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Exploring Connectivity in the Brain's Network of Neurons

By Eric Shea-Brown

The brain has long presented an amazing challenge for applied mathematics: A complete model would require ~10^{11} highly nonlinear neurons, interacting nonlinearly and stochastically through rapid voltage fluctuations (called "spikes") on a complex and dynamic network. But we are in an especially wonderful time for the field, as technical breakthroughs are yielding unprecedented data about both the brain's network architecture and about its activity. There is a major opportunity to integrate this data into a new understanding of how the connectivity among neurons leads to their collective dynamics and, eventually, to their astonishing collective function. Mathematics and scientific computation will play a key role; researchers worldwide, including members of the SIAM community, are hard at work. Our group and our collaborators have the joy of being a part of this effort, and I describe some of our experiences in what follows.

Emerging data on the network of connections among the brain's neurons is revealing a sparse network that is complicated but highly nonrandom (Figure 1). Numerous features of "complexity" in the network have been identified, ranging from heavy-tailed distributions of connection strengths to "small-world" properties [1]. An irresistible question arises: What is the impact of this complex network architecture on the collective dynamics of the network?

In the next breath, we realize that there is no one answer to this question, and that what we find will depend on (at least) (1) the scale at which we characterize the dynamics and (2) assumptions about the dynamics of the individual neurons. Here, I discuss what can be learned when we make the following two choices: (1) For the scale, we consider collective dynamics across the network as a whole. We characterize this by the level of correlation, or *coherence*, C, in the spikes emitted from pairs of neurons, averaged across the network. The value of C can determine, for example, how strongly the spikes of a network of neurons work together to drive spiking in other cells downstream. (2) For the dynamics, we assume that the spiking is stochastic and irregular at each cell. The results described here are due to work by B. Lindner, B. Doiron, and A. Longtin [5] as well as by Stefan Rotter and colleagues [6], and our team—featuring Krešo Josić and two (former) students, James Trousdale and Yu Hu.

We begin with a nonlinear stochastic differential equation for each neuron. The equations are coupled into a network via impulses that occur each time a neuron spikes. When these spikes occur frequently enough, the system can be approximated by a stochastic (point) process, in which the spiking times of each cell are linearly modulated by the past inputs it received.

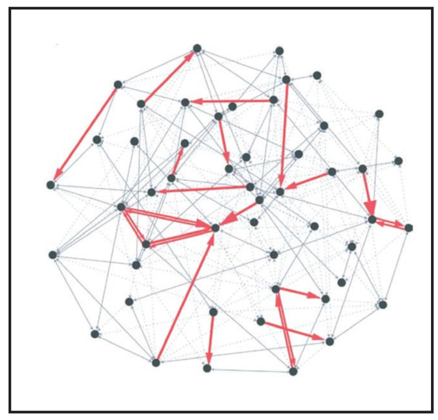

Figure 1. *Statistical reconstruction of sparse network of neurons. From S. Song, P.J. Sjöström, M. Reigl, S. Nelson, and D.B. Chklovskii,* Highly nonrandom features of synaptic connectivity in local cortical circuits, *PLoS Biol., 3 (2005), e68.*

Use of transform methods then yields an explicit solution for the coherence C [2, 6, 7]. This solution involves the connectivity graph W. At first glance, then, it seems that we have what we were after: a link between network architecture and dynamics, which we write $C = f(W)$.

This link, however, is very difficult to use. We rarely have access to the full W for a neural network; with N cells, a direct approach requires ~N^2 measurements to test whether and with what strength each pair is connected, and this is often beyond the limits of experiments. What we need is to isolate the right "summary statistics" of W that successfully predict C.

The first step is to realize that $f(W)$ can be expanded in terms of network connectivity motifs, or small subnetworks embedded within the larger connection graph (Figure 2). Notably, such motifs have been quantified in brain data (see Figure 1), and shown to be over-expressed compared with expectations under random connectivity. This expansion shows that C is determined by the frequency of these motifs—that is, the number of times they occur in the network [6, 7]. Thus, we have a link between the statistics of connectivity and network

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Imaging Sciences 2014

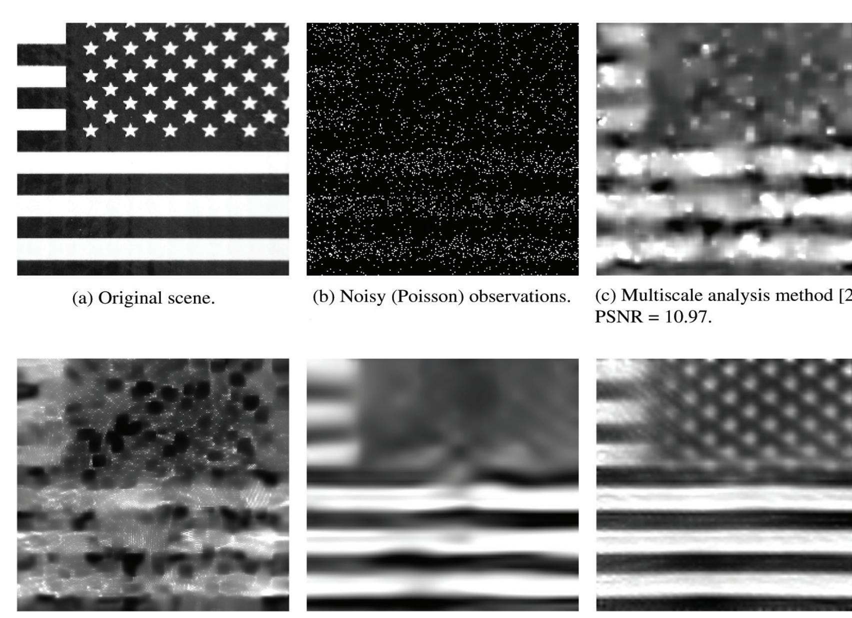
The Dark Side of Image Reconstruction

Emerging Methods for Photon-limited Imaging

By Rebecca Willett

Many scientific and engineering applications rely on the accurate reconstruction of spatially, spectrally, and temporally distributed phenomena from photon-limited data. Night vision, with small amounts of light incident upon a camera's aperture, is a classic example of photon-limited imaging. Photon limitations are also pervasive in fluorescence microscopy. Here, high-quality raw images with large photon counts require (a) long data acquisition times that limit experimental throughput and preclude moving specimens, (b) high laser power that kills living specimens, or (c) large quantities of fluorescent dye that are toxic for living cells. Photon-limited spectral imaging, in which we measure a scene's luminosity at each spatial location for many narrow spectral bands, plays a critical role in environmental monitoring, astronomy, and emerging medical imaging modalities, such as spectral computed tomography.

When the number of observed photons is very small, accurately extracting knowledge from the data requires the development of both new computational methods and new theory. This article describes (a) a statistical model for photon-limited imaging, (b) ways in which low-dimensional image structure can be leveraged for accurate photon-limited image denoising, and (c) implications for photon-limited imaging system design.



(a) Original scene. (b) Noisy (Poisson) observations. (c) Multiscale analysis method [22]; PSNR = 10.97.

(d) BM3D [8]; PSNR = 11.12. (e) Anscombe + BM3D [16]; PSNR = 13.91. (f) Poisson non-local PCA [18]; PSNR = 15.99.

Figure 1. *Photon-limited image denoising. The multiscale method in (c) was representative of the state of the art in the early 2000s; since then, methods like BM3D [8], which exploit low-rank structure among the collection of all patches in an image, have emerged at the forefront of image denoising. Simply applying methods like BM3D (which were designed for Gaussian noise) directly to Poisson images yields poor results, as shown in (d). Applying the Anscombe transform to Poisson data generates approximately Gaussian data that can be processed using tools like BM3D, as demonstrated in (e). Methods designed explicitly for Poisson data can reveal additional structure, as shown in (f).*

Problem Formulation

Imagine aiming a camera at a point in space and observing photons arriving from that location. If you could collect photons for an infinite amount of time, you might observe an average of, say, 3.58 photons per second. In any given second, however, you receive a random quantity of photons, say 3 or 10. The number of photons observed in a given second is a random quantity, well modeled by the Poisson distribution. We would like to estimate this average photon arrival rate based on the number of photons collected during a single second for each pixel in a scene. We can model the observed photons over an arbitrary period of T seconds (cf. [20]):

$$y \sim \text{Poisson}(TAf^*)$$
or
$$y_i \sim \text{Poisson}\left(T\sum_j A_{ij} f_j^*\right), \qquad (1)$$

where $f^* \in \mathbb{R}_+^p$, with $\|f^*\|_1 = 1$, is the true image of interest, with p pixels; $A \in [0, 1]^{n \times p}$ is an operator corresponding to blur, tomographic projections, compressed sensing measurements, or any other linear distortion of the image, where n is the number of photon sensors in our imaging system; and $T \in \mathbb{R}_+$ corresponds to the data acquisition time or overall image intensity.

That is, y_i is the observed number of photons at detector element i, and f_j^* is the average photon arrival rate associated with pixel j in the scene. **Our goal is to estimate f^* from y.**

Accurate estimation requires simultaneously leveraging Poisson statistics, accounting for physical constraints, and exploiting low-dimensional spatial structure within the image.

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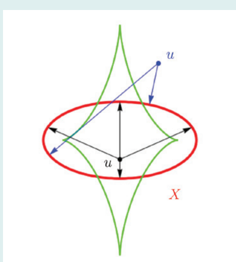
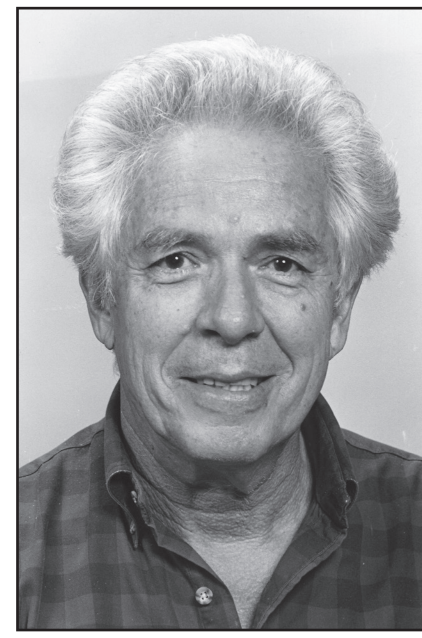


Obituaries



Harold Kuhn (1925–2014). Photo courtesy of Princeton University.

Harold Kuhn, a professor emeritus of mathematical economics at Princeton University and an early president of SIAM, died on July 2. He was 88.

Born in Santa Monica in 1925, to parents of decidedly limited means, Harold Kuhn attended public schools in Los Angeles during the Great Depression. Because of the scarcity of jobs at the time, the chemistry and physics courses at LA's Manual Arts High School were taught by PhD scientists. As a result, Kuhn's talents were quickly recognized, and arrangements were made for him to enter the California Institute of Technology on his graduation, in 1942. There, too poor to pay room and board, he was the only member of the freshman class permitted to live off campus.

Drafted into the army in the summer of 1944, Kuhn was selected for Japanese-language training at Yale. According to his son Nick, now a professor of mathematics at the University of Virginia, Harold discovered on a trip to Japan many years later that he could speak the language well enough to ask questions, but not well enough to understand the answers. In fact, Nick reports that his father was good at foreign languages. On occasion, he gave talks in French and Italian, and as an undergraduate he translated a new novel by Hermann Hesse from German to English as an independent project. While at Yale, Harold used his first extended military leave to visit a friend at Princeton, where he sat in for a day on courses taught by Emil Artin, Claude Chevalley, and Salomon Bochner. He returned to Caltech after leaving the army, determined to win admission for graduate study at Princeton.

