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Mathematical Challenges of Controlling Large-scale Complex Systems

By Paul Davis

An airplane's environmental control system is a complex assembly of components having a broad remit. Passengers should be kept at a Goldilocks temperature — not too hot and not too cold. The plane's electronics must not overheat, and beverages in the galley fridge should be chilled and ready to serve. Identifying an appropriate control architecture for aircraft is but one of the problems described by Andy Sparks of United Technologies in an invited address at the 2017 SIAM Conference on Control and Its Applications, held in Pittsburgh, Pa., this July.

The United Technologies corporate family manufactures significant and complex pieces of modern infrastructure, notably Pratt & Whitney aircraft engines (see Figure 1), Otis elevators, Carrier heating and cooling systems, and multiple sub-assemblies for aircraft. As the Group Leader in Control Systems at the United Technologies Research Center (UTRC), Sparks guides his team in finding better ways to control complicated systems throughout United Technologies' portfolio.

Sparks emphasized that none of his team's target systems are "complex" in the sense of being capable of adaptation or self-organization. Rather, they are large, complicated assemblies that must be engineered to perform specified functions efficiently, safely, and reliably. He and his colleagues develop tools to design, control, and manage these systems and their intricate interactions.

From a control perspective, these complex products are high-dimensional. Hundreds or even thousands of components must work together seamlessly, and the couplings among them are dynamic and complex. Furthermore, each of these products encompasses multiple specialized technical domains, including—but hardly limited to—energy flow, mechanics, electronics of all sorts, thermodynamics, and fluids. To reinforce the depth of this complexity, Sparks observed that many more countries are capable of building a nuclear weapon than are capable of producing a gas turbine aircraft engine!

Control engineers need to model uncertainty. Safety is critical; e.g., elevators are fitted with multiple fail-safe systems to counter the full range of potential internal

Figure 1. *Aerospace is but one source of problems in estimation and control within the United Technologies family. Image courtesy of United Technologies Research Center.*

and external failures that could cause one to fall. Operational limits and stability margins can demand predictive control as well.

In the context of environmental control systems in passenger aircraft, Sparks described two specific challenges his team faces: identifying a control architecture for a given system and selecting gain variables to use in the controller. Topics left for another day include controller cost, estimation, fault detection and diagnosis, computation, communication, security — both cyber and physical, and verification and validation.

These environmental systems are composites of components — air coolers and circulators for the passenger cabin, refrigeration in the galley, thermal protection for some

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Finding Emergent Low-Dimensional Macroscopic Behavior in Large Systems of Many Coupled Dynamical Units

By Edward Ott

Determining and understanding the macroscopic behavior of a large system consisting of many interacting dynamical parts ("units") is of interest across a broad range of science. Examples include brain dynamics (where the units are individual neurons), the beating of animal hearts regulated by cardiac pacemaker cells, synchronous signaling in large animal populations (e.g., yeast cells or insects, such as certain species of fireflies), Josephson junction circuits, and chemical oscillators. The macroscopic behavior that typically results in such systems is often described as "emergent" because it cannot be understood purely on the basis of the microscopic dynamics of individual units, but rather "emerges" as a result of their interactions. Perhaps the simplest type of emergent behavior is synchronization in systems of many coupled oscillators (in this case, the oscillators are the units) [13]. For example, cardiac pacemaker cells synchronize throughout a healthy heart, resulting in its rhythmic beating. Seriously malfunctioning hearts, on the other hand, can exhibit more complex emergent behavior in the form of turbulent-like propagating waves of activity (including spiral waves).

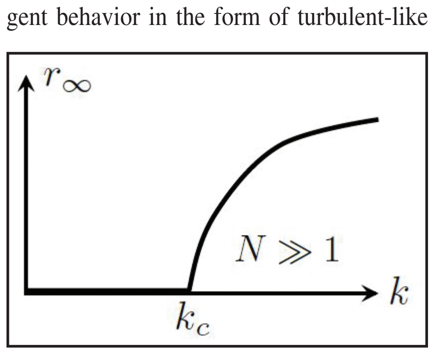

Figure 1. *Attracting fixed point of the $N \to \infty$ macroscopic dynamics of the Kuramoto model, (1), $r_\infty = |R(+\infty)|$ versus k. k_c increases with the spread in the natural frequencies of the oscillators. Image credit: Zhixin Lu.*

While the general subject of emergent behavior is vast, I will focus on a particular (but large) class of models that are often used to address and enhance the understanding of various types of emergent behavior. Somewhat surprisingly, an extremely effective analytical technique uncovers the macroscopic dynamics for the large class of problems to be discussed. I aim to describe this technique here.

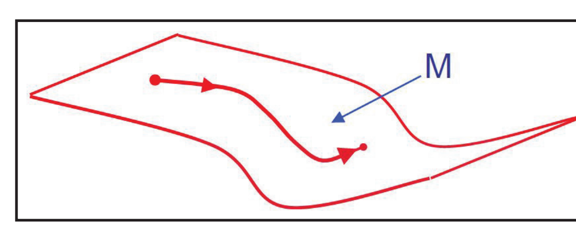

Figure 2. *The invariant manifold M in the space of distributions $f(\theta, \omega, t)$ and an "orbit" of f, starting at a point (the red dot) on M. Image credit: Zhixin Lu.*

To begin, we consider a simple paradigmatic model put forth by Yoshiki Kuramoto in 1975 [2], which provides an excellent illustration of the technique we wish to present. The technique is also capable of treating more complex situations. Kuramoto considers the simplest model of an oscillator, in which the state of oscillator i is given solely by the phase θ_i of its oscillation. The oscillator dynamics are given by $d\theta_i/dt = \omega_i$, where the constant ω_i is the "natural frequency" of the oscillator i. The oscillator population $(i = 1, 2, \ldots, N)$ is heterogeneous, in that each oscillator has a different natural frequency that Kuramoto assumes to be randomly drawn from some unimodal distribution function $g(\omega)$. We note that oscillator heterogeneity is natural, e.g., in biological situations where no two units are expected to be exactly the same. Next, Kuramoto pairwise couples his oscillator states to arrive at the model,

$$\frac{d\theta_i}{dt} = \omega_i + \frac{k}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i), \quad (1)$$

where the second term on the right-hand side represents the coupling between oscillators with a strength given by the coupling constant k. The main questions addressed by this model are whether or not the oscillators synchronize, and if they do, how strong their synchronization is. Before answering these questions, however, Kuramoto's complex order parameter can prove useful:

$$R(t) = \frac{1}{N}\sum_{j=1}^{N}\exp[i\theta_j(t)] \equiv r(t)e^{i\psi(t)}, \quad (2)$$

where $r(t) = |R(t)|$. For large N and a uniform random distribution of θ_j (i.e., no phase synchronization), $r \cong 0$ ($r \to 0$ as $N \to \infty$ in this case), while for perfect phase synchrony (i.e., all θ_j are equal), we have $r = 1$. Thus, r quantifies the amount of synchrony for the oscillator population. We can now provide Kuramoto's main result: for $N \to \infty$, the conflict between the desynchronizing tendency (inherent in the spread of natural frequencies ω_i) and the synchronizing tendency from the coupling between oscillators is resolved by a so-called "dynamical phase transition" (alternatively, a bifurcation of the macroscopic dynamics), as illustrated in Figure 1. The quantity $r_\infty = \lim_{t\to\infty} r(t)$ is the amount of synchronization for the macroscopic attractor of the Kuramoto model, which is 0 when $k < k_c$ and increases continuously as k increases past k_c, approaching complete synchrony $(r_\infty = 1)$ as $k \to \infty$.

See **Macroscopic Behavior** *on page 4*

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There's Something About Yellow

When you think of SIAM, what color comes to mind? Probably certain blues and greens, the colors SIAM uses most often in its materials. SIAM does not have a well-defined color scheme, but is moving towards creating one during development of the new SIAM website.

What other aspects of mathematics are associated with particular colors? Yellow book covers spring immediately to mind. Springer covers are usually yellow, but there are some other striking yellow covers on my shelf: the first edition of *Numerical Recipes* (Cambridge University Press, 1986), the third edition of *LAPACK Users' Guide* (SIAM, 1999), and *Lectures on Finite Precision Computations* (SIAM, 1996).

In fact, yellow is a common color in our daily lives. We make notes on yellow Post-its; we use yellow highlighters on documents; and we write on yellow legal pads, perhaps with yellow Dixon Ticonderoga pencils (if we are based in the U.S.).

In our gardens, yellow is the most omnipresent color after green (especially given that brown is a dark yellow), with yellow flowers in the spring and summer and yellow foliage in the fall.

What is so special about yellow? Part of the answer relates to the human eye. Our retinas have three types of cones, with responses peaking in the red, green, and blue parts of the spectrum, as shown in Figure 1. Yellow almost maximally excites both the green and red cones, thus appearing particularly bright to us.

FROM THE SIAM PRESIDENT
By Nicholas Higham

Yellow is also one of the CMYK colors (cyan, magenta, yellow, and black) universally used for printing. So if the yellow ink has been well-formulated, then yellowish colors should print well (whereas green, for example, must be produced by a nontrivial linear combination of the CMYK inks).

Yellow is a clearly identifiable rainbow color, and occupies a narrow band of the rainbow color map widely used in software. However, this color map is falling out of favor for several reasons, including the difficulty of remembering the order in which its colors appear and the loss of information when a color figure is printed in monochrome. One of the usurpers of the rainbow map is the parula map—introduced in MATLAB in 2014—which passes from blue through green and into yellow. As seen in Figure 2 (on page 4), parula has a fairly uniform luminosity gradient, which allows it to print well in monochrome. It also has a much wider band of yellow than jet, the rainbow color map in MATLAB.

If you write a book for SIAM, you will have the opportunity to provide input on the book's cover design (with more freedom if the book does not belong to a series with a standard cover). One of the biggest decisions is the color choice. For the third edition of *MATLAB Guide*, Des Higham and I wanted a blue version of the cover used for the first two editions. SIAM's graphic designer, Lois Sellers, sent us three versions—in different shades of blue—to choose from. I printed them out on my laser printer so that Des and I could discuss them when we met. We noticed that the printed colors were quite different from the colors displayed on the screen. Indeed, without a color-managed workflow in which the screen and printer are calibrated (with the relevant software using the generated color profiles), we can't assume that either device is telling the truth. My screen is calibrated but my laser printer is not. The cover of the printed book is pretty close to what I saw on the screen.

I mentioned earlier that printing is done in the CMYK color space. Yet when you send in a paper or book to SIAM, you almost certainly send figures as RGB files. Like every publisher, SIAM converts your RGB files to CMYK before printing. Very occasionally, and probably only if you are writing a book with figures where the color accuracy is important, you might want to fine-tune the CMYK conversion yourself. If so, take a look at my blog post,[1] where I report on experiences in producing CMYK figures for *MATLAB Guide*, the third edition of which was produced in color for the first time.

See **Yellow** *on page 4*

Cartoon created by mathematician John de Pillis.

