give the first mathematical construction of twodimensional traveling bore wave solutions to the free boundary incompressible Navier-Stokes equations for a single finite depth layer of constant density fluid. Our construction is based on a rigorous justification of the formal shallow water limit, which postulates that in a certain scaling regime the full free boundary traveling Navier-Stokes system of PDEs reduces to a governing system of ODEs. We find heteroclinic orbits solving these ODEs and, through a delicate fixed point argument employing the Stokes problem in thin domains and a nonautonomous orbital perturbation theory, use these ODE solutions as the germs from which we build bore PDE solutions for sufficiently shallow layers.

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#### MS59

# Traveling Waves for the Viscous Shallow Water System

The viscous shallow water equations can be derived from the free boundary incompressible Navier-Stokes system in the vanishing depth limit. In this talk we will discuss recent work on the construction of traveling wave solutions to this system.

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### MS60

# Data Assimilation in Groundwater Flows and Turbulent Flows

A major difficulty in accurately simulating turbulent flows is the problem of determining the initial state of the flow. For example, weather prediction models typically require the present state of the weather as input. However, the state of the weather is only measured at certain points, such as at the locations of weather stations or weather satellites. Data assimilation eliminates the need for complete knowledge of the initial state. It incorporates incoming data into the equations, driving the simulation to the correct solution. The objective of this talk is to discuss innovative computational and mathematical methods to test, improve, and extend a promising new class of algorithms for

data assimilation in turbulent flows and related systems. We will look at how these techniques can be adapted to yield faster convergence and recover unknown parameters in the context of the Richards equation of groundwater flows, and also in the setting of turbulent flows.

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### MS60

# A Structurally Informed Data Assimilation Approach for Discontinuous State Variables

Data assimilation is a scientific process that combines available observations with numerical simulations to obtain statistically accurate and reliable state representations in dynamical systems. However, it is well known that the commonly used Gaussian distribution assumption introduces biases for state variables that admit discontinuous profiles, which are prevalent in nonlinear partial differential equations. In this talk, we focus on the design of a new structurally informed prior that exploits statistical information from the simulated state variables. In particular, we construct a new weighting matrix based on the second moment of the gradient information of the state variable to replace the prior covariance matrix used for model/data compromise in the data assimilation framework. We further adapt our weighting matrix to include information in discontinuity regions via a clustering technique. Our numerical experiments demonstrate that this new approach yields more accurate estimates than those obtained using ensemble transform Kalman filter (ETKF) on shallow water equations.

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#### **MS60**

### Parameter Reconstruction for Nonlinear Systems

This talk will address recent results in the reconstructability of parameters in nonlinear systems. Several practical approaches are presented, as well as rigorous convergence results associated to them in both finite and infinite-dimensional settings. The results encompass various joint works with Jacob Murri (UCLA), Jared Whitehead (Brigham Young University), Sarah Strikwerda (University of Wisconsin-Madison), and Xiang Wan (Loyola University of Chicago)

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# MS60

# Analysis of Continuous Data Assimilation with Large (or Even Infinite) Nudging Parameters

This talk considers continuous data assimilation (CDA) in partial differential equation (PDE) discretizations where nudging parameters can be taken arbitrarily large. We prove that solutions are long-time optimally accurate for such parameters for the heat and Navier-Stokes equations (using implicit time stepping methods), with error bounds that do not grow as the nudging parameter gets large. Existing theoretical results either prove optimal accuracy but with the error scaled by the nudging parameter, or suboptimal accuracy that is independent of it. The key idea to the improved analysis is to decompose the error based on a weighted inner product that incorporates the (symmetric by construction) nudging term, and prove that the projection error from this weighted inner product is optimal and independent of the nudging parameter. We apply the idea to BDF2 - finite element discretizations of the heat equation and Navier-Stokes equations to show that with CDA, they will admit optimal long-time accurate solutions independent of the nudging parameter, for nudging parameters large enough. Several numerical tests are given for the heat equation, fluid transport equation, Navier-Stokes, and Cahn-Hilliard that illustrate the theory.

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### **MS62**

# Bayesian Inverse Problems on Metric Graphs

A metric graph can be conceptualized as a cobweb of vertices connected by physical wires. Differential equations on metric graphs model diverse phenomena such as traffic flow and wave propagation in thin structures with applications in nanotechnology, mesoscopic systems, and photonic crystals. This talk will discuss the formulation, well-posedness, and numerical solution of Bayesian inverse problems for differential equations on metric graphs. As a prototypical example, we will focus on the inverse problem of recovering the diffusion coefficient of a (fractional) elliptic partial differential equation defined on the metric graph equipped with Kirchhoff vertex conditions from pointwise measurements of the solution.

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# MS62

# X-ray Transforms:Mapping Properties and Bayesian Inversion

Abelian and Non-Abelian X-ray transforms are examples of integral-geometric transforms with applications to X-ray Computerized Tomography and the imaging of magnetic fields inside of materials (Polarimetric Neutron Tomography). Their study uses tools from classical inverse problems (assessments of injectivity, stability and inversions), and mathematical statistics to deal with cases with noisy data. Recent results in nonparametric Bayesian statistics have paved the way towards producing robust statistical

recovery algorithms to deal with noisy data. They are "robust" in the sense that they produce consistent estimators, that further enjoy a Bernstein-VonMises type of theorem. These theoretical guarantees hinge on a refined understanding of the inverse problem at play (stability statements, and new functional contexts where the forward operator is invertible), and this has spurred further progress on accurately addressing the mapping properties of the X-ray transform via the design of non-standard Sobolev scales. I will discuss recent works with R. Nickl (Cambridge) and G.P.Paternain (U Washington), and a collection of further works on the latter topic.

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#### MS62

### A Transport Map Approach for Bayesian Inference of Spatio-Temporal Inverse Problems with Heavy-Tailed Priors

Tackling the immediate challenges that arise from growing model complexities (spatiotemporal measurements) and data-intensive studies (large-scale and high-dimensional measurements collected as time-series), state-of-the-art methods can easily exceed their limits of applicability. Conventional methods based on Gaussian processes (GP) often fall short in providing satisfactory solutions since they tend to offer over-smooth priors. Recently, the Besov process (BP), defined by wavelet expansions with random coefficients, has emerged as a more suitable prior for Bayesian inverse problems of this nature. While BP excels in handling spatial inhomogeneity, it does not automatically incorporate temporal correlation inherited in the dynamically changing objects. In this talk we describe how to generalize BP to a novel spatiotemporal Besov process (STBP) by replacing the random coefficients in the series expansion with stochastic time functions as Q-exponential process which governs the temporal correlation structure. We thoroughly investigate the mathematical and statistical properties of STBP. A transport map representation of STBP is also proposed to facilitate the inference. We demonstrate the utility of our approach on several image reconstruction problems.

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#### **MS62**

# The Measure Evolution of Hyperbolic PDEs with Random Data

We study pointwise statistics in 1D hyperbolic PDEs with random initial datathat is, the distribution of the solution at any given point in space and time. For linear problems, the PDF evolves via the same PDE as the original system. In the nonlinear case, we derive a linear transport equation for the CDF, valid up to shock formation, beyond which a counterexample shows no such equations exist. For sys-

tems, we extend this to multi-point statistics. These equations offer a computationally efficient alternative to Monte Carlo methods and surrogate models. They also yield a priori statistical error bounds for standard Monte Carlo methods.

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#### **MS63**

# Recent Progress in Eigenvalue Problems in Perforated Domain

The eigenvalue problem in a perforated domain was first introduced by J. Rauch in 1974, to study the crashed ice problem, wherein the first eigenvalue of the Laplace problem determines the cooling rate. In this talk, we will discuss the recent progress in the spectral theory in perforated domains, including the optimal error estimates, as well as the convergence rates for corresponding eigenfunctions.

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# MS63

# Active Materials and Reactive Fluids

Active materials or reactive fluids convert and transduce energy from their surrounding into a motion and other mechanical activities. These systems are usually out of mechanical or even thermodynamic equilibrium. In this talk I will present a general theory for active fluids which is the extension of the classical energetic variational approaches for mechanical systems. The methods will cover a wide range of both chemical reaction kinetics and mechanical processes.