Kuhn enrolled at Princeton in the fall of 1947. He wrote his doctoral dissertation in group theory under the direction of Ralph Fox. Concurrently, he joined mathematics professor A.W. Tucker and fellow graduate student David Gale in a hastily organized summer project to study the suspected equivalence between linear programming and matrix game theory. That project, he later wrote, "set the course of my subsequent academic career, which has centered around the applications of mathematics to economics." In 1980, the three shared the John von Neumann Theory Prize of the Operations Research Society of America (now part of INFORMS) for their pioneering work in game theory and optimization.

The late 1940s and early 1950s were the glory years of game theory, both at Princeton and at the newly formed RAND Corporation, where the military applications of game theory were explored in depth. Von Neumann, Oskar Morgenstern, and Tucker from the Princeton faculty, along with Princeton graduate students Kuhn, Gale, Lloyd Shapley, John Nash, Robert Aumann, and Martin Shubik, joined RAND employees Melvin Dresher, Rufus Isaacs, Samuel Karlin, Leonard Berkowitz, and Wendell Fleming in exploring its potential.

Linear programs, it soon emerged, come in pairs. The canonical forms of the "primal" and "dual" problems are:

$$\text{maximize}_x \;\; c \cdot x \text{ subject to } Ax \leq b \text{ and } x \geq 0,$$
$$\text{minimize}_u \;\; b \cdot u \text{ subject to } uA \geq c \text{ and } u \geq 0,$$

in which A is an $m \times n$ real matrix, and b, c, x, and u are conformable real vectors. If the inequality constraints $Ax \leq b$ and $uA \geq c$ were replaced by equalities, the associated Lagrangian functions would be $L = c \cdot x + \lambda \cdot (b - Ax)$ and $\Lambda = b \cdot u + (c - uA) \cdot \mu$, suggesting that the decision variable u in the dual problem be identified with the vector λ of Lagrange multipliers in the primal, and vice versa.

With the introduction of additional conformable real vectors y and v of non-negative "slack variables," the two problems can be rewritten in the standard form,

$$\text{maximize}_x \;\; m = c \cdot x \text{ subject to } Ax - b = -y,$$
$$\text{minimize}_u \;\; M = u \cdot b \text{ subject to } uA - c = v,$$

and/or in Tucker's admirably concise "tableau" format, shown in the following graphic:

	x	−1	
u	A	b	= −y
−1	c	0	= m
	= v	= M	

From this it is immediately clear that $0 \leq v \cdot x + u \cdot y = M - m$ for all feasible solution quadruples x, y, u, v. Moreover, any such quadruple for which $m = M$ must include the optimal decision vectors x^* and u^* for both problems. This is the famous duality theorem of linear programming, which leads more or less directly to all manner of pivoting algorithms for solving linear programs, matrix games, and other computer-friendly problems.

By considering differentials $dm = \sum_i m_{x_i} dx_i$, . . ., the duality theorem can even be used to deduce the famous Kuhn–Tucker conditions of nonlinear programming, the result for which Kuhn is perhaps best known. It turned out later that the same conditions—though unpublished—had been discovered in 1939 by William Karush, a master's-degree candidate at the University of Chicago. As a result, the conditions are more properly known as the Karush–Kuhn–Tucker optimality conditions. After Karush, but before Kuhn and Tucker, Fritz John discovered a slightly more general set of optimality conditions, which hold even when the "constraint qualifications" postulated by Karush, Kuhn, and Tucker are violated. In later years, Kuhn made an extended effort to identify and acknowledge any and all claims of priority.*

Kuhn's favorites among his achievements include the perhaps less well-known "decision-tree" definition of a game. It is little known only because most interested parties—including a surprising number of professional game theorists—are unaware that the definition given by von Neumann and Morgenstern is quite different, being less succinct than Kuhn's. So quickly and completely did Kuhn's definition eclipse the original that the latter is now all but forgotten.

Another of Kuhn's pet results was the so-called "Hungarian method" of solving the "assignment problem," which asks for the allocation of n workers to n tasks that—given that worker i can complete task j at a cost of c_{ij} dollars—will finish the work most affordably. He called the method "Hungarian" because Dénes Kőnig and Jenő Egerváry had used it many years earlier to treat specific problems. In 2004, the editors of the journal *Naval Research Logistics* selected Kuhn's 1955 paper on the method as the paper "best representing the journal since its founding in 1954." "This pioneering paper," they wrote, "set a style for both the content and the exposition of many other algorithms in combinatorial optimization, and also directly inspired the primal–dual algorithm for more general linear optimization problems."

James Munkres revisited Kuhn's algorithm in 1957, observing that the computational complexity is $O(n^4)$; it was improved later by J. Edmonds and R.M. Karp to $O(n^3)$. Then, in 2006, it was discovered that Jacobi (1804–1851) had also solved the assignment problem, and that his solution had been published posthumously (in the original Latin) in 1890. With his accustomed grace, Kuhn responded to the discovery by presenting a seminar at Concordia University titled "The Hungarian Method for the Assignment Problem and How Jacobi Beat Me by 100 Years."

Harold was a charter member of SIAM—which was incorporated on April 12, 1952—and served as the organization's third president (1954–1955). In May 2009, he was named to the inaugural group of Fellows of SIAM, for seminal contributions to game theory and to linear and nonlinear programming, and for leadership of SIAM in its early years.

In the early 1950s, with the encouragement of SIAM founder Ed Block, Harold

*Kuhn's piece, "Nonlinear Programming: A Historical Note," appeared in *History of Mathematical Programming: A Collection of Personal Reminiscences*, edited by J.K. Lenstra, A.H.G. Rinnooy Kan, and A. Schrijver (CWI North Holland, 1991).

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Neural Connectivity

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dynamics. However, the link can be quite complicated. This is because many motifs of increasing size (and hence increasing difficulty to measure) often contribute to the expansion. As a consequence, truncating it to predict the coherence C based on smaller motifs—those motifs that are possible to measure, tabulate, and compare across brain areas—isn't guaranteed to work.

A key advance is to define new network statistics, which we refer to as *motif cumulants* [3, 4]. These isolate the excess frequency of each successively larger motif, above and beyond that which would be predicted from the smaller motifs that make it up (Figure 3). Intriguingly, the expansion of $C = f(W)$ can be resummed explicitly in terms of the motif cumulants. The result *is* possible to truncate—and typically yields quite accurate predictions for C, based on small (and hence measurable) motifs alone. This gives an appealing recipe for contrasting different networks: By counting the motifs that occur and comparing them according to the formula, we can understand why some produce more—or less—coherent dynamics.

That said, the theory is far from complete. One issue is that it isn't really a from-the-ground-up solution of the architecture–dynamics problem: The motifs described above capture "functional" as opposed to purely structural connectivity. For example, cells must be weighted according to their spiking rates when the motif cumulants are measured. A complete solution would predict these rates from the network architecture alone. Moreover, the theory requires a linearization of the point process model, around an operating point given by the spiking rates. This linearization will fail for networks at low spiking rates (rates cannot be negative), or those driven strongly out of equilibrium—inviting the (perturbative?) development of a nonlinear theory.

Another issue arises when we return to the definition of the dynamical coherence C, which describes the average correlation among cell pairs—i.e., given any two cells in the network, how likely they are to produce voltage spikes at similar times. But are these pairwise statistics the end of the story? A subject of resurgent interest is higher-order correlations in the spiking of sets of more than two neurons, beyond what could be predicted from results for the constituent cell pairs. We are working with Michael Buice to identify the network motifs that drive higher-order correlations, using quite beautiful tools developed in theoretical physics.

This connects to a larger question for the field. Just what is the right way to describe population-wide neural activity? The approach we just described predicts successive terms in a hierarchy of statistical moments. But it is not clear that this moment-by-moment approach is the most efficient way to describe the population-wide activity. For example, recent work has identified low-dimensional random process or dynamical systems models that capture population-wide activity via fluctuations in "common modes" of activity. Understanding the origin of this dimensionality reduction, and building a predictive theory of when it will occur, would be a fascinating step forward.

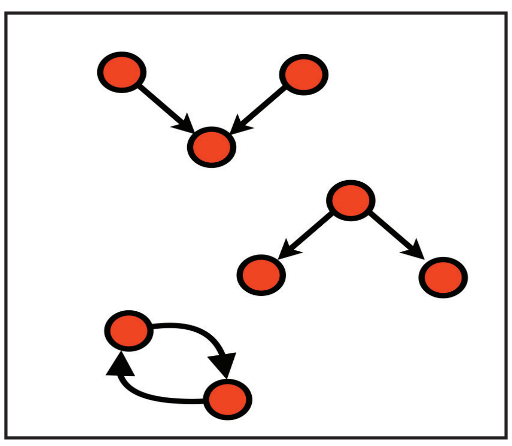

Figure 2 (left). *Network connectivity motifs.*

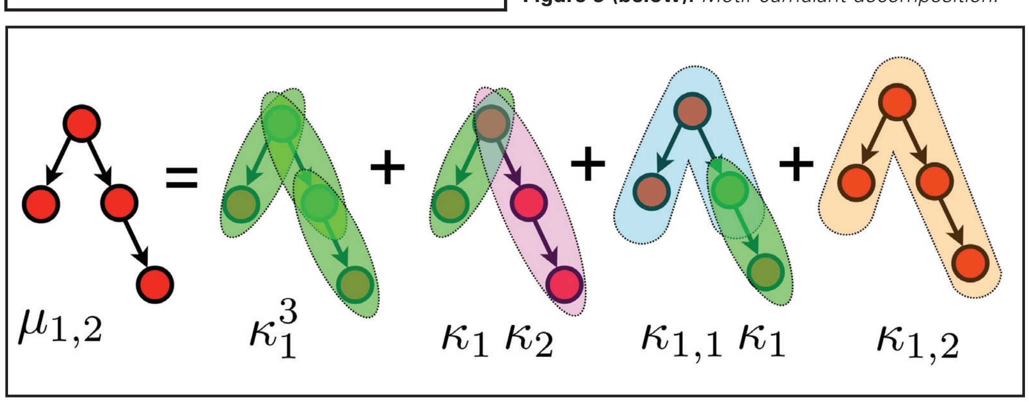

Figure 3 (below). *Motif cumulant decomposition.*

Beyond an understanding of how coherent spiking dynamics arise from network structure lies an even richer question: What do these dynamics mean for the encoding of sensory information? It has become cliché to say that spikes are the language of the brain, but this underlines the point: Patterns of spikes produced in neural circuits are the brain's only tool for making inferences about the sensory world and, eventually, driving behavior. So which features of coherent spiking are relevant to how the spike patterns carry information, and which are superfluous?

Two members of our group, Natasha Cayco Gajic and Joel Zylberberg, are pursuing an answer to this question in work that has a mathematical flavor similar to that of the network dynamics problem described above. Assuming that we know the coherent spiking among cell pairs, how does further coherence among larger cell groups add (or subtract) from levels of encoded information? The answer draws on statistics and information theory, and is sure to be complicated in general, but there appear to be systematic trends in how higher-order coherence, especially when its strength varies as stimuli change, contributes to coding. Thus equipped, Cayco Gajic and Zylberberg are studying whether and how such coherent spiking can emerge from the nonlinear dynamics of neural networks. It is research paths like this that I believe make mathematical neuroscience such an exciting and fun field. It always pushes us to move between mathematical disciplines—in our case, from basic information theory, to dynamics, to stochastic processes—making as many (coherent, we hope!) new connections as we can.