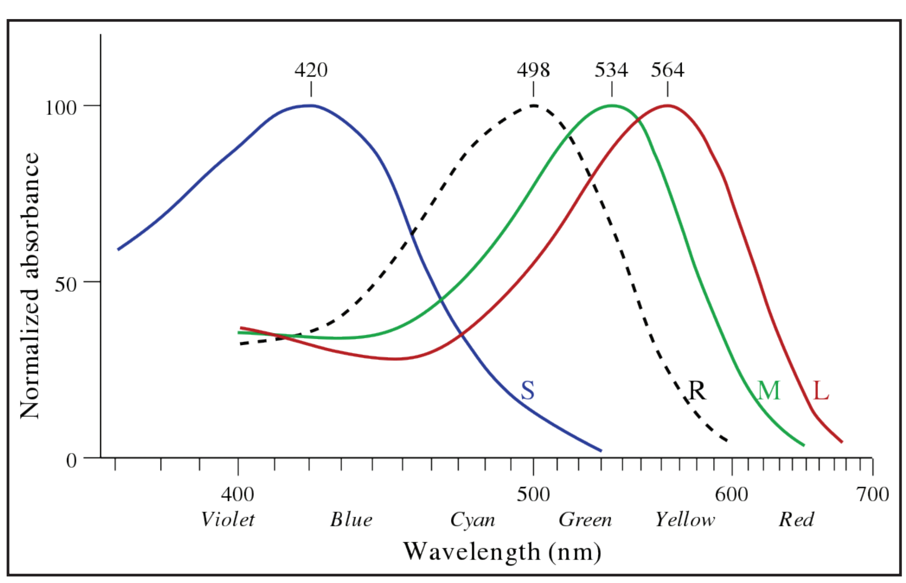

Figure 1. *Response curves for the cones on the retina most sensitive to short (blue), medium (green), and long (red) wavelengths, respectively. Image credit: Wikimedia Commons.*

[1] https://nickhigham.wordpress.com/2017/02/06/preparing-cmyk-figures-for-book-printing/

SIAM is Now a Member of ORCID

ORCID is a central registry of researchers. Authors submitting papers to SIAM journals can log in to the submission system through their ORCID account, or go to their SIAM system profile to connect their ORCID account to their SIAM journals work. If and when papers are accepted and published, an ORCID icon will appear next to the author's name on SIAM Journals Online, which links to his/her ORCID page. If the author chooses, the page can in turn list all published articles.

SIAM encourages authors to take advantage of ORCID's capability, which can be viewed as a DOI for an individual. This unique identifier then pulls together the individual's work and professional activities from across publications, disciplines, and workplaces. The researcher controls, through his or her ORCID account, which work and activities are publicly displayed.

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SIAM Advocates for Applied Math and Computational Science

By Miriam Quintal and Eliana Perlmutter

SIAM is deeply concerned with sustaining federal support for applied mathematics and computational science. It conducts a wide range of activities to encourage Congress's promotion of funding and sound policy in these areas, engage the administration and federal agency officials in informing future programs, and elevate the role of the SIAM community within the Washington, D.C.-based scientific community so that advocacy reflects SIAM priorities. SIAM works with Lewis-Burke Associates, a government relations firm representing nonprofit scientific organizations and research universities, to help facilitate and advise on SIAM's federal advocacy, engagement, and profile-raising.

With support from Lewis-Burke, the SIAM Committee on Science Policy (CSP) conducts advocacy activities and engages with federal agency officials. Every spring, the CSP meets with congressional offices in Washington, D.C., to support funding and robust policy for applied mathematics and computational science. This April, CSP members—who participate in SIAM advocacy on a volunteer basis—conducted 25 meetings with staff on pertinent congressional committees and offices whose members are on committees with jurisdiction over science policy and research funding. At these meetings, CSP members discussed the importance of funding for the National Science Foundation (NSF), Department of Energy (DOE), National Institutes of Health (NIH), and Department of Defense (DoD). They also underscored the value of mathematics and computational science research, and spoke about related education and workforce issues.

In addition to conducting outreach to Congress, the CSP engages with federal agency officials to inform future programs and enhance the profile of SIAM within the scientific community. Twice a year, it convenes agency officials in Washington, D.C., to participate in discussions on current programs relevant to applied mathematics and computational science, and to hear about potential future directions of these agencies. This spring, CSP members met with Frederica Darema, director of the Air Force Office of Scientific Research; Michael Vogelius, director of the Division of Mathematical Sciences at the NSF; and Barbara Helland, associate director for the Advanced Scientific Computing Research (ASCR) program at the DOE Office of Science. Last fall, the committee's federal agency engagement included meeting with leadership from the NIH Division of Biomedical Technology, Bioinformatics, and Computational Biology, and project managers from the Defense Advanced Research Projects Agency.

SIAM also works in collaboration with other associations to advance community efforts advocating for federal support of science and engineering through the Coalition for National Science Funding (CNSF), the Energy Sciences Coalition, and the Coalition for National Security Research. Partnering with these groups allows SIAM to demonstrate broad consensus among scientific associations when advocating to policymakers on behalf of SIAM priorities and the research community's interests. These collaborations elevate SIAM's role within the Washington, D.C.-based scientific advocacy community. Additionally, involvement in the coalitions ensures that communal advocacy efforts reflect SIAM priorities. Recent actions through these coalitions include signing community letters in support of strong funding for basic research. In May, Michael Shelley (New York University) represented SIAM at an annual exhibition hosted by the CNSF in the House of Representatives, which showcased research supported by the NSF.[1]

SIAM communicates priorities to legislators through meetings as well as submission of formal input. To support fiscal year 2018 appropriations for key research agencies, SIAM submitted written testimony to the relevant House and Senate appropriations subcommittees. The testimony endorsed increased funding for the NSF and the DOE Office of Science, and discussed the value of specific programs at the agencies, including the NSF Division of Mathematical Sciences and the Office of Advanced Cyberinfrastructure, the DOE ASCR office and Applied Mathematics program, and associated graduate fellowships and early career researcher activities. Furthermore, Lewis-Burke submitted questions to congressional offices in advance of hearings with agency leadership to prompt the emphasis of SIAM priorities in hearing proceedings.

At the start of the new presidential administration, the CSP drafted white papers containing recommendations focused on the importance of applied mathematics and computational science. These white papers were used in congressional advocacy and shared with the Office of Management and Budget, which plays a central role in the budget process on behalf of the administration. The white papers addressed SIAM priorities at the NSF, DOE, and DoD. SIAM has also responded to destabilizing actions by the administration by releasing a statement in support of the March for Science and participating in coalitions to express opposition to travel bans.

SIAM is always looking to offer its members opportunities to be informed and involved in policy advocacy on behalf of applied mathematics and computational science. If you are interested in receiving updates about science policy and notifications of action alerts, sign up for the science policy electronic mailing list.[2] Additionally, SIAM is launching a new Science Policy Fellowship Program. Each year, the program will offer three to five postdoctoral fellows and early-career researchers training and facilitated opportunities to advocate for federal investments relevant to SIAM priorities. This fellowship will enable participants to continue pursuing their research and teaching while simultaneously gaining experience with the processes that determine science funding and policy decisions. More information about the fellowship—and instructions for applying—are available on the SIAM website.[3] Applications are due September 15th.

Miriam Quintal is SIAM's Washington liaison at Lewis-Burke Associates LLC. Eliana Perlmutter is a Legislative Research Assistant at Lewis-Burke Associates LLC.

[1] https://sinews.siam.org/Details-Page/siam-at-the-cnsf-exhibition

[2] https://www.siam.org/about/science/sci_policy_form.php

[3] http://www.siam.org/about/science/sci-pol-fellowship.php

Complex Systems

Continued from page 1

of the avionics, etc. Since the components do not comprise an organic whole designed from scratch, the overall environmental system controller is commonly an assembly of single-loop, hand-tuned systems.

In an ideal world, a sophisticated multivariable controller would likely outperform such a piecemeal mix. However, the sophisticated option would increase the burdens of system design, cost, and service. For example, a customer might be persuaded that such added investments would pay for themselves in fuel savings over the life of an expensive and highly sophisticated aircraft engine, and those engines do indeed employ state-of-the-art, multivariable controllers. For cabin cooling, however, the perfect technical solution is economically infeasible.

So market forces, abetted by aircraft engineers' well-founded, risk-averse instincts, combine to argue for a synthesis of the tried and true—typically individual proportional-integral controllers—to manage the compressor(s) and circulators that maintain temperatures throughout an aircraft's cabin, in its galley, and within its electronics. Given those realities, Sparks and his colleagues have developed tools for selecting a sparse control architecture that is "efficient, scalable, and robust." That process involved both direct collaboration with the engineers responsible for the product and publication in the open literature [1, 2].

Figure 2 illustrates four possible configurations for a candidate environmental system controller: single-input single-output, block diagonal, sparse, and multiple-input multiple-output (MIMO). Configuring an effective sparse controller requires a rigorous way to identify the variables requiring feedback. Given those variables, graph theory can guide selection of the system architecture. Actual design of the controller completes the job.

The framework for variable selection is a minimization problem with two principal terms. One is a traditional closed-loop control objective function. The other is a weighted "sparsity-promoting term;" e.g., the number of variables in the feedback term. The augmented Lagrangian of that problem is non-convex — the team used an alternating direction method of multipliers to solve the minimization. For the aircraft environmental control problem, this approach began with nine control loops and found six to be optimal. Those six had the same structure as the relative gain array for the corresponding MIMO system, but without the overhead of the extra variables.

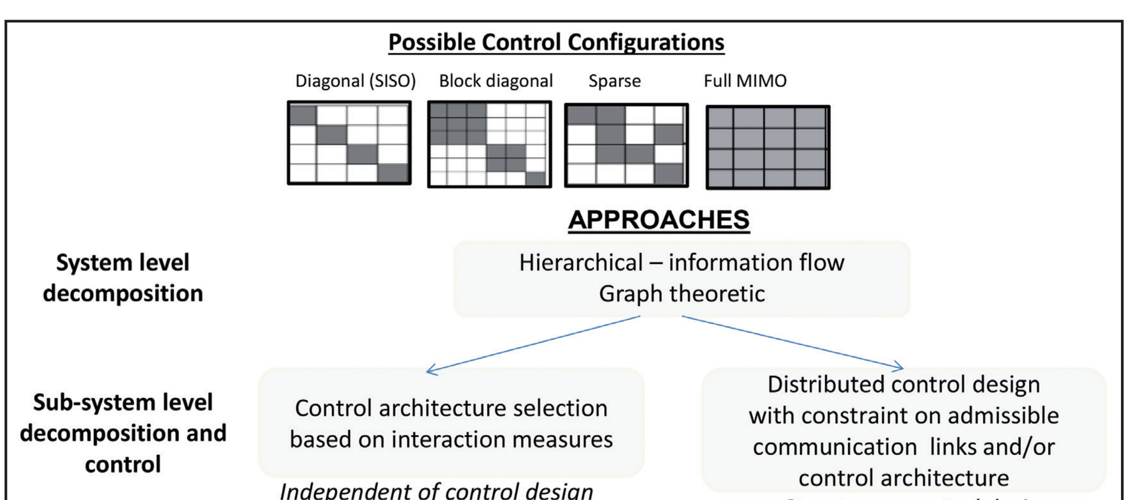

Figure 2. *Four possible configurations for an aircraft's environmental system controller: single-input single-output, block diagonal, sparse, and multiple-input multiple-output (MIMO). Image courtesy of United Technologies Research Center.*

In another application—a 39-bus, 10-generator model of the New England electric power distribution grid—the control objective is to maintain phase angle and frequency throughout the network. Attempting to suppress oscillations using sparse output feedback revealed an explicit trade-off between system performance and the sparsity of the controller. Similar analysis of a toy system containing a hundred or so connected spring-mass oscillators confirmed intuition about sparse control: feedback control must know the behavior of both the local mass and nearby neighbors.