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# **MS63**

Analysis of the Incompressible Navier-Stokes Equations (NSE) Coupled with the Maxwell-Stefan Equations (MSE) Through Interface Membrane Conditions

This talk is centered on the well-posedness of the system of steady-state NSE-MSE equations with the interface/boundary conditions arising in engineering applications involving filtering the desired molecules from the fluid mixture by utilizing man-made membrane whose designed

parameters specify the interface conditions. The boundary conditions are of the inflow/outflow types with explicit pressure terms and hence are non-conventional.

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#### **MS64**

# Recent Results on Plasma Instabilities in Weak Turbulence at High Local Energy

The weak turbulence model, also known as the quasilinear theory in plasma physics, has been a cornerstone in modeling resonant particle-wave interactions in plasmas. Stemming from the Vlasov-Poisson/Maxwell system under the weak turbulence assumption, random phase approximation and ergodicity assumptions, yield a non-local integrodifferential system, describing the dynamics of pdfs and plasmons, as justified by Besse and Bardos(2021). Such interactions between can be treated as a stochastic process, whose transition probability bridges the momentum and spectral space, resembling non-linear and non-local collision forms of kinetic type We establish the existence of global weak solutions to the electrostatic model, suitable for electron beams dynamic models. Our challenge lies on solving a nonlinear non-local system for two unknown functions are defined on two different spaces. We establish the existence of global weak and bounded solutions for the system and show the solutions can be bounded under well prepared data. The method introduces an auxiliary function associating the integral-differential system to a single degenerate inhomogeneous porous medium equation with nonlinear source terms, and advanced techniques to construct weak solutions. This approach opens a novel pathway for analyzing weak turbulence models in plasma physics and offers new tools for tackling related problems in the broader context of non-linear non-local PDEs.

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### **MS64**

## The Hydrodynamic Limit for Prandtls Boundary Laver

We justify Prandtl equations and higher order Prandtl expansion from the hydrodynamic limit of the Boltzmann equations. Our fluid data is of the form shear flow, plus small order term in analytic spaces. The novelty lies in obtaining estimates for the linearized Boltzmann equation with a diffusive boundary condition around a Prandtl layer

flow.

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#### **MS64**

# Constructing Finite Time Singularities for Some Nonlinear PDEs

In this talk I will introduce the implosion blow-up results for the compressible Euler, the compressible Navier-Stokes equations and the defocusing nonlinear Schrdinger equation. We will discuss the existence of self-similar solutions and the stability near those solutions. During the talk I will mention our work with Gonzalo Cao-Labora, Javier Gmez-Serrano and Gigliola Staffilani on the first non-radial implosion result for those three equations.

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# MS65

# Solitary Axisymmetric Capillary Water Waves

We consider steady axisymmetric water waves subject to surface tension, where we study the free-boundary problem for domains close to an infinite cylinder. In the case of linear vorticity in radial direction and no swirl, we are able to prove existence of small solitary solutions of KdV-type. They bifurcate from laminar flows in a flat cylinder and the presence of vorticity is required for their existence. The proof relies on a spatial dynamics approach allowing for a center-manifold reduction, which reduces the problem to a finite-dimensional dynamical system. Homoclinic solutions of this system, which correspond to solitary wave solutions on the cylinder, are found using dynamical systems methods.

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#### MS65

The Benjamin-Feir Instability in KdV-like Equations with General Dispersion and Monomial Non-

#### linearity

Nonlinear waves in dispersive media can exhibit modulation instabilities. We examine a category of scalar equations, with general dispersion and monomial nonlinearity, including a large variety of KdV-like equations. For small-amplitude traveling wave solutions, we provide a complete characterization of the spectrum near the origin of the linear operator obtained from linearizing about periodic traveling waves. We prove rigorously that, when the modulational instability is present, the spectrum connected to the origin consists of curves that invariably form a closed figure eight pattern.

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#### **MS65**

# Computational Method for Calculating Stochastic Wave Speed in Kinematic Equations for Pulse Waves

We present a method for computing the stochastic wave speed of pulse solutions in kinematic equations subject to small stochastic forcing. These kinematic equations arise as the singular limit of sharp pulse solutions in the FitzHughNagumo system, and our approach contributes a new perspective to the growing body of work on stochastic wave propagation in excitable media. The method is based on isochronal phase reduction and yields an effective It process for the waves position. We demonstrate how this reduction enables efficient computation of the wave speed and its variance, without resorting to Monte Carlo simulations.

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### **MS65**

# Global Bifurcation of 3D Gravity-capillary Waves on Beltrami Flows

Most of the current theory on three-dimensional steady water waves assumes irrotational flow. One exception is the construction of a family of small-amplitude doubly periodic gravity-capillary waves on Beltrami flows by Erik Wahln, Lokharu and Svensson Seth from 2020. In the talk I will describe a global continuation of this family. One of the challenges is that the local family is constructed using a multiparameter bifurcation approach, whereas global bifurcation theory usually assumes a single bifurcation parameter. Our theory includes irrotational flow as a special case. This is joint work with Bastian Hilder and Erik Wahln.

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#### **MS66**

# Dynamic Boundary Conditions in General Diffusions

One of the key features in complex fluids is the coupling and competition between the bulk and boundary effects. We will present the derivation of various dynamic boundary conditions through the energetic variational approaches (EnVarA). A few specific applications will be presented as well as analysis and simulations.

#### Chun Liu

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#### **MS66**

# On Patch Solutions to the Generalized SQG Equations

In this talk, I will discuss patch solutions to a class of inviscid generalized surface quasi-geostrophic (gSQG) equations. We show that, under suitable conditions, these solutions develop finite-time singularities when the associated BiotSavart law satisfies an Osgood-type condition. Conversely, if the Osgood condition is violated, the patch solutions remain globally regular. This is joint work with Qianyun Miao, Liutang Xue, and Zhilong Xue.

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#### **MS66**

# Elastic Effects on the Stability of Vortex Sheets and Boundary Layers

Elasticity is important in continuum mechanics with a wide range of applications and is challenging in analysis. In this talk we shall discuss some special elastic effects in elastic fluids, including the stabilizing effect of elasticity on the vortex sheets in compressible elastic flows and on the vanishing viscosity process of compressible viscoelastic flows on the half plane under the no-slip boundary condition.

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# **MS66**

# Stability Results for the Boussinesq Equations with Anisotropic Dissipation

The Boussinesq equations considered in this talk model buoyancy-driven flows, including various atmospheric and oceanographic phenomena as well as RayleighBnard convection (e.g., boiling water). Trivial solutions to these equations may be unstable, even with full dissipation. This talk presents recent stability results for the Boussinesq equations with anisotropic dissipation near two physically

relevant steady states: hydrostatic equilibrium and Couette flow. In the case of hydrostatic equilibrium, the coupling and interaction between the fluid velocity and temperature lead to wave structures that help stabilize the flow. For the Couette flow, the key mechanism is enhanced dissipation induced by a linear, non-self-adjoint operator, which plays a crucial role in enabling nonlinear stability.

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### **MS67**

# On the Spectrum of the Front in a Predator-Prey Model

We consider a predator-prey model with diffusion. There exists a parameter regime when the underlying dynamical system in a singular limit is reduced to a scalar Fisher-KPP equation. The process of the reduction consists of taking limits with respect to two parameters. In this presentation, the stability of these fronts is discussed. In particular, it is focused on obtaining uniform in the singular parameters bounds on the unstable discrete spectrum.

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### **MS67**

### Rate-Induced Tipping in a Moving Habitat

We investigate rate-induced tipping in a model of a moving habitat due to environmental change. We study a scalar reaction-diffusion equation with a non-autonomous reaction term representing a spatially localized, single-species, favorable habitat zone moving from one asymptotic location to another. The movement is characterized by the displacement d and the rate parameter r, which quantifies the time scale on which it occurs. Our main result identifies a critical displacement  $d^*$  and, for  $d>d^*$ , demonstrates the existence of a critical rate  $r^*(d)$  at which the system undergoes rate-induced tipping: for  $r>r^*$  an initially thriving habitat population becomes extinct due to the rapid habitat change.