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Eric Shea-Brown is an associate professor in the Department of Applied Mathematics at the University of Washington, Seattle.

Role for the Math Sciences in Federal BRAIN Initiative

With the launch of the federal Brain Research through Advancing Innovative Neurotechnologies (BRAIN) initiative, neuroscience and cognitive science have become major priorities of the Obama administration, numerous federal agencies, and private partners. As the accompanying article demonstrates, mathematicians have much to contribute toward progress in these fields. The BRAIN initiative seeks to enhance the development and application of new technologies that will revolutionize our understanding of the brain. Three primary agencies are engaged in the initiative: the National Institutes of Health, the Defense Advanced Research Projects Agency, and the National Science Foundation, and there are opportunities for the SIAM community to engage with each.

Each agency is taking a different approach to BRAIN funding, and many of their efforts connect to computational or data issues. At NIH, they are focused on seven goals that range from fundamental understanding of neural circuitry to applications for diseases and disorders. Modeling is central to this effort; in fact, a chapter in the report of the NIH BRAIN Working Group is titled "Theory, Modeling, and Statistics Will Be Essential to Understanding the Brain" (http://www.nih.gov/science/brain/2025/BRAIN2025.pdf). The focus at NSF is also on computational and modeling challenges; themes include innovative technologies to understand and enhance brain function or treat brain disorders; development of cyber tools and standards for data acquisition, analysis, and integration; and multiscale and multimodal modeling to relate dynamic brain activity to cognition and behavior (http://www.nsf.gov/news/newsmedia/sfn_brain_factsheet.pdf). DARPA envisions programs in mathematics that would use scalable computational and modeling brain research tools to analyze brain networks (https://www.fbo.gov/index?s=opportunity&mode=form&id=272b4b9d18e528d33b313352e8122ea9&tab=core&_cview=0).

Given the growing nature of this initiative and the importance of modeling and other mathematical challenges to its success, the SIAM community is encouraged to participate in workshops, symposia, conferences, and proposer's days to shape and compete for new opportunities arising from NIH, NSF, and the Department of Defense agencies. This is a great time to engage, bringing the expertise of the mathematical sciences community to bear on the grand challenge of understanding the brain.—*Miriam Quintal, Lewis–Burke Associates, LLC.*

Obituaries

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took charge of a series of well-attended meetings held in and around Philadelphia. For a SIAM meeting in Pittsburgh at the end of 1954, in conjunction with a joint meeting of the American Mathematical Society, the Mathematical Association of America, and the Association for Symbolic Logic, Harold arranged for three SIAM lectures—two to be delivered by friends of his at no cost to the society—on cutting-edge topics of the day. One of the friends was Herbert Simon, then teaching at Carnegie Tech, who discussed the command and control of industrial operations; Simon later received a Nobel Prize in economics.

In the spring of 1970, following President Nixon's incursion into Cambodia, student unrest came to a head. At Princeton, with round-the-clock teach-ins occurring in the main gymnasium, class attendance declined to the point that final exams had to be cancelled. Harold's ability to listen—together with his longstanding interest in civil liberties—made him an effective liaison between angry elements of the student body and the embattled administration. He went on to serve on the Committee of the Structure of the University, which designed the present-day Council of the Princeton University Community, a forum offering diverse campus groups a voice in the governance of the university.

At about the same time, Harold became a principal in the consulting firm Mathematica. Eventually acquired by Martin Marietta, the firm was located in Princeton (and had nothing to do with the Wolfram product of the same name). As a director, Harold managed projects for the Atomic Energy Commission, the Arms Control Agency, and the Department of Transportation.

Harold's son Nick relates that, perhaps because of preoccupation with matters of national concern, Harold was occasionally guilty of procrastination. Indeed, when Princeton University Press decided to bring back its multi-volume Annals of Mathematics series in "print on demand" format, they had trouble locating a copy of Volume 37: *Lectures on the Theory of Games*, by Harold W. Kuhn. Volumes 36 and 38 had appeared in 1956, but number 37 was nowhere to be found. It turned out that Harold had never returned the page proofs! The book was duly published in 2003, 47 years after its companion volumes. Though records are made to be broken, that one seems likely to last!

Harold was a gifted lecturer whose undergraduate course on linear and nonlinear programming was cross-listed in the mathematics and economics departments at Princeton and was regularly numbered among the best on campus by the (often acerbic) student newspaper. He edited and/or co-edited numerous volumes of conference proceedings and was an active organizer of international conferences. In particular, he teamed with G.P. Szego to direct NATO International Summer Schools on Mathematical Systems Theory (1968) and Differential Games (1970) and directed a NATO Advanced Study Institute on Games with Incomplete Information and Bounded Rationality (Capri, 1987).

He was also asked to convene a special seminar on the work of John Nash in conjunction with the Nobel Award Ceremonies in Stockholm in 1994, having lobbied long, hard, and effectively for Nash to receive the honor. He then served as a technical adviser on the movie *A Beautiful Mind*, which was based loosely—too loosely, in Harold's forthright opinion—on Sylvia Nasar's book of the same name.

He is survived by his wife, Estelle, sons, Clifford, Nicholas, and Jonathan, six grandsons, and one granddaughter. Memorial donations can be made to the American Civil Liberties Union, of which Harold was a longtime supporter. —*James Case, Baltimore.*

Optimization 2014

Euclidean Distance Degree

By Rekha R. Thomas

Many models in the sciences and engineering arise as the set of real solutions to systems of multivariate polynomial equations with real coefficients. In practice, model observations are often noisy and may not satisfy all the equations. In such situations, the problem usually solved is that of finding the maximum likelihood estimate of the noisy sample. If the distribution of noise is Gaussian, the problem reduces to finding the closest point in Euclidean distance from the noisy sample to the model. For example, in the *triangulation problem* in computer vision, which is concerned with the reconstruction of three-dimensional scenes from camera images, the model is the set of possible images of points in three-dimensional space observed by a given set of cameras. This space of images can be described by certain quadratic and cubic polynomials in the coordinates of the images. The task here is to find a set of true images that are closest in Euclidean distance to a given set of noisy images.

The set of real solutions to a system of polynomial equations in n variables, called a *real algebraic variety*, is a subset X of $\mathbb{R}^n$. The application described above is a special instance of the following general optimization problem on X: *Given a data point* $u \in \mathbb{R}^n$, find $u^* \in X$ *that minimizes* $d_u(x) = \|u - x\|_2^2$, *the square of the Euclidean distance from u to X*. Geometrically, u^* is the first point on X that is touched by a ball of growing radius, centered at u (Figure 1).

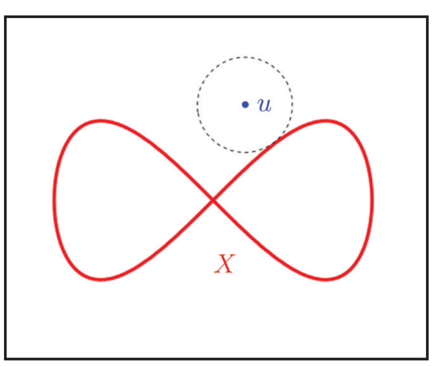

Figure 1. *Minimizing Euclidean distance from u to X.*

As simple as it sounds, this is in general a difficult (NP-hard) problem. The set of *critical points* of the problem (which contains the minimizer) is typically finite; the points are the solutions to a system of polynomial equations associated to those defining X. We are concerned with this set of critical points, whose size is a measure of the algebraic complexity of writing u^* as a function of u.

The critical points of the objective function $d_u(x)$ on X are exactly those points $x \in X$ at which $u - x$ is perpendicular to the tangent space of X at x. Geometrically, these are all the points on X that tangentially meet a growing ball centered at u. The tangent space is well defined only at the smooth points of X, so we consider only that subset.

To simplify the computation, however, we consider all complex solutions to the polynomial equations that define X, rather than just the real solutions. Miraculously, for generic $u \in \mathbb{R}^n$, the number of smooth complex critical points of $d_u(x)$ in X is finite and constant. We call this number the *Euclidean distance degree* (EDdegree) of X. As shown below, EDdegree(X) has surprisingly nice formulas in many instances.

As a quick check, we can consider the circle X in $\mathbb{R}^2$ defined by $x_1^2 + x_2^2 = 1$. For generic $u = (u_1,u_2) \in \mathbb{R}^2$, there are two real points on X at which the tangent is perpendicular to $u - x$, and, indeed, EDdegree(X) = 2. For the special choice of $u = (0,0)$, all points in X are critical. Similarly, the EDdegree of a parabola is 3, that of an ellipse 4.

The plane curves mentioned above carry with them a second curve, called the *ED discriminant,* that breaks $\mathbb{R}^2$ into regions in which the number of real critical points stays constant. The EDdiscriminant of an ellipse is the asteroid curve shown in Figure 2. Inside the asteroid, all four critical points are real. As u passes through the asteroid curve, two of these real points come together and emerge as two complex conjugate points, reducing the number of real critical points to two.

For plane curves, these discriminants (long known under the names *evolute* or *caustic*) appeared as early as 200 BC, in the book of Appollonius. In general, every real algebraic variety X has an EDdiscriminant that is a hypersurface dividing $\mathbb{R}^n$ into regions that correspond to different numbers of real critical points.

■■■

We now come to some of the formulas for EDdegree that arise in applications. In linear regression, we are interested in finding the closest point from a sample to an affine space. In this case, there is a unique critical point and the EDdegree is 1. If X is the set of all $s \times t$ matrices ($s \leq t$) of rank at most r, then EDdegree(X) = C(s,r), where C(s,r) is the number of ways of choosing r items from s items. The Eckart–Young theorem from linear algebra says that given an $s \times t$ matrix U, the closest matrix to U in X is obtained via a singular value decomposition of $U = A\Sigma B$ (where $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_s)$ and $\sigma_1 \geq \sigma_2 \geq \ldots$ are the singular values of U), by setting to zero the singular values $\sigma_{r+1}, \sigma_{r+2}, \ldots$ in the decomposition. While this yields U^*, the remaining critical points correspond to zeroing out any other choice of $s - r$ singular values of U, yielding the formula for EDdegree.

In the triangulation problem from computer vision, a general formula for the EDdegree of the space of images from n cameras is unknown. Based on values for small n, it is conjectured to be $9/2\,n^3 - 21/2\,n^2 + 8n - 4$. An important concern in control theory is the *stability* of a univariate polynomial; $p(t) = \sum_{i=0}^{n} a_i t^i$ is stable if all its complex roots have negative real parts. In this case, the algebraic boundary of the region of stability has EDdegree $4n-7$ if n is odd and $2n-3$ if n is even. This carries important information about the closest stable polynomial to one that is not.