In response to an audience question about redundancy and fault tolerance in a sparse control system, Sparks explained another virtue of the sparsity analysis; in essence, it identifies those components requiring redundancy. Redundancy can be added where it matters.

One of the team's primary current concerns is verification and validation, which requires exhaustive Monte Carlo simulation. The time demanded by these simulations presents a significant barrier to the implementation of new systems. Considerable research is needed to develop more informed, faster random simulations.

When flying in the temperature-controlled comfort of a turbine-engined aircraft to your next SIAM meeting or riding an elevator inside the massive conference hotel, think about the control engineers who coordinated the complexity surrounding you. They are hardly sparse among the SIAM community!

Sparks' presentation is available from SIAM either as slides with synchronized audio, or as a PDF of slides only.[1]

[1] https://www.pathlms.com/siam/courses/4987/sections/7398

Acknowledgments: My thanks to Andy Sparks and Bob LaBarre, Principal Mathematician and Associate Director, Systems in the System Dynamics and Optimization group at UTRC, for helpful conversations about the ways in which the varying needs and expectations of different business units, markets, and products inform scientific and engineering decisions.

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Macroscopic Behavior

Continued from page 1

The order parameter (2), in addition to characterizing the amount of synchrony, also leads to a useful reformulation of the Kuramoto problem. Specifically, noting that $\sin\theta = (e^{i\theta} - e^{-i\theta})/(2i)$, (1) can be written as

$$\frac{d\theta_i}{dt} = \omega_i + \frac{i}{2i}[Re^{-i\theta_i} - R^* e^{i\theta_i}]. \quad (3)$$

Since we are interested in large numbers of units $(N \gg 1)$, in order to proceed further it is useful—as Kuramoto does—to make the approximation $N \to \infty$, in which case we can characterize the oscillator population at time *t* by a distribution function $f(\theta,\omega,t)$, such that $\int_0^{2\pi} f(\theta,\omega,t)d\theta = g(\omega)$. And, since the number of oscillators is conserved, the continuity equation for the oscillator density *f* in $\theta - \omega$ space becomes

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial\theta}\left\{\left[\omega + \frac{k}{2i}\left(Re^{-i\theta} - R^* e^{+i\theta}\right)\right]f\right\} = 0, \quad (4)$$

where

$$R = \int\int_0^{2\pi} f e^{i\theta} d\theta d\omega. \quad (5)$$

Here, (5) comes from noting that, according to (2), *R* is the average of $e^{i\theta}$ over the oscillator population, while (4) results from noting that $d\omega_i/dt = 0$ (the oscillator natural frequencies do not change) and $d\theta_i/dt$ is given by (3).

We now consider a certain class of problems—generalizations of the Kuramoto problem—for which the states of coupled dynamical units are given by a single angle variable (i.e., a variable that can be taken modulo 2π). I wish to point out the following three main results:

Considering the $N \to \infty$ *limit and evolution of the distribution function f, an invariant manifold M exists in the space of distribution functions* [8]. That is, if $f(\theta,\omega,0)$ is initialized on *M*, it continues to evolve on *M*, as shown schematically in Figure 2 (on page 1).

For appropriate $g(\omega)$, *we can explicitly derive the emergent macroscopic behavior for the system dynamics corresponding to f on M, often in the form of a low-dimensional set of ordinary differential equations (ODEs)* [8]. These two points naturally prompt a key question: Given that initial conditions for $f(\theta,\omega,0)$ need not be on *M*, are the two aforementioned points useful? The answer is "yes" because of our third main point [9, 10]: *If* $g(\omega)$ *is analytic, then all solutions for f are attracted to M (i.e., M is an "inertial manifold").*

That is, using an appropriate metric, the distance from *f* to *M* approaches 0 as $t \to +\infty$. This means that we can use the result to analyze all the long-term macroscopic behaviors of these systems, including their attractors and bifurcations.

The specification of the invariant manifold *M* comes from the ansatz that the distribution function is of the special form,

$$f(\omega,\theta,t) = \frac{g(\omega)}{2\pi}\left\{1 + \sum_{n=1}^{\infty}\left[\alpha^n(\omega,t)e^{in\theta} + \alpha^{*n}(\omega,t)e^{-in\theta}\right]\right\}, \quad (6)$$

where α^* denotes the complex conjugate of α; i.e., the n^{th} Fourier coefficient in an expansion of the θ-dependence of *f* is the n^{th} power of $\alpha(\omega,t)$. Substituting (6) into (4), we find that (6) satisfies (4), provided that

$$\partial\alpha(\omega,t)/\partial t = (k/2)\left[R(t)\alpha^2(\omega,t) - R^*(t)\right] + i\omega\alpha(\omega,t) = 0, \quad (7)$$

while (5) yields

$$R^*(t) = \int_{-\infty}^{+\infty} g(\omega)\alpha(\omega,t)d\omega. \quad (8)$$

Determining $\alpha(\omega,t)$ by solving (7) and (8), since the independent variable θ is eliminated, reduces the complexity considerably, as compared to the problem of determining $f(\theta,\omega,t)$ from (4) and (5). But we can make a further significant complexity reduction by considering particular forms for the natural frequency distribution $g(\omega)$. The simplest such form is the Lorentzian

$$g(\omega) = \frac{\Delta}{\pi}\frac{1}{(\omega-\omega_0)^2 + \Delta^2} = \frac{1}{2\pi i}\left\{\frac{1}{\omega - \omega_0 - i\Delta} - \frac{1}{\omega - \omega_0 + i\Delta}\right\}. \quad (9)$$

From (7) it follows that analytically continuing ω into the complex plane yields the results that $\alpha(\omega,t) \to 0$ as $Im(\omega) \to -\infty$, and that we can assume $\alpha(\omega,t)$ to be analytic in the lower half ω-plane. Thus, we can evaluate the integral (8) by contour integration to obtain

$$R(t) = \alpha^*(\omega_0 - i\Delta, t). \quad (10)$$

Using (10) and setting $\omega = \omega_0 - i\Delta$ in (7) yields a single simple ODE for the macroscopic dynamics of the Kuramoto system on *M*, as characterized by the magnitude of the complex order parameter, $r(t) = |R(t)|$,

$$dr/dt + (k/2)(r^2 - 1)r + \Delta r = 0. \quad (11)$$

For $0 < k < 2\Delta$, this equation has a single fixed point attractor at $r = 0$, which, as k increases through the critical point $k_c = 2\Delta$, bifurcates to an unstable fixed point at $r = 0$ and a fixed point attractor at $r = \sqrt{1 - (2\Delta/k)}$. This corresponds to the long well-known result for the Kuramoto model depicted in Figure 1 (on page 1).

While the result (11) is interesting, we have so far not discovered anything new about the Kuramoto model's behavior. However, ansatz (6) and the technique used to obtain (11) also apply to numerous other problems, yielding many significant new results. The following examples illustrate the great diversity of these results:

- Modeling walker-induced shaking of the Millennium Bridge [1] (a pedestrian bridge across the Thames River in London), which occurred when the bridge first opened in 2000.
- Studying why the effect of jet lag can be substantially greater for eastward airplane travel than westward airplane travel by the same number of time zones [6].
- Modeling brain dynamics of macroscopic synchronized behavior for many interacting, excitable, and firing neurons [7, 11]. Ansatz (6) could be particularly valuable in this area, since the number of neurons in the brain is very large, suggesting that macroscopic reduction might be extremely useful.
- Studying dynamics that result from different types of coupling between dynamical units, including the effects of time delays along links [4], various network topological properties [12], and consideration of spatiotemporal dynamics in which oscillators are distributed in space and coupled to other oscillators within their local region [3, 5].

In conclusion, the problem of uncovering emergent macroscopic behavior of complex systems, while old, has received much attention in recent years due to its widening relevance to various applications. We hope that the result described in this article will contribute positively to modern developments in these fields.

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Edward Ott is a Distinguished University Professor in the Department of Physics and the Department of Electrical Engineering at the University of Maryland. He is the recipient of the 2017 Jürgen Moser Lecture. This article is based on the associated lecture delivered in May 2017 at the SIAM Conference on Applications of Dynamical Systems, held in Snowbird, Utah.

Yellow

Continued from page 2

Blue, which we chose for the cover of *MATLAB Guide*, is of course the complementary color to yellow: some linear combination of blue and yellow light produces white. Complementary colors stand out from each other, yet are simultaneously harmonious. The contrast with blue sea is the reason why lifejackets are yellow or orange. Hopefully our blue *MATLAB Guide* will stand out from the many yellow books found on bookshelves!

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Nicholas Higham is the Richardson Professor of Applied Mathematics at the University of Manchester. He is the current president of SIAM.

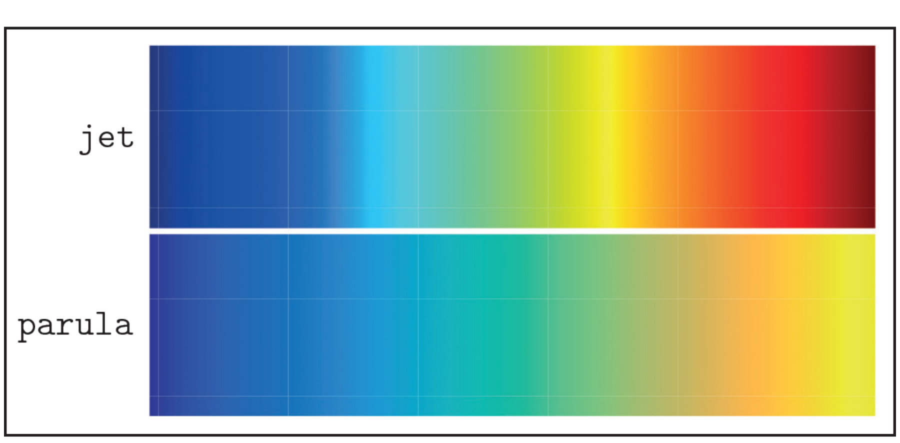


Figure 2. *The MATLAB color maps jet and parula. Image courtesy of [1].*

Congratulations to 2016 SIGEST Authors

Each issue of *SIAM Review* contains the SIGEST section, which spotlights an outstanding paper of general interest that has previously appeared in one of SIAM's specialized research journals; the issues rotate through the journals. The purpose of SIGEST is to make the more than 13,000 readers of *SIREV* aware of exceptional papers published in SIAM's specialized journals.

