MATH MATTERS

Apply It.

The math behind... Financial Derivatives

Technical terms used:

Underlying, derivatives, fair price, hedge, replication, stochastic differential equations, martingales, partial differential equations, numerical methods

Uses and applications:

Financial derivatives—contracts involving a payment depending on the evolution of an underlying entity—can be used to get rid of some risk: A farmer can buy a *forward* today to lock the price at which they will sell their crop in a year; an energy distribution company promising fixed prices to customers can buy an *option* to protect itself from an increase in the price at which it buys this energy on the markets.

How it works:

Banks selling those contracts transfer the risk to themselves. Quantitative analysts (Quants) determine how much should be charged for this and what to do with the inherited "hot potato."

While derivatives are deals whose terms can be nearly anything, the end-result payment can often be *replicated* from simpler products, in the same way that a ready-made meal can be replicated by buying ingredients and following an appropriate recipe, or that a taxi ride from A to B can be replicated by buying public transport tickets and taking an appropriate route.

Say a product costs $100 today. A buyer wants to buy it for no more than $100 next month, when its market price could be $80 or $120. The bank sells an *option* allowing the buyer to buy it for $100 (rather, it will give the buyer $20 if the price goes up). Charging $10 for this, and with an extra $40, the bank invests for $50 in the product. One month later it sells it. If the price went down, it gets $40. If the price went up, it gets $60 and pays $20 to the buyer of the option. In both cases it takes back its $40 and breaks even. With an initial capital of $10 and suitable operations, the bank produces $20 exactly when the option pays $20, and $0 when it pays nothing.

This model is simplistic: In practice, Quants use stochastic differential equations to model the price evolution. And contracts can be more complicated. But the *fair price* and *hedging strategy*—the price of the ingredients and the recipe to follow—can be determined as the solution of some equation. Numerical methods are then used to squeeze out the numbers.

Interesting facts:

- The solution depends only on the evolutions one assumes the price can do, not on the (believed) probabilities that it does one evolution more than the other.
- The first-ever derivatives to appear are thought to be forwards on rice at the Dojima Rice Exchange, in Japan, during the early 18th century.

References:

- S. Shreve, *Stochastic Calculus for Finance I*, Springer, 2004.
- P. Wilmott, S. Howison, and J. Dewynne, *The Mathematics Of Financial Derivatives*, Cambridge University Press, 1995.

Submitted by Arnaud Lionnet, Inria, Fr, Math Matters, Apply it! Contest, February 2015.

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