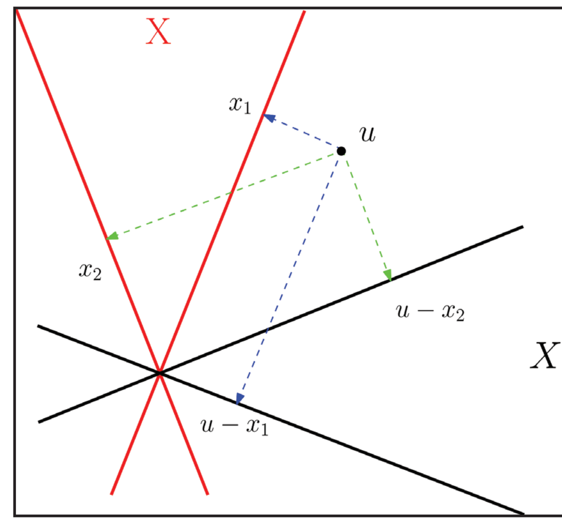

Figure 3. *The bijection between critical points on X and critical points on X*.*

Many additional examples, full proofs, and references can be found in [1], in which we also develop many mathematical properties of EDdegree. A striking one is that EDdegree remains the same under projection. We also develop the notion of average real EDdegree, which counts the average number of smooth real critical points of the squared Euclidean distance

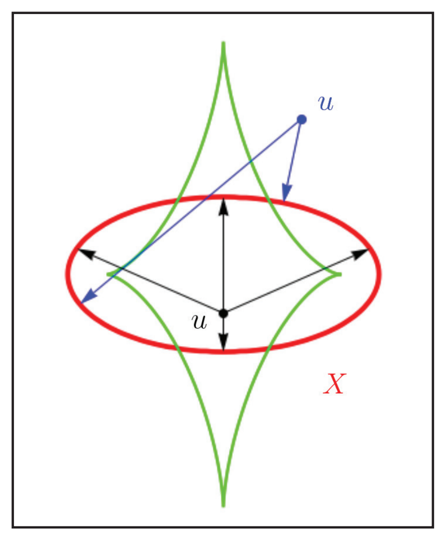

Figure 2. *Real critical points of an ellipse and its EDdiscriminant.*

function with respect to an underlying distribution on the space of data. For instance, if $\mathbb{R}^2$ is equipped with the standard multivariate Gaussian distribution, then the average real EDdegree of the ellipse is 2.8375, which lies between 2 and 4, the possible number of real critical points for the ellipse. We also show how algebraic geometry offers formulas for EDdegree via invariants of X, when X satisfies certain smoothness and intersection conditions. This is at first glance rather surprising—one would think that the variety cannot possibly know about the Euclidean distance function. But this objective function is intimately related to tangent spaces of the variety and hence EDdegree(X) is hardwired into X itself.

The close connection to tangent spaces leads to a natural appearance of duality in the theory of EDdegree. If the polynomials defining our algebraic variety are homogeneous, then the complex variety X has a dual variety defined as the closure of all tangent normals to X. For instance, if X is a linear space, X^* is the orthogonal complement $X^{\perp}$. If X is the set of all $s \times t$ matrices of rank at most r, then X^* is the set of $s \times t$ matrices of rank at most $s - r$. In the first pair, both X and X^* have EDdegree equal to 1, as both are linear spaces. In the second pair, EDdegree(X^*) = C($s,s-r$) by the formula mentioned earlier, but this equals C(s,r) = EDdegree(X). In fact, for any X, EDdegree(X) = EDdegree(X^*) and if $x \in X$ is a critical point of u, then $u - x \in X^*$ is also a critical point of u. Critical points on X and X^* thus come in pairs and are "proximity reversing," in the sense that if x is the closest critical point in X to u, then $u - x$ is the farthest critical point in X^* from u. Figure 3 [1] illustrates these facts for the variety X consisting of a pair of lines.

The work in [1] shows the power of using methods from algebraic geometry to understand optimization problems involving polynomials. Researchers have traditionally studied such problems using analytic techniques, without exploiting the algebraic features of polynomials. Minimizing Euclidean distance is a natural objective function that appears in many applications, and the notion of EDdegree captures the complexity of this problem.

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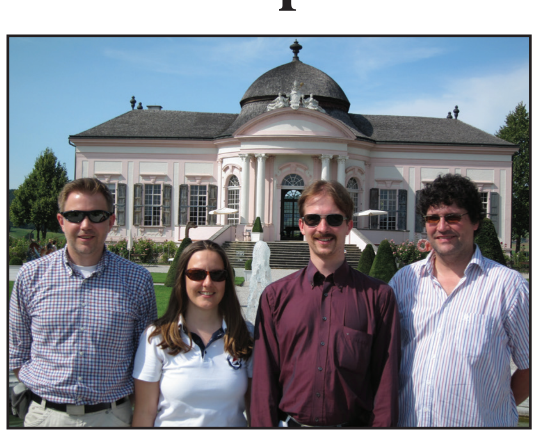
Rekha R. Thomas, a professor of mathematics at the University of Washington in Seattle, works in optimization and computational algebra. This article is based on her joint work in [1] and her talk at the 2014 SIAM Conference on Optimization.

Golub Summer School in Optimization Draws 45 Students to Linz

From August 4 through 15, 45 students representing six continents took part in the Gene Golub SIAM Summer School, held at RICAM, the Radon Institute for Computational and Applied Mathematics, on the campus of Johannes Kepler University in Linz, Austria.

Linz is a small, beautiful, and historic city in Upper Austria. As the home of Johannes Kepler, it provided a germane as well as picturesque setting for the Summer School: The view of its famed Pöstlingberg emerging from morning fog on the hill is not something that any of the participants will forget. A guided city tour on the first day taught participants—locals and visitors alike—about Linz's rich history and cultural significance within central Europe. Another highlight was a visit that evening to



Summer School lecturers, from left, Roland Herzog, Esther Klann, Winnifried Wollner, and Michael Stingl, in the garden of the Melk monastary.

the OK (Offenes Kulturhaus), where interactive installations transported participants and organizers back to childhood—a light-hearted environment perfect for getting to know each other on our first day together.

The technical program consisted of four main lecture tracks, in addition to a guest lecture. We students were given the opportunity to "roll up our sleeves" and, with guidance from the lecturers themselves, tackle both "paper and pencil" and computational exercises designed to complement the lecture material.

Michael Stingl (University of Erlangen-Nürnberg) delivered lectures on topological and material optimization, including an extended example on elastic beam deformation, which provided context for the existence and convergence results presented in the lectures.

Esther Klann (JKU, Linz) introduced students to the study of inverse problems, including regularization methods and shape sensitivity analysis. She gave an extended example from the field of tomography, and her corresponding computational exercises included several real demonstrations supporting her final appeal: "Do not go out there and commit an inverse crime!"

Roland Herzog (TU Chemnitz) discussed optimization subject to complementarity constraints, with topics that included elastoplasticity and the obstacle problem, along with exercises on the energy minimization of cable car configurations given the

See **Summer School** *on page 5*

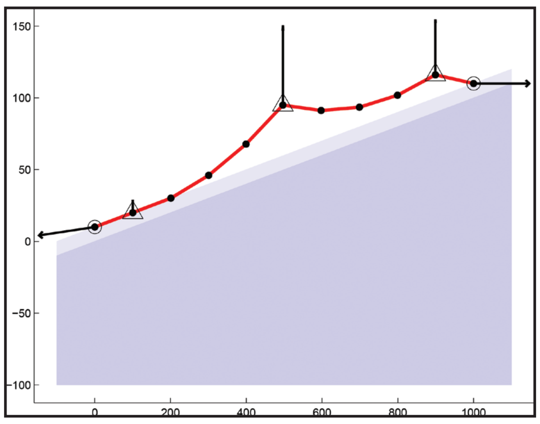
Summer School

continued from page 4

shape of the underlying landscape (inspiring heights of creativity as students designed interesting mountains!).

Winnifried Wollner (University of Hamburg) gave a thorough and self-contained introduction to adaptive finite element methods, with computational demonstrations of both the advantages and—when not thoughtfully implemented—the disadvantages of adaptive meshing; he also offered strategies for ameliorating the most common problems that arise in implementation. Topics included convergence, error estimates, and application to optimization problems.

We were also fortunate to have a guest lecturer—Jakob Søndergaard Jensen (Center for Acoustic-Mechanical Micro Systems, TU Denmark), who devoted his three-hour lecture to the application of topology optimization to dynamics of materials and structures. Along with the classic problems of optimal material design and topology optimization in photonics, he covered recent applications of robust acoustic topology optimization and nonlinear topology optimization.



Real-world applications: Summer School students considered the optimal positions of three pylons (triangles) supporting the weight of a cable car, together with the forces exerted by the pylons. The pylons are positioned in such a way that the vertical components of the forces required at the foot and top of the mountain are minimized.

Over the weekend, we all experienced "science in action" on a visit to Linz's world-famous steel plant, owned and operated by voestalpine AG. Other highlights were a cruise on the Danube in the beautiful Wachau Valley preceding a visit to the Melk monastery.

The thoughtful organization of every aspect of the summer school, technical and social alike, made these two weeks a success. The intense atmosphere brought together students and professionals from a diverse range of areas within computational and applied mathematics, in an environment where discussion and learning—guided by energetic and engaging professors—were paramount. The participants are grateful to the organizers, to all of the lecturers, and to SIAM, for making our summer school experience an unforgettable one.—*Negin Bagherpour, Sharif University of Technology; Peter Gangl, Johannes Kepler University, Linz; Francesco Ludovici, University of Hamburg; Petar Sapun, University of Hamburg; and Erin M. Kiley, Worcester Polytechnic Institute.*

Summer School students and lecturers during a visit to the steel-making facility of voestalpine AG in Linz.

2015 Golub Summer School Slated for Delphi, Greece

Randomization in numerical linear algebra is the topic of the sixth Gene Golub SIAM Summer School, which will be held in Delphi, Greece. This new research area is highly interdisciplinary, with contributions from numerical linear algebra, theoretical computer science, scientific computing, statistics, optimization, data analysis, and machine learning, and such application areas as genetics, physics, astronomy, and Internet modeling. Students will thus be selected from a wide range of backgrounds.

The organizers are Petros Drineas, Rensselaer Polytechnic Institute, USA; Efstratios Gallopoulos, University of Patras, Greece (the host institution); Ilse Ipsen, North Carolina State University, USA; and Michael Mahoney, University of California at Berkeley, USA.

The application deadline is February 1, 2015. Further information will be posted as available at www.siam.org/students/g2s3/.

Dark Side

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Image estimation and linear inverse problems have received widespread attention in the literature, but accounting for Poisson noise explicitly, though critical, is not addressed by most methods. Finding and exploiting low-dimensional structure in photon-limited settings present a myriad of unique computational and statistical challenges.

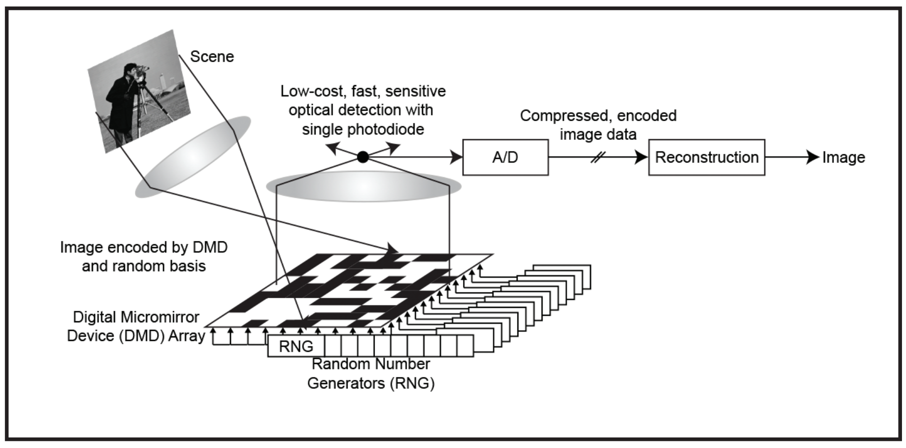


Figure 2. *The single-pixel camera developed at Rice [9].*

Sparsity and Low-dimensional Structure in Image Denoising

We first consider the special case of $A = I_p$, the identity matrix, and $n = p$. In this setting, we essentially observe a "noisy" version of the scene of interest and wish to estimate the underlying image, as depicted in Figure 1(a) and (b). This image corresponds to the flag arm patch worn by US armed forces so that they can readily identify each other using night vision cameras.