We would like to congratulate the following authors of the SIGEST selections that appeared in *SIREV* in 2016:

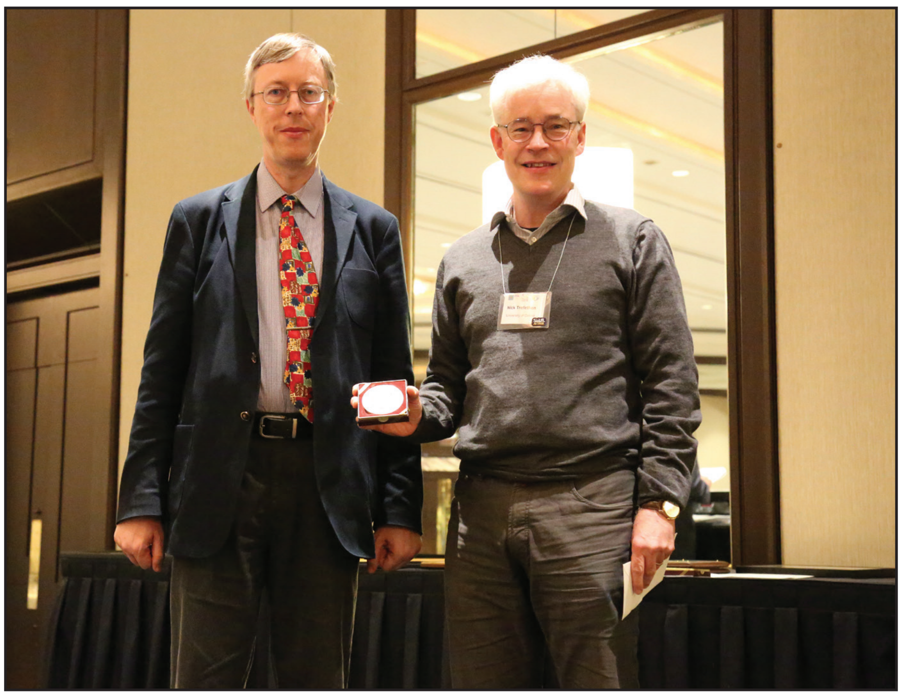
- **Vol. 58, Issue 1**
  A Newton--Galerkin Method for Fluid Flow Exhibiting Uncertain Periodic Dynamics
  By M. Schick, V. Heuveline, and O.P. Le Maître
  (Originally published in the *SIAM/ASA Journal on Uncertainty Quantification)*

- **Vol. 58, Issue 2**
  Diffuse Interface Models on Graphs for Classification of High Dimensional Data
  By Andrea L. Bertozzi and Arjuna Flenner
  (Originally published in *Multiscale Modeling and Simulation*)

- **Vol. 58, Issue 3**
  Hyperspherical Sparse Approximation Techniques for High-Dimensional Discontinuity Detection
  By Guannan Zhang, Clayton G. Webster, Max Gunzburger, and John Burkardt
  (Originally published in the *SIAM Journal on Numerical Analysis*)

- **Vol. 58, Issue 4**
  Optimization in High Dimensions via Accelerated, Parallel, and Proximal Coordinate Descent
  By Olivier Fercoq and Peter Richtárik
  (Originally published in the *SIAM Journal on Optimization*)

Photos from the 2017 SIAM Annual Meeting



SIAM President Nicholas Higham (left) awards Nick Trefethen (University of Oxford) with the George Pólya Prize for Mathematical Exposition during the Prizes and Awards Luncheon at the 2017 SIAM Annual Meeting, held this July in Pittsburgh, Pa. SIAM photo.

AWM President Ami Radunskaya (left) and SIAM President Nicholas Higham (right) present the AWM-SIAM Sonia Kovalevsky Lecture to Liliana Borcea (University of Michigan) at the Prizes and Awards Luncheon during the 2017 SIAM Annual Meeting, held in Pittsburgh, Pa., this July. Borcea's prize lecture was titled "Mitigating Uncertainty in Inverse Wave Scattering." SIAM photo.

The Class of 2017 SIAM Fellows were recognized at the Business Meeting during the 2017 SIAM Annual Meeting, held in Pittsburgh, Pa., this July. From left to right: Andreas Griewank (Yachay Tech University), Zhaojun Bai (University of California, Davis), Ricardo Cortez (Tulane University), Mark A. Lewis (University of Alberta), Lois Curfman McInnes (Argonne National Laboratory), Michael Kwok-Po Ng (Hong Kong Baptist University), Daniel B. Szyld (Temple University), and Carol S. Woodward (Lawrence Livermore National Laboratory). SIAM photo.

SIAM President Nicholas Higham (right) presents The John von Neumann Lecture to Bernard J. Matkowsky (Northwestern University) at the Prizes and Awards Luncheon during the 2017 SIAM Annual Meeting, held in Pittsburgh, Pa., this July. Following the luncheon, Matkowsky presented a talk on "Singular Perturbations in Noisy Dynamical Systems." SIAM photo.

SIAM President Nicholas Higham (right) congratulates Emily Shuckburgh (British Antarctic Survey), who gave the I.E. Block Community Lecture at the 2017 SIAM Annual Meeting, held this July in Pittsburgh, Pa. Shuckburgh's talk was titled "From Flatland to Our Land: A Mathematician's Journey through Our Changing Planet." SIAM photo.

SIAM President Nicholas Higham (left) poses with Lek-Heng Lim (University of Chicago), who received the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing during the Prizes and Awards Luncheon at the 2017 SIAM Annual Meeting, held this July in Pittsburgh, Pa. The following day, Lim spoke about "Tensors in Computational Mathematics." SIAM photo.

SIAM Conference Attendees: Beware of Phishing E-mails

Following a SIAM conference, you may be approached by a third-party publication not affiliated or endorsed by SIAM. Please exercise a high degree of caution when receiving such requests, and reach out to SIAM Executive Director Jim Crowley at JCrowley@siam.org if you are unsure whether a solicitation has been endorsed by SIAM.

Diversity: A Recurring Theme at the SIAM Annual Meeting

By Izzy Aguiar

Diversity is inherent to SIAM meetings. This is particularly true of annual meetings, where the breadth of topics rivals only the geographic, ethnic, and gender representation of attendees. This year's Annual Meeting, held in Pittsburgh, Pa., in July, featured a variety of sessions that specifically addressed issues of diversity in applied mathematics, including biases against female mathematicians, challenges faced by African American researchers, and community efforts to support underrepresented minorities.

One such session was the aptly-named panel on "Celebrating Diversity in Mathematical Sciences." An inspiring and informational panel, it consisted of Richard Tapia (Rice University), Association for Women in Mathematics (AWM) President Ami Radunskaya (Pomona College), Ron Buckmire (National Science Foundation), and Shelby Nicole Wilson (Morehouse College).

Tapia spoke of *diversity* as a bad word, a "cop-out" for universities to bundle all issues of underrepresentation into one easy phrase. He discussed the overwhelming progress of representation and equality for women in science, technology, engineering, and mathematics (STEM) fields, and the discrepancy between this movement and that of other underrepresented racial, ethnic, and socioeconomic minorities. "Gender equity is stepping on the toes of underrepresentation," Tapia said, lauding the successes of the gender movement. He added that, whereas racism in the past was manifested in segregation, the racism of today exists in lower expectations for underrepresented groups. Tapia cautioned that the health and success of the scientific community and the nation depends on adequate minority representation.

Radunskaya referred to diversity as "belonging and not belonging." She identified both visible and invisible signs of belonging and not belonging within a group, namely gender, skin color, and perceptible disabilities, as opposed to socioeconomic status, unseen disabilities, and personal background. Despite the progress in bridging the gender gap, women still experience the feeling of "not belonging," Radunskaya said. She delved into implicit biases and the following stereotypes: women will quit upon having children, women are maternal, and crying is a sign of weakness. Radunskaya observed that talking about being a woman is often easier and more comfortable than talking about the underrepresentation of other groups, and encouraged the audience to engage in difficult dialogues with people from different backgrounds.

Buckmire opened his discussion by quizzing the audience about the percentage of African Americans and Hispanics in the United States. He used the poll as an opportunity to address the fact that non-minorities tend to overestimate the true percentage of minorities. "If you think there are so many more [minorities in the US] and they're not being represented in mathematics, wouldn't that increase the sense of urgency to increase representation?" he asked.

Buckmire compared diversity to a vector space of infinite dimensions. The vast variances of identifying human characteristics exist on these varied axes of identity. Individuals do not prevail on just one but subsist on multiple axes simultaneously; this concept is called intersectionality. People's position on these axes determines their "positionality," or power in certain situations. He urged professors to look for these axes in the classroom, which may not otherwise be readily visible.

From left to right: Richard Tapia, Ami Radunskaya, Ron Buckmire, and Shelby Nicole Wilson address the audience during the panel on "Celebrating Diversity in Mathematical Sciences" at the 2017 SIAM Annual Meeting. Photo credit: Izzy Aguiar.

Buckmire also observed the so-called "cult of objectivity," noting that the mathematics community tends to dehumanize mathematicians, celebrating the words on paper rather than the characteristics of the human that put them there. If we don't value mathematicians as individuals, he argued, we can't understand that these individuals also possess characteristics that define diversity. One of the first steps in increasing diversity is to fight this objectivity and dehumanization.

Wilson shared that during her graduate studies, there was a point at which she realized that while her gender would help her succeed, her race would not. Women receive much support from various groups, programs, and scholarships; black men, on the other hand, lack role models and are often excluded or judged. This realization inspired her to teach at Morehouse College, an all-male institution on the list of Historically Black Colleges and Universities. Wilson emphasized that her students' successes are due to high expectations and professor-student trust. She thus encouraged the audience to foster a similar sense of community and expectation within their own environments.

The AWM panel on "Perspectives from Women in Research" also tackled issues affecting underrepresented groups. Panelist Lenore Cowen studied mathematics at Yale University before attending the Massachusetts Institute of Technology for her master's degree and Ph.D. in applied mathematics. She completed her postdoctoral fellowship at Johns Hopkins University, where she fell in love and got married. Cowen now has two teenage twin girls and researches computational biology in the Department of Computer Science at Tufts University.

Unlike Cowen, Anshu Dubey earned her undergraduate degree in engineering and worked briefly in industry before deciding to attend graduate school, where she received her Ph.D. in computer science. Mentors and colleagues warned Dubey that a zig-zagged path would be detrimental to her career, because others would perceive her decisions as inconsistent, a gendered comment in and of itself. Despite this, she continued pursuing new and exciting opportunities. Because of her research's interdisciplinary nature, Dubey did not fit into any given department and would have been unable to find a tenure-track position in academia. She moved to Argonne National Laboratories—where she currently works—for stability, and emphasized that there are multiple ways to succeed; breadth is just as good as depth.

Fariba Fahroo, a program manager for the Defense Advanced Research Projects Agency (DARPA), majored in physics and applied mathematics, continuing her graduate education in the latter. She then became a professor of applied mathematics at the Naval Postgraduate School, where—as the only female faculty member—she immediately witnessed the gender gap. Fahroo stepped back temporarily from academia to become a program manager in computational mathematics at DARPA, and enjoyed the job so much that she soon transferred permanently to the role.

Alison Marsden, a professor at the Institute for Computational and Mathematical Engineering and in the Departments of Pediatrics-Cardiology, Bioengineering, and Mechanical Engineering at Stanford University, spoke of her interdisciplinary background. She completed her undergraduate studies at Princeton University and her graduate studies at Stanford University, with degrees in mechanical engineering from both. Marsden focused her Ph.D. on applications of fluid dynamics, aerospace engineering, and turbulence modeling with the intention of implementing those tools elsewhere: cardiovascular modeling. While she could not have foreseen her current position in the Department of Pediatrics-Cardiology, she loves how her research spans a large group of people and disciplines. With two children, ages nine and 12, Marsden noted that "work-life balance" is an elusive term.