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#### **MS67**

# Resonances and Solutions of the Abstract Wave Equations

We study the general abstract wave equation with the elastic operator being an unbounded operator generating cosine family, and derive a representation of the solutions to the equation in terms of resonances associated with the operator. This representation generalizes a well known case when the operator is self-adjoint that is described in details in the book by Dyatlov and Zworski on resonances. Our abstract representation is based on the use of Laplace transform rather than the spectral theorem for self-adjoint operators, and involves Frechet spaces rather than spaces of compactly supported functions.

#### Yuri Latushkin

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#### **MS67**

# Asymptotic Stability for Monotone Traveling Kinks of the Viscous Boussinesq Problem

We consider the damped Boussinesq equation (a fourth order damped wave equation). It is well-known that for certain regime of the parameters, there are monotone kink solutions. We show that these kinks are asymptotically stable (up to a translation) in  $L^p, 2 and <math display="inline">\dot{H}^1$  norms.

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#### **MS68**

# What Metric to Optimize for Suppressing Instability in a Vlasov-Poisson System?

Stabilizing plasma dynamics is an important task in green energy generation via nuclear fusion. One common strategy is to introduce an external field to prevent the plasma distribution from becoming turbulent. However, finding such external fields efficiently remains an open question, even for simplified models such as the Vlasov-Poisson (VP) system. In this work, we leverage PDE-constrained optimization to obtain a locally optimal field using different

loss functions. As the stability of the system can be characterized in several different ways, the objective functions need to be tailored accordingly. Through extensive numerical experiments, we show that evaluating the objective function only at the target timewhen stable plasma is desiredyields a highly non-convex landscape, making the global minimum difficult to find. This holds regardless of the loss function used (e.g., KL divergence or electric energy). In contrast, integrating the loss over time produces a landscape with a convex basin near the global minimum, aiding convergence. Moreover, with electric energy as the objective, the landscape outside this basin contains flat, unphysical local minima, making good initialization crucial for effective optimization.

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#### **MS68**

# Learning Dynamics Guided by Mean-field Games Problems

Mean Field Games (MFGs) provide a powerful framework to model the collective behavior of a large population of rational agents. However, in many real-world scenarios, the underlying cost function that governs agent behavior is unknown, making direct modeling infeasible. In this talk, I will present recent progress on the inverse problem of learning agent dynamics when the cost structure is latent. We begin by formulating a bilevel optimization approach to recover MFG-consistent dynamics in low-dimensional settings, where both environmental obstacles and underlying geometric metrics are unknown. Our framework includes theoretical guarantees on local identifiability and a provably convergent alternating gradient algorithm. Building on this, we develop a scalable deep learning approach by connecting the trajectory formulation of MFGs with the structure of normalizing flows, enabling learning in higherdimensional and more complex settings. Numerical experiments validate the effectiveness and flexibility of our proposed methods, offering a new direction for data-driven MFG inference.

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#### **MS68**

# Reconstruction of Heat Conductance Parameter in the Phonon Transport Equation Using Surface Temperature

Phonon transport equation (PTE) is a fundamental model for heat propagation. When two solids are placed side by side, the two PTEs are coupled through the reflective and transmission coefficients of the interface. Surface temperature is measured to infer these parameters in a non-intrusive manner. We formulate this inverse problem into a PDE-constrained optimization, study some analytical properties, and derive the associated Frechet derivative for performing the optimization method. In the small Knudsen number regime, PTE converges to the heat equation, and the surface information is severely damped through the boundary layer. The boundary layer effect and the optimization results will be presented in the numerical examples.

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### **MS68**

### A Policy Iteration Method for Inverse Mean Field Games

Mean-field games (MFGs) model non-cooperative games among large populations of agents and are widely applied in areas such as traffic flow, finance, and epidemic control. Inverse mean-field games address the challenge of inferring environmental factors from observed agent behavior. The coupled forward-backward structure of MFG equations makes solving these problems difficult and adds even greater complexity to their inverse problems. In this talk, I will introduce a policy iteration method for solving inverse MFGs. This method simplifies the problem by decoupling it into solving linear PDEs and linear inverse problems, leading to significant computational efficiency. The approach is flexible, accommodating a variety of numerical methods and machine learning tools. I will also present theoretical results that guarantee the convergence of our proposed method, along with numerical examples demonstrating its accuracy and efficiency.

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### **MS69**

# Convolution Estimates for the Gain Boltzmann Operator with Hard Spheres

We discuss new moment-preserving weighted  $L^p$  convolution estimates for the gain operator of the Boltzmann equation with hard potentials including hard spheres. Our approach relies crucially on a novel collisional averaging mechanism to handle pathological collisions accumulating energy to one particle, and uses purely kinetic tools.

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#### **MS69**

# A Sharp Commutator Estimates for Sub-Coulomb Modulated Energies

The modulated energy plays a central role in deriving mean-field convergence rates for particle systems interacting via Riesz or Coulomb potentials. In particular, for Coulomb and super-Coulomb interactions (d  $-2 \le s < d$ ), the third author established a functional inequality that controls the derivative of the modulated energy along a transport by the modulated energy itself. Combined with a Grönwall-type argument, this yields quantitative rates of

mean-field convergence in the modulated energy distance. This framework was later extended to the full range of Riesz (and Riesz-like) interactions by Q.H. Nguyen and the last two present authors. Building on these developments, we prove improved functional inequalities that are sharp in their additive error terms. As a consequence, we establish the expected convergence rate of  $N^{\frac{1}{6}-1}$  in modulated energy distance for first-order gradient flows with s < d-2. This complements recent work of the second and third authors on the optimal rate for the Coulomb/super-Coulomb case  $(d-2 \le s < d)$ , and therefore completely resolves the Riesz case. Our estimates rely on a novel truncation scheme for the interaction potential, based on a wavelettype integral representation of the Riesz potential, combined with Kato–Ponce-type commutator bounds.

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#### **MS69**

### A Transport Method for Riesz Gases

Coulomb and Riesz gases are an important model in mathematical physics, with applications to random matrix theory, approximation theory and statistical mechanics. Questions regarding the behavior of the gas at small scales are of fundamental importance, and have seen a recent growth in study in the past decade. Many of these questions can be understood via a fine understanding of the partition function of the ensemble. In this talk we develop a transport method for Riesz gases, building on work of Armstrong, Lebl and Serfaty for Coulomb gases, to understand the behavior of the partition function under small perturbations of the potential. Our study involves several questions concerning degenerate, singular elliptic PDE and fractional operators. The transport method we develop allows us to prove local laws and controls on fluctuations of linear statistics down to microscopic length scales, and establish for the first time a corresponding CLT for Riesz gases in dimension d=2. This work is joint with S. Serfaty.

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#### **MS69**

# On the Wave Kinetic Equation and Wave Kinetic Hierarchy

The wave kinetic equation is one of the fundamental models in the theory of wave turbulence and provides a statistical mechanic treatment of the dynamics of weakly nonlinear interacting waves. This talk will address the global in time well-posedness of the spatially inhomogeneous wave kinetic equation for polynomially decaying small initial data by applying techniques inspired by the analysis of the Boltzmann equation another model of statistical mechanics that describes evolution of rarefied gases in which particles undergo predominantly binary interactions. We will also discuss the well-posedness of the wave kinetic hierarchy an

infinite system of coupled equations closely related to the wave kinetic equation. Two essential tools for obtaining these results are the Hewitt-Savage theorem, which allows us to lift the existence result for the equation to the hierarchy level, and the Klainerman-Machedon board game argument, which allows us to control the factorial growth of the Dyson series and consequently prove uniqueness of solutions to the wave kinetic hierarchy. This is a joint work with Ampatzoglou, Miller and Pavlovic.