Fifteen years ago, a large community focused on wavelet-based methods for image denoising. Simple wavelet coefficient shrinkage procedures are effective in Gaussian noise settings because the noise in the wavelet coefficients is uncorrelated, identically distributed, and signal-independent. **These nice noise properties do not hold in the presence of Poisson noise,** so these methods yield estimates with significant artifacts. To address this challenge, many researchers proposed methods designed to account explicitly for Poisson noise [2–5, 14, 15, 19, 21–23]. The result of one of these methods (a state-of-the-art multiscale method, based on a computationally efficient recursive partitioning scheme) is illustrated in Figure 1(c) [22, 23].

Modern denoising methods leverage additional low-dimensional structure within images. For instance, BM3D divides a noisy image into overlapping patches (i.e., blocks of pixels), clusters the patches, stacks all the patches in each cluster to form a three-dimensional array, and applies a three-dimensional transform coefficient shrinkage operator to each stack to denoise the patches [8]. The denoised patches can then be aggregated to form a denoised image. The result of applying BM3D directly to photon-limited image data is shown in Figure 1(d); significant artifacts are present because of the failure to account for the Poisson statistics in the data.

Simply transforming Poisson data to produce data with approximate Gaussian noise via the Anscombe transform [1] can be effective when the photon counts are uniformly high [6, 24]. Specifically, for each pixel we compute the Anscombe transform, $z_i = 2\sqrt{y_i + 3/8}$; for sufficiently large y_i, we know that z_i is approximately Gaussian with unit variance. Hence, we can apply a method like BM3D to the transformed image z_i and invert the Anscombe transform to estimate f^*. Historically, this approach performed poorly at low photon count levels, but recent work on statistically unbiased inverse Anscombe transforms (as opposed to an algebraic inverse Anscombe transform) [16] has resulted in far better estimates, as shown in Figure 1(e).

When photon counts are very low, it is better to avoid the Anscombe transform and tackle Poisson noise directly. For instance, the nonlocal Poisson principal component analysis (PCA) denoising method performs noisy image patch clustering, subspace estimation, and projections for each cluster (similar to the BM3D approach outlined above); within each of these steps, however, the optimization objectives are based on the Poisson log-likelihood and account naturally for the non-uniform noise variance [18]. This approach can yield significant improvement in image quality, as shown in Figure 1(f); further potential improvements have been demonstrated very recently [12].

Photon-limited Problems

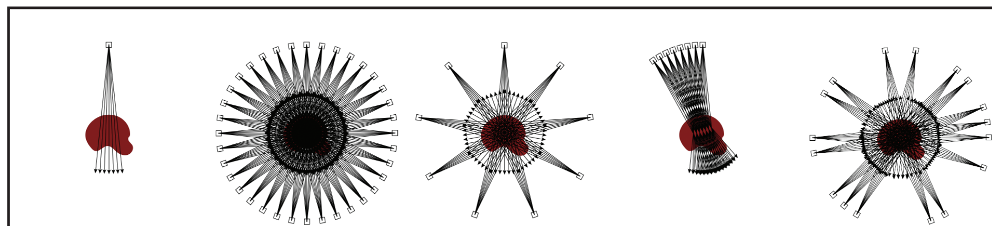
So far, we have seen that sparsity and low-dimensional structure can play a critical role in extracting accurate image information from low-photon-count data. We now address a related question:

Given our understanding of the role of sparsity in image denoising, can we design smaller, more efficient, or more accurate imaging systems?

The preceding section focused on denoising problems, in which A from (1) was the identity operator. In this section we examine inverse problems, where A is a more general (known) operator that describes the propagation of light through an imaging system. Computational imaging is focused on the design of systems and associated system matrices A that will facilitate accurate estimation of f^*.

Consider, for instance, the single-pixel camera developed at Rice University [9] (Figure 2) and the similar single-pixel fluorescence microscope. The idea is to collect successive single-pixel projections of a scene, based on compressed sensing principles, and then reconstruct the high-resolution scene from a relatively small number of measurements. The digital micromirror device at the heart of this design can operate at very high frequencies, allowing the collection of many measurements in a short time. However, given a fixed time budget for data acquisition (T), larger numbers of measurements imply less time (and hence fewer photons) per measurement. Is it better, then, to collect a large number of diverse but low-quality, photon-limited measurements, or a smaller number of higher-quality measurements? How quickly can we acquire such compressive data while ensuring our ability to reconstruct the scene?

Similar questions arise in the context of tomography, as depicted in Figure 3. In applications like pediatric x-ray CT, we operate in photon-limited conditions to ensure patient safety. Given a limited patient dose (which is proportional to the number of photons we can collect and the time budget T), is full-angle tomography with relatively noisy measurements better or worse than measuring only a small subset of the possible projections with higher fidelity?



A set of fan-beam tomographic projections. Full-angle tomography. Sparse-angle tomography. Limited-angle tomography. Random-angle tomography.

Figure 3. *Various tomographic imaging system designs.*

Unfortunately, most work on compressed sensing and inverse problems does not address such tradeoffs. To see why, think of the system matrix entry A_{ij} as corresponding to the probability of a photon from location j in f^* hitting the detector at location i; this interpretation implies that $A_{ij} \in [0, 1]$ and that the columns of A sum to at most 1. Matrices considered in most of the compressed sensing literature (such as those that satisfy the restricted isometry property (RIP) [25]) do not satisfy these constraints! Rescaling RIP matrices to satisfy physical constraints introduces new performance characterizations and tradeoffs; photon-limited compressed sensing thus differs dramatically from compressed sensing in more conventional settings, as summarized in Table 1 [13].

In particular, the analysis in [6] helps address our earlier question, whether it is better to have many photon-limited measurements or few photon-rich measurements. The number of measurements, n, needs to be large enough to ensure that the sensing matrix A will facilitate accurate reconstruction, e.g., A corresponds to a RIP-satisfying matrix that has been shifted and rescaled to reflect the above-mentioned physical constraints. Once this condition is satisfied, n does not control mean squared error performance—suggesting that many photon-limited measurements and few photon-rich measurements are equally informative, and system designers can choose whatever regime

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Compressed sensing, conventional wisdom	Photon-limited compressed sensing
The performance of compressed sensing is independent of the basis in which our image is sparse, provided that the basis is "incoherent" with the sensing matrix.	At low intensities (small T), the geometry of the sparsifying basis and its interactions with various physical constraints impact squared L_2 reconstruction errors; see Figure 4(a).
Once n is sufficiently large to allow A to satisfy something like the restricted isometry property, squared L_2 reconstruction error decays like $1/n$, the number of measurements.	Once n is sufficiently large to allow a shifted and rescaled version of A to satisfy something like the restricted isometry property, reconstruction error is independent of n and instead depends heavily on the data acquisition time T. If T is below a sparsifying basis-dependent threshold (i.e., if the number of observed photons is too small), accurate reconstruction is impossible; see Figure 4(a).
Reconstruction from compressed sensing data is nearly as good as it would be if we knew the locations of the non-zero coefficients and measured them directly.	If we know the locations of the non-zero coefficients, we can compute dramatically better reconstructions from direct measurements; see Figure 4(b).

Table 1. *Compressed sensing in conventional and photon-limited settings.*

Dark Side

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is cheapest, lightest, or easiest to calibrate. We also see that there is a time threshold T_0, and if the data acquisition time T is below T_0, then accurate reconstruction is impossible. Thus, even if hardware advances like faster digital micromirror devices allow us to collect measurements more rapidly, we cannot reduce the total data acquisition time below T_0.

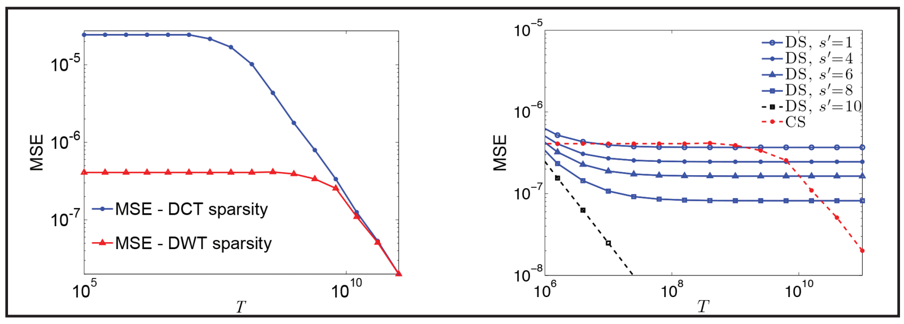

Figure 4. *Photon-limited compressed sensing.*
Left: mean squared error (MSE) as a function of data acquisition time T for two different images, each sparse in a different basis: discrete cosine transform (DCT) and discrete wavelet transform (DWT). For sufficiently large T, the error decays like 1/T, but in low-photon-count regimes we cannot reliably reconstruct the images. The minimum T necessary for reliable reconstruction depends on the choice of sparsifying basis.
Right: mean squared error as a function of data acquisition time T for different sampling schemes. The images here are sparse in a wavelet basis, and s′ is the number of coarse-scale non-zero wavelet coefficients. The total number of non-zero coefficients s is 10. DS refers to downsampling—i.e., measuring a low-resolution version of the image directly. CS refers to compressed sampling with a dense random sensing matrix; the curve is the same regardless of s′. We see that downsampling can significantly outperform compressed sensing at low-photon-count levels.

Conclusions

In general, photon-limited image reconstruction methods and optical system designs that fail to account for the statistics and physical constraints of photon-limited imaging will yield results dramatically worse than what we can achieve by accounting for those constraints. Compressed sensing analyses in particular are often based on methods and assumptions that are not compatible with these constraints, and thus sometimes provide bounds that are not achievable in practical settings. Bounds that take these constraints and statistics into account reveal very different behaviors and, ultimately, could contribute to improved optical sensor design strategies. So become seduced by the dark side of imaging—photon limitations introduce novel challenges and exciting opportunities in imaging science!

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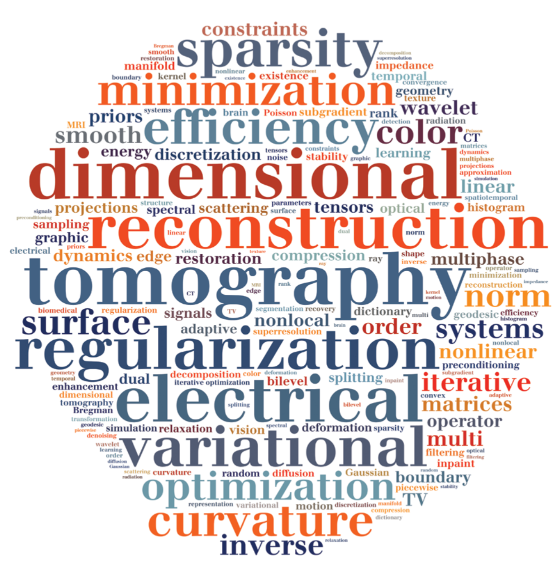
Rebecca Willett is an associate professor in the Department of Electrical and Computer Engineering at the University of Wisconsin–Madison. This article is based on her invited talk at the 2014 SIAM Conference on Imaging Sciences.