After their respective introductions, the panelists collectively shared opinions and advice on childcare, sexism and harassment in the workplace, and advantages women might have in fields where they are underrepresented. Marsden encouraged participation in women's groups, which helped her overcome the feeling of isolation and not belonging. Since becoming a faculty member, she has continued this support system by founding and participating in more groups. Marsden commented that all women experience the same issues regardless of discipline or stage of life, and talking about these issues provides invaluable assistance.

Dubey insightfully noted that the perception of women as "outsiders" has allowed them to develop various survival skills, one of which is clear communication. She has had to pay more attention to the context of dynamics and situations; having to continuously notice and analyze has improved her communication skills. Marsden pointed out that this relative advantage in communication enables success in collaboration, interdisciplinary research, and the management of graduate students. The panelists urged female attendees to take advantage of current opportunities meant to balance the ratio; the odds are so often stacked against women that this should pose no discomfort.

Ultimately, Cowen motivated the room to keep learning from one another, and hailed the women in the audience as the famous mathematicians of the future. Fahroo reassured attendees that no one does everything well, and there's no shame in seeking as much support and help as needed. Dubey commended the power of trusting one's gut to do what's best personally, regardless of what others might say. And Marsden quoted Sheryl Sandberg by saying, "don't quit before you quit," and encouraged women mathematicians to overcome the initial hesitancy of trying new things.

Izzy Aguiar earned her B.S. in applied mathematics and statistics from the Colorado School of Mines and is currently pursuing her M.S. in computer science at the University of Colorado, Boulder.

Funny Faces at the Fellows Reception

By Izzy Aguiar

Dozens of interesting, talented, and prestigious applied mathematicians attended the Fellows Reception at the 2017 SIAM Annual Meeting, held this July in Pittsburgh, Pa. But when asked what *else* interested them, many attendees jumped at the opportunity to discuss other passions.

SIAM President Nicholas Higham (University of Manchester) spoke of his time as a semi-professional keyboardist. W. Randolph Franklin (Rensselaer Polytechnic Institute) has climbed all 115 mountains over 4,000 feet on the East Coast. Chuck Gartland (Kent State University) and Andreas Griewank (Yachay Tech University) reminisced about when they taught together. Griewank also enthusiastically recalled a six-hour drive from New York to Pittsburgh to visit the C.F. Martin Guitar factory. Perhaps most surprisingly, Cynthia Phillips (Sandia National Laboratories) has won the Massachusetts State Taekwondo Championship and was invited to participate in the 1988 Olympic Games.

The distinguished applied mathematicians at the reception are also musicians, "relatively good dancers," baseball fanatics, and linguists — one has even met the Queen of England. They giggled when a friend pulled a funny face and asked engaging questions upon learning something new about a colleague. At a conference dedicated to the collaboration and sharing of knowledge in applied mathematics, it's easy to forget the full lives that exist beyond the convention center walls. Our community can flourish even more if we acknowledge and embrace the unique aspects of each other's lives.

SIAM Fellows have built their careers on their important contributions to the field of applied mathematics. When aspiring towards such a goal, SIAM students and young professors must realize that they are capable of achieving the same greatness. Despite the awards and grants and papers, these mathematicians are also humans, capable of making funny faces.

Izzy Aguiar earned her B.S. in applied mathematics and statistics from the Colorado School of Mines and is currently pursuing her M.S. in computer science at the University of Colorado, Boulder.

Attendees of the Fellows Reception at the 2017 SIAM Annual Meeting joke with one another and speak about their hobbies outside of mathematics. Photo credit: Izzy Aguiar.

FreeFem++: A High Level MultiPhysics Finite Element Software

By Olivier Pironneau

The finite element method (FEM) was invented shortly after computers as a natural framework for solid mechanics; the success of NASTRAN—a finite element analysis program—in the 1960s is well known. Variational methods were then popular among mathematicians in the analysis of partial differential equations (PDEs), which rendered FEM easily adaptable to other domains of physics, such as thermodynamics, fluid mechanics, and electromagnetism. However, the preparation of data at the time was a nightmare, and still is for many 3D applications. Thus, teaching FEM for PDEs was a challenge: one could easily spend half the course on triangulation and graphics methods.

Niklaus Wirth and the Finite Element Method

Dominique Bernardi and I wrote `FreeFem+` in the 1990s at the University of Paris VI to ease the teaching and prototyping of PDE algorithms. Upon Frédéric Hecht's introduction of the advanced automatic mesh generator, I realized the power of that module and asked him to partake in the `FreeFem++` project. We prioritized the user interface's ability to define the PDE and hide the numerical methods. We aimed for the highest possible level, nearest to the mathematical statements. For instance, a Stokes system in

$$-\Delta \mathrm{u} + \nabla p = 0, \quad \nabla \cdot \mathrm{u} = 0,$$

$$\int_\Omega p = 0, \quad \mathrm{u} \text{ given on the boundary},$$

discretized with the Hood-Taylor element and regularization of the pressure constraint, would be defined and solved by the code in Figure 1. The only non-intuitive statement is "`on(3,u=x*(1-x),v=0)`," referring as it does to the numbering convention of the four sides of the square, 3 being the top side. The last instruction generates Figure 2.

For nonlinear, time-dependent multiphysics systems, it is the user's responsibility to specify his/her scheme. For instance, Navier-Stokes equations in a 3D cube—up to time 5, discretized with an implicit Euler scheme, and with semi-linearization of the convection terms—would need a `problem` block—which only defines the PDE—to replace the `solve` block, which defines and solves the PDE, for reusability in a time loop (see Figure 3a, on page 8). This yields 3b. Notice the use of `macro` for clarity and the approach's generality. For instance, adding a temperature equation to simulate natural convection would cost only a few extra lines of code.

Parsing regular expression, such as `x*(1-x)`, requires techniques that are at the core of compilation. *Algorithms and Data Structures* by Niklaus Wirth explains it well [2].

The syntax of `FreeFem++` scripts obeys the rule of LL(1) computer languages. It can be parsed in one pass, generating and interpreting a stack of instructions at execution time, much like Pascal. It does not generate bytecode, meaning it is not possible to embed this Navier-Stokes solver in another application written in MATLAB, for instance, without including the whole `FreeFem++` environment. Incorporating MATLAB into `FreeFem++` may be easier.

The Advantages of C++ to FreeFem++

As the years passed, more and more instructions were added to `FreeFem++`, which now has a considerable subset of C++ instructions in its syntax (useful to open and close files, manipulate matrices, etc.). Actually, the name `FreeFem++` comes from the fact that the software was entirely rewritten in C++ in 2002 and again in 2010. It makes extensive use of operator overloading and templates, something known as generic programming. Consequently, including complex numbers and adding 3D problems—vector problems like Stokes'—was easy, since the syntax is the same as for the heat equation.

SOFTWARE AND PROGRAMMING

`FreeFem++` is a collaborative open-source effort like its cousin `FEniCS`,[1,2] (Ridgeway Scott, the latter's co-creator, is a friend), but Hecht does 95 percent of the development, and I am still wondering how one man can do so much. I'd be tempted to say "try both and tell us which is best," but that is difficult; these are easy to try but time-consuming to master. Both software packages are huge, with `FreeFem++` incorporating 130K lines of code. Most well-known libraries of numerical analysis are integrated and can be called from within `FreeFem++`, including `blas`, `mpi`, `Linpack`, `Arpack`, `GSL`, `PETSc`, `MUMPS`, `UFMPACK`, several optimization modules like `CMAES` and `IPOPT`, and the 3D mesh generator `TetGen`. The documentation contains 418 pages that nobody wants to read, and you truly don't have to for simple cases.

```
load "msh3" ; load "MUMPS"
int[int] ll=[1,1,1,1,1,2]; // puts labels on faces of the cube
mesh3 Th=cube(10,5,10,label=ll);
fespace Vh(Th,P2);
fespace Qh(Th,P1);

macro grad(u) [dx(u),dy(u),dz(u)]//
macro Grad(u1,u2,u3) [grad(u1),grad(u2),grad(u3)]//
macro div(u1,u2,u3) (dx(u1)+dy(u2)+dz(u3)) //
macro Ugradv(u1,u2,u3,v) ([u1,u2,u3]'*grad(v))//
macro UgradV(u1,u2,u3,v1,v2,v3) [Ugradv(u1,u2,u3,v1),Ugradv(u1,u2,u3,v2),Ugradv(u1
    ,u2,u3,v3)]//

real nu=0.01, dt=0.2; // Leads to Reynolds number 100 and 25 time steps
Vh u1=0,u2=0,u3=0,  uh1,uh2,uh3,  uold1,uold2,uold3;
Qh p, ph;

problem  NavierStokes([u1,u2,u3,p],[uh1,uh2,uh3,ph]) =
    int3d(Th)( [u1,u2,u3]'*[uh1,uh2,uh3]/dt
             + UgradV(uold1,uold2,uold3,u1,u2,u3)'*[uh1,uh2,uh3]
             +  nu*( Grad(u1,u2,u3):Grad(uh1,uh2,uh3))
             - div(u1,u2,u3)*ph - div(uh1,uh2,uh3)*p  - 1e-10*ph*p )
  - int3d(Th)( [uold1,uold2,uold3]'*[uh1,uh2,uh3]/dt)
  + on(1,u1=0,u2=0,u3=0)   +  on(2,u1=1,u2=0,u3=0);

for(real t=0; t< 5; t+=dt){
    uold1=u1; uold2=u2;uold3=u3;  NavierStokes;
}
plot(p);
```

Figure 2. `Freefem++` *code for the Navier-Stokes equations in 3D.*

```
// The regularized driven cavity problem on the unit square
mesh Th=square(10,10);   // using a 10x10 triangular grid
fespace Vh(Th,P2);   // defines the P2 Finite Element Space
fespace Qh(Th,P1);
Vh u,v,uh,vh;  // Velocities and test functions are element of Vh
Qh p,ph;   // Pressure and test pressure function are in Qh
solve aa([u,v,p],[uh,vh,ph], solver=CG)   // Variational form
      = int2d(Th)(  dx(u)*dx(uh)+dy(u)*dy(uh)
           + dx(v)*dx(vh)+dy(v)*dy(vh) + 0.001*p*ph
           -(dx(u)+dy(v))*ph - p*(dx(uh)+dy(vh)) )
        + on(3,u=x*(1-x),v=0) + on(1,2,4,u=0,v=0);
 plot([u,v],p, ps="stokes.ps");
```

Figure 1. `FreeFem++` *code for the cavity-driven Stokes problem in 2D.*

[1] https://fenicsproject.org/
[2] https://sinews.siam.org/Details-Page/the-fenics-project

See **FreeFem++** *on page 8*

FreeFem++

Continued from page 7

Strengths and Weaknesses

By comparison to its commercial competitor `COMSOL`,[3] `FreeFem++` is primitive for graphics but interfaced with `paraview` and others. Nonetheless, it does things that `COMSOL` cannot do. Both run on Mac OS, Windows, and Linux in integrated environments; `FreeFem++` runs on smartphones thanks to Antoine Le Hyaric.[4] However, we haven't found much use for this extension yet.