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### **MS70**

# A regularity criterion for the 3D Navier Stokes equation based on finitely many observations.

I will present a new regularity criterion for the threedimensional NavierStokes equations that is formulated entirely in terms of finitely many spatial observations. The starting point is an AOT-style nudging algorithm that evolves an auxiliary velocity field from arbitrary initial data while continuously relaxing it toward coarse measurementseither modal (low Stokes modes), volume-averaged, or nodal samples. I will show that, under a fully explicit sensor-density / feedback-gain condition (linking the observation mesh size h, the nudging parameter  $\mu$ , and the observed data magnitude), the assimilated field is globally regular and exponentially synchronizes to the true solution in  $H^1$ . As a consequence, the underlying NavierStokes solution is itself regular on the same time interval. This yields a criterion that (i) replaces global spacetime norms by operationally realistic measurements at finitely many points or cells, (ii) works for both Type I (modal/volume) and the more delicate Type II (nodal) interpolants, and (iii) provides quantitative guidance on how fine the observations must be. I will compare this perspective with classical results (Prodi-Serrin type), highlight the role of the data-driven stability estimates behind the proof, and outline extensions to coupled systems and discrete-in-time assimilation. Time permitting, I will be briefly addressing the time discretization case as well.

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### MS70

# Large Eddy Simulation Reduced Order Models (LES-ROMs) for Turbulent Flows

In this talk, I will survey our group's work to bridge two distinct research fields, large eddy simulation (LES) and reduced order models (ROMs), for efficient and accurate numerical simulations of engineering and geophysical turbulent flows. The models that we have developed, called LES-ROMs, leverage ideas from both LES and ROMs, as well as data-driven modeling, machine learning, and data assimilation. The main tool used to construct the new LES-ROMs is explicit spatial filtering, which is employed to model the large spatial structures in the underlying turbulent flow. We test the new LES-ROMs in the numerical simulation of the turbulent channel flow, which is a challenging test problem for ROMs.

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### **MS70**

# Global Data Assimilation from Spatially Local Observations

We develop, analyze, and test a global data assimilation/synchronization algorithm based on purely local observations. We assume that the reference flow is analytic with sufficiently large analyticity radius. This underlies nudging algorithms that use data on moving subdomains. Numerical computations are included to demonstrate the effectiveness of this approach.

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### **MS70**

# From Feedback to Enforcement: Synchronization as the Limit of Nudging

In this talk, I explore the relationship between two continuous data assimilation algorithms for the 2D incompressible NavierStokes equations: the AzouaniOlsonTiti (AOT) algorithm and the synchronization algorithm, which is characterized by direct enforcement of the observed quantities in the model. I show that as the nudging parameter in the AOT algorithm becomes very large, the algorithm converges to the synchronization algorithm. I will present numerical experiments supporting these theoretical findings, both in the deterministic setting and in the presence of stochastic observational noise. To address challenges posed by noisy data, I propose a simple adaptive strategy for selecting the nudging parameter, which yields improved performance over fixed-parameter approaches.

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#### **MS71**

# An Equation Error Approach for Inverse Problems with Full Field Interior Data

We present the equation error formulation for inverse problems governed by differential equation models. The formulation is a quadratic optimization problem based on minimizing the residual of the differential equation, and can be solved directly (ie. without iterations) for the unknown parameters. We present two methods to solve the optimization problem, and prove that the formulation is well posed for the inverse heat conduction problem with mild conditions on the data. We demonstrate through computational examples that the Galerkin discretization converges with mesh refinement. Finally, we extend the formulation to an inverse problem governed by a scalar wave equation and show promising computational results.

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# **MS71**

# Fast Methods for the Solution of Inverse Scattering Problems Using Multifrequency Data

In this work, we present a fast, robust, and accurate algorithm for solving the inverse volume scattering problem of reconstructing the sound speed profile of a threedimensional variable medium using multi-frequency data. The problem is recast as an optimization problem. To enhance stability and reduce computational cost compared to solving the full multi-frequency problem, we apply a continuation in frequency strategy. This approach sequentially solves single-frequency inverse scattering problems in order of increasing frequency. Each single-frequency inverse problem is nonlinear and ill-posed. To address the nonlinearity, we apply an iterative optimization method, while the ill-posedness is dealt with by restricting the solution space to a band-limited representation of the sound speed profile. We provide numerical results comparing different optimization strategies and demonstrate the solvers ability to reconstruct both smooth and piecewise-smooth profiles with discontinuities.

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# MS71

Inverse Scattering from Nonlinear Media in Acous-

### tic Waveguides

We discuss in this talk the inverse scattering problem for nonlinear media of Kerr type in a 2D acoustic waveguide. This problem has important applications in physics and engineering, including radar, sonar, and nondestructive testing. We first prove a uniqueness result for the determination of the scatterer via the factorization analysis of the measurement operator. Then we propose an imaging function to numerically reconstruct the scatterer. This imaging function is both fast and stable, with a simple implementation that avoids the need of solving an ill-posed problem. Our numerical studies demonstrate that the proposed imaging function can provide more accurate reconstructions compared that of the direct sampling method. This talk is based on joint work with Nhung Nguyen and Thi-Phong Nguyen.

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#### **MS71**

# Towards Solving Inverse Scattering Problem for the Fractional Helmholtz Equation

There has been a growing interest in the fractional Laplacian operator in the past decades, and in the context of scattering theory, one could ask about what happens when replacing the Helmholtz equation by the Fractional Helmholtz equation, in which the Laplace operator is now fractional. The first step towards solving inverse problem would be the well-posedness of the forward problem. In this talk, we give an explicit computation of the fundamental solution for the fractional Helmholtz equation and its asymptotics. What is interesting is that the fundamental solution appears to be a perturbation in some sense of the fundamental solution for the Helmholtz equation. This leads us to prove, via a limiting absorption principle, that the Sommerfeld radiation condition ensures uniqueness of the solution to the fractional Helmholtz equation  $(-\Delta)^s - k^{2s}$  in weighted Sobolev spaces. Furthermore, this gives a Lippmann-Schwinger equation for inhomogeneous media, which will be helpful for solving inverse problems in future work.

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### MS72

# Data Efficient Kernel Methods for Learning, Solving, and Emulating PDEs

We present a unifying framework for learning equations and their solution operators from scarce measurements using the theory of reproducing kernel Hilbert spaces. We show that our methodology contains many existing ideas in the field of numerical PDEs and inverse problems and leads to accurate and data-efficient algorithms. We also briefly discuss theoretical analysis of the method including quantitative error bounds.

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### MS72

# Scalable Computation of Extreme Event Estimates for Stochastic PDEs Via Precise Large Deviation Theory

Estimating the probability of extreme events is an important problem in many scientific and engineering disciplines. as they are often associated with rare system failures or catastrophes. Asymptotically, rare event probabilities can be estimated with a Laplace approximation, known as precise large deviation theory. The approach involves (i) solving an optimization problem to find the most likely realization of the random parameter leading to a prescribed outcome, and (ii) calculating a determinant to account for Gaussian perturbations around the minimizer. In this talk, I will discuss how to carry out these steps numerically in a scalable way for extreme events in stochastic partial differential equations (SPDEs) with Brownian noise. In particular, I will highlight the necessity to treat the determinant calculation in step (ii) correctly from an infinitedimensional point of view to ensure scalability of the numerical method to high dimensions. This leads to either a Fredholm or Carleman-Fredholm determinant computation, depending on whether the second variation of the noise-to-event map is trace-class or only Hilbert-Schmidt. To illustrate these points, I will consider multiple examples of extreme event estimates for SPDEs: the 1D viscous Burgers equation and 3D Navier-Stokes equations with random forcing, and a 2D random advection-diffusion problem. Joint work with Shanyin Tong, Tobias Grafke, Georg Stadler; in Stat. Comput. 33(6), 137 (2023) and arXiv:2502.20114.

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### MS72

# Adaptive Linear Feature Selection with Dirichlet Energy Minimization

We analyze a machine learning approach that extracts a small set of task-relevant features from a high-dimensional data set and uses the features to make predictions. The features are linear functions of the input coordinates, and the prediction function is a smooth function of the features as measured by the Dirichlet energy. We establish that the linear featurization leading to the smallest Dirichlet energy is given by the square root of the average gradient outer product (AGOP) matrix. Also, we prove the AGOP featurization optimizes the accuracy of nearest neighbor interpolation in the large-data limit. The result leads to a conjecture that the AGOP featurization might similarly optimize the accuracy of a trained neural network.