Hong Kong Hosts 2014 SIAG/IS Conference

The 2014 SIAM Conference on Imaging Science, held at Hong Kong Baptist University, May 12–14, drew approximately 580 scientists from all over the world, including 168 students. Sponsored by the SIAM Activity Group on Imaging Science, the biennial conference gives those working in the field a forum for exchanging research results and addressing open issues in all aspects of imaging science. Barbara Kaltenbacher (University of Klagenfurt, Austria), Michael Ng (Hong Kong Baptist University, China), and Fadil Santosa (University of Minnesota, USA) co-chaired the organizing committee for what was the first conference of a SIAM Activity Group held in Asia.

The program featured six plenary speakers: Antonin Chambolle, Ecole Polytechnique, France; Michael Elad, Technion, Israel; Leo Grady, HeartFlow, USA; Yi Ma, ShanghaiTech University, China; Carola-Bibiane Schölieb, University of Cambridge, United Kingdom; and Rebecca Willett, University of Wisconsin–Madison, USA (see article based on the talk, page 1). Among the topics they discussed were convex representation, graphs in image processing, industrial imaging, low-rank structure in high-dimensional data, optimization models, and photon-limited imaging.



SIAG/IS Prize recipients A.M. Bruckstein (left) and M. Elad; not shown is the third recipient, D.L. Donoho.

The SIAG/IS Prize was awarded to A.M. Bruckstein, D.L. Donoho, and M. Elad for their paper "From sparse solutions of systems of equations to sparse modeling of signals and images" (*SIAM Review*, Vol. 51, No. 1, 2009, pages 24–81). The authors were cited "for their fundamental contribution to the theory and practice of sparse representations and compressed sensing, and their popularization within the imaging science community." Bruckstein gave the prize lecture at the conference.

The conference also honored Vicent Caselles*, who died in August 2013, by awarding the Vicent Caselles Student Award to Vincent Duval (CEREMADE, France); the prize paper was titled "Fine properties of the TVL1 and the TV-G models: Geometry versus oscillations." Eligibility for the prize was limited to candidates with doctoral degrees awarded between January 2010 and January 2014, for work related to contributions and areas of interest to Caselles (e.g., image and video processing, variational formulations, PDEs).

*http://memorialwebsites.legacy.com/vicent-caselles/homepage.aspx.

Sessions at the conference, which in addition to the plenary talks included a minitutorial, 60 minisymposia, 11 contributed sessions, and a poster session, considered the wide variety of mathematical, statistical, and computational methods used in imaging science; among them were transform and orthogonal series methods, nonlinear optimization, numerical linear algebra, integral equations, partial differential equations, Bayesian and other statistical inverse estimation methods, operator theory, differential geometry, information theory, interpolation and approximation, inverse problems, computer graphics and vision, and stochastic processes. The accompanying word cloud depicts the frequency of keywords in abstracts from all the sessions.

The conference concluded with a Chinese banquet at a traditional Cantonese restaurant, featuring a lion dance performance.—*Hongyu Liu and Michael Ng, Department of Mathematics, Hong Kong Baptist University.*

An Invitation from the Research Spotlights Section of *SIAM Review*

The Research Spotlights section of *SIAM Review*, like each of the journal's three other sections, has a specific mission. Research Spotlights articles feature timely research topics or software with broad appeal to the SIREV readership. We invite members of the community to submit articles for the Research Spotlights section.

Articles should be short (about 15–20 pages in published form). Articles of this length can have a broader scope than feasible for research announcements, while being accessible to readers outside the field without major time commitments on their part.

Articles should not be excessively technical but should provide enough background that readers can understand them without exploring outside sources. Statements should be accurate and precise and, if needed, can reference details elsewhere.

Contacting the section editor is always a good idea before writing a SIREV article. All the section editors welcome the opportunity to help prospective authors.—*C.T. Kelley, SIREV Editor-in-Chief, and Evelyn Sander (esander@gmu.edu), Research Spotlights Section Editor.*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DMS Update on Interdisciplinary & Workforce Programs

By Xiaoming Huo, Jennifer Pearl, Henry Warchall, and Michael Vogelius

With this update on funding opportunities at the National Science Foundation, we draw readers' attention to interdisciplinary programs and workforce-development programs of the Division of Mathematical Sciences that should be of particular interest to the mathematical sciences community. While the bulk (roughly 75%) of the DMS investment in mathematical sciences research is through the DMS Disciplinary Research Programs, DMS is also a partner in several crosscutting initiatives. Moreover, DMS itself has a significant workforce-development activity, encompassing four programs: Postdoctoral Research Fellowships (MSPRF), Enhanced Doctoral Training (EDT), Research Experiences for Undergraduates (REU), and Research Training Groups (RTG).

More information about any of these programs is available via www.nsf.gov/DMS.

Mathematical Sciences Innovation Incubator (MSII)

Recognizing that the ideas, tools, and language of mathematics and statistics play important roles in every area of science and engineering research supported by NSF, the Division of Mathematical Sciences recently launched the MSII activity. It is widely acknowledged that interactions between the mathematical sciences and other fields catalyze developments in both. DMS wishes to foster the participation of more mathematical scientists, from every area of mathematics and statistics, in such important interdisciplinary work. In support of this goal, the MSII activity provides funding to catalyze the involvement of mathematical scientists in research areas in which the mathematical sciences do not yet play large roles.

MSII emphasizes scientific research areas of high national priority that would benefit from innovative developments in mathematics and statistics. For example, modern communication, transportation, science, engineering, technology, medicine, manufacturing, security, and finance all depend on the mathematical sciences. Success in meeting crucial challenges currently facing the nation in these areas will rest on advances in mathematical sciences research.

MSII provides support for collaborative research projects in these and other high-priority areas that are managed by NSF programs outside DMS and that involve mathematical scientists in the research. Mathematical scientists are encouraged to consider establishing research collaborations with researchers in other NSF-supported disciplines and to make collaborators aware of the possibility of MSII support for the activity. More information can be found on the MSII web page.

In addition to the MSII activity, DMS manages some special programs that support research in the mathematical sciences with direct applicability in other important areas of emphasis. We briefly describe here the newest of these programs, DMREF and CDS&E-MSS.

Designing Materials to Revolutionize and Engineer our Future (DMREF)

DMREF is the primary program through which NSF participates in the national Materials Genome Initiative (MGI) for Global Competitiveness. MGI recognizes the importance of materials science to the well-being and advancement of society and aims to "deploy advanced materials at least twice as fast as possible today, at a fraction of the cost."

DMREF seeks to promote activities that significantly accelerate the discovery and development of materials by building the fundamental knowledge base needed to progress toward designing and making materials with specific and desired functions or properties from first principles. Also of interest is research that advances fundamental understanding of materials across length and time scales to elucidate the effects of microstructure, surfaces, and coatings on the properties and performance of materials and devices.

Controlling material properties through design requires an understanding of the interrelationships of composition, processing, structure, properties, performance, and process control. The approach envisioned in DMREF to achieve this goal involves modeling, analysis, and computational simulations, validated and verified through measurement, experimentation, or device demonstration. DMREF aims to support collaborative and iterative research wherein theory guides computational simulation, computational simulation guides experiments, and experiments further guide theory.

The topics span efforts in materials science, chemistry, mathematics, statistics, computer science, and engineering to develop new data analytic tools and statistical algorithms; advanced simulations of material properties; advances in predictive modeling that leverage machine learning, data mining, and sparse approximation; and software and data infrastructure that is accessible, extensible, reliable, interoperable, and reusable. The mathematical sciences community has a valuable role to play and much to contribute in these efforts. The recent DMREF program solicitation provides additional details. Readers may also be interested in a special NSF–SIAM minisymposium associated with DMREF that will be held in connection with the SIAM Conference on Computational Science and Engineering in Salt Lake City in March 2015.

Computational & Data-Enabled Science & Engineering in Mathematical and Statistical Sciences (CDS&E-MSS)

The CDS&E-MSS program supports research confronting the host of mathematical and statistical challenges presented to the scientific and engineering communities by the ever-expanding role of computational modeling and simulation on the one hand, and the explosion in the production of digital and observational data on the other. The goal of the program is to promote the creation and development of the next generation of mathematical and statistical theories and methodologies that will be essential for addressing such issues. To this end, the program supports fundamental research in mathematics and statistics whose primary emphasis is on meeting these computational and data-related challenges.

In its first three years, CDS&E-MSS has made awards in a wide range of topics, including stochastic partial differential equations, Lie groups and representation theory, manifold learning, sparse optimiza-

See **DMS Update** *on page 11*

Professional Opportunities

Send copy for classified advertisements to: Advertising Coordinator, SIAM News*, 3600 Market Street, 6th Floor, Philadelphia, PA 19104–2688; (215) 382–9800; fax: (215) 386–7999; marketing@siam.org. The rate is $2.85 per word (minimum $350.00). Display advertising rates are available on request.*

Advertising copy must be received at least four weeks before publication (e.g., the deadline for the December 2014 issue is October 31, 2014).

Advertisements with application deadlines falling within the month of publication will not be accepted (e.g., an advertisement published in the December issue must show an application deadline of January 1 or later).

Students (and others) in search of information about careers in the mathematical sciences can click on "Careers and Jobs" at the SIAM website (www.siam.org) or proceed directly to

www.siam.org/careers

Institute for Advanced Study

School of Mathematics

The School of Mathematics at the Institute for Advanced Study, in Princeton, New Jersey, will have a limited number of memberships with financial support for research during the 2015–2016 academic year. School term dates for 2015–2016 are: Term I, Monday, September 21 to Friday, December 18, 2015; Term II, Monday, January 11 to Friday, April 8, 2016. Applicants should note that the school's Term II begins and ends one week later than the rest of the institute's. The school frequently sponsors special programs; however, these programs comprise no more than one-third of the membership so that a wide range of mathematics can be supported each year.

During the 2015–2016 academic year, the school will have a special program on geometric structures on 3-manifolds. Ian Agol, of the University of California, Berkeley, will be the Distinguished Visiting Professor. Thurston proposed classification of geometric structures on n-manifolds. While the spectacular geometrization theorem classified the geometric structures on 3-manifolds with compact isotropy group, i.e., locally homogeneous Riemannian metrics, there is a cornucopia of other fascinating structures, such as contact structures, foliations, conformally flat metrics, and locally homogeneous (pseudo-) Riemannian metrics. The goal of this program is to investigate these other geometric structures on 3-manifolds and to discover connections between them. Additionally, it is important to forge connections between geometric structures on 3-manifolds and other geometric constructs, such as gauge theory, PD (3) groups, minimal surfaces, cube complexes, geometric structures on bundles over 3-manifolds, and strengthened structures, such as taut foliations, tight contact structures pA flows, convex projective structures, and quasi-geodesic foliations. Many of these do not even have a conjectural classification (in terms of topological restrictions and moduli), and specific examples are still being constructed.

Applicants must have given evidence of ability in research comparable with at least that expected for a PhD degree but can otherwise be at any career stage. Successful candidates will be free to devote themselves full time to research. About half of the school's members will be postdoctoral researchers within five years of receipt of a PhD. The school also expects to offer some two-year postdoctoral positions.