`FreeFem++` has the best open-source module for mesh adaptivity. One can also easily interpolate functions on different meshes, simplifying domain decomposition methods, for example. It has a robust implementation of the characteristic Galerkin method for convection terms, and a large library of triangular finite elements—$P^0 \ldots P^4$, RT, HCT, Edge, mixed—adapted to discontinuous Galerkin methods (DG) and a posteriori mesh refinements. The one thing FreeFem++ does not handle well is conservation laws with shocks. We have not yet been able to write a fast Riemann solver for a general hyperbolic system of any size; one may solve such PDEs with DG but doing so requires considerable expertise, which is not in the spirit of `FreeFem++`.

Many C++/Fortran90 toolboxes are potentially just as powerful, such as `feel++`,[5] deal.II,[6] and `Elmer`,[7] among others.[8] But in general, these require that users have a high level of programming ability. Field-specific languages are coming; the meta language `scala`[9] makes such a promise.

`FreeFem++` has at least 3,000 active users. Ph.D. students worldwide seem to love it; in France, almost all courses on numerical analysis for PDEs use `FreeFem++`. But it is also increasingly popular as a research tool because execution time is comparable with handwritten C++ codes, and development time is so much faster.

For instance, Figure 4a shows the iso-pressure surfaces of blood flow in an aorta using a mesh constructed from an MRI. A simulation of Maxwell equations in 3D (see Figure 4b), entirely written with `FreeFem++`, won the French supercomputing Atos-Fourier prize[10] because it scales perfectly on a parallel machine up to 10,000 cores. The next challenge is hiding parallel computing instructions from the user and automatically employing the computer's resource. Frédéric Nataf and Pierre Jolivet are working hard at it, with Hecht's help, of course!

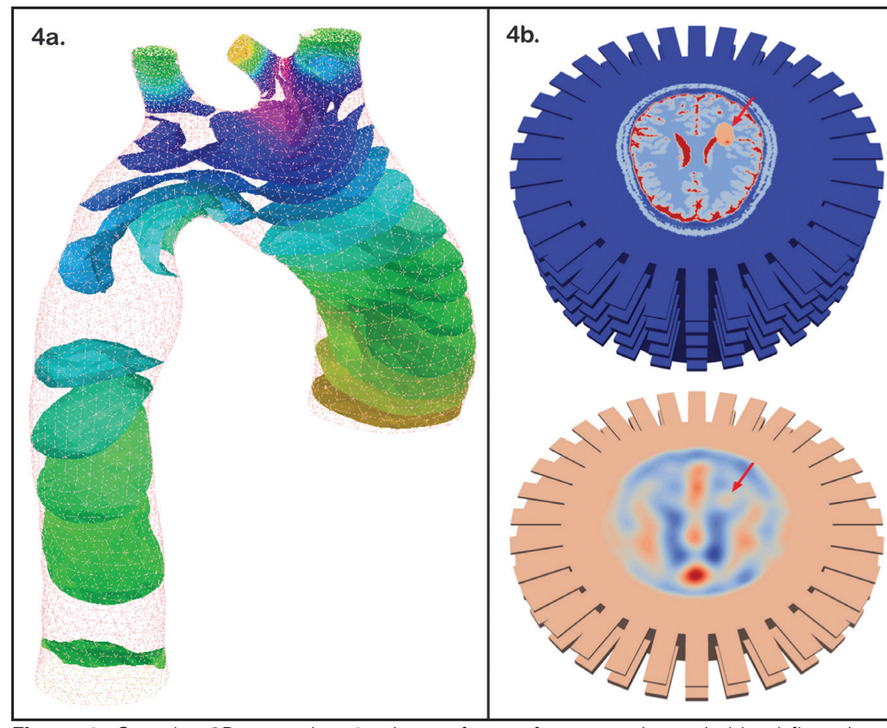


Figure 4. *Complex 3D examples.* **4a.** *Iso surfaces of pressure in aortic blood flow. Image credit: Olivier Pironneau. More details in [1].* **4b.** *Navier-Stokes flow: iso-pressure surfaces. Blood flow reconstruction in the brain by simulation of Maxwell equations. Top: Target permitivity of the brain generated by high-precision imaging. Bottom: Computer-reconstructed image from noisy microwave measurements. The arrows indicate the region of stroke. Image credit: Frédéric Nataf.*

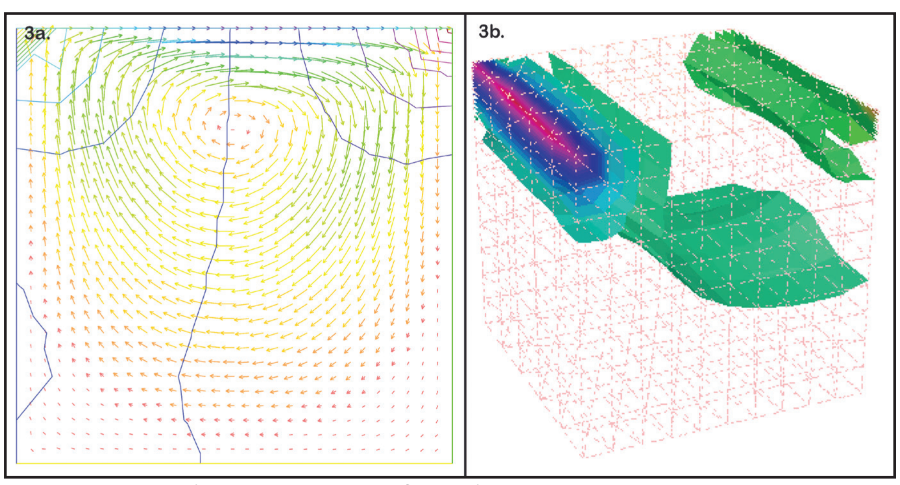


Figure 3. *Results of Figures 1 and 2.* **3a.** *Stokes flow: velocity vectors and pressure lines.* **3b.** *Navier-Stokes flow: iso-pressure surfaces. Image credit: Olivier Pironneau.*

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Olivier Pironneau is emeritus professor at the Université Pierre et Marie Curie in Paris, France. He is also a member of the French Académie des Sciences and co-author of eight books, including Computational Methods for Option Pricing (SIAM, 2005). Visit his website[11] for more information.

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[3] www.comsol.fr
[4] www.ljll.math.upmc.fr/lehyaric/ffjs
[5] www.researchgate.net/project/Fee
[6] www.dealii.org
[7] www.csc.fi/web/elmer
[8] https://en.wikipedia.org/wiki/List_of_finite_element_software_packages
[9] www.scala-lang.org
[10] https://www.ljll.math.upmc.fr/nataf/presentationLongue.pdf
[11] https://www.ljll.math.upmc.fr/pironneau/

Celebrating Mathematical Greats

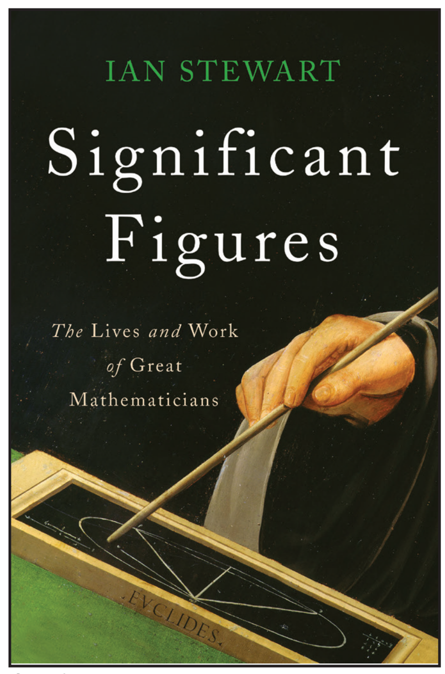
Significant Figures: The Lives and Work of Great Mathematicians. *By Ian Stewart. Basic Books, New York, NY, September 2017. 328 pages. $28.00.*

BOOK REVIEW
By James Case

Ian Stewart's latest venture into the realm of popular mathematics should meet, for many years to come, the need for an update of E.T. Bell's 1937 classic *Men of Mathematics*. Both books offer chaptered accounts describing the lives, times, and work of one or more great mathematicians. For obvious reasons, neither volume can include all who fit that description. Bell, for example, never speaks of Camille Jordan, and mentions Pafnuty Chebyshev and Siméon-Denis Poisson only in passing. Stewart says nothing of Hermann Minkowski, George David Birkhoff, or Norbert Wiener, and devotes no chapter to John von Neumann or Alexander Grothendieck. Both authors confine their attention to mathematicians deceased at press time, which explains Bell's neglect of David Hilbert along with Stewart's of (say) Stephen Smale, Andrew Wiles, and Grigori Perelman.

After brief introductions, both authors commence—naturally enough—with chapters on Archimedes. But Stewart follows with accounts of three other ancients (Liu Hui, Muhammad ibn Musa al-Khwarizmi, and Madhava of Sangamagrama) whose works were almost certainly unknown to Bell, and another on Gerolamo Cardano, whose formulas Bell mentions without attribution. The latter serves as Stewart's segue into the Renaissance/Enlightenment revival of scholarship, which he covers in significantly less detail than Bell. Following an obligatory chapter on Évariste Galois, Stewart introduces the first of three female mathematicians: Augusta Ada King, Countess of Lovelace. Why King, rather than Maria Gaetana Agnesi, whose book on differential calculus contains an early discussion of the curve known as "the witch of Agnesi;" Sophie Germain, whose contributions to number theory and elasticity were of lasting significance; or perhaps the ancient Hypatia of Alexandria?

King's claim to fame rests on her collaboration with Charles Babbage, inventor of the difference and analytical engines. She translated into English the published version of notes taken by Luigi Federico Menabrea at one of Babbage's lectures on the analytical engine. At Babbage's suggestion, King added commentary of her own, until the comments quite eclipsed the notes.


Significant Figures: The Lives and Work of Great Mathematicians. By Ian Stewart. Courtesy of Basic Books.

Her comments illustrated the machine's capabilities with a series of examples, the most ambitious of which involved the Bernoulli numbers, best known as the coefficients of $x^n/n!$ in the Taylor series expansion of $x/(1-e^{-x})$. They explained, at least in principle, how one could program the machine with punched cards—of the sort employed in Joseph Jacquard's then-celebrated loom—to function as a universal computer capable of carrying out any imaginable computation. Though one historian finds "not a scrap of evidence that Ada ever attempted original mathematical work," none can deny her importance as Babbage's interpreter and publicist. In later life, she partook excessively of wine, indulged in opium, had numerous lovers, and left gambling debts in excess of £2,000 upon her death from cancer at age 37. Yet nothing she ever did could dim Babbage's fascination with the woman he called "the Enchantress of Numbers."

Stewart's second leading lady of mathematics is Sofia Kovalevskaya. His account begins when Kovalevskaya turned up in Karl Weierstrass' Berlin office, hoping to become his pupil. She quickly astonished him with the originality of her solutions to problems he assigned, including a demonstration that solutions of the initial value problem for the backward heat equation are not unique. She possessed, as he later put it, "the gift of intuitive genius."