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#### MS73

Optimal Multimaterial Composites: Quasiconvex Envelope of Multiwell Lagrangians, Piecewise Constant Fields and Rank-One Envelope

The paper discusses a problem of exact bounds for the ef-

fective properties of multimaterial composites. The new bounds extend and refine Translation bounds in the region of parameters where the last ones are loose. We find bounded regions of supporting fields in multiwell Lagrangians. We demonstrate that these fields vary within restricted domains and may lie on their boundaries. Because all structures are compared, the problem can be reduced to a finite-dimensional mathematical programming problem. With this, we modify the Translation method and obtain new exact bounds. Unlike two-material composites, different volume fractions of components in multi-material mixtures correspond to different expressions for bounds and topologically different types of optimal structures. The optimal laminate structures represent the Rank-One envelope of a multiwell Lagrangian.

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#### **MS73**

# Constitutive Thermodynamic Modeling and Homogenization for Heterogeneous Dielectrics

We delineate a multiscale and multiphysics electromechanical system modeling the response of dielectric elastomer and quantify its macroscopic behavior. Our approach involves formulating a precise theoretical framework within the main laws of thermodynamics. In the initial stage, constitutive laws are derived through the Coleman-Noll procedure. Subsequently, the constitutive laws are refined using a linear approximation of the response function and homogenization theory. This assumes rapidly oscillating space-charges as a source term in a moderately strong electric field. The approach involves an indirect method that approximates the quasistatic Maxwell's equations in free space, combined with the periodic homogenization asymptotic procedure. Employing the periodic unfolding operator and the averaging operator facilitates the transition to the homogenization limit, revealing the pivotal role of correctors in bridging the gap between microscopic and macroscopic aspects by quantifying disparities between heterogeneous and homogenized solutions.

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### **MS73**

# Multidimensional Analysis of the Adhesion Model for Mass Transport

The adhesion model is a multidimensional model of mass transport in cosmology in a perturbed Einstein-de Sitter universe, which reduces to sticky particle flow in 1D. Velocity is given by a Hopf-Lax formula and concentrates mass on sheets, filaments and points, but it is known that momentum is not conserved in general. Work in the astronomy literature by Brenier, Frisch and collaborators relates the problem of reconstructing the primeval velocity distribution to an optimal mass transport problem. We carry out a mathematical analysis of the initial value problem, establishing a number of basic facts, including the uniqueness of the propagated mass measure and a number of relations between the Lagrangian flow and a natural Monge-Ampre transport map.

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#### **MS73**

# MultiscaleAnalysis for Surface-Energetic Composites in the AntiplaneSetting

The talk is concerned with the periodic homogenization problem in antiplane elasticity setting for a composite reinforced by arbitrary-shaped inclusions, periodically distributed in the matrix material. The inclusions and the matrix are joined through an imperfect interface described by the Gurtin-Murdoch model of surface elasticity. The main focus of this talk is on deriving a homogenized or effective response of this heterogeneous material in the limit when the size of the microstructure tends to zero. First, starting with the formal asymptotic expansions, we establish the strong formulation for the homogenized equation provided that the solution possesses sufficient regularity. Next, using the rigorous mathematical theory of homogenization, we derive the homogenized equation that describes the influence of the interfaces on the overall behavior of the effective properties of the heterogeneous material.

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# **MS74**

# Solutions to the Euler Equations in $\mathbb{R}^2$ with Random Vortex Patch Initial Data

We study the (deterministic, unforced) Euler equations in  $\mathbb{R}^2$  with random initial vorticity chosen to be +1 or -1 on every unit square with integer lattice vertices (i.e. an random, infinite checkerboard). We show that there exists a deterministic time T > 0 such that with probability 1, the solution exists and is regular up to time T, and provide a bound on the rate at which solutions can grow near infinity.

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### **MS74**

# Phase Mixing and Landau Damping for the Hartree Equation

The Hartree equation is a mean-field model describing many-body quantum systems. In this talk, I will focus on the stability of translation-invariant equilibria for the Hartree equation. In the short-range regime, I will present a criterion for Penrose type stability and prove phase mixing estimates for the density associated with the perturbation from the Penrose stable equilibrium. In the long-range regime, we establish that the Penrose type stability fails for every equilibrium. Furthermore, in the Coulomb potential case, we show that the leading behavior of the density for the linearized Hartree equation is dominated by the Klein-Gordon type dispersion relation.

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#### **MS75**

### Selection of Pushed Pattern-Forming Fronts

Complex coherent structures in physical systems often form after a homogeneous background state becomes unstable. When the transition out of the unstable state is seeded by a spatially localized perturbation, this perturbation grows and forms an invasion front, which propagates into the unstable state and selects a new state in its wake. The marginal stability conjecture asserts that the propagation speed is the unique speed for which the associated invasion front solution is marginally spectrally stable. In many cases, propagation at a fixed speed combines with oscillatory dynamics in either the leading edge or the wake of the invasion process to generate a spatially periodic pattern. Universal wavenumber selection laws predict the wavelength of this pattern through an appropriate combination of the selected speed and the frequency of the temporal oscillations. We explore this phenomenon in the FitzHugh-Nagumo system, a prototypical model for large amplitude pattern formation. In this setting, we give the first rigorous proof of the marginal stability conjecture and associated wavenumber selection laws for any patternforming invasion process. Our proof relies on a nonlinear stability analysis of a pushed front, which is in many ways similar to a generic source defect.

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#### **MS75**

# On the Spectrum of Schrdinger Operators Interacting at Two Distinct Scales

We study Schrdinger operators of the form  $\Delta-W$  on  $L^2_{\rm rad}(\mathbb{R}^3)$ , the space of radially symmetric square integrable

functions. The potential W is taken to be radially symmetric (i.e. W(x) = W(|x|)) and to decompose into two components with distinct spatial scales:  $W = V_0 + V_{1,\varepsilon}$ . The second component  $V_{1,\varepsilon}(x) = \varepsilon^2 V_1(\varepsilon x)$  represents a scaled potential that becomes increasingly delocalized as  $\varepsilon \to 0$ , while  $V_0$  contributes at a different characteristic scale. We will assume that both potentials  $V_0(x), V_1(x)$  exhibit rapid decay as  $|x| \to \infty$ , though such assumptions can be relaxed somewhat. We establish rigorous eigenvalue counts in the spectral gap between the essential spectrum  $(-\infty,0]$  and the smallest eigenvalue of  $\Delta - V_0$ . This then gives that the total number of eigenvalues of  $\Delta - W$  is the sum of the number of eigenvalues of  $\Delta - V_0$  and  $\Delta - V_1$ . Our analysis combines dynamical systems techniques with a separation of scales argument, providing a broadly applicable framework for studying spectral properties of differential operators where multiple spatial scales interact.

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### MS75

# Traveling Waves Solutions of an Epidemiological Model

In a diffusive epidemiological model with saturating treatment, we demonstrate the existence of traveling waves. When the diffusion rate of the infected population is significantly larger than the susceptible population, we show that the underlying dynamics of these patterns is governed by a modified version of the BurgersHuxley equation.

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# MS75

# Existance and Stability for the Travelling Waves of the Benjamin Equation

In the seminal work of Benjamin in the late 70's he derived the ubiquitous Benjamin model, which is a reduced model in the theory of water waves. It contains two parameters in its dispersion part and under some special circumstances, it turns into the celebrated KdV or the Benjamin-Ono equation. During the 90's, Benjamin studied the existence of solitary waves, followed by works of Bona-Chen and Albert-Bona-Restrepo, as well as Pava, who showed the existence of travelling waves, mostly by variational or bifurcation methods. Some results about the stability became available, but those were restricted to either small waves or Benjamin model, close to a distinguished (i.e. KdV or BO) limit. Quite recently, Abdallah, Darwich and Molinet, proved existence, orbital stability and uniqueness results

for these waves, but only for some values of the parameters. We present an alternative constrained maximization procedure to construct these waves in the full range of the parameters and prove their spectral stability. Moreover, we extend this construction to all  $L^2$  subcritical cases. Finally, we propose a different procedure, based on a specific form of the Sobolev embedding inequality, which works for all powers p¿2, but produces some unstable waves for large p.