Applications are invited for up to eight von Neumann Fellowships that are available each academic year; to be eligible for a von Neumann Fellowship, applicants should be at least five, but no more than 15, years after receipt of a PhD. Applicants can also apply for Veblen Research Instructorships, which are three-year positions that were in established in partnership with the Department of Mathematics at Princeton University. These instructorships are offered each year to candidates in pure and applied mathematics who have received a PhD within the last three years. Usually, Veblen research instructors spend their first and third years at Princeton University; these years will carry regular teaching responsibilities. The second year is spent at the institute and dedicated to independent research of the instructor's choice. Applicants interested in a Veblen instructorship position can apply directly at the IAS website, https://applications.ias.edu or through MathJobs, https://www.mathjobs.org/jobs. Applicants applying through MathJobs must also complete an application form at https://applications.ias.edu; however, they do not need to submit a second set of reference letters. Applicants who have questions about the application procedure can e-mail applications@math.ias.edu.

Applications are also invited for two-year postdoctoral positions in computer science and discrete mathematics to be offered jointly with one of the following: Department of Computer Science at Princeton University, http://www.cs.princeton.edu; DIMACS at Rutgers, The State University of New Jersey, http://www.dimacs.rutgers.edu; or the Intractability Center, http://intractability.princeton.edu. For a joint appointment applicants must apply to the IAS, as well as to one of the listed departments or centers, indicating their interest in a joint appointment. The deadline for all applications is December 1, 2014.

The Institute for Advanced Study is committed to diversity and strongly encourages applications from women and minorities.

Rutgers University–New Brunswick

Department of Mathematics

Subject to availability of funding, the Department of Mathematics at Rutgers University–New Brunswick invites applications for an opening at the level of tenured associate professor or tenured full professor in numerical analysis/scientific computation, starting in September 2015. Applicants must have a PhD, show a strong record of research accomplishments, and have a concern for teaching. The normal annual teaching load for research-active faculty is two courses for one semester and one course for the other semester.

Review of applications begins immediately and will continue until the opening is filled. Applicants should go to https://www.mathjobs.org/jobs first and fill out the AMS Cover Sheet electronically and completely, including naming the position being applied for (namely, TAP), giving the AMS Subject Classification number(s) for area(s) of specialization, and answering the question about how materials are being submitted. Applicants should submit a curriculum vitae (including a publication list) and arrange for four letters of reference to be submitted, one of which must evaluate teaching. Online applications are strongly preferred. If necessary, however, application materials may instead be mailed to Search Committee, Dept. of Math., Hill Center, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019.

Updates on this position will appear on the Department of Mathematics' webpage, http://www.math.rutgers.edu.

Rutgers is an affirmative action/equal opportunity employer.

Dartmouth College

Department of Mathematics

New or recent PhD graduates with research interest in applied and computational mathematics may apply for instructorships in these areas for terms of two to three years. Successful candidates will teach three 10-week courses spread over three terms. Appointments are for 26 months, with a possible 12-month renewal. Positions offer a monthly salary of $5,202, which includes a two-month research stipend for instructors in residence during two of the three summer months; if an instructor is not in residence, the salary will be adjusted accordingly.

To initiate an application, applicants should go to http://www.mathjobs.org and find position ID IACM #6022. The application can also be accessed at http://www.math.dartmouth.edu/activities/recruiting/. General inquiries can be directed to Tracy Moloney, Administrator, Department of Mathematics, tfmoloney@math.dartmouth.edu. Applications completed by January 5, 2015, will be considered first.

Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

Dartmouth College

Department of Mathematics

The Department of Mathematics anticipates a senior opening with an initial appointment in the 2015–2016 academic year. The successful applicant will have a research profile with a concentration in computational or applied mathematics, will be appointed at the level of full professor, and is expected to have an overall record of achievement and leadership consonant with such an appointment.

Applicants should apply online at www.mathjobs.org (position ID: PACM #6023). Applications received by December 15, 2014, will receive first consideration. For more information about this position, please visit http://www.math.dartmouth.edu/activities/recruiting/.

Dartmouth is committed to diversity and encourages applications from women and minorities.

Dartmouth College

Department of Mathematics

John Wesley Young Research Instructorships are available for two to three years for new or recent PhD graduates whose research overlaps a department member's. Successful candidates will teach three 10-week courses spread over three terms. Appointments are for 26 months, with a possible 12-month renewal; the monthly salary is $5,202, including a two-month research stipend for instructors in residence during two of three summer months. If an instructor is not in residence, the salary will be adjusted accordingly.

To initiate an application go to http://www.mathjobs.org (position ID: JWY #6021). The application can also be accessed through a link at http://www.math.dartmouth.edu/activities/recruiting/. General inquiries can be directed to Tracy Moloney, Administrator, Department of Mathematics, tfmoloney@math.dartmouth.edu. Applications completed by January 5, 2015, will be considered first.

Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

Dartmouth College

Department of Mathematics

The Dartmouth College Department of Mathematics is pleased to announce a tenure-track opening for the academic year 2015–2016. There is a preference for a junior appointment, but appointment at higher rank, with tenure, is possible. The successful candidate will have a research profile with a concentration in applied or computational mathematics.

Applicants should apply online at www.mathjobs.org (position ID: APACM #6024). Applications received by December 15, 2014, will receive first consideration. For more information about this position, please visit http://www.math.dartmouth.edu/activities/recruiting/.

Dartmouth is committed to diversity and encourages applications from women and minorities.

National University of Singapore

Department of Mathematics

The Department of Mathematics at the National University of Singapore (NUS) invites applications for tenured, tenure-track, and visiting positions at all levels, beginning in August 2015.

NUS is a research intensive university that provides quality undergraduate and graduate education. The Department of Mathematics has about 65 faculty members and teaching staff whose expertise covers major areas of contemporary mathematical research.

The department seeks promising scholars and established mathematicians with outstanding track records in any field of pure and applied mathematics. The department, housed in a newly renovated building equipped with state-of-the-art facilities, offers an internationally competitive salary with start-up research grants, as well as an environment conducive to active research, with ample opportunities for career development. The teaching load for junior faculty is kept especially light.

The department is particularly interested in, but not restricted to, considering applicants specializing in any of the following areas:

- partial differential equations and applied analysis;
- computational science, imaging and data science;
- operations research and financial mathematics;
- probability; or
- combinatorics.

Application materials (as PDF files) and enquiries should be sent to the Search Committee via email: search@math.nus.edu.sg.

Please include the following supporting documentation in the application:

1. NUS Personal Data Consent for Job Applicants: http://www.nus.edu.sg/careers/potentialhires/applicationprocess/NUS-Personal-Data-Consent-for-Job-Applicants.pdf;
2. an American Mathematical Society Standard Cover Sheet;
3. a detailed CV, including publications list;
4. a statement (max. of three pages) of research accomplishments and plan;
5. a statement (max. of two pages) of teaching philosophy and methodology. Please attach an

See **Professional Opportunities** *on page 11*

INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES

The *Institute for Computational Engineering and Sciences (ICES)* at *The University of Texas at Austin* is searching for exceptional candidates with expertise in computational science and engineering to fill several Moncrief endowed faculty positions at the Associate Professor level and higher. These endowed positions will provide the resources and environment needed to tackle frontier problems in science and engineering via advanced modeling and simulation. This initiative builds on the world-leading programs at ICES in Computational Science, Engineering, and Mathematics (CSEM), which feature 16 research centers and groups as well as a graduate degree program in CSEM. Candidates are expected to have an exceptional record in interdisciplinary research and evidence of work involving applied mathematics and computational techniques targeting meaningful problems in engineering and science. For more information and application instructions, please visit: www.ices.utexas.edu/moncrief-endowed-positions-app/. This is a security sensitive position. The University of Texas at Austin is an Equal Employment Opportunity/Affirmative Action Employer.

THE UNIVERSITY OF TEXAS AT AUSTIN

INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES

POSTDOCTORAL RESEARCH FELLOWSHIPS

Research areas include, but are not limited to, inverse analysis, differential equations, kinetic theory, remediation of groundwater contaminants, tidal surges in coastal environments, drug design, damage and failure of composite materials, patient-specific surgical procedures, dynamics of polar ice, and the human ear.

Annual Stipend of $60,000 plus benefits for up to two years

DEADLINE: **JANUARY 5, 2015**

Apply at: www.ices.utexas.edu/programs/postdoc/

DMS Update

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tion, data assimilation, partially observed Markov processes, and high-dimensional learning. Among the many emerging methodologies proposed are efficient parallel iterative Monte Carlo methods, accelerated Monte Carlo schemes, solution of large-scale eigenvalue problems, and measurement model specification search. Some projects deal with newly emerged datasets—for example, algebraic, geometric, and computational tools for data clouds and data arrays, LiDAR point cloud data, and data with network structure. The widely ranging application areas include tumor microenvironments, genetic association, brain connectivity, coastal ocean modeling, and subsurface imaging. The online award abstracts reflect the broad spectrum of research projects supported by the program.

CDS&E-MSS is part of the NSF-wide CDS&E program. There are differences: If the proposed work emphasizes mathematical or statistical foundations, CDS&E-MSS may be a fit. If the proposed work is driven more by particular scientific and/or engineering applications, the NSF-wide CDS&E program may be more suitable. The NSF-wide CDS&E program has varying proposal deadlines, depending on the NSF division to which the proposal is submitted; the next submission window for CDS&E-MSS is November 25–December 9, 2014. Investigators are encouraged to contact the cognizant program directors before preparing proposals.

■■■

The primary mission of DMS is the support of research in the mathematical sciences; students and postdoctoral associates receive training as frequent participants in these research projects. DMS also supports other activities by the community to enhance the training of the next generation of U.S. mathematical sciences researchers. Much of this additional support is provided through the DMS Workforce activity, which comprises the four programs centered on training through research involvement mentioned at the beginning of this article. Two of those programs—REU and Postdoctoral Research Fellowships—are of long standing and are not discussed further here. We briefly describe the new Enriched Doctoral Training in the Mathematical Sciences program and summarize updates to the long-running Research Training Groups in the Mathematical Sciences.

Enriched Doctoral Training in the Mathematical Sciences (EDT)

The EDT program supports efforts to enrich research training in the mathematical sciences at the doctoral level by preparing PhD students to recognize and find solutions to mathematical challenges arising in other fields and in areas outside today's academic setting. Graduate research training activities supported by EDT will prepare participants for a broader range of mathematical opportunities and career paths than has been traditional in U.S. mathematics doctoral training.

The long-range goal of the EDT program is to strengthen the nation's scientific competitiveness by increasing the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences and in other professions in which expertise in the mathematical sciences plays an increasingly important role. The program supports efforts by academic institutions or other qualified organizations to train doctoral students in the mathematical sciences who will be well equipped to recognize opportunities for the development of mathematics and statistics in problems from other disciplines, and who can effectively apply advanced mathematics and statistics to solve problems originating outside the traditional academic mathematical sciences setting. The program will support projects that include training in areas supplementary to students' dissertation research themes and that are instrumental for connections with business, industry, government, and the non-profit sector; among such activities are internships, research projects, consulting, and participation in complementary courses or summer schools. Projects are expected to train students to work in teams to refine, attack, and solve problems that are open-ended, not initially sharply formulated, and that originate outside the academic mathematical realm.