Although she published only 10 mathematical papers in her lifetime, Kovalevskaya's discoveries amazed the leading figures of her day. In partial differential equations, mechanics, and the diffraction of light by crystals, her work was penetrating, original, and technically proficient. She was appointed professor ordinarius at Stockholm University in 1889—a paid, tenured position—becoming the first woman to hold such a post in a northern European university. Soon thereafter, she was elected to occupy a chair in the Russian Academy of Sciences, another first. Kovalevskaya was arguably the foremost female scientist of her generation, eclipsed only by Marie Curie some two decades later. She died of influenza in 1891, less than a month after her 41st birthday.

The third of Stewart's trio of great female mathematicians is Amalie Emmy Noether. Born to a prosperous Jewish family in the sleepy Bavarian city of Erlangen, she was drawn to mathematics by her father, an accomplished geometer in his own right. Noether was indeed fortunate that her hometown university changed its rules in 1904, allowing women to matriculate on the same basis as men. Having attended lectures by her father and a few others for several years, she was able to obtain her degree that very year and decamp for Göttingen and the study of invariant theory under the eminent Paul Gordan. Though she was awarded her Ph.D. *summa cum laude* in 1907, Noether found herself ineligible to even apply for a teaching position at a German university, since women were not allowed to pursue habilitation. She therefore returned to Erlangen, where she helped her father with his courses and continued her own research. Her work paid off, and after seven years David Hilbert and Felix Klein summoned her back to Göttingen, then the center of the mathematical universe.

Noether is perhaps best known for a theorem she proved soon after returning to Göttingen, associating conservation laws with symmetries in the laws of nature: Time translation symmetry yields conservation of energy, space translation symmetry yields conservation of momentum, rotational symmetry yields conservation of angular momentum, and so on. Albert Einstein praised the theorem—proven in 1915—as a piece of "penetrating mathematical thinking." It has since become a standard meta-theorem of mathematical physics. In its original version, the theorem applies to Lagrangian motion.

Consider a particle moving on a line with Lagrangian $L(q,\dot q)$, where q is its position on the line and $\dot q$ is the velocity. The momentum of the particle is $p=\partial L/\partial \dot q$, while the applied force is $F=\partial L/\partial \dot q$. The so-called Euler-Lagrange equation then asserts that the rate of change of momentum $\dot p$ equals the force F.

Next, suppose the Lagrangian L to be invariant under a one-parameter family $\{T_s\}$ of symmetries, so that $L(T_s q, T_s \dot q)$ remains constant. Then $C=p\cdot d(T_s q)/ds$ is a conserved quantity, since $\dot C$ vanishes identically. It does so because

$$\dot C = \dot p \cdot \frac{d(T_s q)}{ds} + p \cdot \frac{d(T_s \dot q)}{ds},$$

which becomes

$$\dot C = \frac{\partial L}{\partial q}\cdot\frac{d(T_s q)}{ds} + \frac{\partial L}{\partial \dot q}\cdot\frac{d(T_s \dot q)}{ds} = \frac{dL(T_s q, T_s \dot q)}{ds} \equiv 0$$

upon substitution of $\partial L/\partial \dot q$ for p, $\partial L/\partial q$ for $\dot p$, and appeal to the chain rule.

There is, of course, nothing special about a particle traveling on a line. One can easily generalize the proof (via subscripts) to collections of particles in higher-dimensional space, or even on manifolds. However, it only works if the symmetries in the group $\{T_s\}$ are time-independent, so that

$$\frac{d}{dt}\left[\frac{d(T_s q)}{ds}\right] = \frac{d}{ds}[T_s \dot q].$$

More complicated versions of Noether's theorem are required to handle groups of time-dependent symmetries, such as the Lorentz transformations of special relativity.

In 1919, the University of Göttingen finally capitulated to Hilbert's nagging pressure, first approving Noether's habilitation and then conferring the rank of Privatdozent. Soon thereafter she changed fields, taking up the study of abstract algebra and bringing it—in collaboration with Bartel Leendert van der Waerden, among others—to roughly the condition described in his 1931 text *Modern Algebra*. Noether later developed an interest in topology, becoming a founder of algebraic topology.

She remained quite happily at Göttingen until 1933, when the Nazis dismissed all Jewish people from university positions and obliged her to move to Bryn Mawr College near Philadelphia, Pa. The new location was admirably convenient to the then-new Institute for Advanced Study in Princeton, N.J., where Noether was regularly invited to speak. She died in 1935, of complications following a cancer operation.

David Hilbert, Kurt Gödel, Alan Turing, Benoit Mandelbrot, and William Thurston are among the mathematicians who lived too long for inclusion in Bell's volume. Stewart devotes a chapter to each. Though Srinivasa Ramanujan died in 1920, his work was not yet sufficiently well-enough known (or widely-enough understood) in 1937 to justify inclusion by Bell. Stewart rectifies the omission.

Because nearly all the aforementioned mathematicians have been the subjects of recent biographies, there seems no need to recap Stewart's summaries here. Suffice it to say that Stewart has written a worthy successor to Bell's far-from-outdated classic — one that may in time incline an even greater number of young readers to pursue careers in mathematics. Meanwhile, working professionals curious about the lesser-known masters profiled in the book, yet lacking the time or inclination to digest an entire biography, will find *Significant Figures* both informative and entertaining.

James Case writes from Baltimore, Maryland.

Preparing for a Prosperous Career in Industry

AN17 Professional Development Panel Offers Advice for Students and Recent Ph.D.s

By Lina Sorg

CAREERS IN MATHEMATICAL SCIENCES

Upon graduation, recent applied mathematics Ph.D.s are routinely confronted with what often feels like a career-defining choice: academia or industry? Some choose to pursue an industry-based career at a company, government institution, or nonprofit, while others work towards a tenure-track position at a university. At the 2017 SIAM Annual Meeting, held in Pittsburgh, Pa., this July, a professional development session entitled "Beyond Research Training: Critical Skills for Careers in Industry" offered attendees advice for becoming more competitive in industrial positions. Drawing from their own professional experiences, the five panelists shared examples of industrial careers and proposed tips on valuable skills, industry trends, and opportunities for growth.

Employment at U.S. national laboratories is a common non-academic path, with positions for both postdoctoral researchers and early and mid-career level professionals. Lois Curfman McInnes (Argonne National Laboratory) and Carrie Manore (Los Alamos National Laboratory) espoused the versatility of research at laboratories. "It's a really fun, interdisciplinary environment for applied math and computational research," McInnes said.

Manore, who focuses primarily on disease modeling, echoed this thought, adding that the national labs work on a remarkably wide variety of projects in chemistry, biology, and other fields. "Don't count yourself out of the national labs because you don't feel like you fit exactly with what they're doing," she said. "If you have the math and programming skills, they're open to you switching directions. And once you get there, there are a lot of opportunities to do so."

Manore encouraged attendees to apply for summer workshops and programs at the labs, which match students with potential future mentors. "It's a good way to get experience that could apply to other labs, but also for the lab to get a feel as to whether it's a good fit," she said. Labs are often willing to take a chance on students and postdoctoral fellows because they are fairly inexpensive. Like other facets of industry, labs are characteristically focused on deliverables, and put a considerable emphasis on applicants' published papers or self-written software.

Recently retired from AstraZeneca, Jeff Saltzman spent 18 years at Los Alamos and 13 years at Merck, where he developed an applied mathematics group. He provided input on valuable resources for a career in pharmaceuticals research, such as mathematical biology courses, and cautioned that job-seekers should be careful with titles. "If you're looking for jobs in the pharmaceutical industry, you'll rarely see a job labeled 'applied mathematician,'" Saltzman said. "Yet there's a rich array of work that is applied. When looking at job boards, look beyond jobs labeled 'applied mathematics.'" Instead, he suggested searching for jobs requiring biology or math experience.

Unsurprisingly, attendees were particularly interested in the differences between academic and industry-based positions. Though academia is normally thought to offer more freedom, Saltzman indicated that the two are not necessarily as dissimilar as they appear. "Industry seems highly constrained, like you have to work on a specific project," he said. "But you end up steering the work to the area that you want to work on." Because there is less pressure to publish in industry, Saltzman finds the publications to be more meaningful. While transitioning between the two settings is possible, he suggests establishing a personal timeline and deciding in advance how many—and what types—of publications would be necessary for a switch. "It's not going to be a fast transition," he warned. "But I know a lot of people who transitioned from industry and academics and they were very deliberate about it. Many academic departments are looking for people with some industry experience so their students can learn something practical."

Henry Warchall initially planned to spend only two years in a temporary rotator position as a program director at the National Science Foundation (NSF), but enjoyed the work so much that he stayed on. "The Division of Mathematical Sciences at the NSF is actively promoting the idea that graduate students in mathematics particularly, and statistics as well, should be exposed to at least the possibility that they could pursue a non-academic career," he said. "When you're thinking of applying for jobs in industry, you should bear in mind that you have something rare. You have the ability to approach an open-ended problem and potentially solve it."

Panelists encouraged the audience to dabble in software, as industry employers look favorably upon programming experience. "There's a lot of work done with interpretive language these days," Saltzman said, citing Python, R, MATLAB, and Mathematica as frequently-used programming languages. He urged students to have patience, be flexible, and experiment with different software. "It's not so hard to learn R if you've learned Python," he said. "In the end, it's all about finding the right code with which you can share and communicate your thoughts."

Efficient use of software is particularly useful in collaborative industrial settings, which bridge diverse fields increasingly linked by big data. "I believe software is really a foundational part of science and engineering as a means to collaborate across interdisciplinary boundaries and sustain those collaborations," McInnes said.

One trend in the discipline replete with opportunities is big data. Saltzman touched on the changing nature of high-performance computing, given the explosion of data in nearly every field. "What you're seeing in companies is cloud services like Amazon," he said. "There's a huge demand for people to process information, especially genomics and medical information, at a level that demands them to really understand."

Kristin Bennett (Rensselaer Polytechnic Institute) reiterated Saltzman's sentiments about data processing and spoke of the value mathematicians bring to the new age of big data. "Industry wants you, pharma wants you, everybody wants you," she said. "Virtually we are collecting data in every industry — for finance, for drug discovery, for government, for roads. Data [utilization] is a fundamental skill that we all need to use in the future."

Data processing has applications in healthcare, disease mediation, finance, portfolio and investment management, and marketing, in addition to computer science and machine learning. Internships and workshops offer students opportunities to work with data, and

See **Industry** *on page 12*

Parabola of Safety and the Jacobian

MATHEMATICAL CURIOSITIES
By Mark Levi

I would like to describe an alternative (to the standard) way of finding the parabola of safety, i.e., the boundary of the set of points vulnerable to a projectile launched with a given initial speed v from a fixed point (see Figure 1).

A conventional derivation goes by maximizing—over all launching angles—the elevation y above a given x. The result happens to be

$$y = \frac{v^2}{2g} - \frac{g}{2v^2}x^2. \qquad (1)$$

Alternatively, one can employ the standard way to find an envelope of the family of parabolic trajectories, parametrized by the angle of launch.