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### **MS76**

# Qualitative Experimental Design for Kinetic Parameter Reconstruction Problems

Inverse problems related to kinetic PDEs seek to reconstruct their parameters, e.g. an absorption cross section or an interaction kernel, from experimental data and appear in a plethora of applications, such as optical tomography or model fitting. The success of such inference relies heavily on the quality of the collected data, which is affected - amongst others - by the choice of the experimental design (sensor locations, preparation of source data etc.). Especially when experiments are expensive, it is crucial to thoroughly plan experiments that yield informative data. In this talk, we demonstrate how an analytic tool developed for the theoretical analysis of inverse problems for transport equations can be exploited to derive practically feasible experimental designs that facilitate the reconstruction of a discretized parameter. We demonstrate this strategy on a specific inverse problem that aims to reconstruct a velocity jump kernel in a linear kinetic model which describes the directed motion of bacteria in mathematical biology, termed chemotaxis.

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#### MS76

# Advances in Plasma Microturbulence Understanding Using GPU-accelerated Vlasov-Poisson Simulations

Microphysics plays a critical role in collisionless plasma transport and affects macroscopic properties like resistivity. A kinetic description is required to model these anomalous transport phenomena. The challenge of using kinetic simulations is the computational cost associated with tracking the species distribution functions in phase space, along with significant numerical stiffness present in

multi-species transport with large particle species mass ratios. By utilizing GPU accelerated supercomputers with compute-efficient numerical algorithms the scope of kinetic simulations accessible can be increased to handle realistic proton-electron mass ratios. We present theoretical bounds on numerical stability and scalable multi-GPU parallel algorithms for a multi-species fourth-order accurate explicit finite-volume Vlasov-Poisson solver. We then use the improved computational throughput to characterize nonlinear collisionless lower hybrid drift instabilities in pulsed power inertial confinement fusion experiments. This new predictive capability is applied to develop reduced models for anomalous collisionless transport, and show how theoretical linear instability growth rates can be used as a predictor for nonlinear quantities such as the anomalous collision frequency. We also demonstrate how having access to high resolution realistic kinetic simulations enables the validation of quasi-linear kinetic models. Prepared by LLNL under Contract DE-AC52-07NA27344.

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#### **MS76**

# An Explicit, Energy-Conserving Particle-in-Cell Method for the Vlasov Equation and Related Kinetic Plasma Models

The particle-in-cell (PIC) algorithm has been a standard numerical method for the Vlasov equation and its relatives for over 50 years. It combines a Lagrangian representation of the distribution function with an Eulerian representation of the electromagnetic fields to mitigate the curse of dimensionality. However, PIC has long suffered from so-called grid heating, in which the energy in the system grows secularly when certain (potentially irrelevant) physical scales are under-resolved. This problem has been solved in the context of implicit PIC methods over the last decade, but implicit methods come with additional costs. We present an explicit PIC method that eliminates grid heating by enforcing exact energy conservation. This is achieved by the analytic solution of an optimization problem for each particle at each time-step. In addition to successful application to standard test problems, we will report on ongoing work extending the algorithm to include the Lenard-Bernstein-Dougherty collision operator and relativistic effects.

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# MS76

### Control of Plasma Through an External Field

Plasma instabilities pose significant challenges in plasma science, impacting applications from particle accelerators to nuclear fusion reactors. In this talk, we explore the potential for controlling these instabilities by introducing external fields into Vlasov-type equations. Through a combination of theoretical analysis and computational methods, we demonstrate that appropriately chosen external fields can fully suppress plasma instabilities when both the equilibrium distribution and perturbation are known. We also examine various control strategies that could be implemented in practical plasma simulations.

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#### **MS77**

# Irregularity in the Multi-D Derivative Martingale

Branching Brownian motion describes a growing swarm of particles that move and multiply stochastically. In multiple dimensions, the frontier of this population is governed by an associated "derivative martingale," which converges to a random field on the sphere. In this talk, we will explore the irregularity of this limiting field: it is almost surely discontinuous almost everywhere in  $d \geq 3$ .

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#### **MS77**

# Phase Transitions and Linear Stability for the Mean-Field Kuramoto-Daido Model

We study the McKean-Vlasov equation

$$\partial_t q = \frac{1}{2} \partial_\theta^2 q - K \partial_\theta (q \partial_\theta W * q),$$

on the one-dimensional torus, where K>0 is the interaction strength. The bimodal interaction potential

$$W(\theta) = \cos \theta + m \cos 2\theta, \quad m \ge 0,$$

defines the Kuramoto-Daido model, while m=0 reduces to the Kuramoto model. We fully characterize the phase transition threshold  $K_c$  by comparing it to the linear stability threshold  $K_{\#} = \min(1, m^{-1})$  of the uniform distribution. When  $m \leq 1/2$ ,  $K_c$  coincides with that of the Kuramoto model, i.e.  $K_c = 1$ . On the other hand, for  $m \geq 2$ , we show  $K_c = m^{-1}$ . As a byproduct, we completely identify the regimes in which the phase transition is continuous or discontinuous by the value of m. Furthermore, we analyze the linear stability of the non-uniform stationary solution q. Our approach extends the Dirichlet form method of [Bertini-Giacomin-Pakdaman, Dynamical Aspects of Mean Field Plane Rotators and the Kuramoto Model, 2010 from the Kuramoto model to the Kuramoto-Daido setting. In particular, for  $m \le 8.568 \times 10^{-4}$  in the supercritical regime K > 1, we establish an explicit lower bound on the spectral gap of the linearized McKean-Vlasov operator at q. To our knowledge, this is the first rigorous stability analysis for bimodal interactions.

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### **MS77**

Some Results on the Analysis of Wave Kinetic

#### Equations

Kinetic equations can be used to describe the dynamics of nonlinear classical and quantum waves out of thermal equilibrium, as well as the propagation of waves in a random medium. In this talk, I will present some of our recent results on the kinetic theory of waves. I will discuss the analysis of those kinetic equations for waves. Next, I will focus on the numerical schemes we have been developing to resolve those equations.

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### **MS78**

# Well-posedness of the Free Boundary Incompressible Porous Media Equation

We consider the free boundary incompressible porous media equation which describes the dynamics of a density transported by a Darcy flow in the field of gravity, with a free boundary between the fluid region and the dry region above it. For any stratified density state, we identify a stability condition for the initial free boundary. Under this condition, we prove that small localized perturbations of the stratified density lead to unique local-in-time solutions in Sobolev spaces.

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## **MS78**

# On the Stability of the Compacton Waves for the Degenerate KdV and NLS Models

This talk is based on the degenerate semi-linear Schrödinger and Korteweg-de Vries equations in one spatial dimension. We construct variationally special solutions of the two models, that is, standing wave solutions of NLS and traveling waves for KDV, which turn out to have compact support, hence compactons. We show that the compactons are unique bell-shaped solutions of the corresponding PDEs and for appropriate variational problems as well. We also provide a complete spectral characterization of such waves for all values of p. Namely, we show that all waves are spectrally stable for 2 , while a single mode instability occurs for <math>p > 8. This extends the previous work of Germain, Harrop-Griffiths, and Marzuola, who have previously established orbital stability for some specific waves, in the range p < 8.

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#### **MS78**

### Random Fput Lattices

We consider a FermiPastaUlamTsingou lattice with randomly varying coefficients. We discover a relatively simple condition which when placed on the nature of the randomness allows us to prove that small amplitude/long wavelength solutions are almost surely rigorously approximated by solutions of Kortewegde Vries equations for very long times. The condition is an instance of "hyperuniformity" and without this condition numerical simulations no such KdV approximation should be possible. The key ideas combine energy estimates with homogenization theory and the technical proof requires a novel application of autoregressive processes.