DMS intends that the collection of projects funded will benefit students whose dissertation topics lie in all areas of the mathematical sciences, and we are hopeful that a wide spectrum of departments will submit proposals for this program. The intent is, funding permitting, to have fifteen or more EDT projects running by the third year of the program.

Research Training Groups in the Mathematical Sciences (RTG)

The REU, EDT, and postdoctoral fellowship programs support enhanced training through research involvement at the undergraduate, doctoral, and postdoctoral levels. The RTG program spans all these levels, supporting efforts to improve research training by involving undergraduate students, graduate students, postdoctoral associates, and faculty members in structured groups centered on a common research theme. Research groups supported by the RTG program must include vertically integrated activities that span this entire spectrum of educational levels.

The potential of such vertically integrated activities to enhance engagement, accelerate progress, and improve recruitment and retention in the discipline has been indicated by several reviews, as described in the RTG program solicitation. These observations reveal that well-implemented vertically integrated research groups can generate enormous enthusiasm, high motivation, and accelerated research progress among participants at all levels.

The RTG program aims to further the adoption of this research group model in mathematical sciences programs that conduct training spanning this entire spectrum of educational levels. The new RTG solicitation (re)-emphasizes the essential importance of vertical integration and strong training plans in successful RTG proposals.

We end this brief review of recent developments in the DMS portfolio by encouraging the mathematical sciences community to continue to submit strong proposals to the DMS Disciplinary Research Programs and to take advantage, when appropriate, of the additional opportunities outlined here. More information about all these opportunities is available through the program pages and program solicitations accessible via the DMS home page (www.nsf.gov/DMS). As always, questions can be addressed to the program directors listed on the program pages for the various funding opportunities.

Xiaoming Huo and Jennifer Pearl are program directors in NSF's Division of Mathematical Sciences. Henry Warchall and Michael Vogelius are deputy director and director, respectively, of DMS.

Opportunities

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evaluation on teaching from faculty members or students of applicant's current institution, where applicable; and

6. at least three letters of recommendation, including one which indicates the applicant's effectiveness in and commitment to teaching. Reference letters should be sent directly to search@math.nus.edu.sg

The review process will begin on October 15, 2014, and will continue until positions are filled.

For further information about the department, please visit http://www.math.nus.edu.sg.

Georgia Institute of Technology

School of Mathematics

The School of Mathematics at Georgia Tech is accepting applications for faculty positions at all ranks and in all areas of pure and applied mathematics and statistics.

Applications by highly qualified candidates, especially those from groups underrepresented in the mathematical sciences, are particularly encouraged. See www.math.gatech.edu/resources/employment for more details and application instructions.

University of Michigan

Center for the Study of Complex Systems

The Center for the Study of Complex Systems at the University of Michigan invites applications for a tenure-track position of assistant professor of complex systems. Candidates must have a demonstrated research agenda focusing on complex systems. This may involve theoretical or applied research on complexity, including (but not limited to) mathematical and computational models in areas such as networks, computation, emergence, large events and robustness or applications where complexity lies at the core such as quantitative modeling of social systems, soft condensed matter physics, evolutionary or ecological dynamics, epidemiology and disease transmission, artificial life, neuroscience, and cognition. Preference will be given to candidates with a track record of working across disciplines.

The appointment will begin September 1, 2015. This is a university-year appointment. Information about the Center can be found at www.cscs.umich.edu.

All application materials must be uploaded to http://tinyurl.com/UMCSCS. Applicants must submit a current CV, statement of current and future research plans, a statement of teaching philosophy and experience, evidence of teaching excellence (if any), and one writing sample. At least three letters of recommendation are required and must be uploaded to the same website. Applications will be reviewed starting October 17, 2014. Applications will be accepted until the position is filled.

Women and minority candidates are encouraged to apply. The University of Michigan is an equal opportunity/affirmative action employer and is supportive of the needs of dual career couples.

University of Delaware

Department of Mathematical Sciences

The Department of Mathematical Sciences at the University of Delaware invites applications for a tenure-track position at the rank of assistant professor. The appointment is expected to start in Fall 2015.

An innovative leader in research and teaching, the University of Delaware combines a rich historic legacy with a commitment to undergraduate education and the latest in advanced technology. With external funding exceeding $200 million, the university ranks among the top 100 universities in federal R&D support for science and engineering. Enhanced by state-of-the-art facilities, research is conducted across all seven colleges and numerous interdisciplinary institutes and centers. The main campus in Newark, Delaware, provides the amenities of a vibrant college town with convenient access to the major cities of the East Coast.

The department encourages applications from outstanding candidates in all areas of mathematics, particularly those related to the research interests of current faculty. The department especially encourage applications from those working in the following areas:

• additive/extremal/probabilistic combinatorics with connections to analysis
• data science
• stochastic models, computation, and analysis.

Required qualifications for the position include a PhD in mathematics or a related field; evidence of a significant, growing, research program with world-class potential; and a firm commitment to maintaining an integrated program of teaching and research. More information about the department and the position can be found at www.math.udel.edu.

A cover letter, curriculum vitae, four letters of recommendation (one addressing teaching), a research statement, and a teaching statement should be submitted at www.MathJobs.org. Upon receipt of a complete application, applicants will receive a confirmation email directing them to apply online at http://www.udel.edu/002295 by supplying a detailed letter of interest and curriculum vitae (all in one document). The deadline for applications is November 14, 2014.

Employment offers will be conditioned upon successful completion of a criminal background check. A conviction will not necessarily exclude a candidate from employment.

The University of Delaware is an equal opportunity employer that encourages applications from minority group members, women, individuals with disabilities, and veterans. The University's Notice of Non-Discrimination can be found at http://www.udel.edu/aboutus/legalnotices.html.

Announcements

Send copy for announcements to: Advertising Coordinator, SIAM News, *3600 Market Street, 6th Floor, Philadelphia, PA 19104–2688; (215) 382–9800; marketing@siam.org. The rate is $1.85 per word (minimum $275.00). Announcements must be received at least one month before publication (e.g., the deadline for the December 2014 issue is October 31, 2014).*

A Solution to the 3x + 1 Problem?

I believe I have solved this very difficult problem and am looking for an academic mathematician to check the solution and then, if he or she believes it is correct, to help me prepare the paper for publication. I will offer a generous consulting fee, and/or shared authorship if the mathematician's contribution merits it, and/or generous mention in the Acknowledgments. The paper is "A Solution to the 3x + 1 Problem" on occampress.com.

Contact: Peter Schorer, peteschorer@gmail.com.

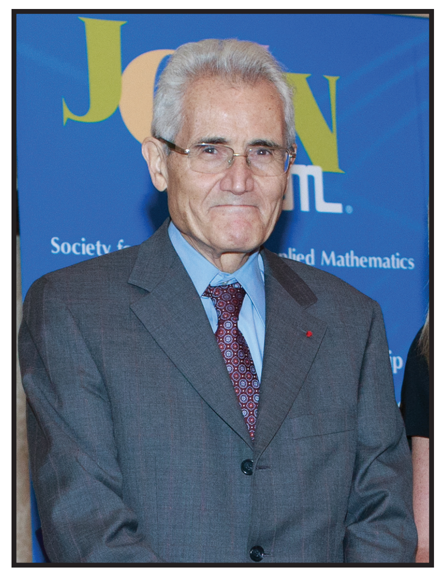
Irene Gamba (center) of the Institute for Computational Engineering and Sciences at the University of Texas at Austin gave the 2014 AWM–SIAM Sonia Kovalevsky Lecture at the SIAM Annual Meeting in Chicago. Gamba, shown here with AWM president Ruth Charney and SIAM president Irene Fonseca, titled her lecture "The Evolution of Complex Interactions in Non-Linear Kinetic Systems." Honored "for her significant contributions to analytical and numerical methods for statistical transport problems in complex particle systems," she was also cited for her outstanding record of service to the applied mathematics community.

Given every four years since 2002, the Julian Cole Lectureship was awarded in 2014 to John Lowengrub of the University of California, Irvine. The prize committee cited his "seminal contributions to fluid dynamics, materials science, and computational biology through the development of mathematical models, computational methods, and numerical simulations of free-boundary problems and tumor growth." Lowengrub's prize lecture was titled "Growth, Patterning, and Control in Nonequilibrium Systems."

SIAM Prizes Chicago, 2014 Annual Meeting

Other 2014 prize recipients were featured in the September issue of *SIAM News*; still others will appear in upcoming issues.

Given to a junior scientist in recognition of outstanding research in applied mathematics, the biannual Richard C. DiPrima Prize was awarded in Chicago to Thomas Trogdon, an NSF postdoctoral fellow at NYU's Courant Institute of Mathematical Sciences. Selection is based on candidates' doctoral dissertations. Trogdon, who received a PhD in 2013 from the University of Washington, wrote his dissertation, "Riemann–Hilbert Problems, Their Solution and the Computation of Nonlinear Special Functions," under the supervision of Bernard Deconinck. Trogdon's dissertation, according to the prize committee, "made outstanding contributions to the theory of and numerical methods for Riemann–Hilbert problems and their application to integrable systems, nonlinear partial differential equations, including the KdV and nonlinear Schrödinger equations, and special functions."



"For his fundamental contributions in partial differential equations, deterministic as well as stochastic, including applications in filtering, stochastic and impulsive control, optimal stopping, variational inequalities and mean field game theory," Alain Bensoussan received this year's W.T. and Idalia Reid Prize. He currently divides his time between the University of Texas, Dallas, and City University of Hong Kong. A longtime member of the faculty of the University of Paris Dauphine, Bensoussan was president of INRIA from 1984 to 1996, followed by a term as president of the Centre National d´Etudes Spatiales (until 2003); he chaired the council of the European Space Agency from 1999 to 2002. He gave a Reid Prize lecture titled "On the Master Equation in Mean Field Theory" in Chicago.

Work in Progress

Recent visitors to the website of *SIAM News* might have a sense of shifting ground—and justly so.

Having worked with a web design/implementation firm, we introduced a new format with our September 2014 issue. Shortly afterward, we transformed all of our existing 2014 issues into the new format. You can see what we're talking about at sinews.siam.org.

For the moment, you will be able to select "Current Issue" and view the content of the current print issue. You will also be able to select "All Issues," which actually means "all issues converted so far to our new format." *SIAM News* was founded in 1953; "All Issues" will never really mean "all issues." We've been online since 1998 and, working backward, plan to convert issues from 1998 to the present, as time permits. During the transition, "All Issues" will continue to take you to issues in the new format; "*SIAM News* Archive" will call up issues from 1998 to where we are in our conversion effort.

A few observations: First, we hope that the changes are more than cosmetic (although we do like the look of our new online version). Mainly, the experience should be more intuitive, like selecting articles to read in a newspaper, from a clearly presented home page. You can access the new *SIAM News* online from any mobile device.

Both readers and prospective authors should be aware that we can embed video and, of course, link to related websites. We're now exploring the possibility of slightly different print and web versions of the same article—perhaps with expanded reference lists, additional illustrations, or links to other supplementary materials in the online version.

What we strive to do—in print and online—is present content that is relevant, interesting, and appealing to the SIAM community, from senior researchers in areas covered by the articles to graduate students in the process of settling on their interests. Beyond questions of design, aesthetics, and ease of web access, we would like to have your input (to siamnews@siam.org) on the level of our articles (accessibility to you and your students), the balance of areas of applied/computational math covered, and topics you would like to read about in the future. Be judicious in making suggestions: You might be recruited to write an article!