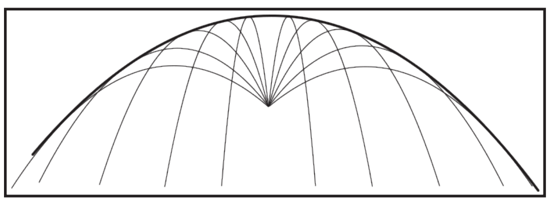


Figure 1. *The parabola of safety, given by (1).*

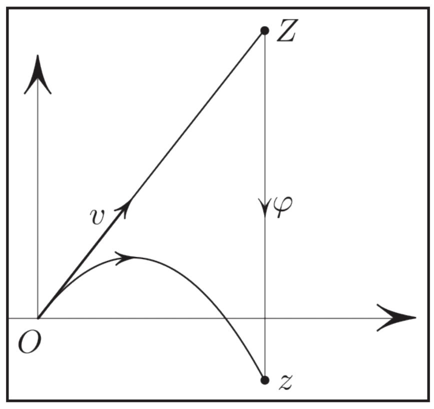


Figure 2. *The "sagging" map.*

I would like to present perhaps an unconventional approach which has some advantages, one of them being added geometric insight.

The key idea is to consider the "sagging map" $\varphi := Z \mapsto z$, illustrated in Figure 2 and defined as follows. For any given $Z \in \mathbb{R}^2$, we consider the *gravity-free* motion with initial speed v (fixed throughout), which hits Z. Let z be the point hit by a gravity-laden trajectory (with the same initial velocity) in the same travel time as Z is hit. In short, φ assigns the gravity-laden destination z to a gravity-free destination Z, the two trips sharing the initial conditions and travel time. Under the map φ, straight lines sag to become parabolas. The main point (explained subsequently in more detail) is this: *the zero set of the Jacobian of φ is mapped under φ onto the safety parabola* (see Figure 3). This zero set happens to be a straight line.

Denoting $Z = (X, Y)$, $z = (x, y)$, the map φ is given by $x = X$, $y = Y - \frac{gt^2}{2}$, where $t^2 = (X^2 + Y^2)/v^2$, so that φ is given by

$$\begin{cases} x = X \\ y = Y - \dfrac{g}{2v^2}(X^2 + Y^2). \end{cases} \qquad (2)$$

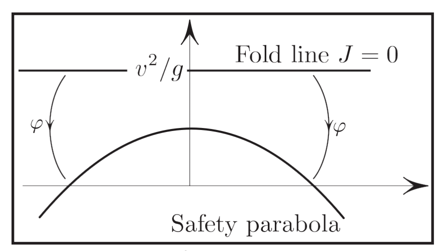


Figure 3. *The safety parabola is the image of the straight line under the "sagging map."*

The Jacobian J of φ computes to be

$$J = 1 - \frac{gY}{v^2},$$

and $J = 0$ gives $Y = v^2/g$, the horizontal straight line at twice the maximum rise of the projectile. And the safety parabola (1) is simply the image of this line under φ (see Figure 3). Inserting $Y = v^2/g$ into (2) confirms this. What justifies the statement of Figure 3? In Figure 4, the ray *OB* and the line $J = 0$ are transversal at *A*. But their images are tangent at

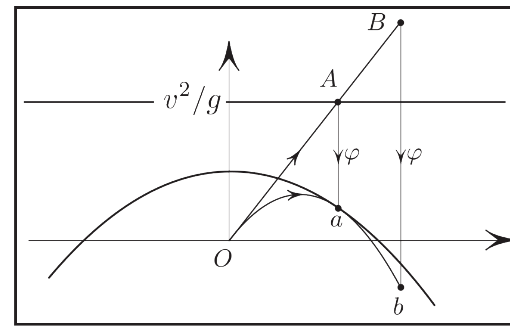


Figure 4. *What is the φ-image of the set $J = 0$ the envelope of trajectories? Two transversal at A lines map to two tangent at a lines due to the vanishing of J at A.*

$a = \varphi(A)$ thanks to the vanishing of J, which explains why the image of $J = 0$ is the envelope of the family of trajectories.

As a direct elementary observation, apart from Jacobians, φ is a composition of two simple maps: first, a fold around the line $J = 0$, namely $(x, y) \mapsto (x, y - ay^2)$ (with $a = g/2v^2$, see (2)), followed by the parabolic shear $(x, y) \mapsto (x, y - ax^2)$, see (2). Under the first map, the rays from the origin turn into parabolas tangent to the same horizontal line $y = v^2/2g$. Under the second map, the latter line becomes a parabola, to which the images of the trajectories are tangent — hence, the safety parabola.

As a concluding item, Figure 5—together with its caption—offers a puzzle.

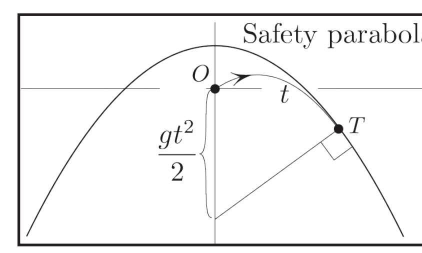


Figure 5. *A puzzle. Show, without calculation, that the normal to the safety parabola at the point of tangency T intersects the vertical at the distance $gt^2/2$ from the launch point O, where t is the time of flight from O to T.*

The figures in this article were provided by the author.

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G2S3 Participants Study Data Sparse Approximation and Algorithms

By Gitta Kutyniok, Jörg Liesen, and Volker Mehrmann

Students at the eighth annual Gene Golub SIAM Summer School battled the forces of nature when they paddled through headwinds across two large lakes in Berlin, Germany. Photo credit: Carlos Echeverría.

Many large-scale problems stemming from the most diverse areas of science and engineering have become tractable only because of the existence of data-sparse representations or approximations, and their exploitation in suitable computational methods. In this exciting and rapidly-developing area, researchers try to exploit the fundamental observation that data—as well as functions and operators—can often be represented, or highly accurately approximated, by only a small number of relevant features. Groundbreaking advances in this context have been seen in recent years using, for instance, tensor-based methods, as well as low-rank and low-order representations and approximations.

The eighth Gene Golub SIAM Summer School, held in late May, focused on this emerging area and was titled, "Data Sparse Approximation and Algorithms." The Akademie Berlin-Schmöckwitz, on the southeastern tip of Berlin, Germany, hosted the international and diverse group of participants, consisting of 45 highly-qualified master's and Ph.D. students from 18 different countries. The school was held in conjunction with the SIAG/LA International Summer School on Numerical Linear Algebra.

Students partook in four one-week courses, with lectures in the morning and exercise sessions in the afternoon. Deanna Needell (University of California, Los Angeles) discussed topics in optimization that rely on sparsity, with a wide range of applications that include medical imaging, sensor network monitoring, and video. Bernhard G. Bodmann (University of Houston) taught a course devoted to sparse recovery and the geometry of high-dimensional random matrices. He led the students from the restricted isometry property of Emmanuel Candès, Justin Romberg, and Terence Tao to results on the randomized construction of sensing matrices due to Mark Rudelson and Roman Vershynin. Lars Grasedyck (RWTH Aachen University) introduced participants to low-rank tensor formats for the data-sparse representation of higher-order tensors and multivariate functions. The students learned how to apply representations like Canonical Polyadic, Tucker, Tensor Train, and Hierarchical Tucker in model reduction, uncertainty quantification, high-dimensional partial differential equations, and analysis of big data. Serge Gratton (University of Toulouse, INP-IRIT) dealt with numerical methods for inverse problems in the geosciences. His lectures focused specifically on reducing computational complexity by considering problem structures.

Fittingly, the school was held in the spirit of the late Gene Golub, with many interactions between lecturers and participants. More than 20 students exhibited their own work in the poster session. Ample opportunities for joint activities existed due to the Akademie's location and the beautiful early-June weather. Volleyball and swimming in the lake were especially popular among the students. The "Gene Golub Class of 2017" also mastered challenges in real-world engineering and with the forces of nature. Participants built rafts from scratch with provided materials during one of the activities. When the moment of truth arrived, all five rafts built by the group floated smoothly on the water. During a canoe trip in the second week, the students paddled through some serious headwinds when crossing two large lakes. One certainly does not have to worry about the skills and determination of this new generation of computational scientists!

45 master's and Ph.D. students from 18 different countries participated in the eighth Gene Golub SIAM Summer School, held in Berlin, Germany, this summer. Here they pose—along with the organizers and lecturer Bernhard Bodmann—in front of the "Herrenhaus," the main building of the Akademie Berlin-Schmöckwitz. Photo credit: Carlos Echeverría.

In addition to the generous funds provided by SIAM out of Gene Golub's bequest, the school was supported by the U.S. National Science Foundation, the Einstein Center for Mathematics Berlin, the Berlin International Graduate School in Model and Simulation based Research, and MathWorks.

Gitta Kutyniok, Jörg Liesen, and Volker Mehrmann, all from the Institute of Mathematics at the Technical University of Berlin, co-organized the 2017 Gene Golub SIAM Summer School.

Industry

Continued from page 10

a significant portion of math industry workshops are data-focused. "Right now, there's a huge shortage of people who can do data analytics," Bennett said. "I'm encouraging you to get even a little bit of data experience. This can be a differentiating factor to get your foot in the door with companies and open up opportunities in the future. If you can do data, you can do anything."

Communication is also a critical skill for mathematicians in industry. Warchall emphasized the importance of understanding your audience, both when interviewing and during employment. "When you're interviewing for a job, you have to prepare to talk to people who are not in your field," he said. "They might not even be mathematicians. I've seen a lot of certifiably brilliant mathematicians who have failed miserably in public talks." Warchall then introduced a helpful analogy. "Pretend it's like teaching," he said. "Think about what students know and don't know, and plan your lessons from there. Mathematicians tend to assume an awful lot about their audience, and you really can't. There are lots of things that people don't know that you know really well."

Once securing a position, strong communication skills remain just as imperative. "Often times when you're working in science projects in Department of Energy labs, you'll be working with people who have complementary skills or expertise," McInnes said. While learning each other's strengths and developing joint skills takes time, it is undoubtedly rewarding — especially because industry and business jobs generally emphasize interdisciplinary work. Each discipline also commonly has a slightly different communication technique, which Manore admitted can be off-putting at first. "Be prepared for those differences in communication styles and try not to be offended," she said.

Upon establishing oneself in a given area, there are naturally opportunities for growth. Upward mobility in industry is typically first lateral, in that one moves to a different job before directly advancing. In some cases, industry Ph.D.s will ultimately find themselves in managerial positions. "I think it's almost impossible to avoid management in any of these fields," Warchall said. "That's not true without exception, but I think the expectation is that you'll eventually turn into a leader." Saltzman clarified that he encountered different levels of leadership during his time at Los Alamos, given practical constraints in the number of managers an organization can have. "I thought everything was management in industry," he said. "But it turns out there are parallel tracks as well. For every manager, you should have 10 people who are doing work."

The panel concluded with a brief discussion on funding. "I think there's an illusion that working for the government, it's easier to get funding than working for industry," Saltzman observed. "It's not. If you think you're going to make a decision based on work environment, the variation within any particular area is much greater than across." When asked about industry job security given the lack of tenure, Bennett had advice pertinent to almost any career field. "You should go for the position of what interests you," she said. "Not what you think will be most secure."

Lina Sorg is the associate editor of SIAM News.