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#### **MS78**

### Existence of Asymmetric Grain Boundaries

Physically, a grain boundary is a linear defect that is formed as an interface between roll solutions of different orientations. While the existence of small-amplitude symmetric grain boundaries has been rigorously verified in various frameworks, grain boundaries observed in physical experiments and numerical simulations are often asymmetric. This work is to establish the existence of asymmetric grain boundaries (AGB) in the framework of SHE. As in the symmetric case, the construction of the asymmetric grain boundaries also takes the path via spatial dynamics and center manifold reduction to reduce a PDE problem to an ODE one, where the existence of AGB solutions boils down to the existence of a heteroclinic orbit in the reduced ODE. We establish such an existence via variation of calculus techniques in the normal form ODE and show the persistence via a Lyapunov-Schmidt scheme.

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# MS79

# Accuracy and Stability of the Ensemble Kalman Filter for Certain Dissipative Geophysical Systems

We develop a unified framework for analyzing several well-known and efficient data assimilation methods derived from Gaussian approximations of Bayesian filtering for dissipative systems with quadratic nonlinearities, such as those

in geophysical fluid dynamics. We establish rigorous timeasymptotic accuracy and stability results under general assumptions on the covariance and observation operators. To our knowledge, this is the first such result for EnKF and EnSRKF in this general PDE setting. A key insight is a previously unexploited cancellation property between the ensemble covariance and observation operator. Together with covariance inflation and localization, this structure is essential to ensuring filter accuracy and stability. Our analysis also clarifies connections between approximate-Bayesian and control-theoretic approaches through determining functionals. We study the 2D incompressible Navier-Stokes equations with observations given by noisy spatial averages or spectral/modal measurements. In this context, continuous-time data assimilation methodsNudging (AOT), 3DVar, EnKF, and EnSRKFreduce to stochastically forced Navier-Stokes equations. We derive new conditions for the accuracy and stability of EnKF and En-SRKF, showing how filter performance depends on the resolution of the observation space and the use of covariance inflation and localization strategies.

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#### **MS79**

# Control of Dissipative and Non-Dissipative Dynamical Systems with Partial Measurements

One of the fundamental challenges of accurate simulation of turbulent flows is that initial data is often incomplete, which for said flows is a strong impediment to accurate modeling due to sensitive dependence on initial conditions. A continuous data assimilation method, known as the Azouani-Olson-Titi (AOT) or Continuous Data Assimilation (CDA) algorithm, introduced a linear feedback control term to dissipative systems, giving a simple and rigorous deterministic method by which to understand the underpinnings of more complex data assimilation algorithms used in the geosciences for, e.g., weather and climate prediction. The ability of the AOT algorithm to recover solutions in non-dissipative systems has only recently been explored, demonstrating the criticality of the dissipation assumption. Inspired by previous works on regularity as a proxy for short-time dissipation, we will discuss in what scenarios we can effectively apply the AOT algorithm to non-dissipative systems (in particular the passive scalar and Euler equations), obtaining decay on the error between the reference solution and the data assimilation solution in the Sobolev and analytic norms for short time given analytic data. If time permits, I will also discuss work with Andrew Stuart when the large-time behavior is more regular than simply being dissipative, and how this affects the conditions for convergence.

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#### **MS79**

Leveraging the Finite Dimensionality and Disjointedness of Attractors to Infer Parameters of Dissi-

#### pative Systems

In this talk, we will discuss the inverse problem of determining parameters of a differential equation from observations of the solution. We will discuss the well-posedness of this inverse problem, and some numeric results. Our method makes use of continuous data assimilation techniques, which can determine the solution exactly using finite dimensional data, provided the system is dissipative. As an example, we will consider the Navier–Stokes equations in 2D (with periodic boundary conditions), which are known to possess a global attractor, and trajectories on the attractor are determined by finitely many modes (called determining modes). When the viscosity is unknown, we will show that trajectories on the attractor are still determined by finitely many modes, and discuss the problem of recovering the viscosity from the determining modes.

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#### **MS79**

# Parameter Recovery of the Simplified Bardina Model via Continuous Data Assimilation

In this study, we investigate a continuous data assimilation algorithm applied to the Simplified Bardina model with an unknown parameter alpha. We analyze the long-time error between the true solution and the assimilated solution arising from discrepancies between the true and estimated values of alpha. Furthermore, we develop a parameter recovery algorithm that operates alongside the continuous data assimilation scheme to recover both the true solution and the correct value of alpha.

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#### MT1

# Nonlocal Phase Transitions, Minimal Surfaces, and Crystal Dislocations

Input your abstract, including TeX commands, here. The abstract should be no longer than 1500 characters, including spaces. Only input the abstract text. Don't include title or author information here.

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#### MT1

# Nonlocal Phase Transitions, Minimal Surfaces, and Crystal Dislocations

Input your abstract, including TeX commands, here. The abstract should be no longer than 1500 characters, including spaces. Only input the abstract text. Don't include title or author information here.

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#### MT2

Dynamics for Sampling: From Markov Chains to

#### Gradient Flows and Interacting Particles

Input your abstract, including TeX commands, here. The abstract should be no longer than 1500 characters, including spaces. Only input the abstract text. Don't include title or author information here.

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#### PP1

## On the Study of the Long-Time Statistics of the Stochastically Forced Boussinesq System for Rayleigh-Bnard Convection

The ergodicity for stochastic partial differential equations has been studied by several mathematicians recently, and Harris' theorem provides a result concerning the question of stability of long-time statistical properties for dynamical systems. In [Glatt-Holtz and Mondaini, Long-term accuracy of numerical approximations of SPDEs with the stochastic Navier-Stokes equations as a paradigm, '24], the authors obtained a weak form of Harris theorem, showing that the Markov semigroup is a contraction in a suitable Wasserstein distance, which implies existence uniqueness and exponential rates of convergence towards the invariant measure. In this poster, we apply this result to the Boussinesq approximation for Rayleigh-Bnard convection, a model for the evolution of a fluid flows velocity field, temperature field, and pressure under buoyancy-driven flow caused by thermal gradients. Our main result concerns the stability of the long-time statistics with respect to the Prandtl and Rayleigh numbers. We show that the invariant measure of the Markov semigroup associated to the dynamics of the system depends continuously on these parameters. Our proof consists of two crucial steps: firstly, we show that the underlying Markov semigroup satisfies a contraction estimate, in a suitable Wasserstein distance; and secondly, we obtain a finite-time pointwise continuity estimate. This is a joint work with Cecilia Mondaini (Drexel University) and Nathan Glatt-Holtz (Indiana University).

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#### PP1

# Image Segmentation and Data Approximation under Geometric Constraints Using a Level Set Method

We give a new model combining image segmentation under constraints and data approximation. We define the geometric constraints and we give a minimization problem leading to a variational equation: for any function  $\xi:\Omega\to\mathbb{R}$ , we consider the set

$$S_{\xi} = \{(x, y, z) \in \Omega; \xi(x, y, z) = 0\}.$$
 (1)

We introduce the energy functional  $E(S_{\xi})$ :

$$E(S_{\xi}) = \frac{\alpha}{meas(S_{\xi})} \int_{S_{\xi}} d_D(s) ds + \beta \int_{S} g(\|\nabla I\|)(s) ds, (2)$$

where d is the distance from the front to a given set of geometric data (a set of of points or a curve given by the user),

g is the usual potential  $g(\|\nabla I\|) = \frac{1}{1 + \|\nabla I\|^2}$ ,  $meas(S_{\xi})$  is

the measure of  $S_{\xi}$ ,  $\alpha$  and  $\beta$  are strictly positive,  $\alpha$  controls the fidelity criterion to the dataset D and  $\beta$  controls the attraction force of S linked to the potential  $g(\|\nabla I\|)$ . We give a level set formulation of (2), we define a variational equation and a finite element discretization. Numerical results are given on medical image segmentation and data approximation, with a suitable choice of the coefficients. Funding This work is funded in parts by Rgion Normandie and ERDF fund via the I-demo SCALE Op project under convention 00152289.

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#### PP1

# Elliptic Problems Involving Mixed Local-Nonlocal Operator in the Hyperbolic Space

We have explored the existence of solutions for a class of nonlinear elliptic equations involving a mixed localnonlocal operator of the form  $-\Delta_{\mathbb{B}^N} + (-\Delta_{\mathbb{B}^N})^s$ , with 0 < s < 1, set in the hyperbolic space  $\mathbb{B}^N$ . Specifically, we consider the equation

$$\left\{ -\Delta_{\mathbb{B}^N} u + (-\Delta_{\mathbb{B}^N})^s u - \lambda u = |u|^{p-1} u + |u|^{2^* - 2} u, \quad u \in H^1 \right\}$$

where  $\Delta_{\mathbb{B}^N}$  is the LaplaceBeltrami operator and  $(-\Delta_{\mathbb{B}^N})^s$  is the fractional Laplacian of order 0 < s < 1. The parameter  $\lambda < \frac{(N-1)^2}{4}$ , the bottom of the  $L^2$ -spectrum of  $-\Delta_{\mathbb{B}^N}$ , and  $1 for <math>N \ge 3$ . A key step in our approach is to establish that  $H^1(\mathbb{B}^N)$  embeds continuously into the fractional Sobolev space  $H^s(\mathbb{B}^N)$ , which allows us to define a suitable energy functional and apply critical point theory to obtain nontrivial solutions.

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### PP1

## Polynomial Complexity Sampling from Multimodal Distributions Using Sequential Monte Carlo

A computationally challenging problem that arises in many situations is to sample from the Gibbs distribution when the configuration space is large. When the energy function is convex, there are several algorithms that perform well even in high dimensions. When the energy landscape is complicated, the cost of existing algorithms is exponential in the inverse temperature, and are impractical when the temperature is small. We present an algorithm (Rebalanced Annealing) whose cost is (a low degree) polynomial in the inverse temperature. This is joint work with G. Iyer and D. Slepcev.

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#### PP1

# On Generalizing the Induced Surface Charge Method to Heterogeneous Poisson-Boltzmann Models for Electrostatic Free Energy Calculation

The induced surface charges (ISC) method, which computes the induced charges on the molecular surface of macromolecules and uses them via Coulomb's law to calculate the polar solvation energy, was shown to be a robust and almost grid independent approach for electrostatic analysis based on the sharp-interface Poisson-Boltzmann (PB) model. Besides being physically intuitive, the ISC method avoids using the potential near the point charges, which is singular at each atom center. However, the ISC method cannot be physically generalized to heterogeneous dielectric PB models, due to the non-existence of a dielectric boundary. In this work, a novel far-field (FF) method is proposed to calculate the polar solvation free energy, which is derived through reformulating the energy functionals of nonlinear PB potential in solvent and vacuum states. Built upon a rigorous mathematical analysis, the FF method reconstructs the free energies by using far-field solutions outside the solute so that the self-energy terms generated by the singular charges are avoided, just as in the ISC method. Being valid for both sharp-interface and heterogeneous PB models, the performance of the proposed FF method has been validated by considering diffuse interface, Gaussian and super-Gaussian PB models for Kirk- $\left\{-\Delta_{\mathbb{B}^N} u + (-\Delta_{\mathbb{B}^N})^s u - \lambda u = |u|^{p-1} u + |u|^{2^*-2} u, \quad u \in H^1(\mathbb{B}^n) \text{ spheres and various protein systems. Comparison with grid-energy cancellation and regularization methods}\right\}$ is also considered.

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### PP1

# Robust Impulse Control Under Uncertainty: Applications in Quantitative Finance

We study a robust impulse control problem in mathematical finance, where an investor optimally adjusts a risky portfolio at discrete times while facing ambiguity about the assets drift. The problem is modeled as a stochastic differential game between the investor and nature, where nature distorts the reference probability measure using Girsanovs theorem. The resulting optimization leads to a nonlinear Hamilton-Jacobi-Bellman quasi-variational inequality with fixed and proportional transaction costs. We solve the HJB-QVI numerically using methods tailored to the impulse control structure. In addition, we propose a reinforcement learning algorithm that approximates the value function and the associated policy directly from simulated trajectories, enabling data-driven control under partial model knowledge. We compare both approaches and analyze the sensitivity of the optimal intervention thresholds to the level of model uncertainty.

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#### PP1

# Patterns in the Wake of a Parameter Ramp

We study the formation of patterns in the light-sensitive CDIMA system in the presence of a slowly-varying opaque mask which induces a Turing instability as it moves in one spatial dimension. Previous work used a sharp mask to control the formation of patterns. Here the parameter ramp induces an asymptotically constant front from which Turing patterns form in the wake of the ramp. Through numerical simulation and continuation we consider the impact of the slow ramp on wavenumber selection curves. We then study complex Ginzburg-Landau equation as a prototypical pattern-forming equation to observe these phenomena. For the cGL equation, we find that a slow passage through an absolute instability governs the leading order front position and asymptotic wavenumber. We then seek to rigorously establish existence of pattern-forming fronts through a GSPT framework and observe a novel slowpassage phenomenon through a Hopf-like singular point.

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## PP1

# Motion of Elastic Thin Films by Evaporation-Condensation in the Dewetting Regime

In this work, we show the short-time existence of solutions of the evolution equations that represent the solid state dewetting of thin films through evaporation-condensation as a two dimensional sharp interface variational model. The evolution law is established as the  $L^2$  gradient flow of surface energies in the presence of epitaxial strain. The main novelty is the presence of moving contact lines when the film formation is governed by evaporation-condensation

method.

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## PP1

### Perturbation of Physical Systems and Probabilistic Forecasting Using Flow Matching

The modelling of dynamical systems is essential in many fields, but applying machine learning techniques is often challenging due to incomplete or noisy data. This study introduces a variant of stochastic interpolation (SI) for probabilistic forecasting, estimating future states as distributions rather than single-point predictions. We propose a technique to physically perturb a complex high-dimensional state. Again, we explore its mathematical foundations and demonstrate its effectiveness on various dynamical systems, including the challenging Weather-Bench dataset.

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### PP1

# A Harris Theorem for Enhanced Dissipation, and An Example of Pierrehumbert

In many situations, the combined effect of advection and diffusion greatly increases the rate of convergence to equilibrium – a phenomenon known as enhanced dissipation. Here we study the situation where the advecting velocity field generates a random dynamical system satisfying certain Harris conditions. If  $\kappa$  denotes the strength of the diffusion, then we show that with probability at least  $1-o(?^N)$  enhanced dissipation occurs on time scales of order  $|\log \kappa|$ , a bound which is known to be optimal. Moreover, on long time scales, we show that the rate of convergence to equilibrium is almost surely independent of diffusivity. As a consequence we obtain enhanced dissipation for the randomly shifted alternating shears introduced by Pierrehumbert '94.

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# PP1

# Analysis of Predator-Prey Dynamics with Taxis and Diffusion: Existence, Uniqueness, and Stability

This poster presents a comprehensive analysis of a reaction-diffusion system modeling predator-prey interactions. The model uniquely incorporates taxis phenomena and our study addresses fundamental mathematical questions for this coupled predator-prey modeling framework. We rigorously establish the existence of weak solutions using the powerful Schauder fixed-point theorem. To further characterize these solutions, their uniqueness is proven through a sophisticated duality technique. Beyond existence and uniqueness, we delve into the system's long-

term behaviour by performing a detailed linearized stability analysis around its equilibrium points. This work offers a robust mathematical foundation for understanding complex ecological dynamics driven by both local interactions and directional movement.

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#### PP1

# Traveling Wave Solutions of a Reaction-Diffusion System Modeling Social Outbursts

Despite legal recognition of protest activity, the historical interaction between the government and protesters has been complex. Motivated by the "wave-like dynamics of rioting activity during the French riots in 2005, a system of equations is introduced to model the interplay between the dynamics of protesters, social tension, and law enforcement. We establish the existence and stability of traveling wave solutions in this system, supported by both theoretical analyses and numerical simulations. Changes in certain parameters cause a qualitative difference in the solution to the system, leading to situations that correspond to different protest management strategies. We delve into the impact of two distinct protest management approaches on the qualitative and semi-quantitative characteristics of the traveling waves.

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